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Quest for understanding hadrons at low energies: Monte Carlo tools vs. experimental data

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1 Introduction

introduction [1]

2 Luminosity

luminosity [2]

2.1 Motivations

Written by a theorist!

The luminosity $\mathcal{L}$ of a collider is the proportionality constant between the event rate dN/dt and the corresponding cross section σ of a given process, according to the relation

$$\frac{dN}{dt} = \mathcal{L}\sigma, \qquad N = \sigma \int dt\mathcal{L} = \sigma L \tag{1}$$

where L is the luminosity integrated during a given period of data taking.

Because, for a given process, an experiment counts, in practice, the number of events observed within the detector acceptance, the experimental cross section can be quoted making use of (1) as

$$\sigma = \frac{N}{L} \tag{2}$$

Therefore, it is clear that to fully exploit the experimental information contained in N, the error affecting the luminosity L must not exceed (possibly, be smaller than) the experimental error affecting N. In principle, the collider luminosity can be inferred from machine and beam parameters, from which, however, the luminosity depends in a highly non-trivial way. Furthermore, the rather limited knowledge of the luminosity parameters would lead to a measurement of the luminosity limited by an error (at the some per cent level?), which should be completely unsatisfactory in the light of a program of precision physics. To by-pass this problem, the strategy adopted at modern e^+e^- accelerators to monitor luminosity consists in identifying a process (or more processes) not influenced by too many and too large uncertainties, and determining the luminosity through the "data-theory driven" relation

$$L = \frac{N}{\sigma_{\rm th}} \tag{3}$$

where N and σ are the number of events and the theoretical cross section of the chosen reference process, respectively. Because of the relation (3), the total luminosity error will be given by the sum in quadrature of the relative experimental and theoretical uncertainty, i.e.

$$\frac{\delta L}{L} = \frac{\delta N}{N} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} \tag{4}$$

where the second term in the r.h.s. is the experimental (systematic and statistical) error affecting the measurement of the normalization process, and the second one is the relative theoretical uncertainty of the cross section calculation. It follows that, to maintain small the total error $\delta L/L$, the reference process must be a reaction with clean topology, high statistics and calculable with high theoretical accuracy. In particular, the latter requirement implies that the predictions must include the contribution of all the relevant higher-order corrections beyond the lowest-order approximation of perturbation theory.

For example, at high-energy accelerators LEP/SLC running in the '90s around the Z pole to perform precision tests of the Standard Model, the process of e^+e^- production (Bhabha scattering [**?**]), with the final-state leptons detected at small scattering angles, was used because

[a] *Present address:* Insert the address here if needed

dominated by the electromagnetic interaction and, therefore, calculable in perturbation theory, at least in principle, with very high theoretical accuracy. A total (experimental plus theoretical) precision of $\sim 0.05 \div 0.1\%$ was achieved at the end of LEP/SLC operation [?,?,?], thanks to the work of different theoretical groups and the excellent performances of precision luminometers.

At presently running flavour factories, the normalization reaction primarily used is the large-angle Bhabha process [?]. Actually, at all flavour factories, the final–state leptons are detected at wide scattering angles, within a typical central acceptance region of $\sim 40° \div \sim 140°$, because of the absence of dedicated luminosity counters, for example, at small scattering angles. Usually, the two photon and muon pair production processes are used as a cross check of the Bhabha results. At CLEO–c all the the three QED channels $e^+e^- \to e^+e^-, \gamma\gamma, \mu^+\mu^-$ are considered and the luminosity is derived as an average of the measurements of the three processes.

It is worth noting that the comparison between theoretical predictions and luminosity data necessarily require the development of sophisticated Monte Carlo (MC) programs, to pass the generated events through the experimental software environment, to enable the implementation of cuts as flexible as possible (to resemble realistic data taking conditions) and allow the simulation of differential cross sections, of strong experimental interest. Two examples of comparisons between luminosity data and MC expectations in shown in FIgs... for the KLOE experiment at DAΦNE and CLEO–c at CESR.

The goal of flavour factories is to perform measurements of the collider luminosity with a total uncertainty better that 1% and, possibly, even better, at the one per mille level. Such a high-precision luminosity measurement is of utmost importance to perform accurate measurements of the $e^+e^- \to$ hadrons cross section, which is, in turn, a key ingredient in high–precision calculations of the running of $\alpha_{\rm QED}$ and lepton $g-2$.

Here we should include something more about exp. issues, such as trigger, efficiency... as in EPJ C47 (2006) 589 by KLOE or in CLEO-c Appendix C of PRD... Federico, Achim, others?.

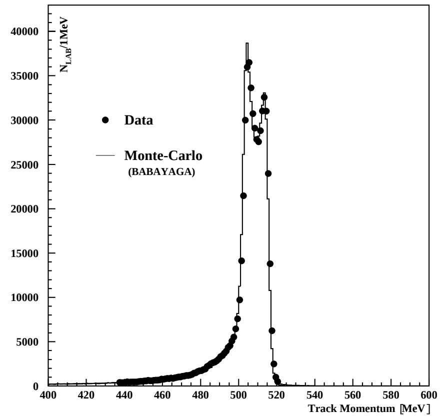

Fig. 1. Data-theory comparison for the energy distribution of the e^+e^- final state of the Bhabha process at KLOE.

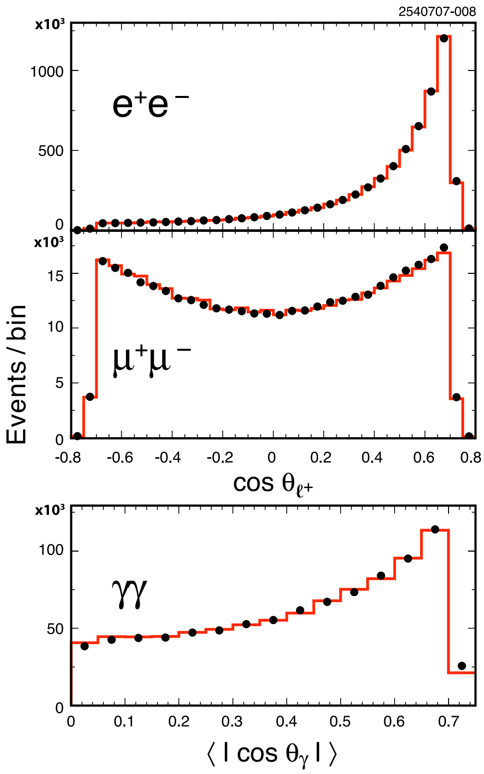

Fig. 2. Data-theory comparison for the angular distributions of the three QED channels at CLEO-c.

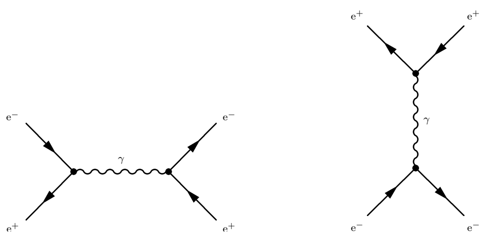

Fig. 3. Lowest-order Feynman diagrams for the Bhabha process in QED

2.2 LO cross sections and NLO corrections

by A. Arbuzov in coll. with myself and BabaYaga authors

Formulae for Born cross sections of the three QED processes used in the luminosity measurement

1. $e^+e^- \to e^+e^-$
2. $e^+e^- \to \gamma\gamma$
3. $e^+e^- \to \mu^+\mu^-$

and review of the status of NLO corrections. State that pure weak contributions are irrelevant at flavour factories, with the exception of γ-Z interference (about 0.1%) at the B-factories. A couple of questions:

1. Should we include a plot for the QED cross section scaling $1/s$ as a function of the c.m. energy in the 1-10 GeV range, in the LO and NLO approximations,

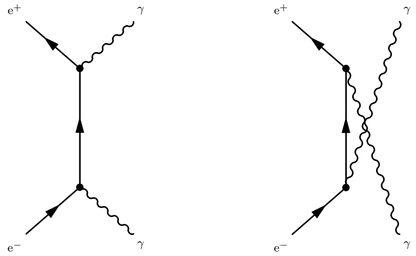

Fig. 4. Lowest-order Feynman diagrams for the process $e^+e^- \to \gamma\gamma$

to show the typical values of QED cross sections and NLO radiative corrections in typical experimental set-up? Personally, I vote yes

2. Should we include a few examples of Feynman diagrams for NLO (real/virtual) corrections, to give the reader an idea of which corrections we are speaking about? My answer is yes, I think it can help...

2.3 NNLO corrections

jointly prepared by the two main groups of mostly up-to-date two-loop calculations, i.e. S. Actis, R. Bonciani, M. Czakon, A. Ferroglia, J. Gluza, P. Mastrolia, A. Penin, E. Remiddi, T. Riemann and J.J. van der Bij. I will check the exact list of contributors with Roberto and Tord, if further contributions come.

A few questions, emerged from exchange of emails with people above:

1. Should we include one-two examples of Feynman diagrams contributing to NNLO corrections? I think so, useful for a review.
2. Should we include *here* plot(s) about the size of NNLO effects or should we postpone them in the final section on present th. accuracy of generators? Personally, I vote for the second option. For sure, we will include numerical results on NNLO corrections, to quantify their typical size.
3. Light pairs, in particular $e^+e^- \to e^+e^-e^+e^-$. We understand that such a contribution is included in Bhabha event sample, when requiring at least two tracks in the detector (at least for KLOE). Is the same true for other experiments? Is there substantial interest for a very precise evaluation of such an effect, beyond the soft + virtual approximation? Presently, light pairs are not included in MCs (yes in MCGPJ?) and are a source of th. uncertainty at the 0.05% level.

Since Bhabha scattering involves stable charged leptons both in the initial and in the final states, it can be measured experimentally with very high precision. At LEP, the experimental error in the luminosity measurement has been reduced to 0.4 permille [**?**] and it is expected to be even smaller at the ILC: the goal of the TESLA forward calorimeter collaboration is to reach the experimental accuracy of 0.1 permille in the first year of run [**?**]. Finally, at the low-energy accelerators DAΦNE and VEPP-2M the cross section of the large-angle scattering is measured with the accuracy of about 1 permille [**?**,**?**].

The theoretical error on the Bhabha scattering differential cross section directly affects the luminosity measurement precision. This is the reason whyin recent years a significant effort was devoted to the calculation of the perturbative corrections to this process (for the next-to-leading order corrections see Section **??**).

The calculation of the full NNLO corrections to the Bhabha scattering cross section requires three types of ingredients: *i)* the two-loop matrix elements for the $e^+e^- \to e^+e^-$ process; *ii)* the one-loop matrix elements for the $e^+e^- \to e^+e^-\gamma$ process, both in the case in which the additional photon is soft or hard-collinear[1]; *iii)* the tree-level matrix elements for $e^+e^- \to e^+e^-\gamma\gamma$, with two soft, two hard-collinear or one soft and one hard-collinear photons, and of $e^+e^- \to e^+e^-e^+e^-$, with the second e^+e^- pair to be considered as soft or hard-collinear. Dependent on the kinematics, other final states like e.g. $e^+e^-\mu^+\mu^-$ or those with hadrons are also possible. Moreover, the fixed-order corrections should be matched with a parton shower generator that takes into account the logarithmically enhanced contributions of soft and collinear photons at all orders in perturbation theory (see Section **??**).

The first complete one-loop prediction in the Standard Model was [**?**], followed by [**?**] and several others, and the leading virtual NNLO corrections from the top quark in [**?**,**?**,**?**]. The complete electroweak two-loop corrections are available in form of few form factors [**?**,**?**], but they are not implemented for Bhabha scattering so far. While further ~~only~~ the logarithmically enhanced terms of the two-loop electroweak corrections were calculated [**?**], in pure QED the situation is considerably different. The advent of new calculational techniques and a deeper understanding of the IR structure of unbroken theories, such as QED or QCD, made in such a way that nowadays the complete set of two-loop QED corrections is available. The calculation of these corrections will be presented in section 2.3.1.

We consider now the one-loop matrix elements with three particles in the final state. The diagrams involving the emission of a soft photon are known and they were included in the calculations of the two-loop matrix elements, in order to remove the IR soft divergencies. However, although the contributions due to a hard-collinear photon are taken into account in logarithmic accuracy by the parton shower, a full calculation of the diagrams involving a hard-photon in a general phase-space configuration is still missing. In section 2.3.2, we will comment on the possible strategies which can be adopted in order to calculate these corrections.

Concerning the tree level graphs with four particles in the final state, we can distinguish once more between the soft-photon contributions, which are included in the two-loop calculations, and the contributions involving one hard or two hard photons, which are taken into account in loga-

[1] By hard-collinear we mean a hard photon which is collinear to a charged particle within a given acollinearity angle imposed by the experimental cuts.

rithmic accuracy by the parton shower in the Monte Carlo event generators. The production of a soft e^+e^- pair was also studied, but only with logarithmic accuracy and it is included in the two-loop calculation (see section 2.3.1). However, the production of a hard-collinear pair of e^+e^- is not included in the current calculation of the NNLO corrections. Further comments on final states with four particles will be given in section 2.3.3.

Finally, in ~~on~~ section **??** we draw our conclusions.

2.3.1 Virtual corrections for the $e^+e^- \to e^+e^-$ Process

The calculation of the virtual two-loop QED corrections to the Bhabha scattering differential cross section was carried out in the last 10 years. This calculation was made possible by an improvement of the techniques employed in the evaluation of multiloop Feynman diagrams. An essential tool used to manage the calculation is the Laporta algorithm [**?**], which allows to reduce a generic combination of dimensionally-regularized scalar integrals to a combination of a small set of independent integrals called the "Master Integrals" (MIs) of the problem under consideration. The calculation of the MIs is then pursued by means of a variety of methods. Particularly important are the differential equations method [**?**] and the Mellin-Barnes techniques [**?**]. quote also here: [**?**,**?**,**?**,**?**,**?**,**?**,**?**,**?**]

(Some comments about the two methods??). Both methods proved to be very useful in the evaluation of virtual corrections to Bhabha scattering because they show there strength in preoblems with a small number of different kinematical parameters. They both allow one to obtain an analytic expression for the integrals, which must be written in terms of a suitable functional basis. A basis which was extensively employed in the calculation of multiloop Feynman diagrams of the type discussed here is represented by the Harmonic Polylogarithms [**?**] and their generalizations. Another fundamental achievement which allowed to complete the calculation of the QED two-loop corrections was an improved understanding of the IR structure of QED. In particular, the relation between the collinear logarithms in which the electron mass m_e plays the role of a natural cut-off and the corresponding poles in the dimensionally regularized massless theory was investigated [**?**,**?**,**?**].

A first complete diagrammatic calculation of the two-loop QED virtual corrections to Bhabha scattering can be found in [**?**]. However, this result was obtained in the fully massless approximation ($m_e = 0$), by employing dimensional regularization (DR) to regulate both soft and collinear divergencies. Today, the complete set of two-loop corrections to Bhabha scattering in pure QED have been evaluated using m_e as a collinear regulator, as required in order to include these fixed-order calculations in available Monte Carlo event generators. The Feynman diagrams involved in the calculation can be divided in three gauge independent sets: *i)* diagrams without fermion loops ("photonic" diagrams), *ii)* diagrams involving a closed electron loop, and *iii)* diagrams involving a closed loop of a fermion heavier than the electron or hadrons. These three sets are discussed in more details in the following of this section.

Photonic Corrections

A large part of the NNLO photonic corrections can be obtained in a closed analytic form, retaining the full dependence on m_e [**?**], by using the Laporta algorithm for the reduction of the Feynman diagrams to a combination of MIs, and then the differential equations method for the MIs analytic evaluation. With this technique it is possible to calculate, for instance, the NNLO corrections to the form factors [**?**,**?**]. However, a calculation of the two-loop photonic boxes retaining the full dependence on m_e seems to be ~~is~~ beyond the reach of this method. This is due to the fact that the number of Master Integrals belonging to the same topology after the reduction process in some cases is large. Therefore, one has ~~should be able~~ to solve analytically large systems of first-order ordinary linear differential equations, which is not easy or even not possible. Alternatively, in order to calculate the different MIs involved, one could use the Mellin-Barnes techniques, as shown in [**?**,**?**,**?**,**?**], or a combination of both methods. The calculation is very complicated and a full result is not yet available[2]. However, the full dependence on m_e is not phenomenologically relevant. In fact, the physical problem exhibits a well defined mass hierarchy. The mass of the electron is always very small compared to the other kinematic invariants and it can be safely neglected everywhere, with the exception of the terms in which it acts as a collinear regulator. The ratio of the photonic NNLO corrections to the Born cross section is the following

$$\frac{d\sigma^{(2,\text{PH})}}{d\sigma^{(\text{Born})}} = \frac{\alpha^2}{\pi^2} \sum_{i=0}^{2} \delta^{(\text{PH},i)} L_e^i + \mathcal{O}\left(\frac{m_e^2}{s}, \frac{m_e^2}{t}\right) , \quad (5)$$

where $L_e = \ln(s/m_e^2)$ and where the coefficients $\delta^{(\text{PH},i)}$ are functions of the scattering angle θ. The approximation given by Eq. (5) is sufficient for a phenomenological description of the process[3]. The coefficients of the double and single collinear logarithm in Eq. (5), $\delta^{(\text{PH},2)}$ and $\delta^{(\text{PH},1)}$, were obtained in [**?**,**?**]. However, the precision required for luminosity measurements at e^+e^- colliders demands the calculation of the non-logarithmic coefficient, $\delta^{(\text{PH},0)}$. The latter was obtained in [**?**] by reconstructing the differential cross section in the $s \gg m_e^2 \neq 0$ limit from the dimensionally regularized massless approximation [**?**]. The main idea of the method developed in [**?**] is outlined below. As far as the leading term in the small electron mass expansion is considered, the difference between the massive and the dimensionally regularized massless Bhabha scattering can be viewed as a difference between two regularization schemes for the infrared divergences. With the

[2] For the planar double box diagrams, all the MI integrals are known [**?**] for small m_e, while the MIs for the non-planar double box diagrams are not completed.

[3] It can be shown that the terms suppressed by a positive power of m_e^2/s do not play any phenomenological role already at very low c.m. energies, $\sqrt{s} \sim 10$ MeV. Moreover, the terms m_e^2/t (or m_e^2/u) become important in the extremely forward (backward) region, unreachable for the experimental set ups.

known massless two-loop result at hand, the calculation of the massive one is reduced to constructing the *infrared matching* term which relates the two above mentioned regularization schemes. To perform the matching an auxiliary amplitude is constructed, which has the same structure of the infrared singularities but is sufficiently simple to be evaluated at least in leading order in the small mass expansion. A particular form of the auxiliary amplitude is dictated by the general theory of infrared singularities in QED and involves the exponent of the one-loop correction as well as the two-loop corrections to the *logarithm* of the electron form factor. The difference between the full and the auxiliary amplitudes is infrared finite. It can be evaluated by using dimensional regularization for each amplitude and then taking the limit of four space-time dimensions. The infrared divergences, which induce the asymptotic dependence of the virtual corrections on the electron and photon masses, are absorbed into the auxiliary amplitude while the technically most nontrivial calculation of the full amplitude is performed in the massless approximation. The matching of the massive and massless results is then necessary only for the auxiliary amplitude and it is straightforward. Thus the two-loop massless result for the scattering amplitude along with the two-loop massive electron form factor [?] are sufficient to obtain the two-loop photonic correction to the differential cross section in the small electron mass limit.

A method based on a similar principle was subsequently developed in [?,?]; the authors of [?] confirmed the result of [?] for the NNLO photonic corrections to the Bhabha scattering differential cross section.

Electron Loop Corrections

The NNLO electron loop corrections arise from the interference of two-loop Feynman diagrams with the tree-level amplitude as well as from the interference of one-loop diagrams, as long as one of the diagrams contributing to each term involves a closed electron loop. This set of corrections presents a single two-loop box topology, and it is therefore technically less challenging to evaluate with respect to the photonic correction set. The calculation of the electron loop corrections was completed a few years ago [?,?]; the final result retains the full dependence of the differential cross section on the electron mass m_e. The MIs involved in the calculation were identified by means of the Laporta algorithm and evaluated with the differential equation method. As expected, after UV renormalization the differential cross section presented only residual IR poles which were removed by adding the contribution of the soft photon emission diagrams. The resulting NNLO differential cross section could be conveniently written in terms of 1- and 2-dimensional Harmonic Polylogarithms (HPLs) of maximum weight three. Expanding the cross section in the limit $s, |t| \gg m_e^2$, the ratio of the NNLO corrections to the Born cross section can be written as in Eq. (5):

$$\frac{d\sigma^{(2,\mathrm{EL})}}{d\sigma^{(\mathrm{Born})}} = \frac{\alpha^2}{\pi^2} \sum_{i=0}^{3} \delta^{(\mathrm{EL,i})} L_e^i + \mathcal{O}\left(\frac{m_e^2}{s}, \frac{m_e^2}{t}\right) . \quad (6)$$

Note that the series now starts with a cubic collinear logarithm. This logarithm appears, with an opposite sign, in the corrections due to the production of an electron-positron pair (the soft-pair production was considered in [?]). When the two contributions are considered together in the full NNLO, the cubic collinear logarithms cancel, and the cross section exhibits again at most a double logarithm, as in Eq. (5).

The explicit expression of all the coefficients $\delta^{(\mathrm{EL,i})}$, obtained by expanding the results of [?] was confirmed by two different groups [?,?]. In [?] the small electron mass expansion was performed within the soft-collinear effective theory (SCET) framework, while the analysis in [?] employed the asymptotic expansion of the Master Integrals.

Heavy-Flavor and hadronic Corrections

Finally, we consider the corrections originating from two-loop Feynman diagrams involving a heavy flavor fermion loop[4]. Since this set of corrections involves one more mass scale with respect to the corrections analyzed in the previous sections, a direct diagrammatic calculation is in principle a more challenging task. Recently, in [?] the authors applied their technique based on SCET to Bhabha scattering and obtained the heavy flavor NNLO corrections in the limit in which $s, |t|, |u| \gg m_f^2 \gg m_e^2$, where m_f^2 is the mass of the heavy fermion running in the loop. Their result was very soon confirmed in [?] by means of a method based on the asymptotic expansion of Mellin Barnes representation of the Master Integrals involved in the calculation. However, the results obtained in the approximation $s, |t|, |u| \gg m_f^2 \gg m_e^2$ cannot be applied to the case in which the $\sqrt{s} < m_f$ (as in the case of a tau loop at $\sqrt{s} \sim 1$ GeV), and they apply only to a relatively narrow angular region perpendicular to the beam direction when $\sqrt{s}$ is not very much larger than m_f (as in the case of top-quark loops at ILC). It was therefore necessary to calculate the heavy flavor corrections to Bhabha scattering assuming only that the electron mass is much smaller than the other scales in the process, but retaining the full dependence on the heavy mass, $s, |t|, |u|, m_f^2 \gg m_e^2$.

The calculation was carried out in two different ways: in [?,?] it was done analytically, while in [?,?] it was done numerically with the dispersion relations.

The technical problem of the diagrammatic calculation of Feynman integrals with four scales can be simplified by considering carefully, once more, the structure of the collinear singularities of the heavy-flavor corrections. The ratio of the NNLO heavy flavor corrections to the Born cross section is given by

$$\frac{d\sigma^{(2,\mathrm{HF})}}{d\sigma^{(\mathrm{Born})}} = \frac{\alpha^2}{\pi^2} \sum_{i=0}^{1} \delta^{(\mathrm{HF,i})} L_e^i + \mathcal{O}\left(\frac{m_e^2}{s}, \frac{m_e^2}{t}\right) , \quad (7)$$

[4] Here by "heavy flavor" we mean a muon or a tau-lepton, as well as an heavy quark, like the top, the b- or the c-quark, depending on the c.m. energy range that we are considering.

where now the coefficients $\delta^{(i)}$ are functions of the scattering angle θ and, in general, of the mass of the heavy fermions involved in the virtual corrections. It is possible to prove that, in a physical gauge, all the collinear singularities factorize and can be absorbed in the external field renormalization [?]. This observation has two consequences in the case at hand. The first one is that box diagrams are free of collinear divergencies in a physical gauge; since the sum of all boxes forms a gauge independent block, it can be concluded that the sum of all box diagrams is free of collinear divergencies in any gauge. The second consequence is that the single collinear logarithm in Eq. (7) arises from vertex corrections only. Moreover, if one chooses on-shell UV renormalization conditions, the irreducible two-loop vertex graphs are free of collinear singularities. Therefore, among all the two-loop diagrams contributing to the NNLO heavy flavor corrections to Bhabha scattering, only the reducible vertex corrections are logarithmically divergent in the $m_e \to 0$ limit[5]. The latter are easily evaluated even if they depend on two different masses. By exploiting these two facts, one can obtain the NNLO heavy-flavor corrections to the Bhabha scattering differential cross section assuming only that $s, |t|, |u|, m_f^2 \gg m_e^2$. In particular, one can set $m_e = 0$ from the start in all the two-loop diagrams with the exception of the reducible ones. This procedure allows one to effectively eliminate a mass scale from the two-loop boxes, so that these graphs can be evaluated with the techniques already employed in the diagrammatic calculation of the electron loop corrections[6]. In the case in which the heavy flavor fermion is a quark, it is straightforward to modify the calculation of the two-loop self-energy diagrams to obtain the mixed QED-QCD corrections to Bhabha scattering [?].

An alternative approach to the calculation of the heavy flavor corrections to Bhabha scattering, is based on dispersion relations.

(Tord, could you please comment on that?).
TR: Yes, here I will write a short text piece.
text .. text .. text.. text
text .. text .. text.. text

2.3.2 Fixed-Order calculation of the Hard Photon Emission at One Loop

The one-loop matrix element for the process $e^+e^- \to e^+e^-\gamma$ is one of the contributions to the complete set of NNLO corrections to Bhabha scattering. Its evaluation requires the non-trivial computation of one-loop tensor integrals associated to pentagon-diagrams.

According to the standard Passarino-Veltman (PV) approach [?], one-loop tensor integrals can be expressed in terms of scalar integrals, called Master Integrals (MI's), with trivial numerators that are independent of the loop variable, each multiplied by a Lorentz structure depending only on combinations of the external momenta and the metric tensor. The achievement of the complete PV-reduction amounts to solve a non-trivial system of equations. TR: The next statement is not clear to me. I try to modify, please check if it is ok with you. ~~Due to its size, it is reasonable replacing the analytic techniques by numerical tools.~~ The difficulty encountered with the numerical implementation of PV-reduction, which is in principle trivial, is indeed due to the subtlety of Gram determinants: naturally arising in the procedure of inverting a system, they can vanish or be numerically unstable in special configurations of the phase space, thus requiring proper modifications of the reduction algorithm, in combination with the strategy not to perform the complete analytical reduction to simple PV masters [?,?,?,?,?,?,?]. A viable solution for the complete algebraic reduction of tensor-pentagon (and tensor-hexagon) integrals has been ~~recently~~ formulated in [?,?,?] by exploiting the algebra of signed minors [?], and the cancelation of powers of inverse Gram determinants in this approach was performed recently in [?,?].

(please Tord, add any further comment if you whish to expand this point.)
TR: done, maybe not complete? Please have a close look!
Generally I find the text a bit long for having no numerical results ... but why not ...

The computation of the one-loop 5-point amplitude $e^+e^- \to e^+e^-\gamma$ can be alternatively performed by using generalized-unitarity cutting rules (see [?] for a detailed compilation of references). In the following, we propose two ways to achieve the result, respectively *via* an analytical and a semi-numerical method. The application of generalized cutting-rules as an on-shell method of calculation is based on two fundamental properties of scattering amplitudes: *i)* analyticity, according to which any amplitude is determined by its own singularity structure [?]; *ii)* and unitarity, according to which the residues at the singularities are determined by products of simpler amplitudes. Turning these properties into a tool for computing scattering amplitudes is possible because of the underlying representation of the amplitude in terms of Feynman integrals and their PV-reduction, which grant the existence of a representation of any one-loop amplitudes as linear combination of MI's, each multiplied by a rational coefficient. In the case of $e^+e^- \to e^+e^-\gamma$, pentagon-integrals ~~should be ultimately~~ may be expressed, though PV-reduction, to a linear combination of 17 MI's (including 3 boxes, 8 triangles, 5 bubbles, and 1 tadpole). Since the required MI's are analytically known [?,?,?,?,?,?,?], the determination of their coefficients is what is needed for reconstructing the amplitude as a whole. To this aim, one may use the Mathematica program hexagon [?,?]. Also, matching the generalized cuts of the amplitude against the cuts of the MI's provides an efficient way to extract their (rational) coefficients out of the amplitude itself. In general, the fulfillment of multiple-cut conditions requires loop momenta with complex components. The effect of the cut-conditions is to freeze some of its components, when not all, accord-

[5] Additional collinear logarithms arise also from the interference of one-loop diagrams in which at least one vertex is present.

[6] The necessary MIs can be found in [?,?].

ing to the number of the cuts. With the *quadruple*-cut [?] the loop momentum is completely frozen, yielding the algebraic determination of the coefficients of n-point functions with $n \geq 4$. In cases where fewer than four denominators are cut, like *triple-cut* [?,?,?], *double-cut* [?,?,?,?], and *single-cut* [?], the loop momentum is not frozen: the free-components are left over as phase-space integration variables.
For each multiple-cut, the evaluation of the phase-space integral would generate, in general, logarithms and a non-logarithmic term. The coefficient of a given n-point MI finally appears in the non-logarithmic term of the corresponding n-particle cut, where all the internal line are on-shell (while the logarithms correspond to the cuts of higher-point MI's which share that same cut). Therefore all the coefficients of MI's can be determined in a *top-down* algorithm, starting from the quadruple-cuts for the extraction of the 4-point coefficients, and following with the triple-, double-, and single-cuts, for the coefficients of 3-, 2- and 1-point, respectively. The coefficient of an n-point MI ($n \geq 2$) can be also obtained by specializing to the case at hands the generating formulas given in [?] for general one-loop amplitudes.

Instead of the analytic evaluation of the multiple-cut phase-space integrals, it is worth considering the feasibility of computing the process $e^+e^- \to e^+e^-\gamma$ with a seminumerical technique by-now known as `OPP`-reduction [?,?], based on the decomposition of the numerator of any one-loop integrand in terms of its denominators [?]. Within this approach, the coefficients of the MI's can be found simply by solving a system of numerical equations, and avoiding any explicit integration. The `OPP`-reduction algorithm exploits the polynomial structures of the integrand when evaluated at values of the loop-momentum fulfilling multiple cut-conditions: *i)* for each n-point MI, one considers the n-particle cut obtained by setting all the propagating lines on-shell; *ii)* such a cut is associated to a polynomial in terms of the free components of the loop-momentum, which corresponds to the numerator of the integrand evaluated at the solution of the on-shell conditions; *iii)* the constant-term of that polynomial *is* the coefficient of the MI.
Hence, the difficult task of evaluating one-loop Feynman integrals is reduced to the much simpler problem of polynomial fitting, recently optimized by using a projection-technique based on the Discrete Fourier Transform [?].

In general the result of a dimensional-regulated amplitude in the 4-dimensional limit, being D $(= 4-2\epsilon)$ the regulating parameter, is expected to contain (poly)logarithms, often referred to as the *cut-constructible term*, and a pure *rational term*. In a remarkable over-emphasized? paper [?] which completed the `OPP`-method, the rising of the rational term was attributed to two potential sources (of UV-divergent integrals): one, defined R_1, due to the D-dimensional completion of the 4-dimensional contribution of the numerator; a second one, called R_2, due to the (-2ϵ)-dimensional algebra of Dirac-matrices. Therefore within the `OPP`-approach the calculation of the one-loop amplitude $e^+e^- \to e^+e^-\gamma$ can proceed through two computational stages:

1. the coefficients of the MI's that are responsible both for the cut-constructible and for the R_1-rational terms can be determined by applying the `OPP`-reduction discussed above [?,?,?];
2. the R_2-rational term can be computed by using additional tree-level-like diagrammatic rules, very much resembling the computation of the counter terms needed for the renormalization of UV-divergencies [?].

The numerical influence of the radiative loop diagrams, including the pentagon diagrams, is expected to be not large and not being logarithmically enhanced (??). Nevertheless, we expect some predictions for them in the near future.

2.3.3 Four Particles in the Final State

Here, as a minimum, some comment close to what Guido wrote in his emails.
Maybe something more?
Volunteers?

2.4 Multiple photon effects and matching with NLO corrections

by myself with BabaYaga authors (for the QED Parton Shower and matching recipe as in BabaYaga@NLO) + A. Arbuzov and L. Trentadue for QED Structure Functions and theory underlying MCGPJ. I will prepare a more detailed text, including also YFS exponentiation as in BHWIDE, and I will circulate it for corrections/improvements. Hope S. Jadach, B. Ward and W. Placzek will review the part on YFS.... They told me to be very busy but willing to contribute with some text and in the review of the numerical results involving BHWIDE.

2.4.1 Universal methods for leading logarithmic corrections

I think it will be important to emphasize that the methods on the grounds of the generators today employed at flavour factories have been already used, *widely and successfully*, in the 90s at LEP for electroweak tests of the SM, especially for high-precision predictions to luminosity through small-angle Bhabha scattering and Z^0 physics. Give references to original papers and some relevant examples of precision codes used by LEP collaborations. This should make clear that the theory we are using today at low energies is particularly robust, having passed the stringent tests of very accurate LEP data.

Mostly popular and standard approach: QED electron Structure Function $D(x, Q^2)$. It can be obtained by solving the Altarelli-Parisi equation in the non-singlet approximation

- either analytically, in the strictly collinear approximation (as implemented in MCGPJ). Just a question: what is the Q^2 scale in MCGPJ? I guess $Q^2 = s$, to check.
- or numerically, through a Parton Shower MC algorithm, including photon $p_\perp$ (as implemented in BabaYaga). Here $Q^2 = st/u$, to resum the leading terms due to initial-final-state interference, which is the correct choice for non-resonant processes.

Alternative method: Yennie-Frautschi-Suura exponentiation (as in BHWIDE).

Generator	Processes	Theory	Accuracy
Bagenf	e^+e^-	$O(\alpha)$	0.5%
BabaYagav3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$0.5 \div 1\%$
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$O(\alpha)$ + PS	$\sim 0.1\%$
BKQED	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$O(\alpha)$	1%
MCGPJ	$e^+e^-, \mu^+\mu^-...$	$O(\alpha)$ + SF	$< 0.2\%$
BHWIDE	e^+e^-	$O(\alpha)$ YFS	$\sim 0.5\%$(LEP1)

Table 1. MC generators used for luminosity monitoring.

2.4.2 Matching higher-order and NLO corrections

Common solution for most precise generators: BHWIDE, BabaYaga@NLO and MCGPJ, albeit different in the implementation aspects.

Here I propose that, rather giving too many th. details and too lengthy formulae, it will be important to stress that *i)* it is possible to match NLO and multiple photon corrections consistently, avoiding double counting of LL contributions *ii)* the convolution of NLO corrections with h.o. effects allows to include, *even if approximately*, the dominant part of NNLO corrections, given by infrared-enhanced $\alpha^2 L$, L collinear log, contributions. This can greatly help and make more clear the discussion on the generators' accuracy in the last section *iii)* that BabaYaga@NLO and BHWIDE implement a fully factorized matching recipe, while MCGPJ include some terms in additive form. This can give rise to some (small) differences, which can show up in the presence of particularly severe cuts (and seen in the comparisons done by A. Sibidanov, see below).

Do you agree on this scheme?

2.5 Monte Carlo generators

by myself. I will circulate the draft asking to MCs' authors for corrections.

MC programs used in early measurements of luminosity at flavour factories (and sometimes still used in recent experimental publications) include generators such as Bagenf, BabaYaga v3.5 and BKQED. However, the above generators either are based on a fixed NLO calculation (such as Bagenf and BKQED) or include corrections to all orders in perturbation theory, but in the leading log approximation only (as for BabaYaga v 3.5). Therefore, the precision of these codes can be estimated to lye in the range 0.5-1%, depending on the adopted experimental cuts.

The increasing precision reached on the experimental side during the last few years led to the development of new, dedicated precision tools, such as BabaYaga@NLO and MCGPJ, and the adoption of already well-tested codes, such as BHWIDE, the latter extensively used at high-energy LEP/SLC colliders for simulation of the large-angle Bhabha process. All these three codes include NLO corrections in combination with multiple photon contributions and have therefore, as it will be further emphasized in the following, a precision tag of 0.1%.

The basic theoretical features of the different generators are summarized in the following (*include web address, if available*).

1. Bagenf/BKQED –
2. BabaYaga v3.5 – It is a MC generator developed by Pavia group at the starting of DAΦNE operation [?] using a QED Parton Shower (PS) approach for the treatment of leading log QED corrections to luminosity processes and later improved according to Ref. [?] to account for the interference of radiation emitted by different charged legs in the generation of the momenta of the final-state particles. The main drawback of BabaYaga v3.5 is the absence of $O(\alpha)$ non-log contributions, resulting in a theoretical precision of 0.5% for large-angle Bhabha scattering and of about 1% for $\gamma\gamma$ and $\mu^+\mu^-$ final states [?].
3. BabaYaga@NLO – It is the presently released version of BabaYaga, based on the matching of exact $O(\alpha)$ corrections with QED PS, as described in detail in Ref. [?]. The accuracy of the current version is estimated to be at 0.1% level, as detailed in the following, for large-angle Bhabha scattering, two-photon and $\mu^+\mu^-$ [7] production.
4. MCGPJ – It is the generator developed by a Dubna-Novosibirsk collaboration [?] and used at VEPP collider. This program includes exact $O(\alpha)$ corrections supplemented with higher-order leading logarithmic contributions related to the emission of collinear photon jets and taken into account through collinear QED Structure Functions (SF) [?]. The theoretical precision is estimated to be better than 0.2%.
5. BHWIDE – It is a MC code realized in Krakow-Knoxwille at the time of LEP operation and described in Ref. [?]. In this generator, exact $O(\alpha)$ corrections are matched with the resummation of soft and collinear logarithms through the Yennie-Frautschi-Suura (YFS) exponentiation approach. According to the authors, the precision is estimated about 0.5% for LEP1. This accuracy estimate was derived through detailed comparisons of the

[7] At present, finite mass effects in the virtual corrections to $e^+e^- \to \mu^+\mu^-$, which should be taken into for precision simulations at the Φ-factories, are not included in BabaYaga@NLO.

BHWIDE predictions with those of other LEP tools in the presence of pure weak corrections. However, since the latter are phenomenologically unimportant at e^+e^- accelerators of moderately high energies and the QED theoretical ingredients of BHWIDE are very similar to the formulation of both BabaYaga@NLO and MCGPJ, one can argue that BHWIDE accuracy for physics at flavour factories is at the level of 0.1%.

2.6 Numerical results

Before showing the results and presenting arguments which enable to settle the technical and theoretical accuracy of generators, it is worth discussing the impact of the various sources of radiative corrections implemented in the programs used in the experimental analyses. This allows to understand which corrections are strictly necessary to achieve a precision at the per mille level, for both the calculation of integrated cross section and simulation of more exclusive distributions.

2.6.1 Cross sections

From Tab. 2, it can be seen that $\mathcal{O}(\alpha)$ corrections decrease the Bhabha cross section of about 15% at the Φ–factories and of about 20–25% at the B–factories. Within the full set of $\mathcal{O}(\alpha)$ corrections, non–log terms are of the order of 0.5%, almost independently of the centre-of-mass (c.m.) energy, as expected, and with a mild dependence on the angular acceptance cuts, as due to box/interference contributions. The effect of higher–order corrections due to multiple photon emission is about 0.5-1% at the Φ–factories and reaches 1–2% at the B–factories. The contribution of (approximate) $\mathcal{O}(\alpha^2 L)$ corrections is not exceeding the 0.1% level, while the vacuum polarization increases the cross section of about 2% around 1 GeV and of about 5–6% around 10 GeV. Concerning the latter correction, the non–perturbative hadronic contribution to the running of α is included in BabaYaga@NLO both in the lowest–order and one–loop diagrams through the HADR5N routine [**?**], that returns a data driven error, thus affecting the accuracy of the theoretical calculation. Analogous results about the size of radiative corrections have been obtained recently [**?**] for the process $e^+e^- \to \gamma\gamma$, also of interest for precision luminosity studies at flavour factories. *Specify effects for τ-charm !*

We can include a similar table for $\gamma\gamma$ production. What do you think?

2.6.2 Distributions

We can include similar plots for $\gamma\gamma$ production. What do you think?

As a whole, these results indicate that both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarization) are necessary for 0.1% theoretical precision, for both cross sections and distributions.

$\sqrt{s}$(GeV)	1.02	4.	10.
δ_α	-17.16	-17.34	-24.35
$\delta_\alpha^{\mathrm{non-log}}$	-0.66	-0.68	-0.70
δ_{HO}	0.93	1.25	1.76
$\delta_{\alpha^2 L}$	0.09	0.09	0.11
δ_{VP}	2.43	4.46	6.03

Table 2. Relative size of different sources of correction (in per cent) to the large-angle Bhabha cross section for typical selection cuts at Φ, τ-charm and B factories.

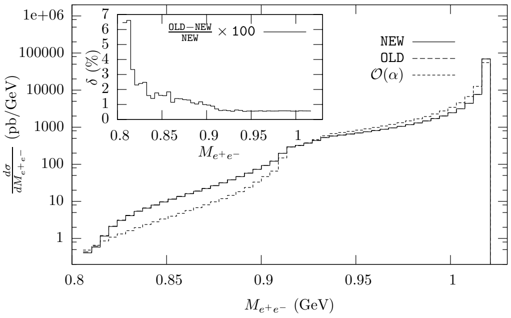

Fig. 5. Invariant mass distribution of the Bhabha process at KLOE, according to BabaYaga v3.5 (OLD) and BabaYaga@NLO (NEW). The inset shows the relative effect of NLO corrections, given by the difference of BabaYaga v3.5 and BabaYaga@NLO predictions

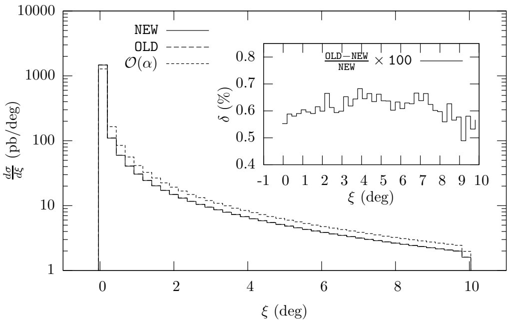

Fig. 6. The same as Fig... for the acollinearity distribution

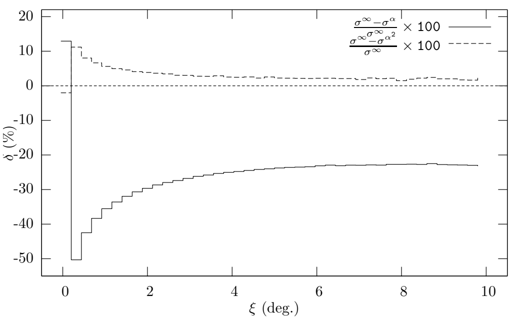

Fig. 7. Relative effect of higher-order corrections on the acollinearity distribution

angular acceptance	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$20^\circ - 160^\circ$	6086.6(1)	6086.3(2)	0.005
$55^\circ - 125^\circ$	455.85(1)	455.73(1)	0.030

Table 3. Cross section predictions [nb] of BabaYaga@NLO and BHWIDE for the Bhabha cross section corresponding to two different angular acceptances, for KLOE experiment at DAΦNE, and their relative differences

2.7 Tuned comparisons

The typical procedure followed in the literature for establishing the technical precision of a given generator is to perform tuned comparisons between independent predictions, using the same set of input parameters and experimental cuts. This strategy was initiated during the CERN workshops for precision physics at LEP and is still in use nowadays when considering processes of interest for physics at hadron colliders, such as single W and Z production, demanding particularly accurate theoretical formulations. The tuning procedure is a key step in the validation of generators, because it allows to check that the different details entering the complex structure of generators themselves, e.g. implementation of radiative corrections, event selection routines, MC integration and event generation, are under control and to fix possible bugs.

The tuned comparisons discussed in the following have been obtained

- taking into account *realistic* event selection cuts. *We will include the cuts used to produce numerical results*
- switching off the vacuum polarization correction to the Bhabha scattering cross section. Actually, the generators implement the non-perturbative hadronic contribution to the running of α according to different parameterizations, which differently affect the cross section prediction. Hence, this simplification is introduced to avoid possible bias in the interpretation of the results and allows to disentagle the effect of pure QED corrections.

2.7.1 Φ-factories

- *DAΦNE: here I included the results prepared by BabaYaga group for the comparisons between BabaYaga@NLO and BHWIDE, using cuts of KLOE selection.*
- *VEPP-2M: here I put the new results by G Fedotovich and A Sibidanov for CMD-2. The BabaYaga@NLO, BHWIDE and MCGPJ Bhabha results for the integrated cross section with cuts agree within 0.1%. Differences of a few per mille show up for very small acollinearity cut, as shown in the figures. I'll ask precise numbers to be given in a Table.*

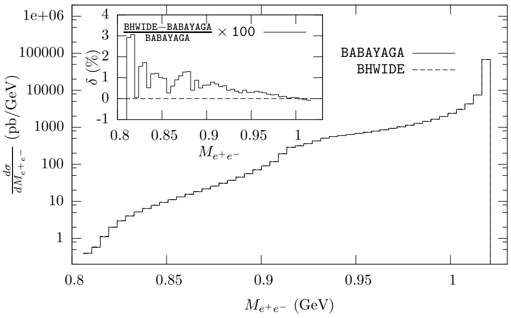

Fig. 8. Invariant mass distributions according to BHWIDE and BabaYaga@NLO, for KLOE experiment at DAΦNE, and relative differences of the codes' predictions (inset).

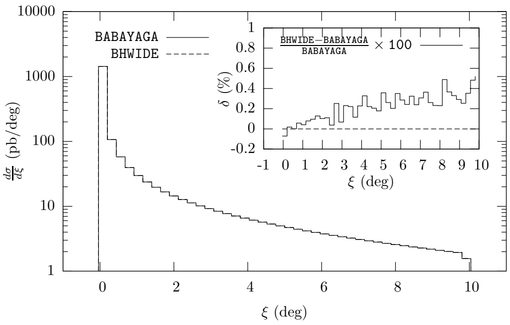

Fig. 9. The same as Fig... for the acollinearity distributions.

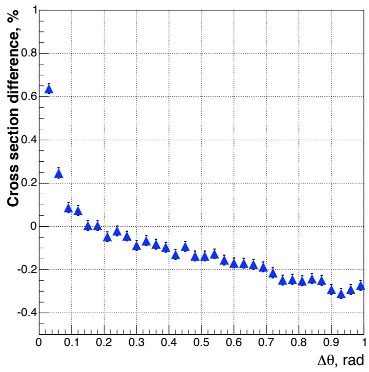

Fig. 10. Relative differences between BHWIDE and MCGPJ Bhabha cross sections as a function of the acollinearity cut, for CMD-2 experiment at VEPP-2M

2.7.2 τ-charm factories

Here we have nothing concret, at present... We received an email past week from Ronggang PING of BES, who is making comparisons at $\sqrt{s}$ = 3.686 GeV, but he sees too large differences between BabaYaga v3.5 and BabaYaga@NLO-BHWIDE (that agree!). I'm confident we will solve the issue, and it would be nice to put the results of his inspection here.

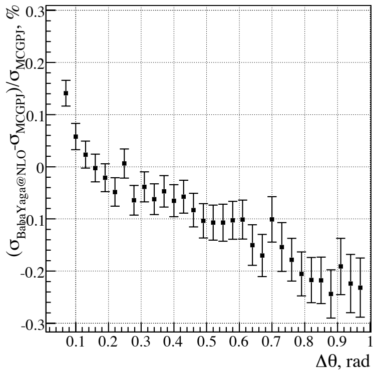

Fig. 11. Relative differences between BabaYaga@NLO and MCGPJ Bhabha cross sections as a function of the acollinearity cut, for CMD-2 experiment at VEPP-2M

angular range (c.m.s.)	`BabaYaga@NLO`	`BHWIDE`	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$30^\circ \div 150^\circ$	24.17(2)	24.22(2)	0.086
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	1.928(2)	1.931(3)	0.141
$70^\circ \div 110^\circ$	3.554(6)	3.549(3)	0.155
$80^\circ \div 100^\circ$	0.824(2)	0.822(1)	0.243

Table 4. Cross section predictions [nb] of BabaYaga@NLO and BHWIDE for the Bhabha cross section as a function of the angular selection cuts, for Babar experiment at PEP-II, and their relative differences

2.7.3 *B*-factories

PEP-II: here I put the Baba results presented by Andreas Hafner and Achim at previous meetings, and present in the PhD thesis of Andreas. I don't have anything for Belle... Simon, should we do to try something?

The main tuned conclusions are

- The cross sections predictions of BabaYaga@NLO, BHWIDE and MCGPJ generally agree within 0.1%. If larger differences show up (for particularly severe cuts). they can be understood in terms of the slightly different implementation of sub-leading $O(\alpha^2)$ contributions.
- Also distributions agree well, with relative differences at a few mille level. Larger discrepancies are only seen in sparse populated phase-space regions corresponding to very hard photon emission and which does not influence the luminosity measurement

2.8 Theoretical accuracy

I'l write it, in collaboration with two-loop people. Here we can put plots of the effects presently neglected in generators and a summary table about th. precision

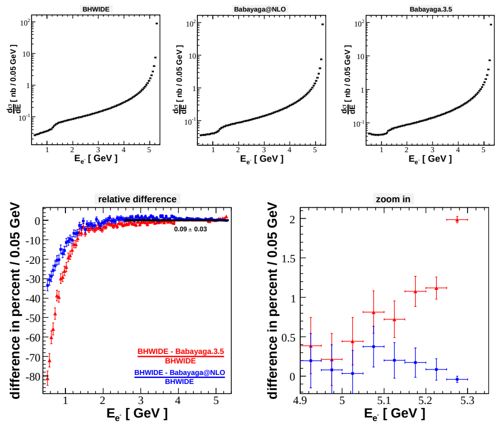

Fig. 12. Electron energy distributions according to BHWIDE, BabaYaga@NLO and BabaYaga v3.5 for Babar experiment at PEP-II, and relative differences of the codes' predictions.

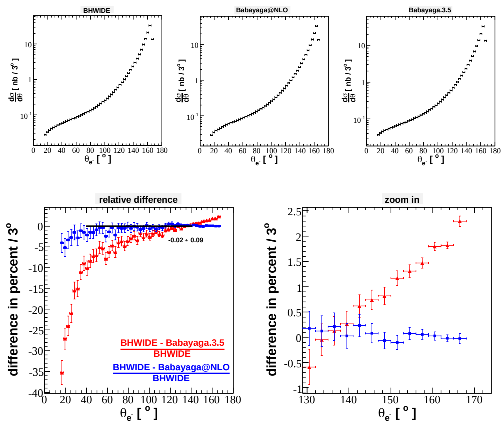

Fig. 13. The same as Fig. 5 for the electron scattering angle distribution

2.9 Total luminosity error

Do you agree on a (also short) section like this? I.e. where do we are and can we realistically arrive in the total luminosity budget

3 Scan

scan

4 Radiative return

radiative return

5 Vacuum polarization

vacuum polarization

6 Tau physics

tau physics

7 Summary

summary

References

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2. G. Balossini, C.M. Carloni Calame, G. Montagna, O. Nicrosini, F. Piccinini, Nucl. Phys. Proc. Suppl. **162**, 59 (2006), hep-ph/0610022