Coherent X-radiation of a relativistic electron on Nano-scale multilayer structure in Laue scattering geometry

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The coherent X-radiation of relativistic electron crossing a periodic multilayer artificial structure is considered in Laue geometry. The expressions describing spectral-angular characteristics of the radiation are derived. The considerable increase of possibility of the radiation photon yield with change of the asymmetry of the relativistic electron coulomb field reflection with respect to multilayer target entrance surface is shown.

• Introduction

- Traditionally the relativistic particle radiation in periodic lamellar structure would be considered in Bragg scattering geometry, when the reflecting layers are situated parallel to entrance surface that is in symmetric case. The radiation would be considered as resonance transition radiation [1-2]. A progress is observed recently in the description of the radiation in such a media [3] is represented as a sum of diffraction transition radiation (DTR) and parametric X-radiation (PXR) by analogy with the relativistic electron coherent radiation in crystal media [4-7].
- In the present work the coherent X-radiation in artificial periodic structure for the first time is considered in Laue scattering geometry under arbitrary asymmetry of relativistic electron coulomb field reflection with respect to target surface by analogy with the radiation in single crystal media.

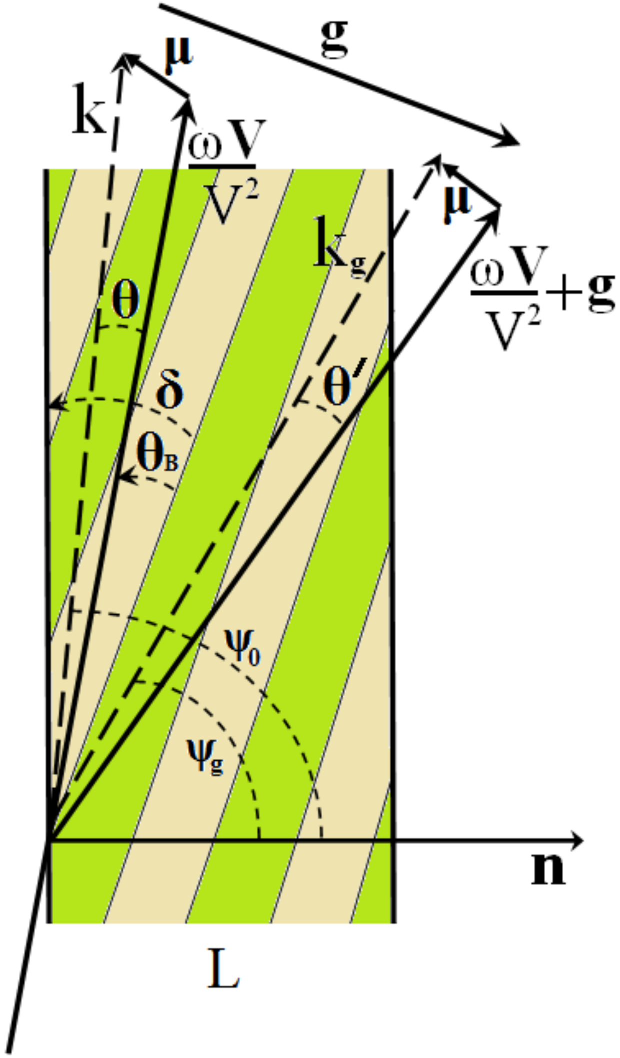


Figure 1. Scheme of the radiation process of relativistic electron in multilayer structure (**k** and $\mathbf{k_g}$ -incident and diffracted pseudo photons)

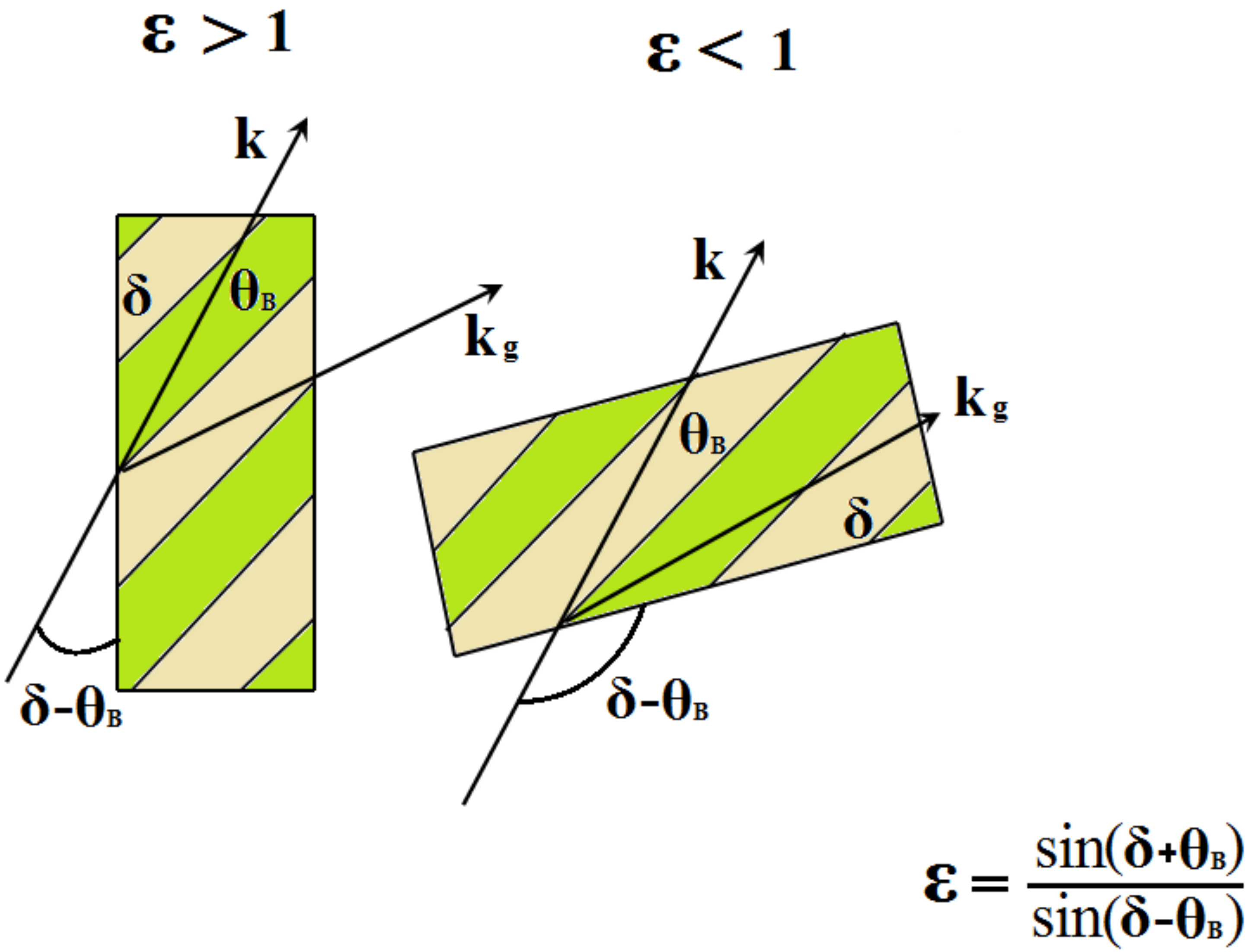


Figure 2. The radiation process under different reflection asymmetry

The system of equation for the Fourier transform images of electromagnetic field in two-wave approximation of dynamic theory of diffraction has the following view [9]:

$$\begin{cases} (\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-\mathbf{g}}C^{(s,\tau)}E_{\mathbf{g}}^{(s)}=8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega-\mathbf{kV}), \\ \omega^2\chi_{\mathbf{g}}C^{(s,\tau)}E_0^{(s)}+(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)}=0, \end{cases} \tag{1}$$

It is not too difficult to show that the magnitudes of the Fourier coefficients χ_a and χ_b have such a view:

$$\chi_0(\omega)=\frac{a}{T}\chi_a+\frac{b}{T}\chi_b \qquad \chi_{\mathbf{g}}(\omega)=\frac{\exp(-iga)-1}{igT}(\chi_b-\chi_a)$$

Where *a* and *b* are the thickness of the first and second layers, respectively χ_a and χ_b are their appropriate dielectric susceptibility,

$T=a+b$ the structure period, reciprocal lattice vector $g=\frac{2\pi}{T}n$, $n=0,\pm1,\pm2,\ldots$

RADIATION AMPLITUDE

- When solving the dispersion equation

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)-\omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}C^{(s)^2}=0$$

- following from (1) by a standard methods of dynamic theory [10], we will find expressions for и $k_{\mathbf{g}}$:
- Where the dynamic addition agents are

$$k=\omega\sqrt{1+\chi_0}+\lambda_0 \qquad k_{\mathbf{g}}=\omega\sqrt{1+\chi_0}+\lambda_{\mathbf{g}}$$

$$\lambda_{\mathbf{g}}^{(1,2)}=\frac{\omega}{4}\left(\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right) \qquad \lambda_0^{(1,2)}=\omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)$$

$$\beta=\alpha-\chi_0\left(1-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\right) \qquad \gamma_0=\cos\psi_0 \qquad \gamma_{\mathbf{g}}=\cos\psi_{\mathbf{g}} \qquad \alpha=\frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)$$

- The solution of the combined equations (1) gives us the field of the relativistic particle radiation:

$$E_{\mathbf{g}}^{(s)Rad}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{L}{\gamma_g}\right]}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)}\right)}\left[\left(\frac{\omega}{-\chi_0\omega-2\lambda_0^*}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}\right)}\right)\times\right.$$

$$\left.\times\left(1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right)-\left(\frac{\omega}{-\chi_0\omega-2\lambda_0^*}+\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}\right)}\right)\left(1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right]$$

(2)

where $\lambda_0^*=\omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right)$ $\lambda_{\mathbf{g}}^*=\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$ $\gamma=1/\sqrt{1-V^2}$

- The terms in square brackets in (16) correspond to two branches of solution for X-ray waves excited in the multilayer structure. For the further analyses of the radiation let us present the dynamic addition agent as $\lambda_g^{(1,2)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2}\left(\xi^{(s)} - \frac{i\rho^{(s)}(1-\varepsilon)}{2} \pm\right.$

$$\left.\pm\sqrt{\xi^{(s)^2} + \varepsilon - 2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)} + \kappa^{(s)}\varepsilon\right) - \rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4} + \kappa^{(s)^2}\varepsilon\right)}\right)$$

, where

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1-\varepsilon}{2\nu^{(s)}}$$

$$\nu^{(s)} = \frac{C^{(s)}\operatorname{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}}{\chi'_0} \equiv \frac{2C^{(s)}\sin\left(\frac{ga}{2}\right)}{g}\frac{\chi'_b - \chi'_a}{a\chi'_a + b\chi'_b}$$

$$\rho^{(s)} = \frac{\chi''_0}{\left|\operatorname{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C^{(s)}} \equiv \frac{a\chi''_a + b\chi''_b}{|\chi'_b - \chi'_a|C^{(s)}}\frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|}$$

$$\kappa^{(s)} = \frac{\chi''_{\mathbf{g}}C^{(s)}}{\chi''_0} \equiv \frac{2C^{(s)}\sin\left(\frac{ga}{2}\right)}{g}\frac{\chi''_b - \chi''_a}{a\chi''_a + b\chi''_b}$$

Radiation spectral-angular density

- The expression for radiation field amplitude in (2) can be presented as sum of two components:
- $E_{\mathbf{g}}^{(s)Rad} = E_{PXR}^{(s)} + E_{DTR}^{(s)}$ where

$$E_{PXR}^{(s)} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2 \chi_{\mathbf{g}} C^{(s)}}{8\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2} \frac{\gamma_{\mathbf{g}}}{\gamma_0}}} \cdot \frac{1}{\lambda_0^*} \times$$

$$\left[\left(\beta + \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2} \frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}} L\right)}{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}\right) - \right. \qquad (3)$$

$$\left. - \left(\beta - \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2} \frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}} L\right)}{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}\right)\right] \exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right],$$

- Formula (3) expresses the radiation field amplitude analogous to one for the parametric X-radiation in a crystal.

$$E_{DTR}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\chi_{\mathbf{g}} C^{(s)}}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}} \left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right) \times$$

- $$\times\left[\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right) - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right]\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right]. \quad (4)$$

- . Formula (4) expresses the amplitude of the field that appears as the result of diffraction on the periodic structure of transition radiation arising on entrance surface of the multilayer target.

Substituting (3), (4) to known expression for spectral-angular density of X-radiation

$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E^{(s)Rad} \right|^2$$

we will find the expressions describing the PXR and DTR contributions to spectral-angular density of the radiation and the summand being the result of these radiation mechanisms interference:

$$\omega \frac{d^2 N_{PXR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \frac{\theta^2}{(\theta^2 + \gamma^{-2} - \chi_0')^2} R_{PXR}^{(s)}$$, where

$$R_{PXR}^{(s)} = \left(1 - \frac{\xi}{\sqrt{\xi^2 + \varepsilon}}\right)^2 \frac{1 + \exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right) - 2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)^2 + \rho^{(s)2}\Delta^{(1)2}}$$

$$\omega \frac{d^2 N_{DTR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \theta^2 \left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0'}\right)^2 R_{DTR}^{(s)},$$ where

$$R_{DTR}^{(s)} = \frac{4\varepsilon^2}{\xi^2 + \varepsilon} \exp\left(-b^{(s)}\rho^{(s)} \frac{1 + \varepsilon}{\varepsilon}\right) \times$$
$$\times \left[\sin^2\left(b^{(s)} \frac{\left(\sqrt{\xi^2 + \varepsilon}\right)}{\varepsilon}\right) + sh^2\left(b^{(s)}\rho^{(s)} \frac{(1 - \varepsilon)\xi^{(s)} + 2\varepsilon\kappa^{(s)}}{2\varepsilon\sqrt{\xi^2 + \varepsilon}}\right)\right],$$

$$\omega \frac{d^2 N_{INT}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \theta^2 \left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0'} \right) \frac{1}{\theta^2 + \gamma^{-2} - \chi_0'} R_{ИНТ}^{(s)},$$

where

$$R_{ИНТ}^{(s)} = -\frac{2\varepsilon}{\xi^{(s)2} + \varepsilon} \mathrm{Re} \left(\left(\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon} \right) \frac{1 - \exp\left[-ib^{(s)} \left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right) - b^{(s)} \rho^{(s)} \Delta^{(1)} \right]}{\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} - i\rho^{(s)} \Delta^{(1)}} \times \right.$$

$$\left. \times \left(\exp\left[ib^{(s)} \left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right) - b^{(s)} \rho^{(s)} \Delta^{(1)} \right] - \exp\left[ib^{(s)} \left(\sigma^{(s)} + \frac{\xi^{(s)} + \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right) - b^{(s)} \rho^{(s)} \Delta^{(2)} \right] \right) \right).$$

- The main parameters, used in the expressions for the radiation:

$$\Delta^{(2)}=\frac{\varepsilon+1}{2\varepsilon}+\frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}+\frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}$$

$$\Delta^{(1)}=\frac{\varepsilon+1}{2\varepsilon}-\frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}-\frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}$$

$$\sigma^{(s)}=\frac{1}{\left|\chi'_{\mathbf{g}}\right|C^{(s)}}\left(\theta^2+\gamma^{-2}-\chi'_0\right)\equiv\frac{1}{\nu^{(s)}}\left(\frac{\theta^2}{\left|\chi'_0\right|}+\frac{1}{\gamma^2\left|\chi'_0\right|}+1\right)$$

$$b^{(s)}=\frac{\omega\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C^{(s)}}{2}\frac{L}{\gamma_0}=\frac{1}{2\sin\left(\delta-\theta_B\right)}\frac{L}{L_{ext}^{(s)}}$$ electrons' path in multilayer target expressed in extinction length

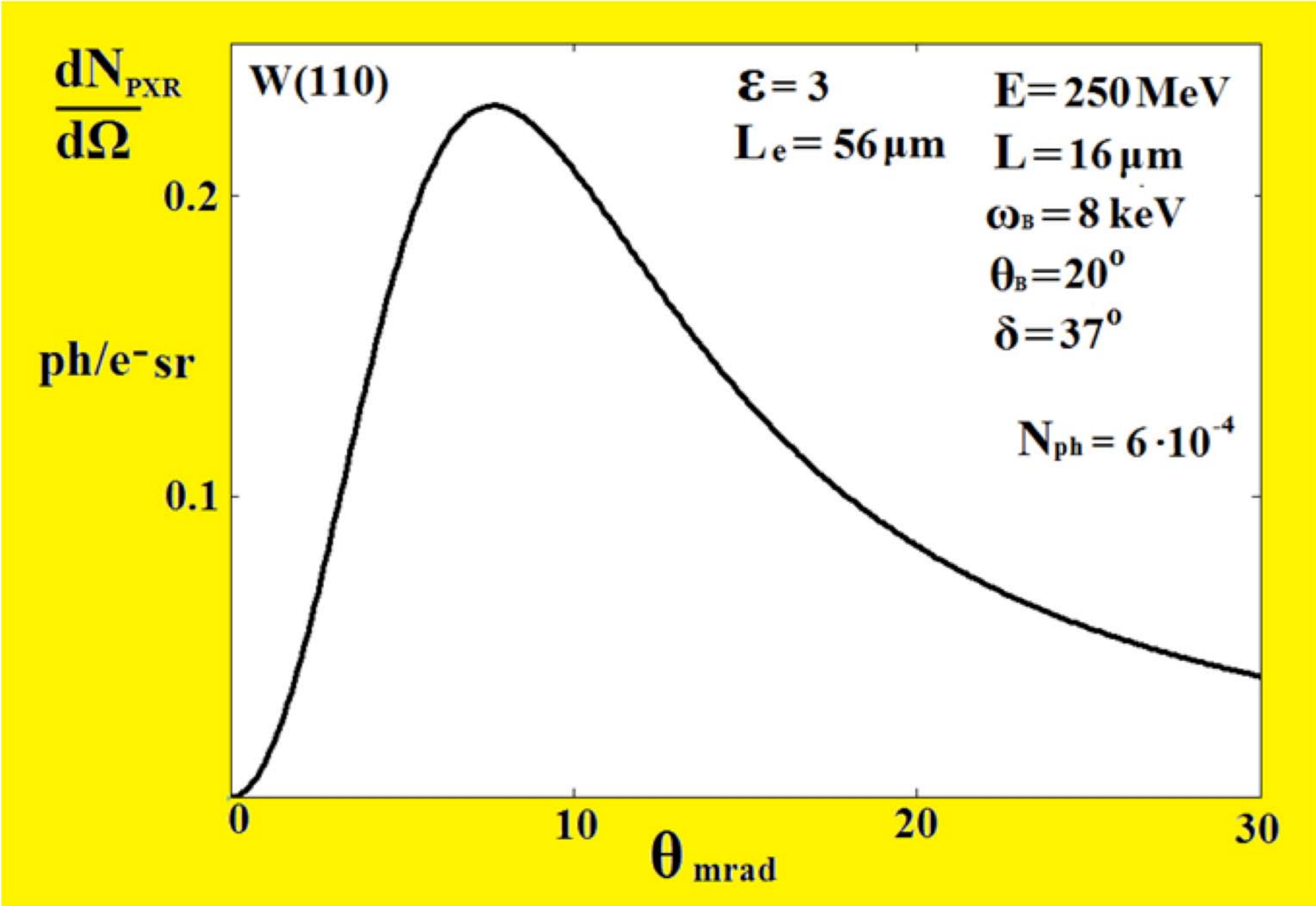
$$L_{ext}^{(s)}=\frac{1}{2C^{(s)}\omega}\frac{gT}{\left|\sin\left(\frac{ga}{2}\right)\right|\left|\chi'_b-\chi'_a\right|}$$

Angular density

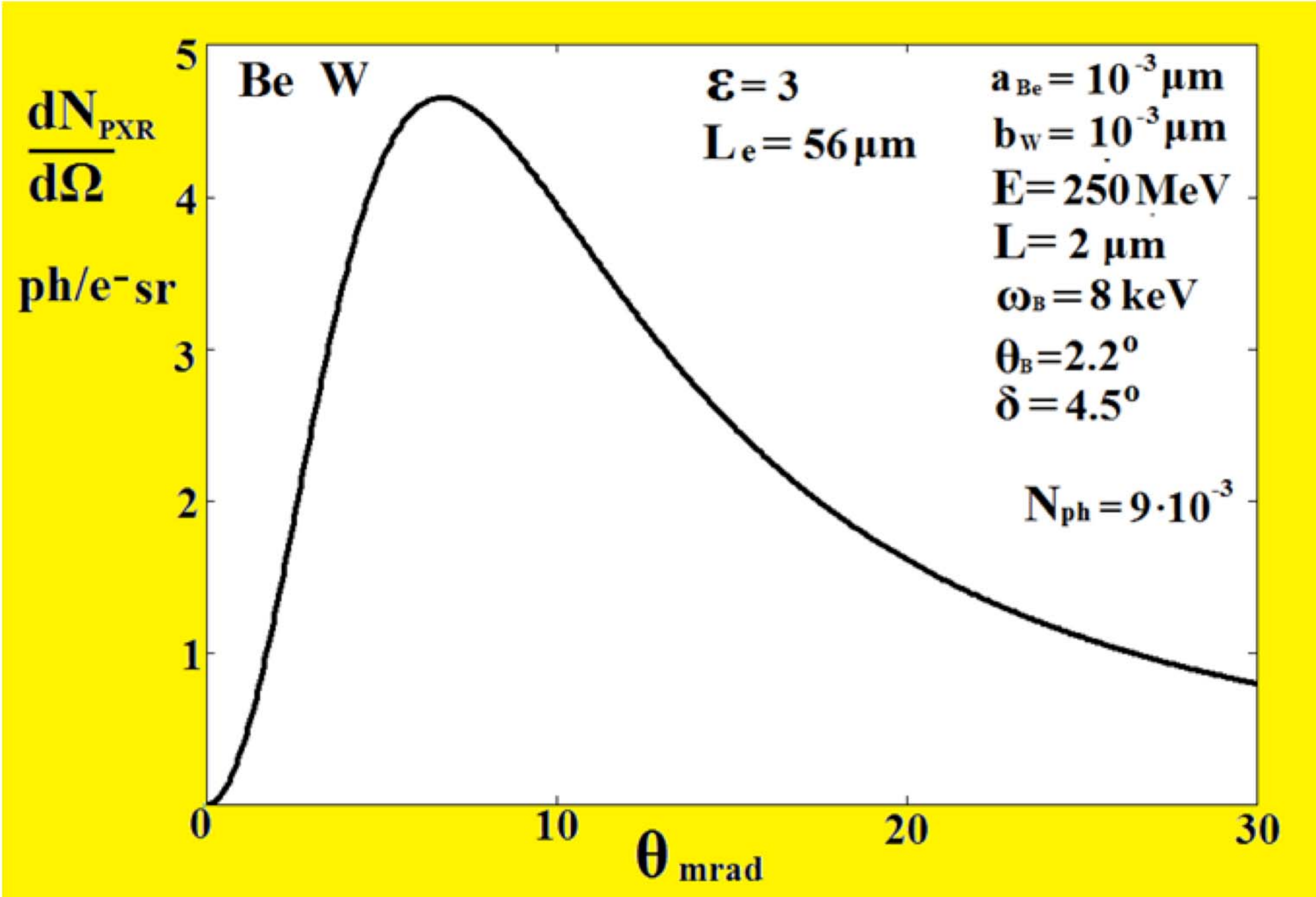
$$\frac{\mathrm{dN}_{\mathrm{PXR}}^{(\mathrm{s})}}{\mathrm{d}\Omega}=\frac{\mathrm{e}^2\mathrm{P}^{(\mathrm{s})^2}\nu^{(\mathrm{s})}}{8\pi^2\sin^2\theta_{\mathrm{B}}}\frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|}+\dfrac{1}{\gamma^2|\chi_0'|}+1\right)^2}\int_{-\infty}^{+\infty}\mathrm{R}_{\mathrm{PXR}}^{(\mathrm{s})}\mathrm{d}\eta^{(\mathrm{s})}(\omega),$$

$$\frac{\mathrm{dN}_{\mathrm{DTR}}^{(\mathrm{s})}}{\mathrm{d}\Omega}=\frac{\mathrm{e}^2\mathrm{P}^{(\mathrm{s})^2}\nu^{(\mathrm{s})}}{8\pi^2\sin^2\theta_{\mathrm{B}}}\frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|}+\dfrac{1}{\gamma^2|\chi_0'|}+1\right)^2}\int_{-\infty}^{+\infty}\mathrm{R}_{\mathrm{DTR}}^{(\mathrm{s})}\mathrm{d}\eta^{(\mathrm{s})}(\omega),$$

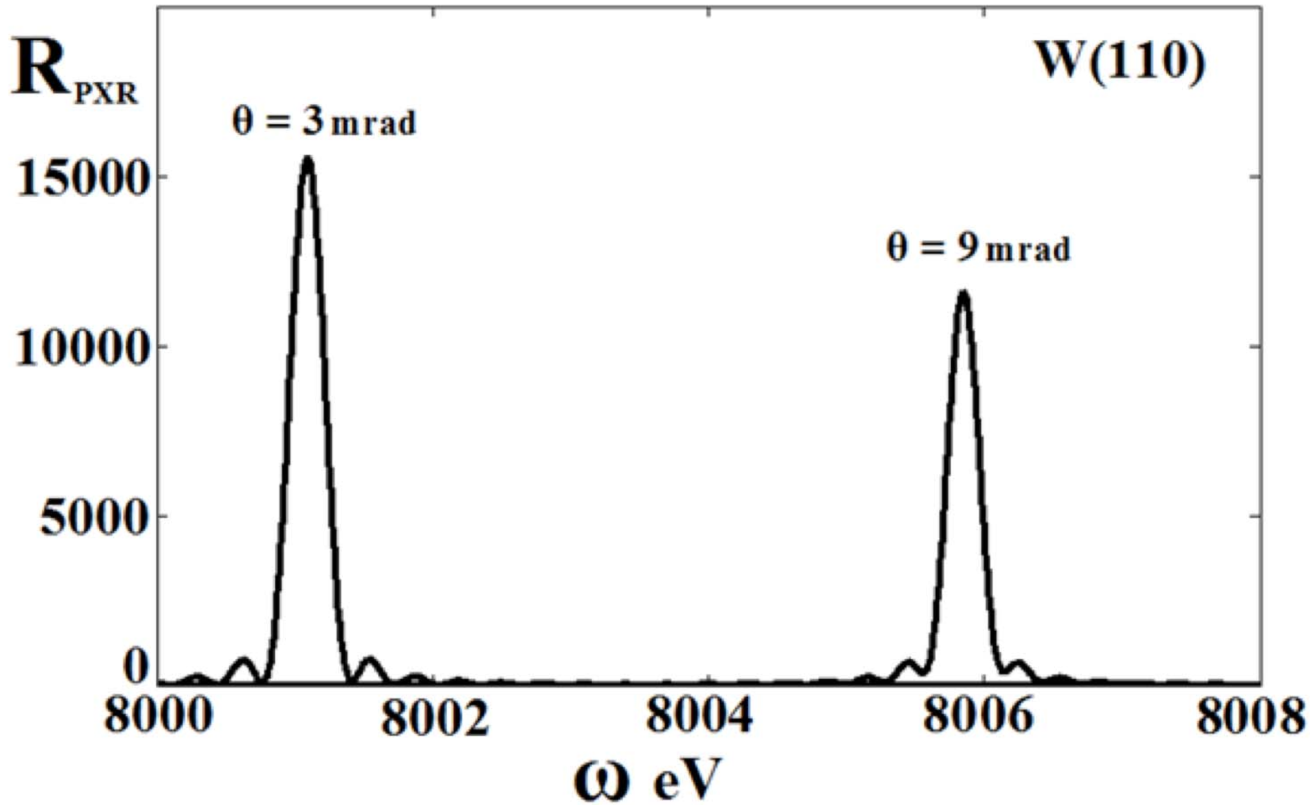
$$\frac{dN_{INT}^{(s)}}{d\Omega}=\frac{e^2P^{(s)^2}\nu^{(s)}}{8\pi^2\sin^2\theta_B}\frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|}+\dfrac{1}{\gamma^2|\chi_0'|}+1\right)^2\left(\dfrac{\theta^2}{|\chi_0'|}+\dfrac{1}{\gamma^2|\chi_0'|}\right)}\int_{-\infty}^{+\infty}R_{INT}^{(s)}d\eta^{(s)}(\omega),$$



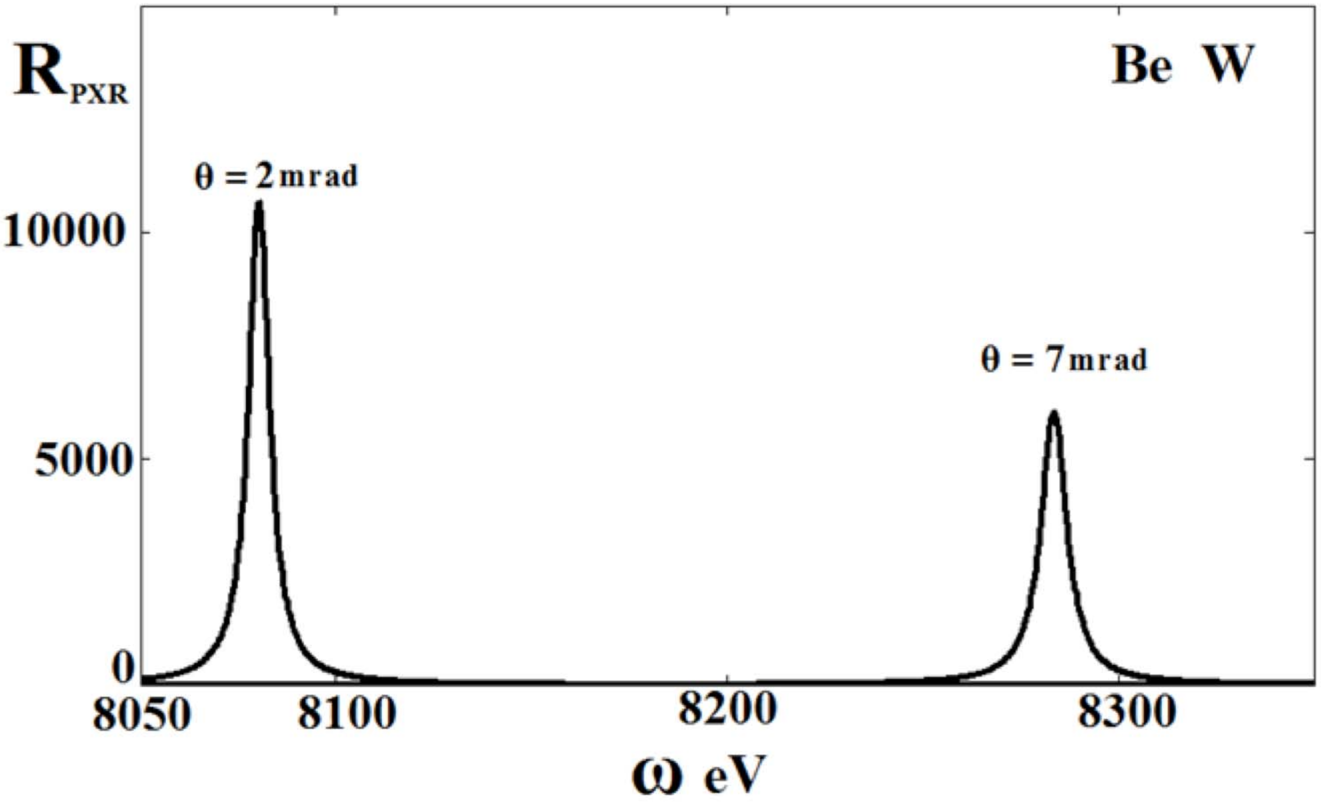
- Figure 3. Angular density of PXR of relativistic electron in W single crystal target.



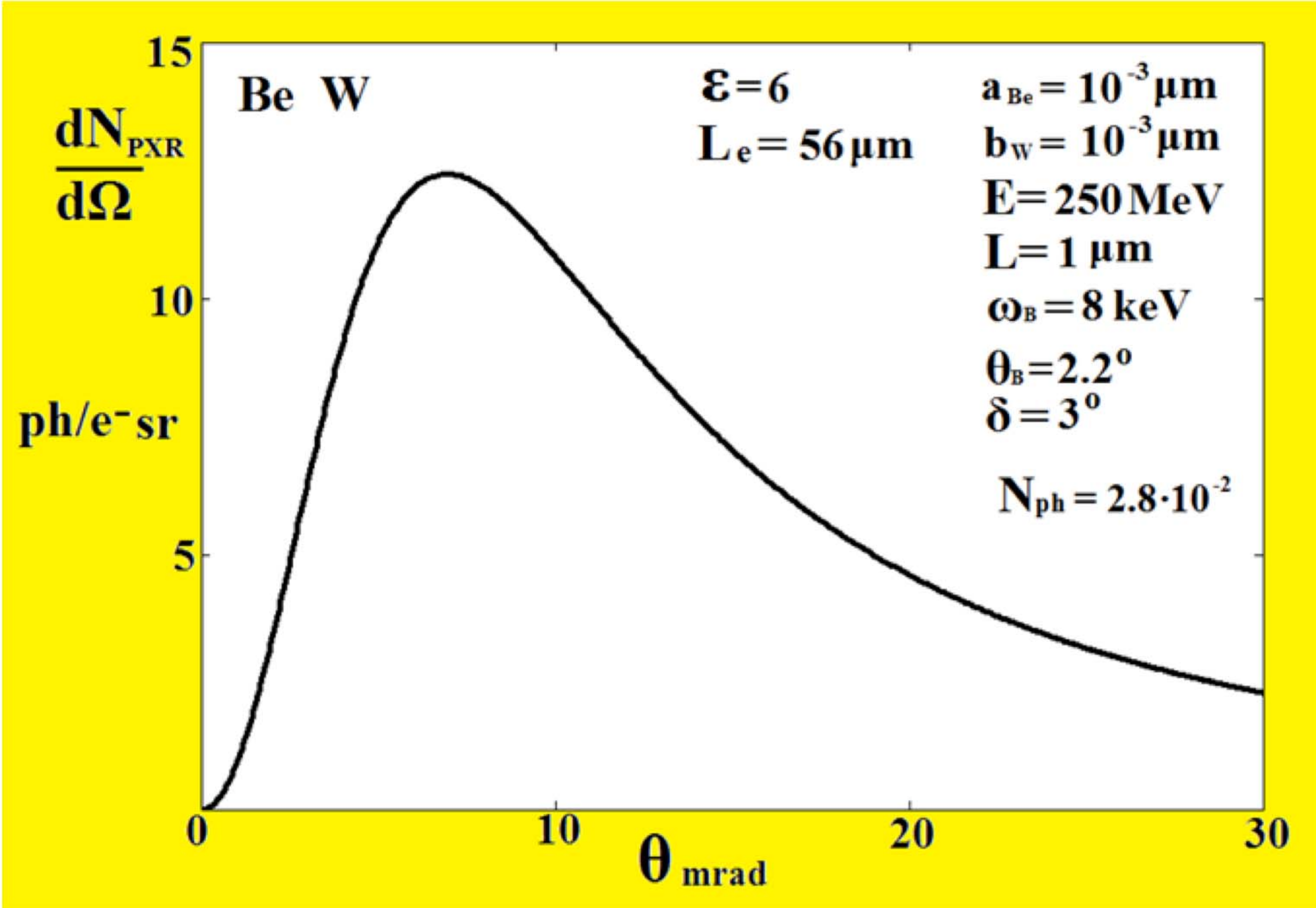
- Figure 4. Angular density of PXR of relativistic electron in multilayer Be-W target.



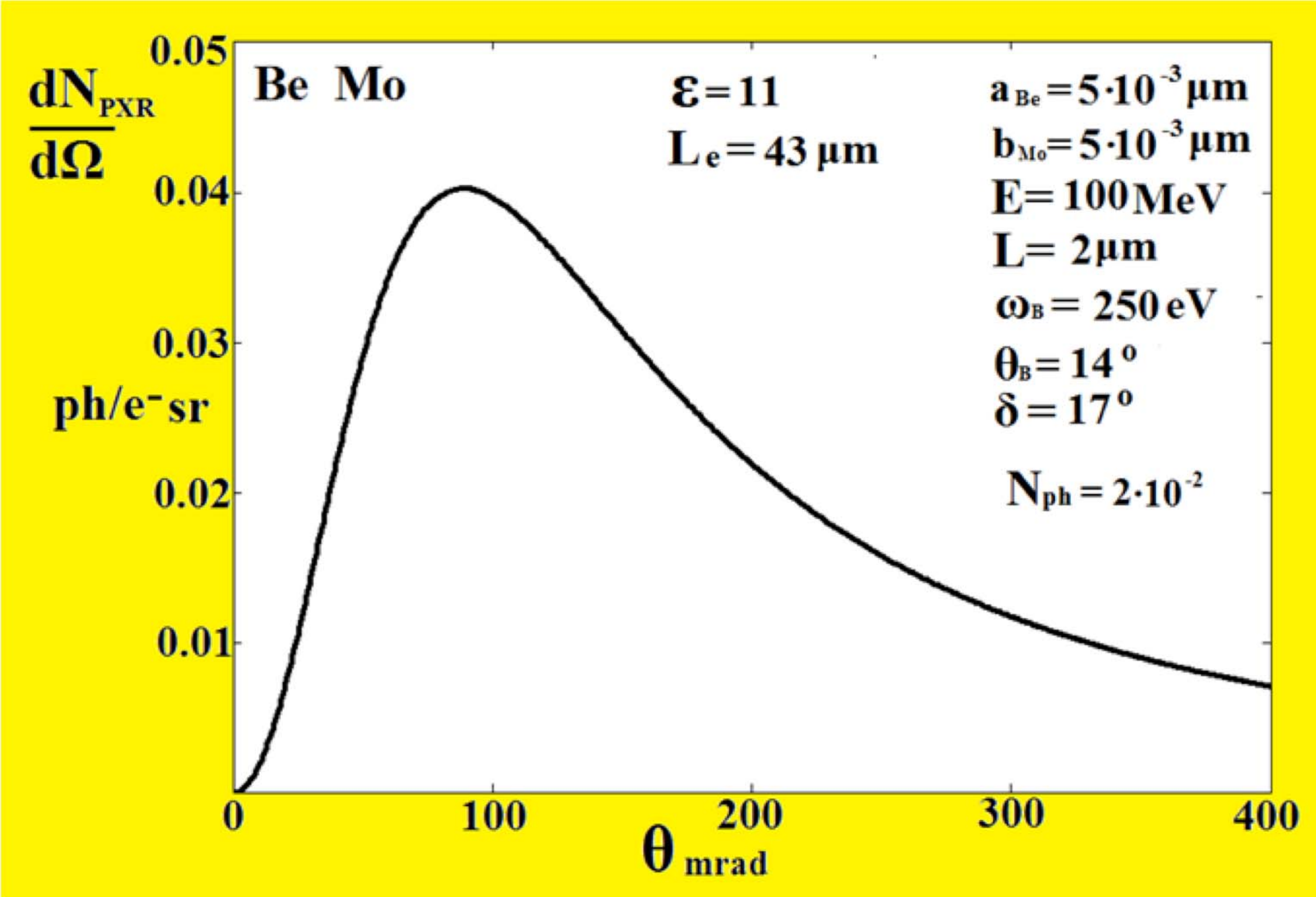
- Figure 5. Spectrum of PXR in W single crystal target for different observation angles



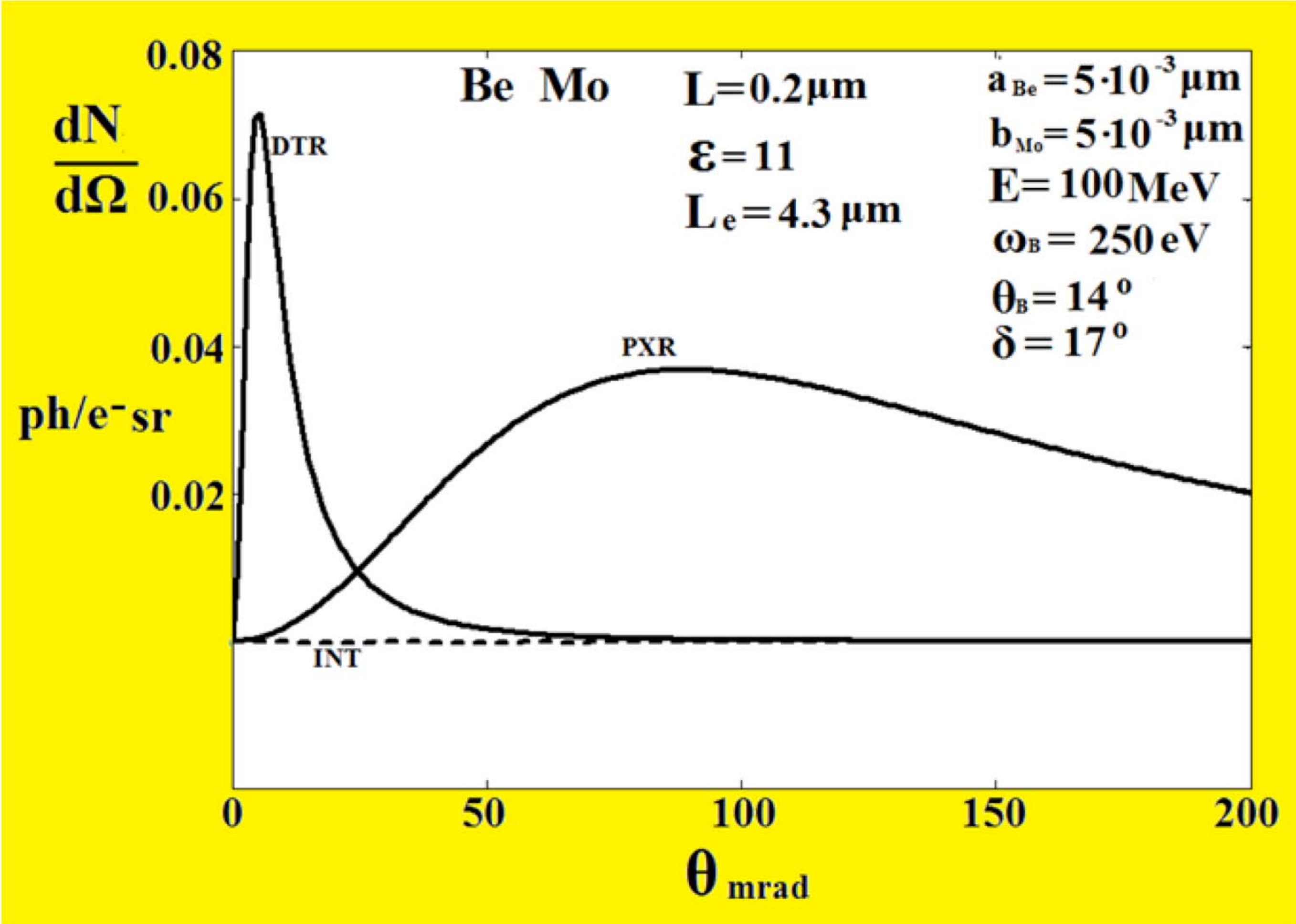
- Figure 6. The same as in Fig.5 for case of Be-W multilayer target



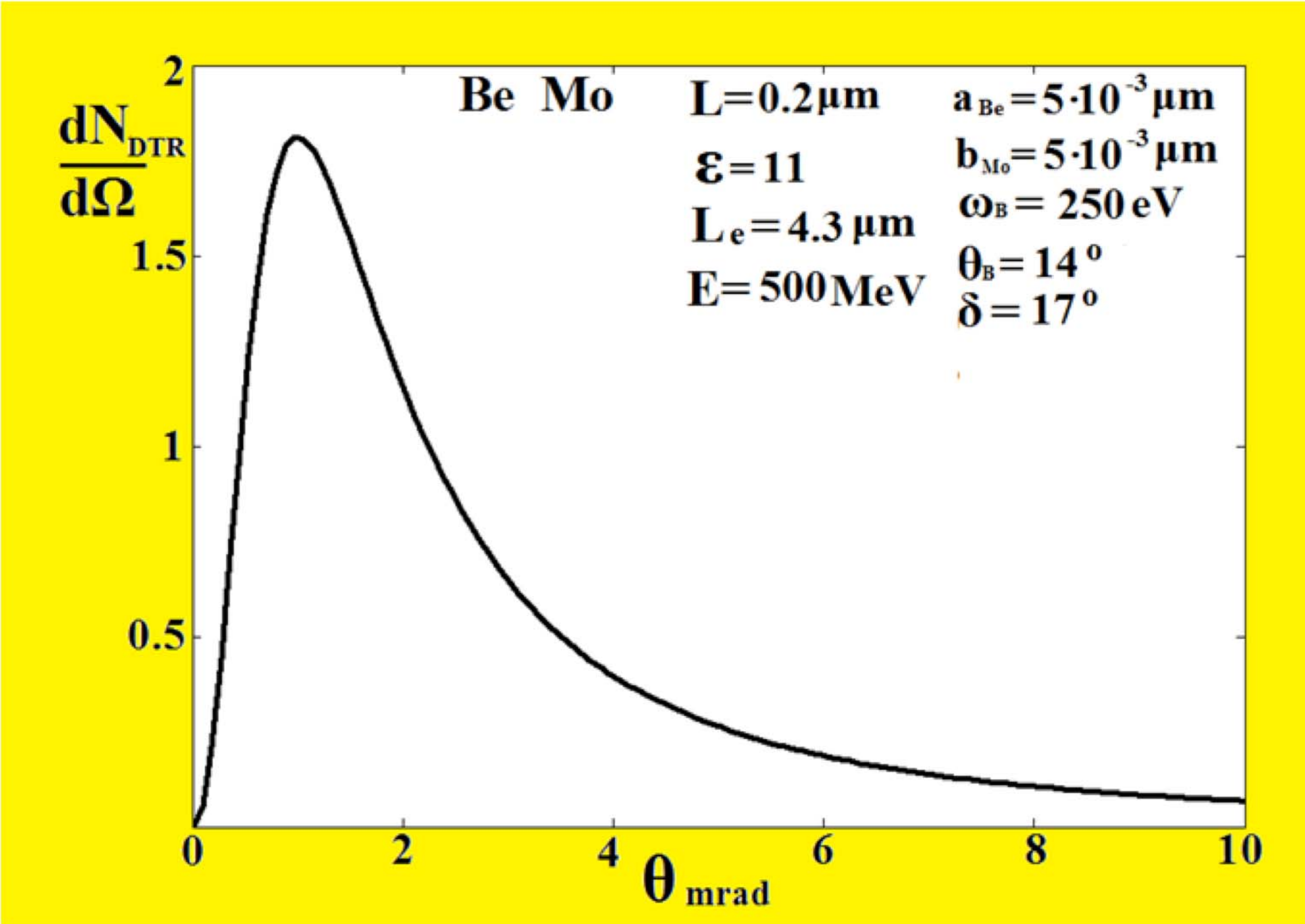
- Figure 7. The same as in Fig.4 for bigger reflection asymmetry



- Figure 8 . Angular density of the relativistic electron PXR in multilayer Be-Mo target.



- Figure 9. Angular density distributions for DTR and PXR of the relativistic electron



dN_DTR / dΩ
Be Mo
L=0.2μm
ε=11
L_e=4.3 μm
E=500MeV
a_Be=5·10^-3 μm
b_Mo=5·10^-3 μm
ω_B= 250 eV
θ_B=14°
δ=17°
θ mrad
0
2
4
6
8
10
0.5
1
1.5
2

- Figure 10.

CONCLUSIONS

Based on two-wave approximation of the dynamic scattering theory, coherent radiation of a relativistic electron is investigated in a nano-scale multilayer structure in Laue scattering geometry.

Analytical expressions for the spectral-angular distribution of PXR and diffracted transition radiation (DTR) are derived in general case of asymmetric reflection. It is shown that under fixed Bragg angle and path length of relativistic electron in multilayer structure the radiation yield considerable exceed the yield of the radiation in the crystal medium.

The yields of PXR and DTR also strong depends on the angle between reflected layers and inlet surface of the multilayer target, i.e. on reflection asymmetry.