

The background of the slide is a reproduction of the painting 'The Starry Night' by Vincent van Gogh. It depicts a swirling, turbulent night sky filled with bright, glowing stars and a large, luminous moon. Below the sky, a dark, silhouetted cypress tree stands on the left, and a small village with a church spire is visible in the distance. The overall mood is one of cosmic wonder and mystery.

Mathematics, our language to unveil the structures of the Universe

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- **Gravitational interaction among two particles**
 $(m_1 m_2)/r^2$
Principia of Newton (1687)
- **Gravitational interaction is long range: any particle of the system influences the motion of any other!**
- **Gravitational interaction diverges when $r \rightarrow 0$ serious problem when you use computers!**

**Our image of the univers began to be clear only
50-60 years ago:**

- **Galileo was the first to resolve in the “*celestial fluid*” single stars similar to our Sun**
- **Immanuel Kant: the galaxy could be a disk. And why not other similar objects, “universe islands”?**
- **construction of greater telescopes.. discovery of the interstellar medium.. birth of spectroscopy..**

**our personal
star:
the Sun**



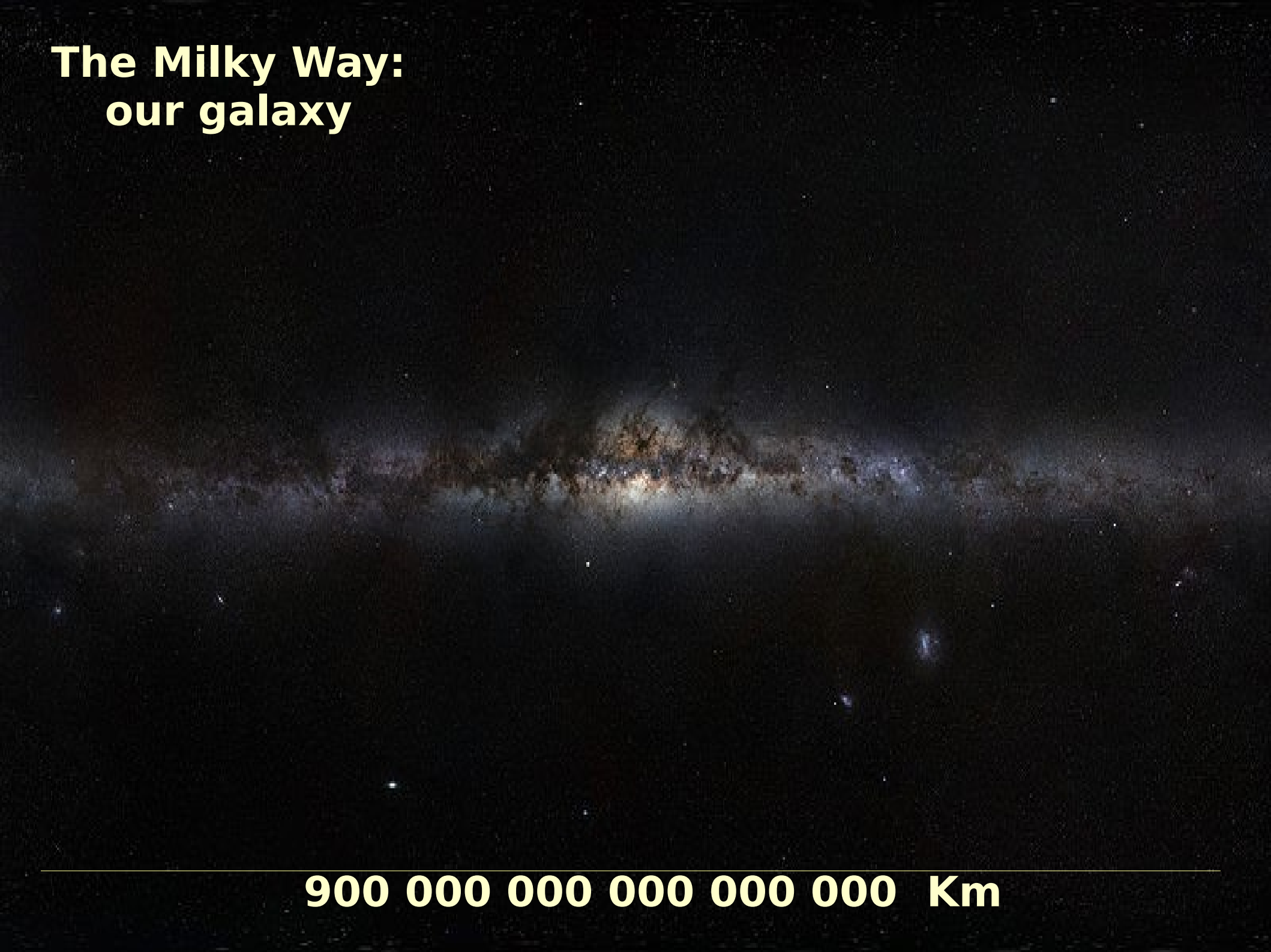
700 000 Km

**Globular star
cluster
M13
Hercules**



80 000 000 000 000

The Milky Way: our galaxy

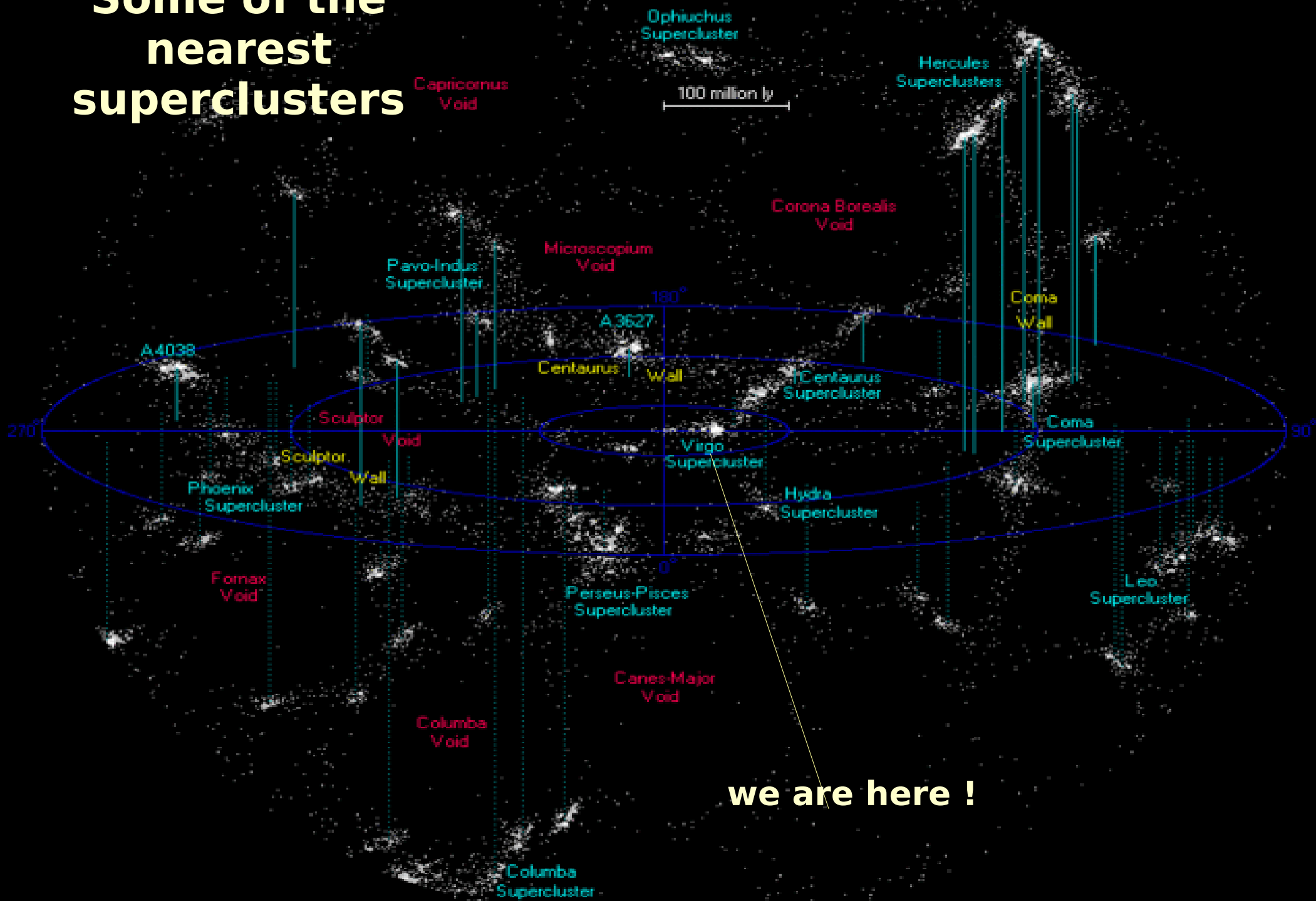


900 000 000 000 000 000 Km

Virgo cluster of galaxies

4.4 Mpc = 10^{20} Km **1 pc = 10^{13} Km**
= 3 light years

Some of the nearest superclusters



we are here !

What we learned?

Gravitational force extends to **all the scales** of the universe!!

Gravitation is the force **responsible for the formation of structures** in the universe
it shapes the matter giving rise to complex entirety of particles

*planets, stars, solar systems, clusters of stars,
galaxies, clusters of galaxies, superclusters of
galaxies ...*

...and so on

*but **WHY GRAVITY ??***

Let us make a step backward to clarify two main concepts:

FORCE and ENERGY

***when a particle moves in the free space
(absence of grip) it has just two possibilities ..***

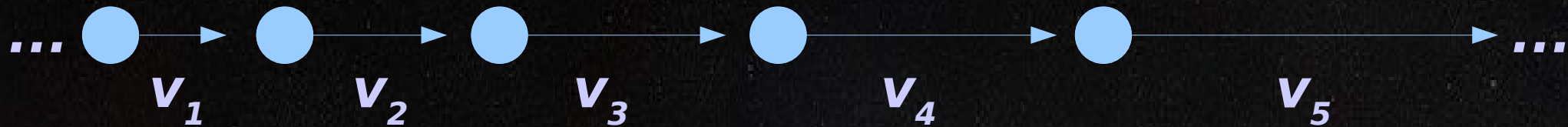
.. FIRST POSSIBILITY the particle moves with **constant velocity** (which could be zero, that is the particle is at rest)



This means that **NO FORCE** is acting on our particle!! Or the particle is **NOT INTERACTING** with any other body.

This case the particle is endowed with an energy due to the simple fact it is moving with a certain **CONSTANT** velocity, named **KINETIC ENERGY**

.. SECOND POSSIBILITY the particle moves with a **velocity which changes** at any instant of time



This means that **SOME FORCE** is acting on our particle!! Or the particle is **INTERACTING** with another body.

This case the particle has of course **KINETIC ENERGY** (just because the fact it has a velocity)

BUT it **also has** an energy caused by the fact that something external is impressing a force on her..
..this is known as **POTENTIAL ENERGY**

FOUR fundamental ***FORCES*** are known to govern
our universe

***WEAK
FORCE
STRONG
FORCE***

***Just act on particles
at microscopic
scales***

FOUR fundamental FORCES are known to govern our universe

***ELECTROMAGNETIC
FORCE***

$$\frac{(Q_1 Q_2)}{R^2}$$

But each macroscopic body has about the same number of protons and electrons so the total charge of the body is $Q = 0$.

At large scale the electromagnetic force is null !!

FOUR fundamental FORCES are known to govern our universe

***GRAVITATIONAL
FORCE***

$$\frac{(M_1 M_2)}{R^2}$$

***The mass of each object can just be positive!!!
that is the sum of the masses of the particles
componing a body can NEVER be zero
that is GRAVITATIONAL FORCE IS NEVER NULL and
DOMINATES THE UNIVERS AT ANY SCALE!***

***Once established the role of gravitational force
let us move to..***

..GRAVITATIONAL ENERGY

This nice picture represents the giant elliptical galaxy Virgo A



apart the different dimensions (pc instead of cm) does not look like a gas of stars



A crystal ball on a wooden stand against a starry night sky background. The crystal ball is positioned in the center, and the text is overlaid on it. The background is a deep blue night sky filled with numerous small, bright stars of varying colors (white, yellow, red, green). The crystal ball itself is clear, reflecting the stars and the text. The wooden stand is dark and appears to be made of polished wood.

A big difference is immediate:

***if you remove the container which encloses the
particles of a common gas, this will move away
pushed by pressure
expanding in all the space at their disposal***

while if we remove the bubble surrounding Virgo A ?

...the stars do not move away, why this difference?

Because the mutual (long range) gravitational force of each star respect to all the others makes the system stable ..

*this is a **SELFGRAVITATING SYSTEM***

Let us consider the galaxy Virgo A (of mass M_1 and radius R) and the cluster of galaxies to which belongs (of mass M_2), and let R_{12} be their distance

Gravitational energy of interaction of Virgo A with the cluster

Gravitational selfenergy of Virgo A


$$M_1 M_2 / R_{12} \approx M_1 M_1 / R$$

If we made the same count referred to a bubble of gas (1m of radius) with respect to the body to which belongs, that is the earth we would have found..

Gravitational energy of interaction of a bubble of gas with the earth

Gravitational selfenergy of the bubble of gas

$$M_1 M_2 / R_{12} \approx 10^{17} M_1 M_1 / R$$

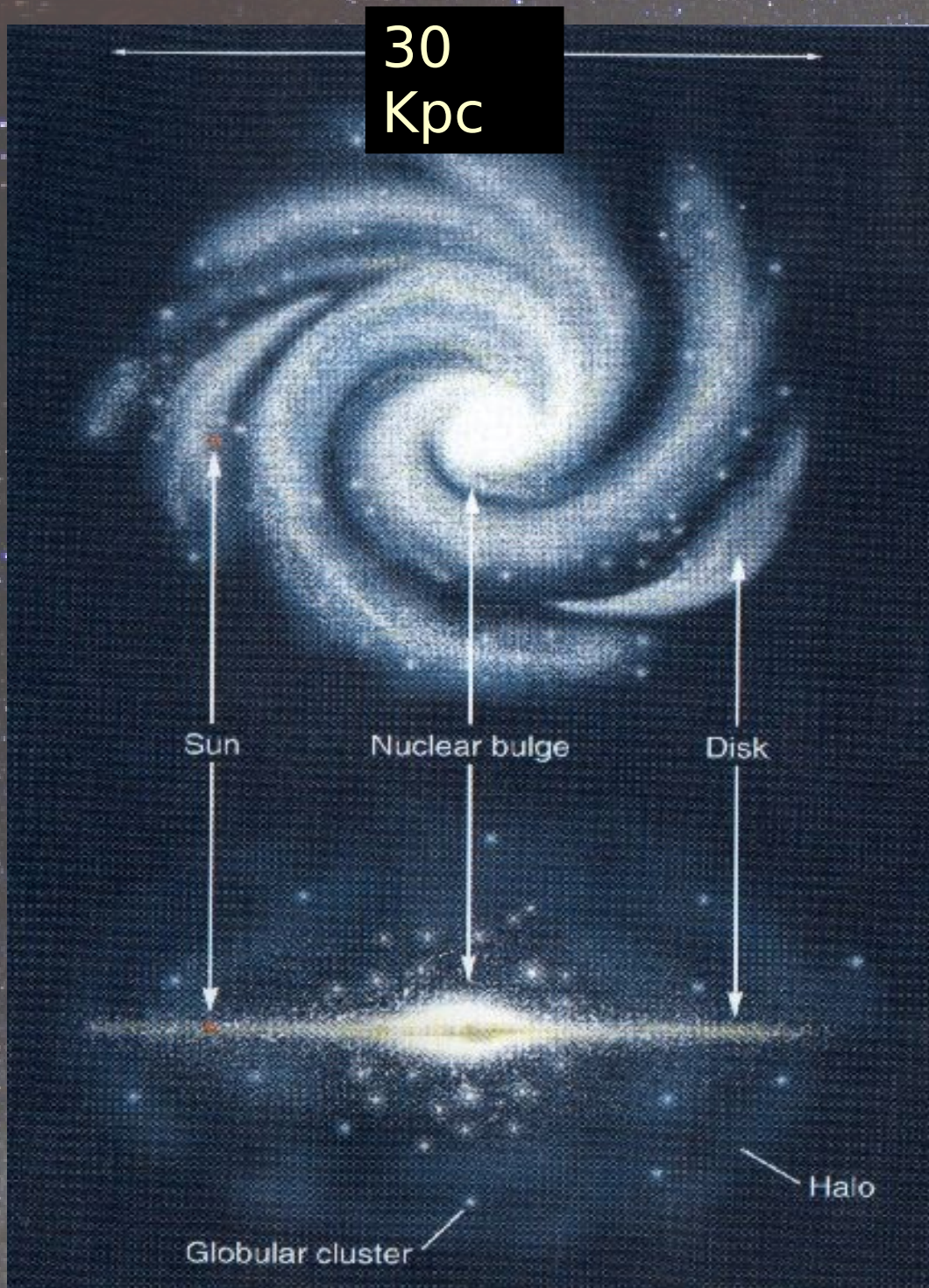
INFINITELY SMALLER !!

*...this is why **galaxies, stars etc.. are**
selfgravitating systems
while common gases are not.*



Structure of our galaxy





Main components:
disk (30 Kpc)
bulge (5 Kpc)
spiral arms

all surrounded by..

halo (30 Kpc)
spherical crown
(100 Kpc)

**containing about
150
globular clusters**

30 Kpc

250
Km/s

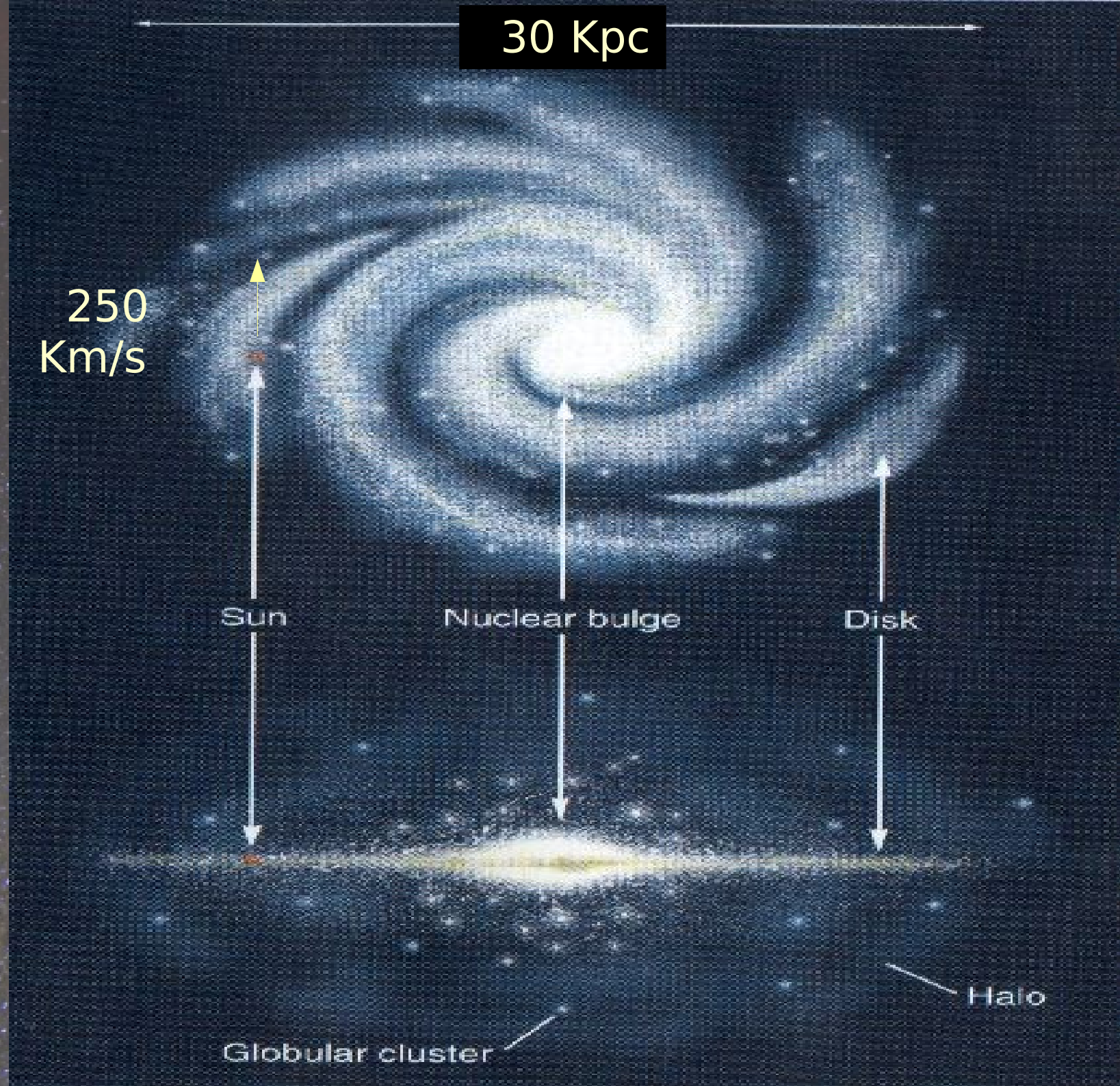
Sun

Nuclear bulge

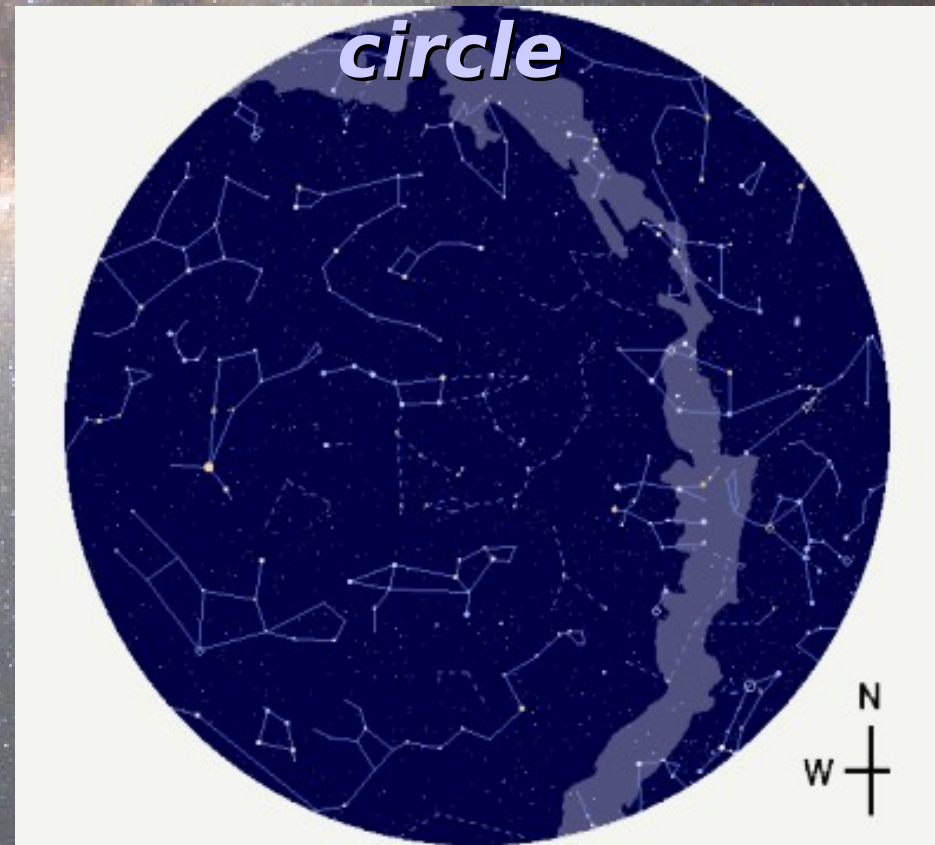
Disk

Halo

Globular cluster



*stars and nebulae
surrounding us appear
projected on the
celestial sphere as a
luminous maximum
circle*



The situation looks complicated..

*but it's even **MORE COMPLICATED***

the Galaxy is just one of billions galaxies which populates the universe. These looked to first observers as diffuse nebulae..

10% of matter in our Galaxy is in the form of diffused matter.

Clouds of matter in our galaxy (75% H 25% He) (expelled by supernova explosions as we will see..)

illuminated by near stars can appear as galaxies.

The first classification of galaxies is due to Hubble, also known
DIAPASON

**NORMAL
SPIRAL
GALAXIES**

**IRREGULAR
GALAXIES**

**BARRED
SPIRAL
GALAXIES**

**ELLIPTICAL
and
LENTICULAR
R
GALAXIES**



The number proportions of the various types are:

***ELLIPTICAL
and
LENTICULAR
GALAXIES***

20%

***SPIRAL
GALAXIES***

65%

***IRREGULAR
GALAXIES***

5%

MAIN DIFFERENCES:

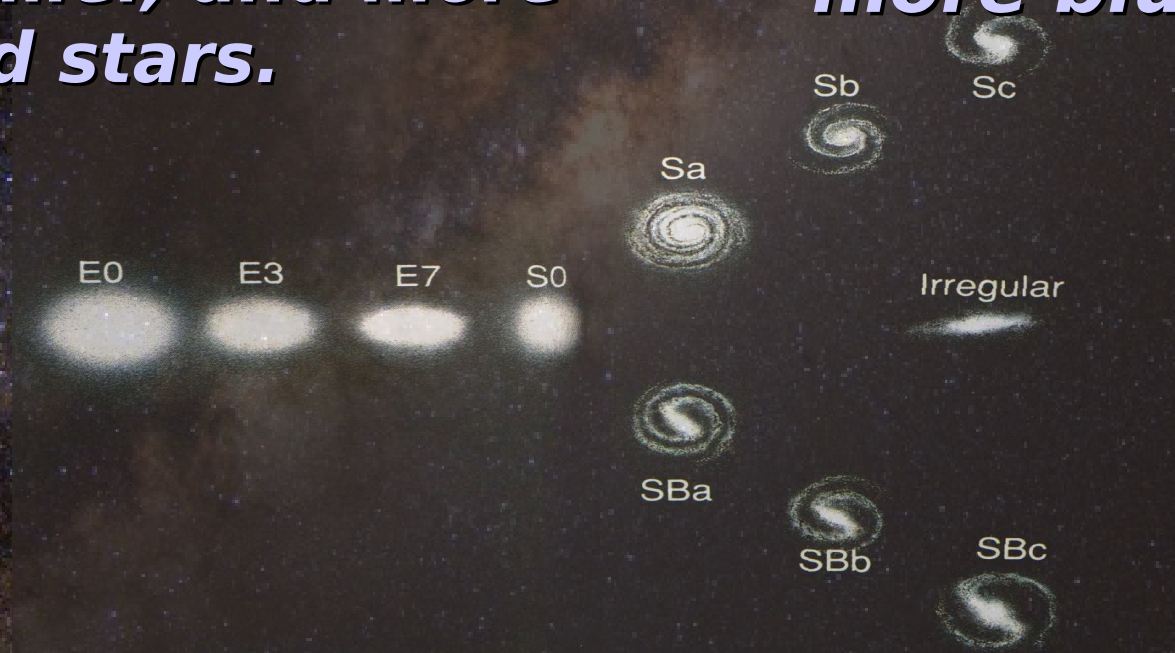
ELLIPTICAL GALAXIES

elliptical galaxies show almost spherical symmetry.

They are characterized by older, smaller, and more red stars.

SPIRAL GALAXIES

their bulge is similar to an elliptical galaxy. The disc is reach of jounge, massive, and more blu stars.



All the galaxies has the same age: 10^{10} years

Searching for a model of galaxy formation gravitational instability

The first physician who faced the problem of the formation of gravitational structures was Jeans at the beginning of nineteenth century.

He imagined an infinite, uniform, homogeneous and static gas of particles which self-interact via gravitational force.

*The prototype for this gas are **interstellar medium** (75% H, 25% He) very low density (about 100 particles per cm^3) or **HII regions**, formed by ionized hydrogen*

Searching for a model of galaxy formation **gravitational instability**

Let us now imagine that something causes in this gas a small local perturbation, which modifies density, pressure and velocity of the particles

*Jeans discovered that such **perturbation will evolve like an exponentially growing spherical wave** if the mass of the gas exceeds the critical value:*

$$M_j \propto 1/\rho^{0,5}$$

this give srise to the shrinking of the matter inside a radius λ_j and it's density grows and grows up..

Searching for a model of galaxy formation gravitational instability

*The critical mass for a static interstellar medium, or
an HII
region is of the order of $10^6 M_{\odot}$*

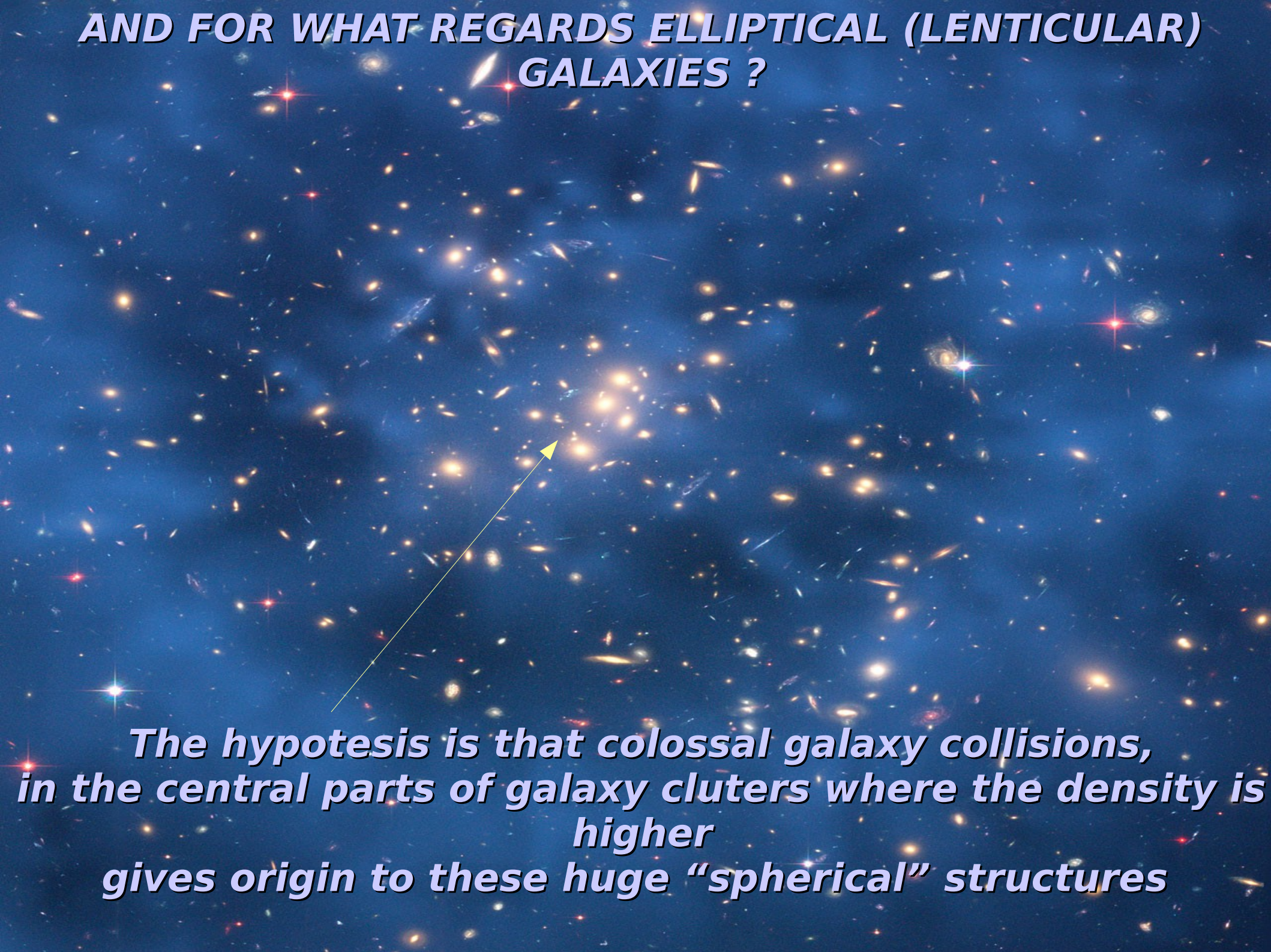
one million times greater than the mass of the sun !!

*but becomes much greater for a rotating gas such
as the medium
in which spiral protogalaxies form.*

To conclude the formation of a spiral galaxy follows these steps:

- 1) a protogalactic rotating cloud, of spherical shape undergoes gravitational instability**
- 2) angular momentum conservation makes the contraction follow a disk shape and not spherical**
- 3) fragments of the protogalactic cloud, shot from the central body, contract to form globular clusters**
- 4) friction in turbulent motions gives rise to a bulge**
- 5) perturbations in the disk originates the spiral arms.**

AND FOR WHAT REGARDS ELLIPTICAL (LENTICULAR) GALAXIES ?



***The hypothesis is that colossal galaxy collisions,
in the central parts of galaxy clusters where the density is
higher
gives origin to these huge “spherical” structures***

The phases of a colossal galaxy collision



Formation and evolution of stars

(one of the main achievements of human mind)

- *Theory of stellar structure: selfconsistent theory based on*

balance between pressure and gravity

balance between generated energy and emitted energy

energy transportation across the star

- *Totally confirmed by the study of the Sun, and star populations in globular clusters (globular clusters are the consequence of one single formation impulse, all has the **SAME AGE** but **DIFFERENT INITIAL MASSES!!!**)*

REPRESENT A SNAPSHOT OF ALL POSSIBLE CONFIGURATIONS

- *Explains the origin of **ALL the chemical known elements**
BIG BANG just created **H** and **He***

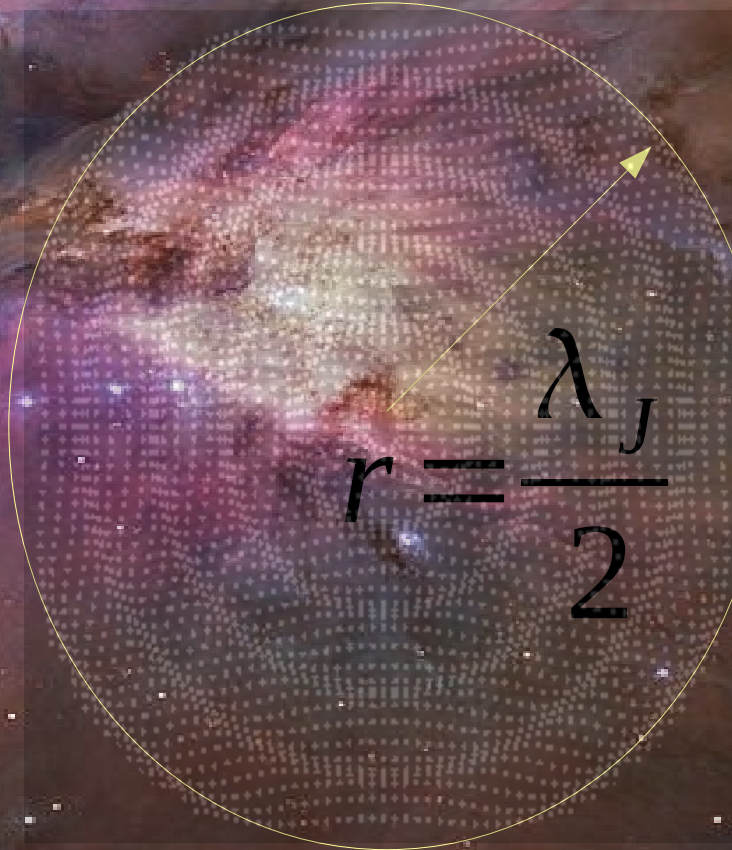
Orion nebula

*pillars of contracting gas
experience **gravitational
instability**
giving origin to new stars*

*young massive
stars*



**interstellar medium becomes unstable respect to the propagation of a density perturbation, if the radius exceeds a critical value:
the JEANS RADIUS**



**this corresponds
to a mass of $10^6 M_{\odot}$
1 MILLION TIMES
THE SUN**

**This explains *globular clusters*.. but not the formation of single stars like the Sun!
What is the solution to the paradox??**

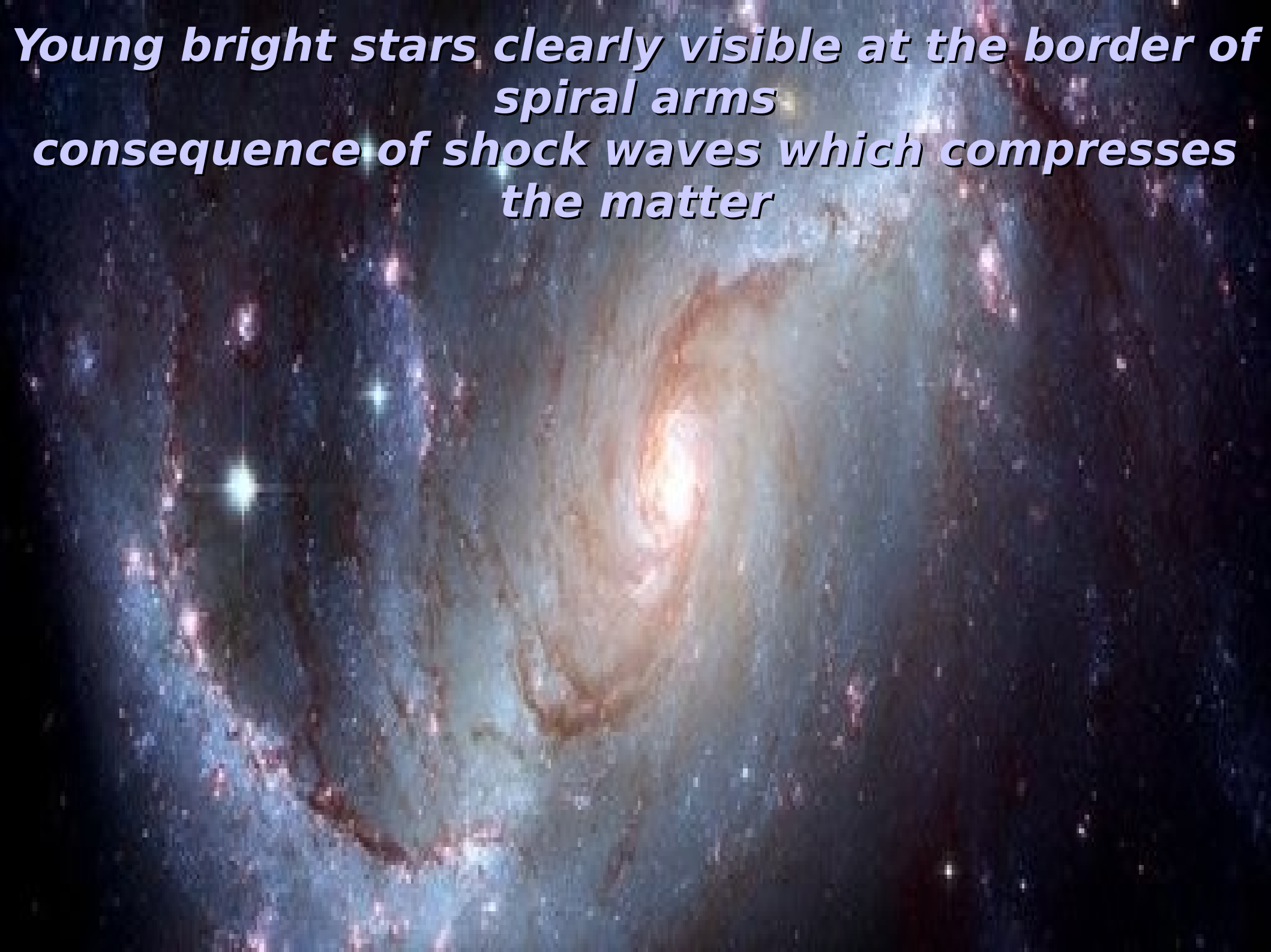
Crab nebula:
expanding gas as a result of **supernova explosion**

supernova explosions or cloud collisions create
SHOCK WAVES

$$M_j \propto 1/\rho^{0,5}$$

the denser compressed matter has a **LOWER**
CRITICAL MASS

***Young bright stars clearly visible at the border of
spiral arms
consequence of shock waves which compresses
the matter***



**..the cloud collapses, radius decreases
gravitational energy grows**

$$W = -\frac{1}{2} GM^2/r$$

kinetic energy

$$K = \frac{1}{2} GM^2/2r$$

gravitational energy

**BUT energy does not disappear, it transforms in kinetic
energy**

that is heat..

$$K = \frac{3}{2} k T$$

kinetic energy is temperature

the temperature begins to grow

**equilibrium between gravity and pressure
begins to establish indeed..**

$$P V = N k T$$

**The protostar develops a dense and hot nucleus ($M_{\text{nuc}} = \frac{1}{2} M_{\text{star}}$)
and reaches a stage named **zero age sequence****

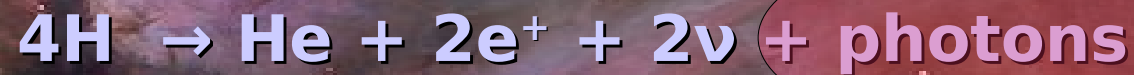
**the time passed from the beginning is
 $\tau_k = 10^7$ years**

**if the star mass is $M > 1/10 M_{\odot}$ something extraordinary happens that
represents the **birth of the star ...****

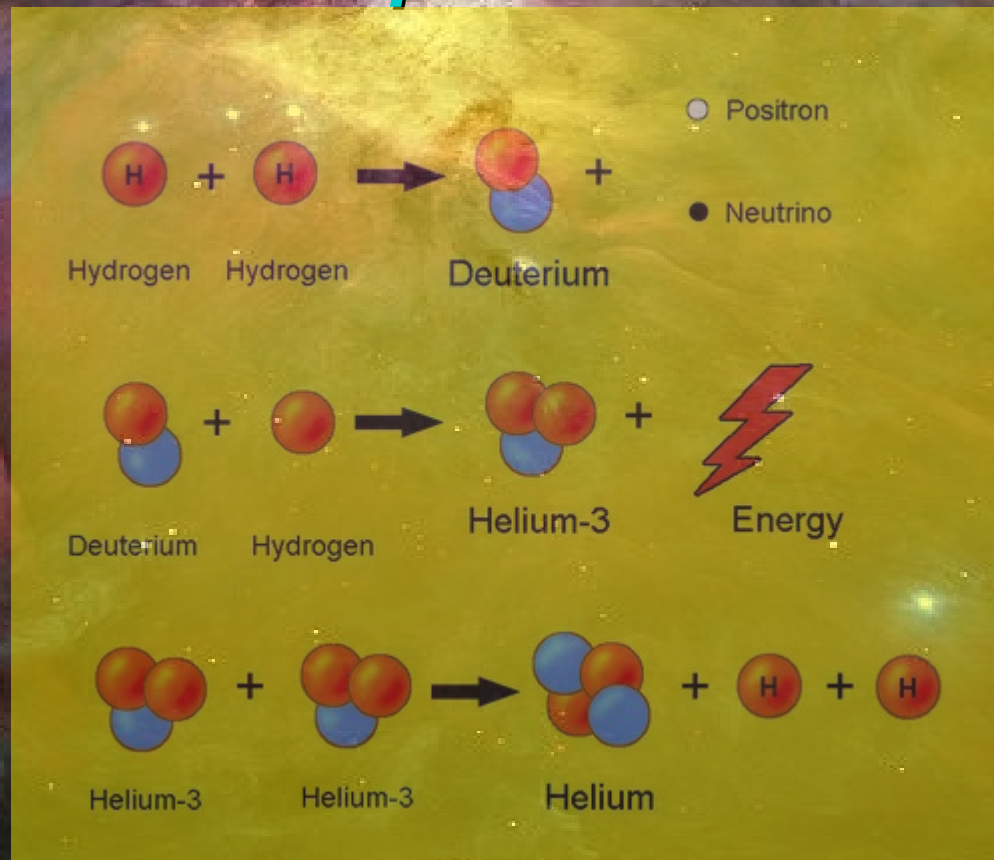


*When the nucleus temperature reaches 10^7 °K
nuclear fusion of hydrogen in helium begins*

ENERGY



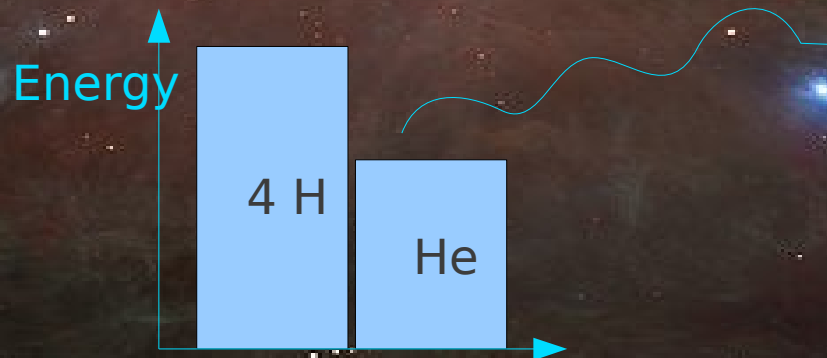
this can happen through various reactions, one of this is **p-
p chain**



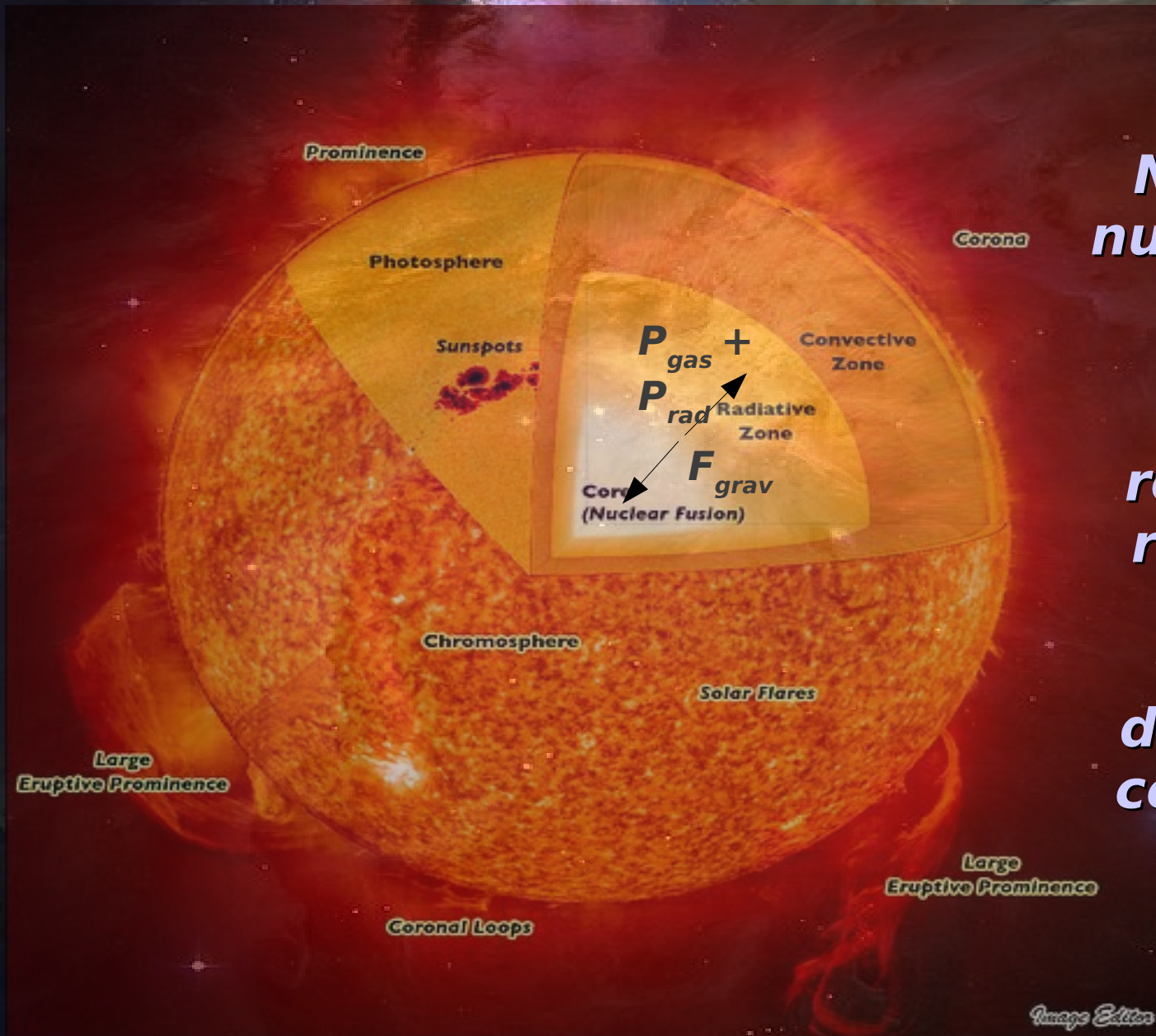
Each system of interacting objects in the universe naturally evolves towards the condition of minimum energy

the total energy of four separated hydrogen atoms is greater than the total energy of the helium nucleus

the energy difference is emitted in the form of particles and mainly photons



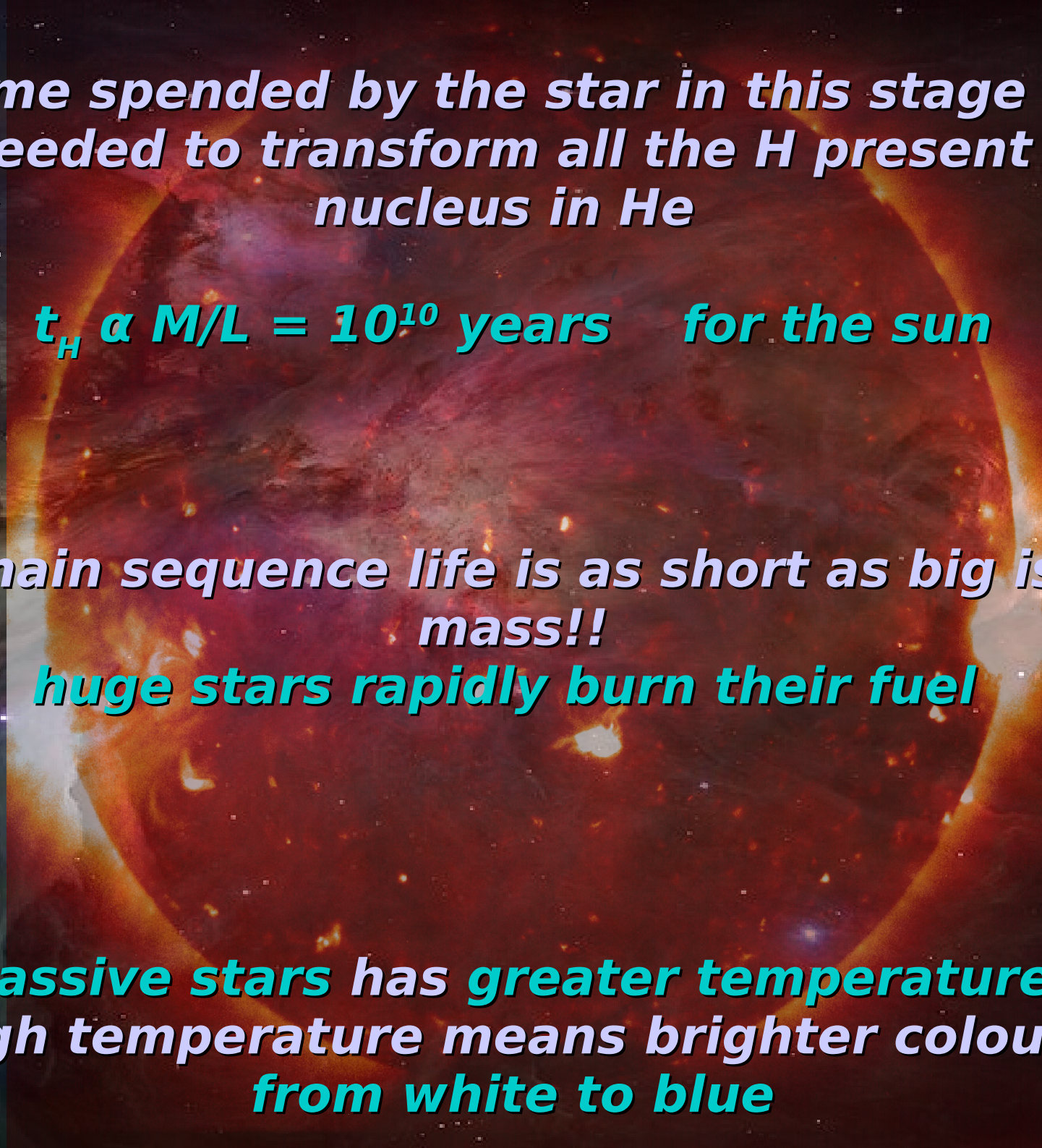
The protostar evolution is finished!
nuclear contraction stops because the **balance of gravity and pressure**



NUCLEUS: where nuclear fusion takes place

ATMOSPHERE: responsible for the radiation emitted

T , P and ρ decrease from the center to the edge



The time spented by the star in this stage is the time needed to transform all the H present in the nucleus in He

$t_H \propto M/L = 10^{10}$ years for the sun

the main sequence life is as short as big is the mass!!

huge stars rapidly burn their fuel

***massive stars has greater temperatures
high temperature means brighter colours
from white to blue***

If instead the mass of the protostar is too small

$$M < 0.1 M_{\odot}$$

*the temperature reached by the nucleus is not
sufficient ..*

*the star continues to contract
it's temperature decreases
the star ends it's life as a **black dwarf***



**When all the H present in the core is converted into He
H fusion shifts in a shell surrounding the He nucleus**

the central engine is turn off → gravity is not substaained

**the nucleus
collapses and the
central temperature
rises to**

$$T = 10^8 \text{ } ^\circ\text{K} !!$$

**the fusion of He in C
begins**

**trigger of He will
coincide with a
violent
FLASH!**



***The transformation of a star with a mass similar to Sun
will be
magnificent and catastrofic..
..as shown by this nice picture of a star in the fase of
GIANT***



CREDIT: NASA / STScI / AURA

***The transformation of a star with a mass similar to Sun
will be***

magnificent and catastrophic..

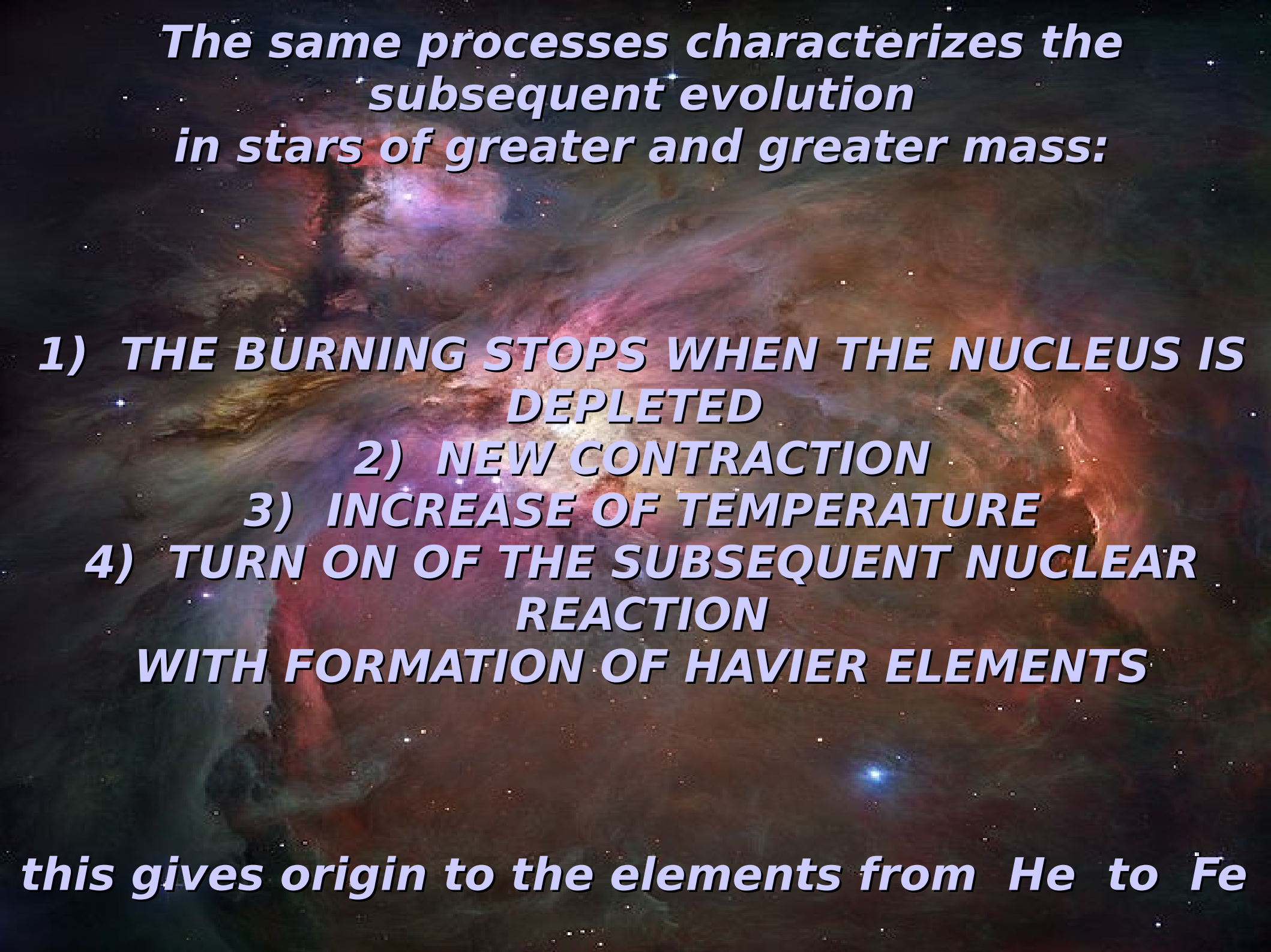
***the contraction of the nucleus will cause the expansion of
the envelop and the atmosphere***

Mercury, and Venus will be completely destroyed ...



***The transformation of a star with a mass similar to Sun
will be
magnificent and catastrofic..
as the nucleus expands the envelop and the atmosphere
expand
the Earth itself will not survive***





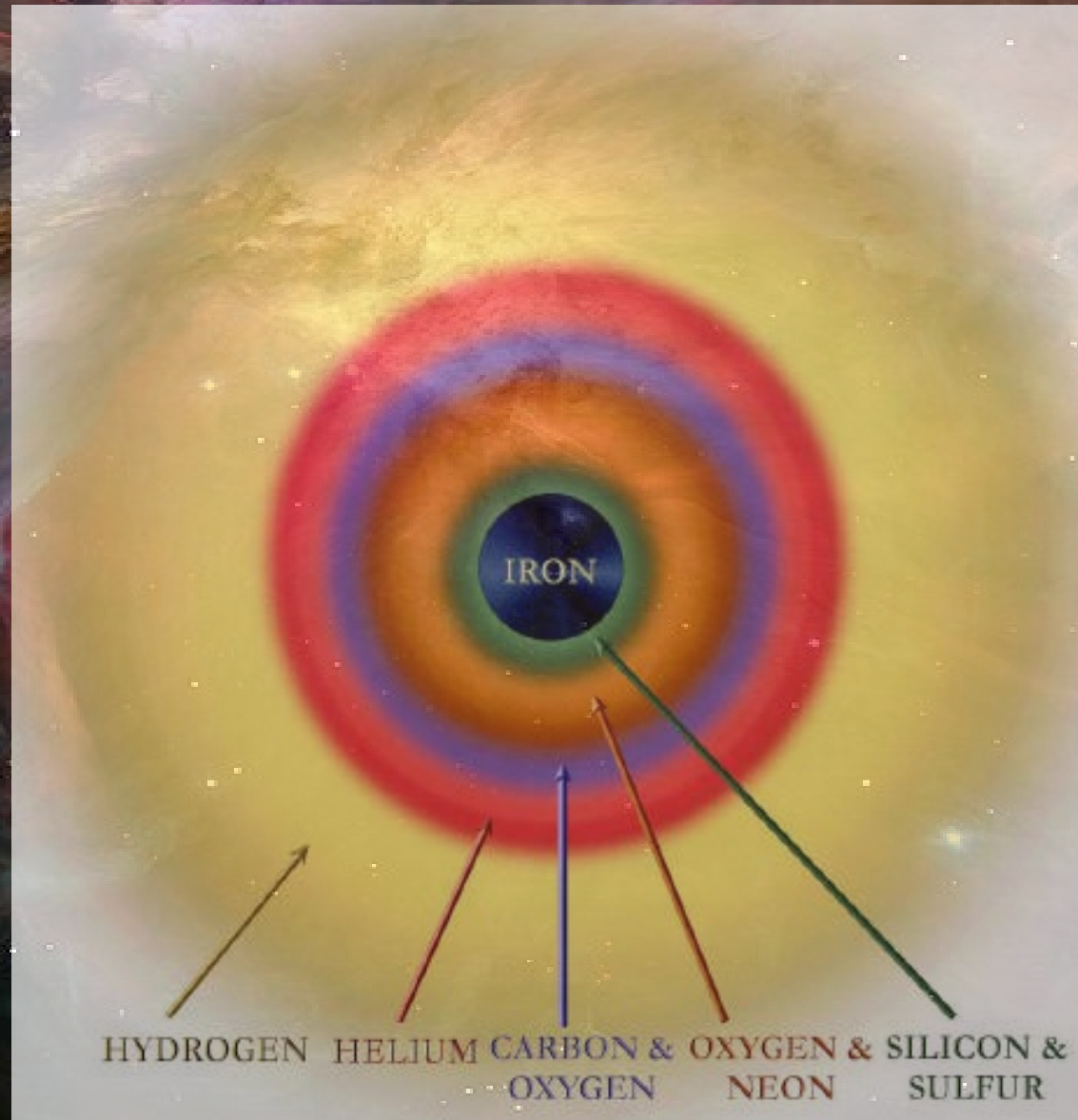
***The same processes characterizes the
subsequent evolution
in stars of greater and greater mass:***

- 1) THE BURNING STOPS WHEN THE NUCLEUS IS
DEPLETED***
- 2) NEW CONTRACTION***
- 3) INCREASE OF TEMPERATURE***
- 4) TURN ON OF THE SUBSEQUENT NUCLEAR
REACTION
WITH FORMATION OF HAVIER ELEMENTS***

this gives origin to the elements from He to Fe

Structure of a heavy star at the end of the process:

- 1) a core of iron***
- 2) concentric layers with nuclear reactions of lighter elements***



If the star has a mass greater than $12 M_{\odot}$ it's final fate is really violent:

**THE INTERNAL PRESSURE IS NOT ENOUGH TO SUBSTAIN
THE WEIGH OF SUPERFICIAL LAYERS**

**A CATASTROPHIC IMPLOSION CONDENSES THE MATTER TO
 10^{14} g/cm^3
(the same density you could obtain shrinking the moon in
a room)**



*The free falling matter bounces on this compressed core
producing a **shock wave** which **destroys the star***

SUPERNOVA EXPLOSION



***During the deflagration the elements heavier than iron
are produced
this concludes the nucleosynthesis of ALL the KNOWN
ELEMENTS!***





What is the fate of such new formed elements ??

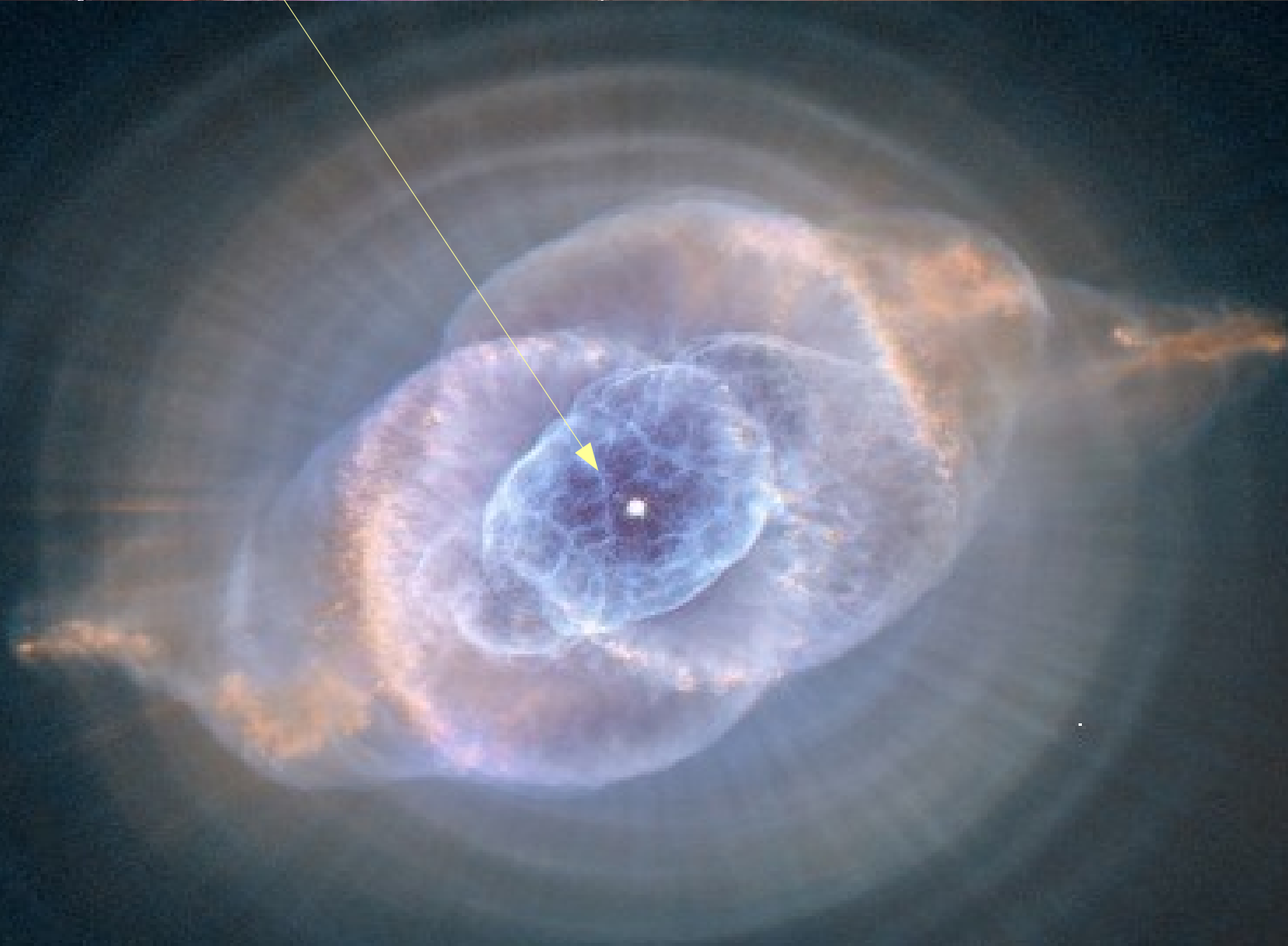
***they will form second generation stars, such as
the Sun,***

solar systems and ourselves !!

This is the deep meaning of the sentence:

“ we are stardust ”

The **central object** which survives the supernova explosion can be..




1) a NEUTRON STAR if $M < 2 M_{\odot}$: an extremely compact object (radius of 10 Km !!!) in rapid rotation about it's axis. Such objects are too faint in the visible light but are powerful X ray sources



2) a **BLACK HOLE** if $M > 2 M_{\odot}$: if the mass exceeds this limit the **gravitational collapse can not be stopped** at all !!
The intensity of the gravitational field becomes so huge that **even light can not escape**.

Inside the radius of this object (**the horizon of events**) the density is such extreme that space and time lose significance.





***“ The starry heavens above me
and the moral law within me”***

***Critique of Practical Reason
Immanuel Kant***



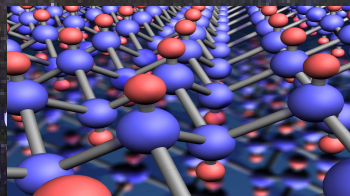
Astrophysical origin of Dark Matter

the curve of rotation of galaxies

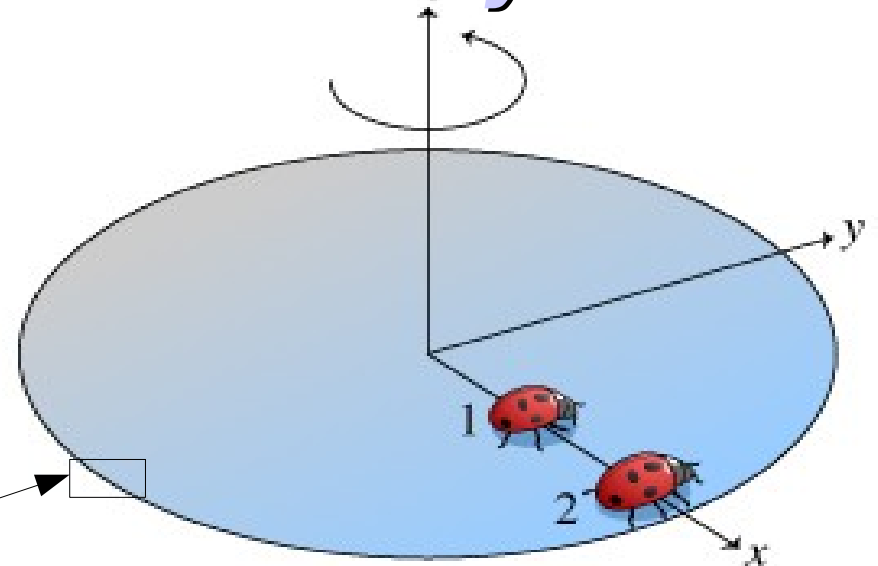


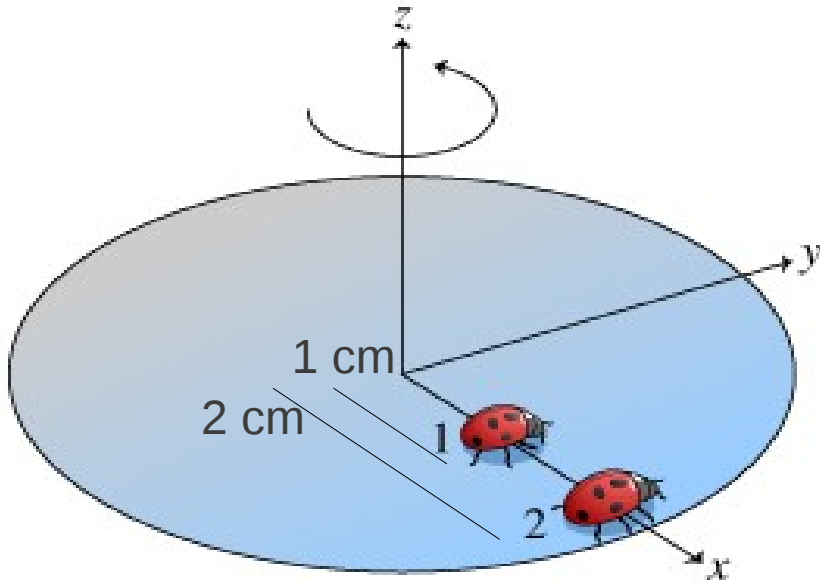
the disc rotates around the center, the velocity of the circular motion of stars depends on the distance from the center..

***let us consider a rigid disk
(for example a CD)
whose atoms maintain
FIX
positions in***



..but why?





***Ladybug 1 covers $2\pi \times 1\text{cm} \approx 6\text{ cm}$
in a time t***

***Ladybug 2 covers $2\pi \times 2\text{cm} \approx 12\text{ cm}$
in the SAME TIME t***

***velocity of ladybug 1 $v_1 =$
 $6\text{cm}/t$
in a time t***

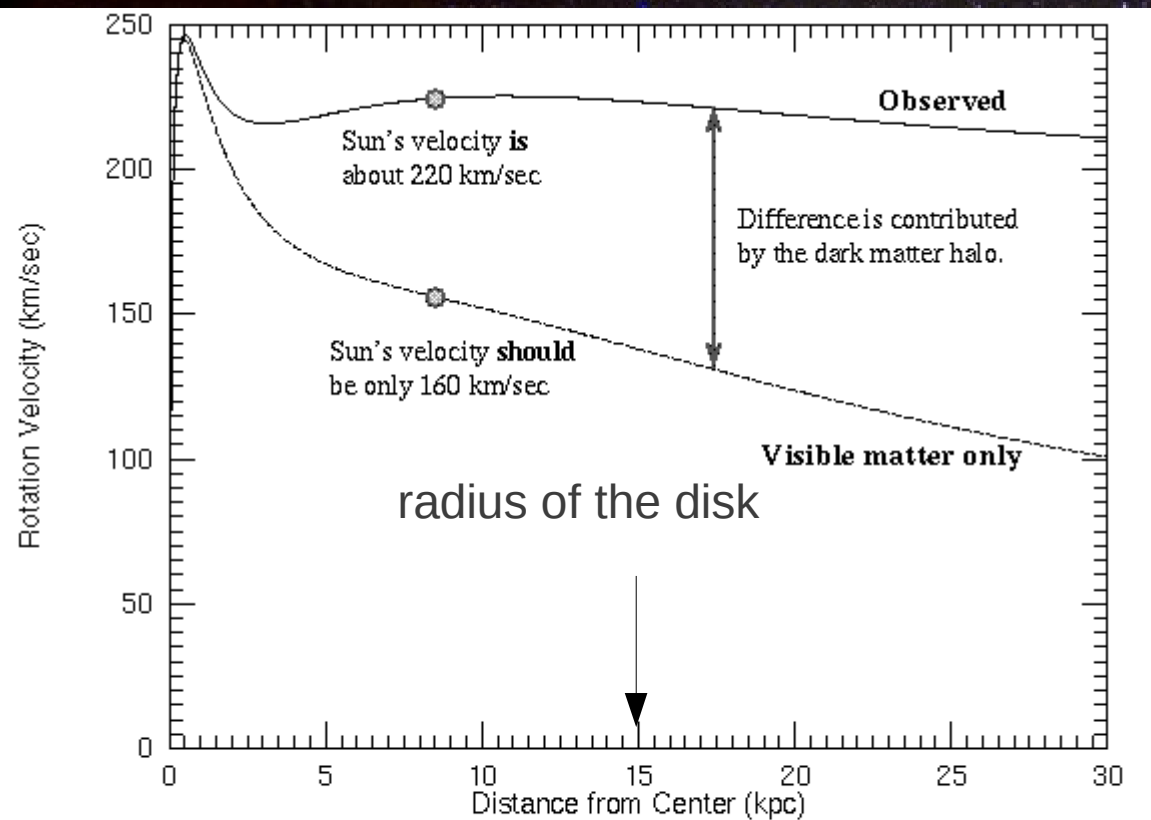
***velocity of ladybug 2 $v_2 =$
 $12\text{cm}/t = 2 \times v_1$***



***but the disk of the
Galaxy is not rigid..
stars does not form a
crystal
they are held together
by the mutual
gravitational
interaction!!***

***the velocity of a star at distance r
from the center is caused by the
gravitational force acting on the
star:***

$$v = \sqrt{(r F_{\text{grav}})}$$



The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo**.

The velocity far away the center is TOO HIGH!!

the mass of the observed matter is not enough to explain the observed velocity

the Galaxy is surrounded by a spherical crown of matter which does not emit light: DARK MATTER

(small stars, planetoids or new exotic