EXtreme X-Ray Focusing

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Lay-out

- Fundamentals of x-ray optics
- Fresnel Zone Plates
- Mirrors
- Waveguides (1-D 2-D)

Basic properties of x-rays

Index of refraction in the x-ray region:

$$r = n - i\beta = 1 - \delta - i\beta$$

$\delta = \lambda^2 (N_0/A)\rho_m\, r_0(Z+f')$ $\beta = (\lambda/4\pi)\mu$

$\delta > 0 \Rightarrow n < 1$ **for all materials**

$$1-n \approx 10^{-5} - 10^{-6}$$

Very difficult to make optics for x-rays

Type of x-ray optics

- **Diffractive optics**

(e.g. Fresnel Zone Plates, Bragg-Fresnel ZP, reflective ZP)

- **Refractive optics**

(e.g. Compound Refractive Lens, Alligator lens, Clessidra…)

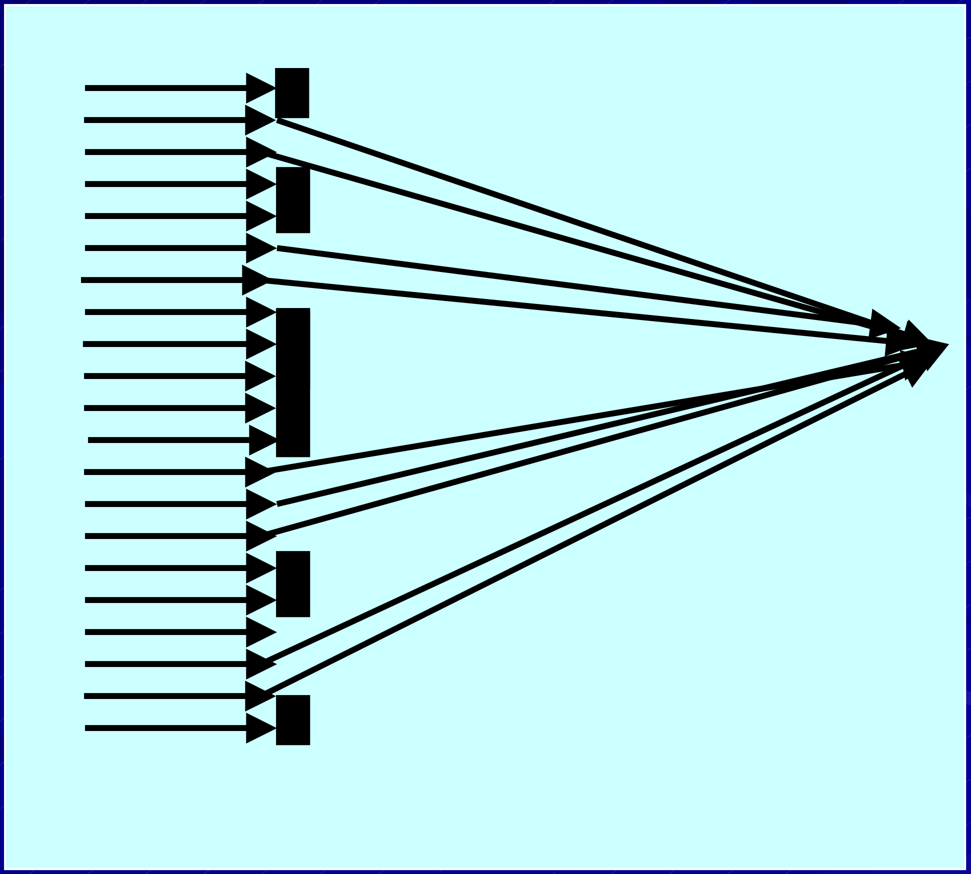
- **Reflective optics**

(e.g. Mirrors, Capillaries, Waveguides,…)

In each type of optics there are examples of μ–meter (in some cases nanometer) beam production

Diffractive optics: Fresnel Zone Plates

- Alternating transparent and opaque rings such that the diffracted rays interfere constructively in the focus



Basic equations for FZP

Relation between zone radii r_n and focal distance f:

$$f^2 + r_n^2 = (f+n\lambda/2)^2 \quad \Rightarrow \quad r_n^2 \approx nf\lambda$$

Width dr_n of the last zone

$dr_N \approx \lambda f/2r_N \approx \sqrt{(\lambda f)} /(2 \sqrt{N})$ N = number of zones

Distribution of intensity from a FZP

$$I_1(\vartheta) = I_0 N^2 \left| 2J1(kr_N \vartheta)/kr_N \vartheta \right|^2$$

ϑ = angle of obs.; J1 = Bessel function; k = wavevector

$I_1(\vartheta)$ correspond to the Airy function

Spatial resolution Δr (Rayleigh criterion):

$\Delta r_{Rayl} = 0.610 \lambda / N.A.$ $N.A. = m\lambda/(2\, dr_N)$

$$\Delta r_{Rayl} = 1.22\, dr_N/m$$

In a plane wave approximation

For soft x-rays: $\Delta r_{Rayl} \cong 20$ nm

Real spot dimension

FZP as a lens: $1/f = 1/p + 1/q$

(q = object-FZP ; p = FZP – image)

If source at a finite distance (no plane wave)

or beam non perfectly monochromatic:

$$\Delta r_m = (\Delta r^2_{Rayl} + \Delta^2_s + \Delta^2_c)^{1/2}$$

$\Delta_s = s\, p/L$ $\Delta_c = 2\, r_N\, \Delta E/E$

S = source dimension; L source-FZP dist. ; p = FZP-focus dist.

$\Delta E/E$ = beam monochromaticity

Efficiency

Amplitude Zone Plates: transparent and opaque zones

$I_m = |c_m|^2 \, Io$ $\qquad$ $c_m = \sin(m\pi/2) / m\pi$

Efficiency: $\eta_m = I_m/Io$

$\eta = 0.25$ $m=0$; $\quad \eta = 1/m^2\pi^2$ m odd; $\quad \eta = 0$ m even

Maximum efficiency in first order : ~ 10 %

Phase Zone Plates: instead of opaque zones,

zones where the phase changes of π

$$\eta_m = (1/m^2\pi^2)\,(1 + e^{-2\nu\phi} - 2\,e^{-\nu\phi}\cos\phi)$$

$$\Phi = 2\pi\delta t/\lambda \qquad \nu=\beta/\delta$$

t = thickness of phase zones

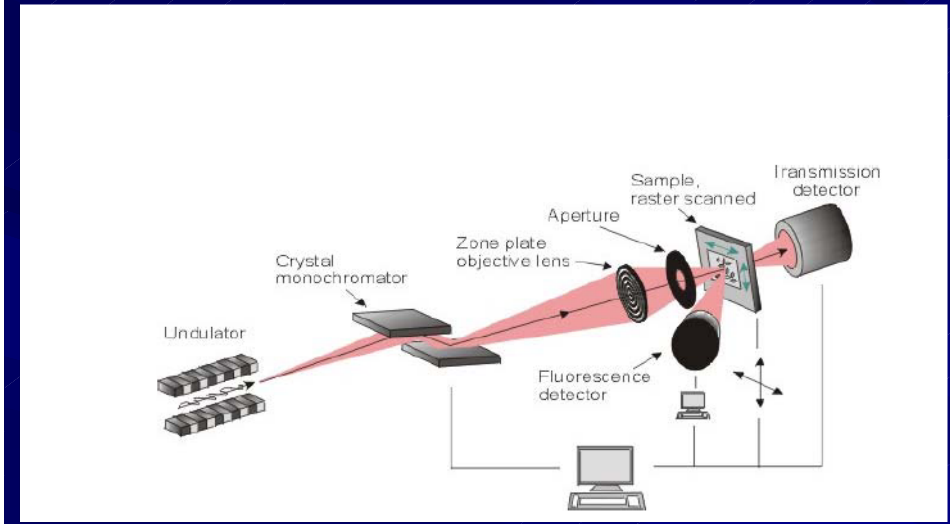
t larger for higher energy

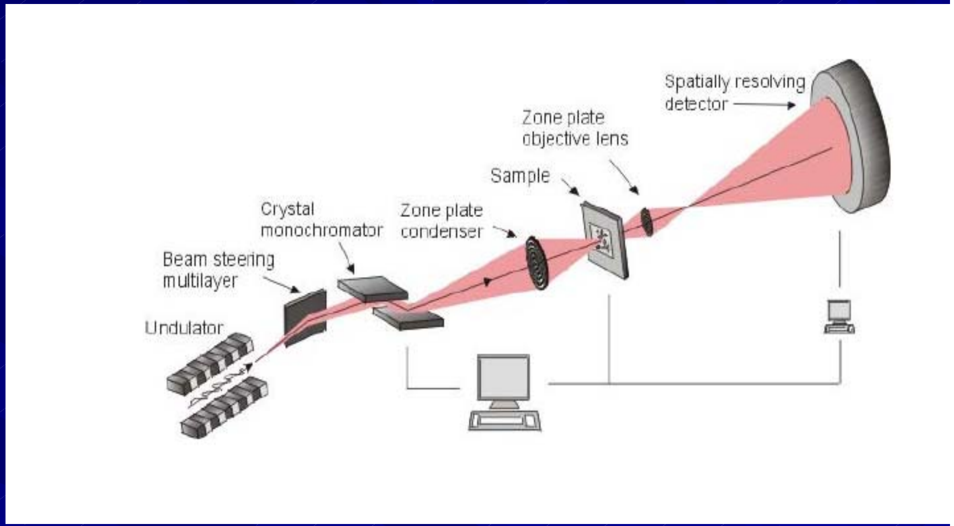
Maximum efficiency in first order : 40.5 %

Thickness t has an influence on minimum zone width:

$$d_{min} = (m\lambda t)^{1/2}$$

Modes of operation in x-ray microscopy





Scanning microscopy

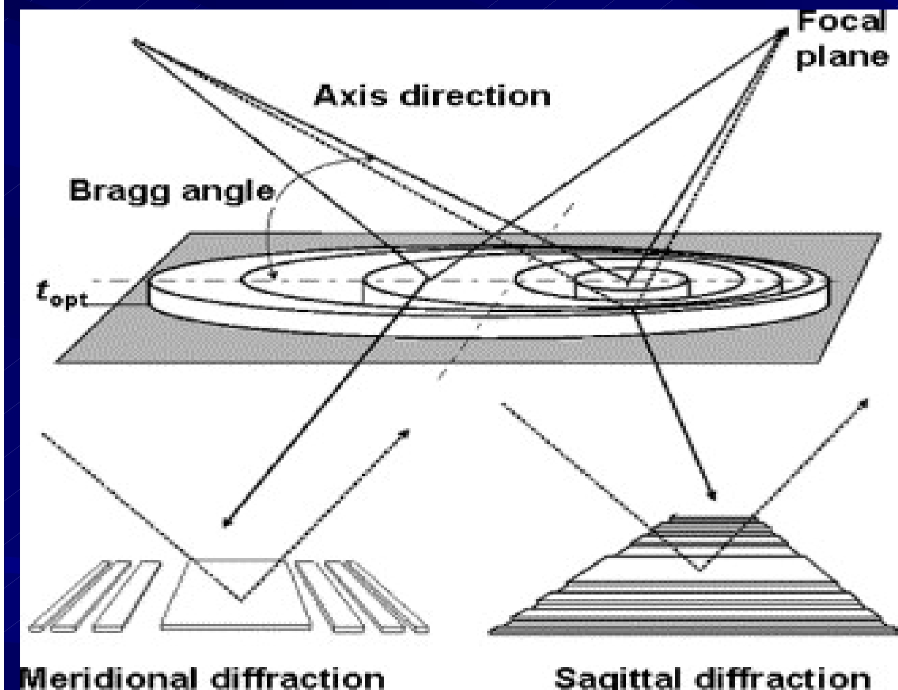
Beam is focused and sample is scanned. Slow, but different detection schemes are allowed

Full field microscopy

The entire sample is illuminated and magnified. Faster, but requires intense source.

Modified Reflection Zone Plates

Reflecting substrate with elliptical phase-shifting Fresnel structure patterned on the surface



Higher resolution limit

(in the meridional direction)

and thinner structures

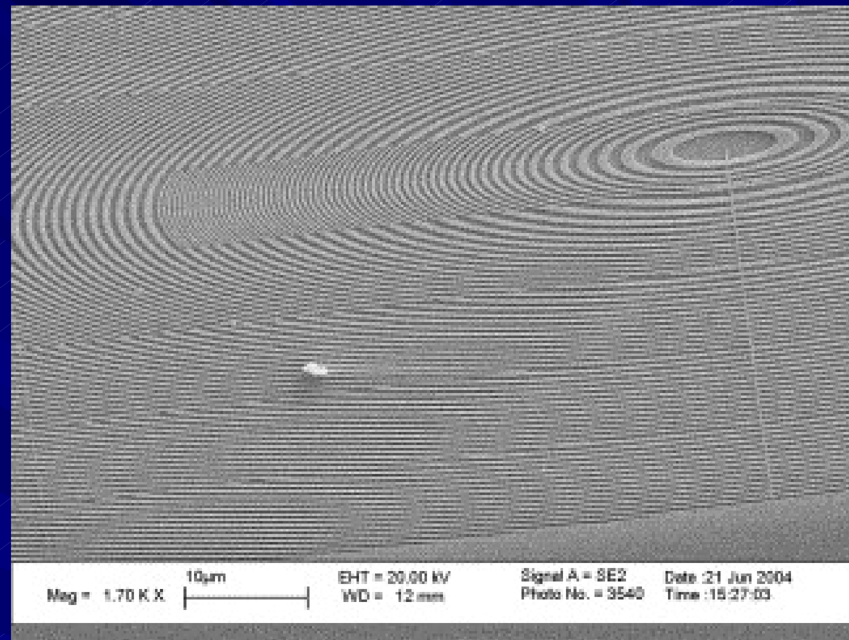
$$\Delta_{trans} = 1.22\,\lambda\,(\Phi/2\pi\delta)^{1/2}$$

$$\Delta_{refl} = 1.22\,\lambda\,(\Phi\sin 2\theta/4\pi\delta)^{1/2}$$

From Michette et al., Opt. Comm., 2005

Different possible substrates: crystal, multilayer,reflecting surface

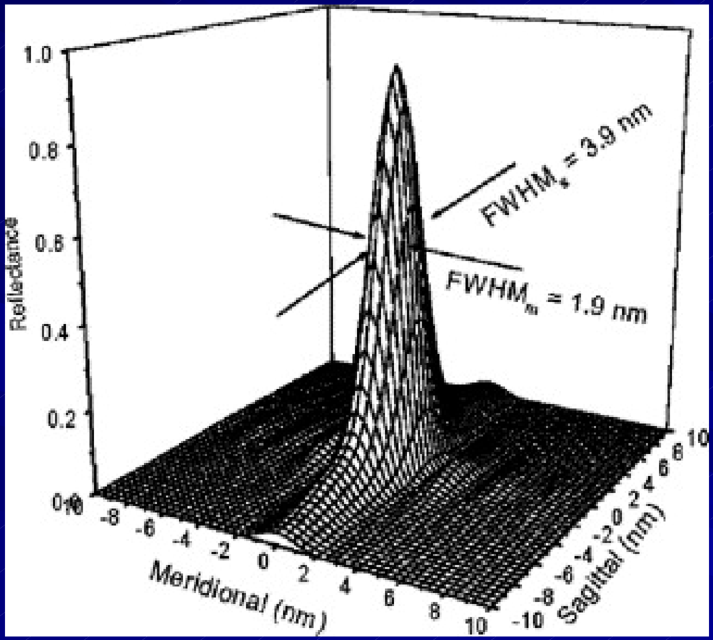
To improve resolution also in sagittal direction:



Focusing of different diffraction orders in the same focus

Diffraction order	Relative efficiency	Relative area	% of total flux in focal spot
1	100	1.2	29.3
3	11	9.8	26.0
5	4	19.7	18.7
7	2	29.6	14.3
9	1.2	39.5	11.5

Calculated intensity distribution in the focal plane with diffraction order up to the ninth in the sagittal direction (min. z.w. 15 nm), and first order in meridional direction (min. z.w. 115 nm)



Advantages: better resolution

power load distributed in larger area

Disadvantages: lower numerical aperture

Distortion of time structure:

Difference in optical path from direct beam and peripheral one:

$\Delta L = N\lambda/2$ (N = number of zones)

For N =5000 and λ = 5 nm

(r = 200 μ and dr_N = 20 nm)

ΔL = 12.5 μ ~ 40 fs

If pulse is shorter no interference takes place.

Possible problems with time structure

of the SASE FEL

Power load consideration (from Bessy TDR):

10^{11} ph/pulse at 500 eV cause:

$\Delta T \sim 1000$ °K if concentrated on 0.1 mm

$\Delta T \sim 10$ °K if concentrated on 1 mm

$\Delta T \sim 100$ °K is acceptable

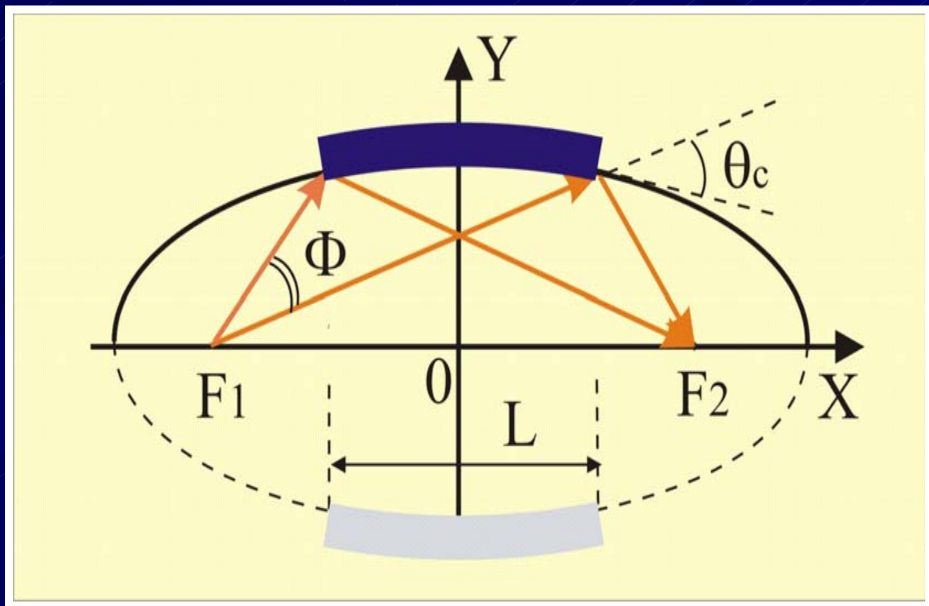
Focusing optics with high efficiency: mirrors

- ✓ Sub-micrometer spot size achieved
- ✓ High efficiency (R ~ 1)
- ✓ Figured mirrors with minimum slope error (< 1 μrad) and minimum roughness (< .1 nm)
- ✓ Possibility of 2-D focusing in the Kirkpatrick-Baez geometry (two crossed mirrors)
- ✓ Practically no effect on pulse time structure
- ✓ High cost
- ✓ Limited angular acceptance for high magnification

Reflecting elliptical surface

Optimization of parameters with analytical approach

I.N.Bukreeva, S.Dabagov, S.Lagomarsino, "On the efficiency of elliptically shaped x-ray mirror", Appl.Opt., 2004



Ellipse: best geometrical figure for point-to-point focusing.

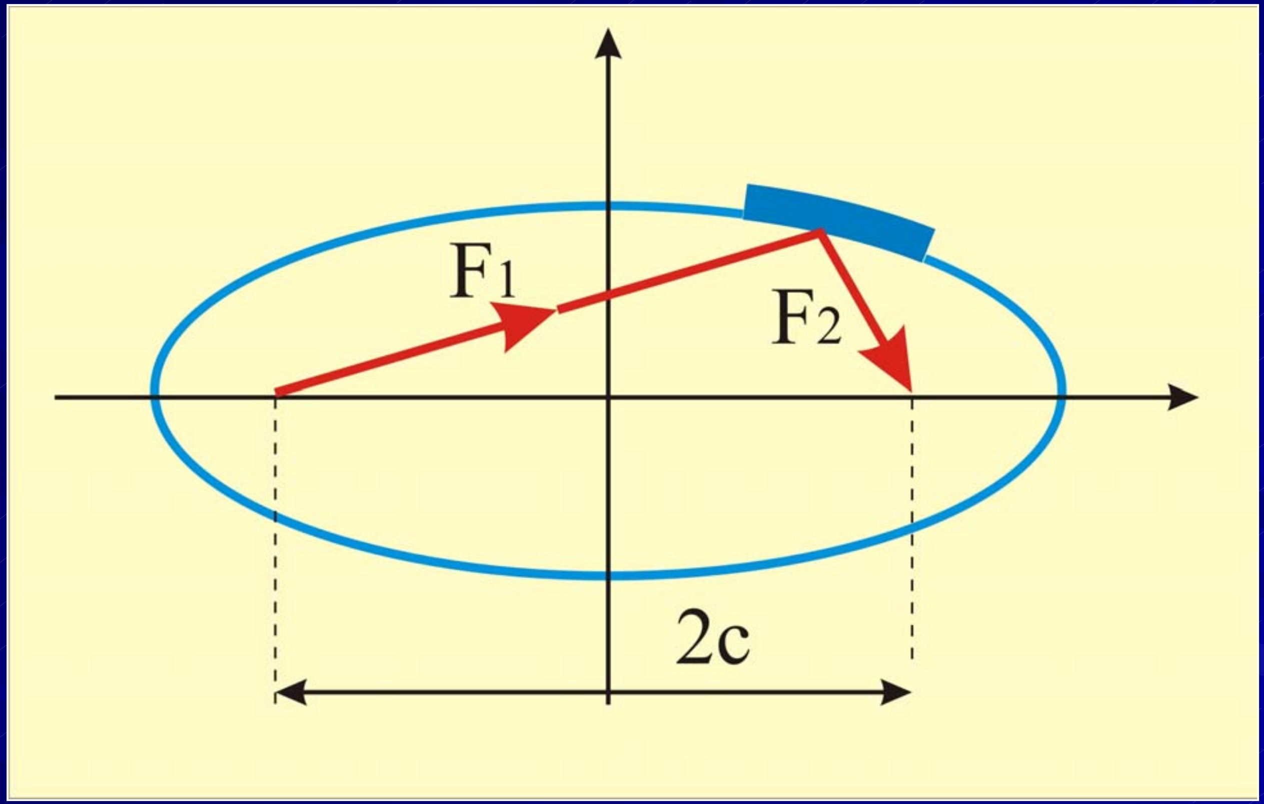
Parameters to optimize for highest possible acceptance angle Φ:
e = eccentricity
L = length of mirror

For effective reflection incidence angle must always be less then critical angle θ_c

$$1 - e < \frac{\theta_c^{\,2}}{2} = \delta$$

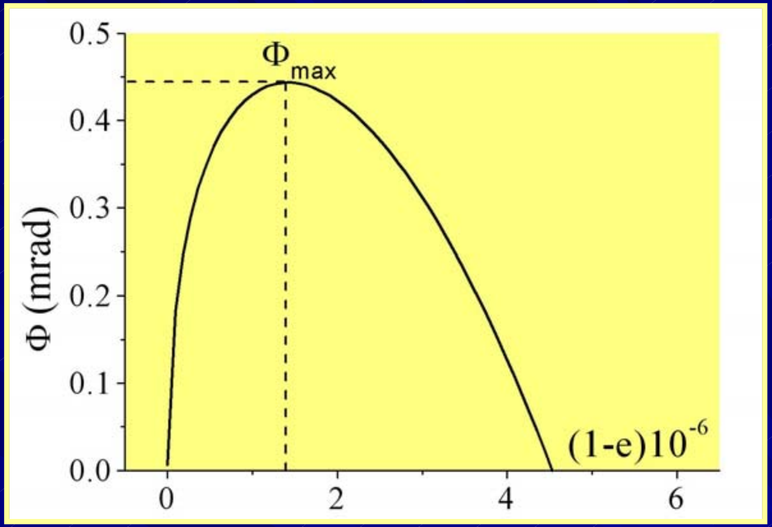
For extreme focussing source image is highly demagnified:

$$M = F_2/F_1 << 1$$



Optimal parameters of the focusing mirror

Maximum of acceptance angle Φ as a function of eccentricity



$$e_{opt} \sim 1 - \theta_c^2 M / 2$$

Optimal parameters:

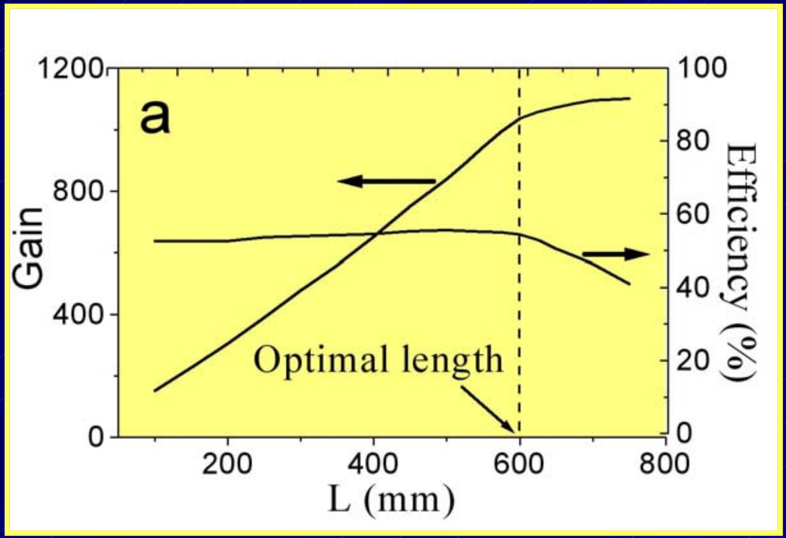
$$\Phi_{max} \sim (5/6)\,\theta_c M$$

$$L_{opt} \sim 3cM$$

M	$(1\text{-}e)/\theta_c^2$	$\Phi/\ \theta_c$	L/2c
10^{-1}	$4.55\cdot10^{-2}$ ($\sim 5.0\cdot10^{-2}$)	$8.11\cdot10^{-2}$ ($\sim 8.33\cdot10^{-2}$)	$1.35\cdot10^{-1}$ ($\sim 1.5\cdot10^{-1}$)
10^{-2}	$4.95\cdot10^{-3}$ ($\sim 5.0\cdot10^{-3}$)	$8.31\cdot10^{-3}$ ($\sim 8.33\cdot10^{-3}$)	$1.48\cdot10^{-2}$ ($\sim 1.5\cdot10^{-2}$)
10^{-3}	$5.0\cdot10^{-4}$ ($\sim 5.0\cdot10^{-4}$)	$8.33\cdot10^{-4}$ ($\sim 8.33\cdot10^{-4}$)	$1.5\cdot10^{-3}$ ($\sim 1.5\cdot10^{-3}$)
10^{-4}	$5.9\cdot10^{-5}$ ($\sim 5.0\cdot10^{-5}$)	$8.33\cdot10^{-5}$ ($\sim 8.33\cdot10^{-5}$)	$1.5\cdot10^{-4}$ ($\sim 1.5\cdot10^{-4}$)

Estimation of normalized eccentricity $(1\text{-}e)/\theta_c^2$, maximal acceptance angle $\Phi/\ \theta_c$, and optimal length L/2c for different magnification values M. In parenthesis are reported the approximate values.

Computer simulation



Synchrotron radiation source

Size 100 μm e_{opt} =1.16·10^{-7}

λ= 0.1 nm L_{opt}=600 mm

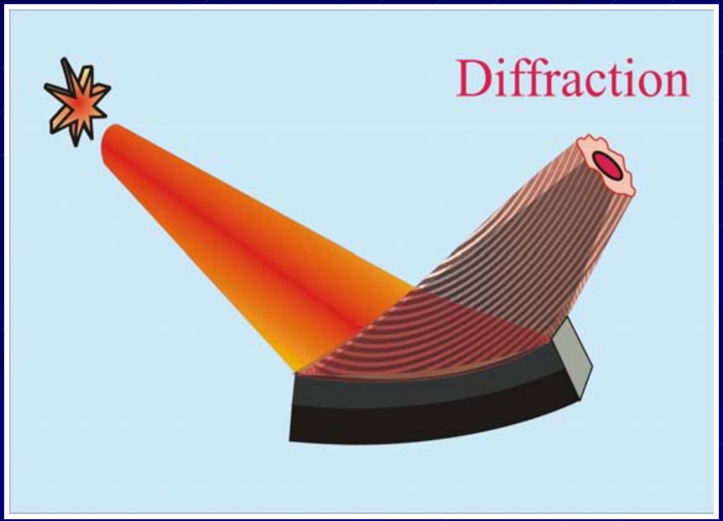
M=10^{-2}

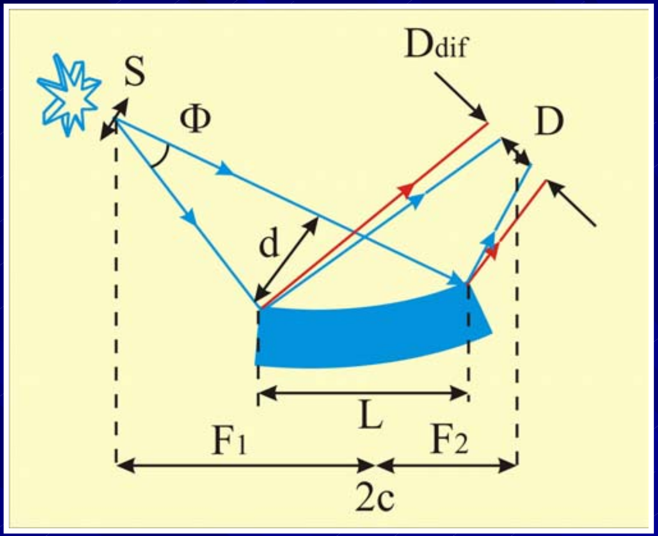
2c=40 m

Efficiency is the ratio between the number of photons in the focal spot and the number of photons captured by the mirror.

Gain is the ratio between the flux density in the focal spot and the flux density in the incoming beam.

Diffraction-limited focal spot size





Diffraction-limited focal spot size:

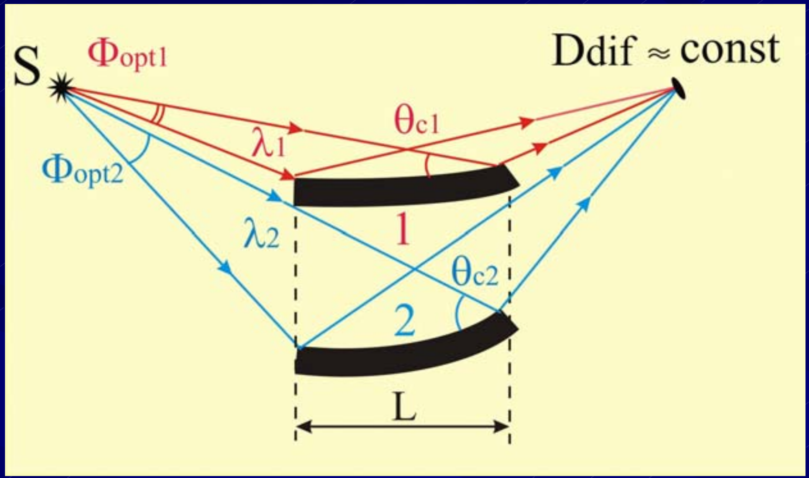
$$D_{dif} \sim \frac{\lambda}{\theta_c}$$

- approximately does not depend on wavelength for same material
- for different materials has different values (ratio 1-3).

$$\theta_c = \sqrt{2\delta} \approx \lambda\, [2(N_0/A)\rho_m\, r_0(Z+f\,')]^{1/2}$$

$$D_{dif}(nm) \approx \text{Const} \sim \frac{180}{(1\text{-}3)\pi}(nm)$$

Parameters of the optimal mirror near diffraction limit



$$G \sim R\frac{d}{D_{dif}} \qquad Const = \frac{180}{(1-3)\pi}(nm)$$

$$G_{opt} \sim \frac{2c}{S} \cdot R\frac{\lambda}{Const}$$

$$D_{dif} \sim Const$$

$$M \sim \frac{1}{S} \cdot Const$$

$$L_{opt} \sim \frac{3c}{S} \cdot Const$$

$$\Phi_{opt} \sim \lambda / S$$

$$1 - e_{opt} \sim \frac{1}{2S} \cdot \frac{\lambda^2}{Const}$$

Wavelength is constant, material is different:

- acceptance angle is constant,
- length of mirror, size of spot and Gain depends on material (ratio 1-3).

Material is the same, wavelength is different:

- length of mirror and size of spot are constant,
- acceptance angle and Gain are proportional to wavelength.

Parameters of the optimal mirror near diffraction limit

Material	C	
S, µm	200	
2c, m	20	
λ, nm	6	30
θ_c, rad	0.116	0.6
M	$2.5\cdot10^{-4}$	
D_{dif}, nm	52	50
L_{opt}, mm	7.5	
Φ_{opt}, rad	$2.5\cdot10^{-5}$	$1.3\cdot10^{-4}$
G_{opt}	$10^4\ R_c$	$6\cdot10^4\ R_c$

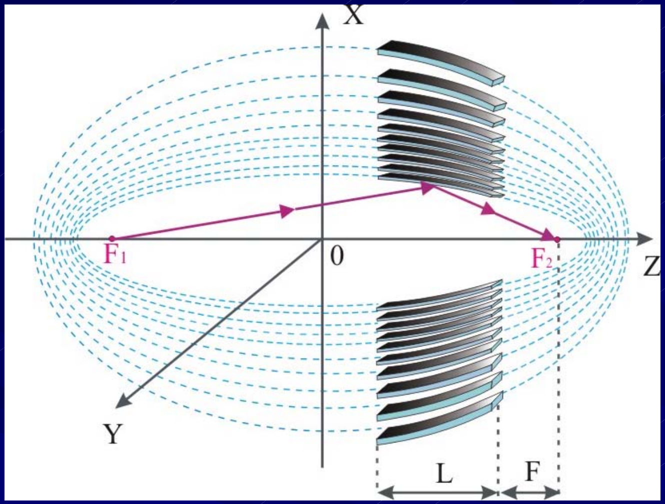
Material	C	Pt
S, µm	100	
2c, m	40	
λ, nm	0.15	
θ_c, rad	$3.7\cdot10^{-3}$	10^{-2}
M	$4\cdot10^{-4}$	$1.5\cdot10^{-4}$
D_{dif}, nm	40	15
L_{opt}, mm	24	9
Φ_{opt}, rad	$1.25\cdot10^{-6}$	
G_{opt}	$1.3\cdot10^3\ R_c$	$4\cdot10^3\ R_{Pt}$

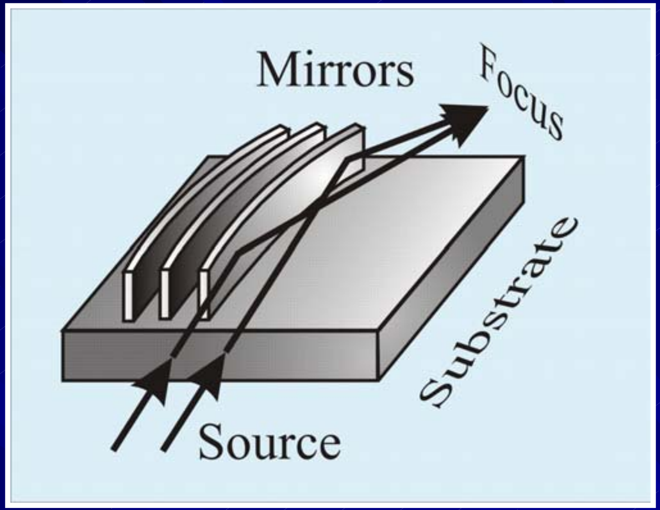
To increase acceptance angle and gain:

- Increase mirror length is not a good way (an optimal length exists for any exp. condition)
- Alternative: nested mirrors (like in astronomy)
- Possible solution: microfabrication tools.

e-beam, optical and x-ray lithography used for system fabrication

System of confocal elliptical mirrors





Advantages

- Possibility to make any shape of mirror.
- Higher flux gain due to nested surfaces.
- Compact and flexible system
- Simpler and cheaper preparation

Possible problems

- Accuracy in shape
- Roughness
- Preservation of spatial coherence and time structure

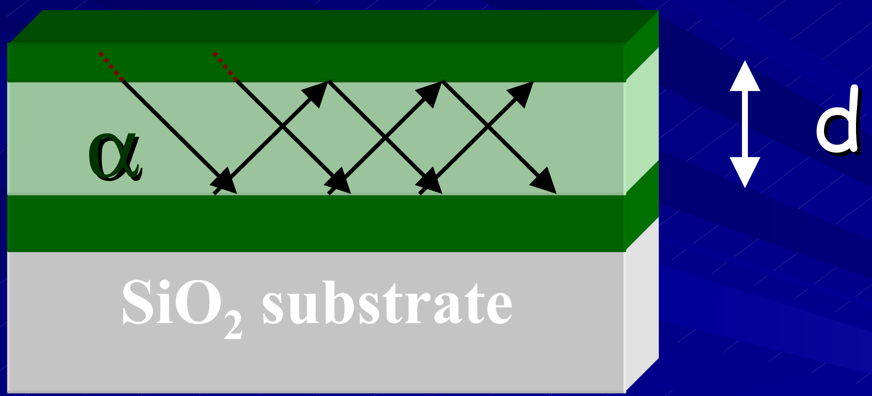
Elliptical mirror does not introduce significant time broadening.

Power load can be dealt minimizing incident angle and using suitable material for mirror (or for coating) such as diamond or berillium

What is a Waveguide?

- A structure where well defined resonance modes can propagate:

Guiding layer between cladding layers

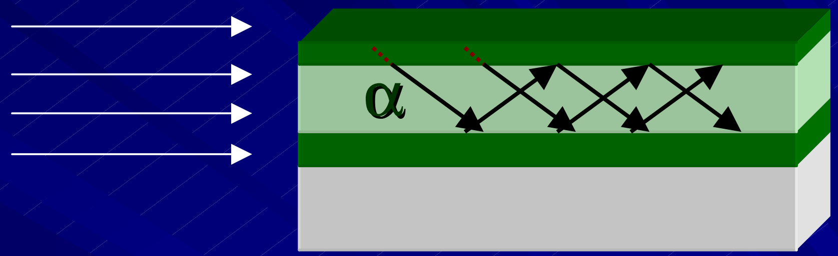


Conditions for resonance:

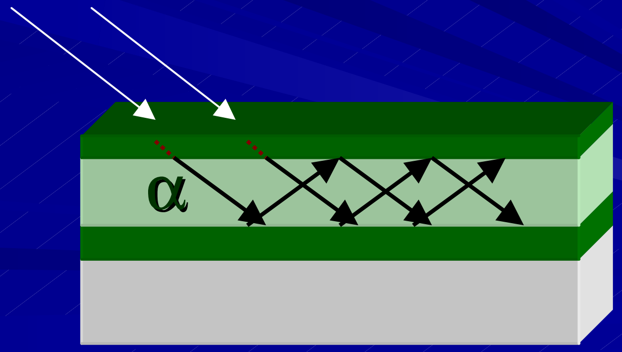
$n_{WG} > n_{cl}$ $m\lambda = 2\ d \sin(\alpha)$

Different Coupling modes

- Front Coupling



- Resonant Beam Coupling



Front Coupling suitable also for soft x rays (guiding layer: vacuum)

Many modes are excited at the same time (if resonance conditions are met)

Resonances higher than TE_0 can be chosen selectively

Main features of beam from WG:

Nanometer beam size. Diffraction limit $\approx$ 10 nm

Fully coherent beam. A gaussian beam with tailored waist can be formed, allowing high resolution microscopy in projection mode and phase contrast.

Two inherently correlated beams can be formed

Interesting dispersion relations for short pulses

(See presentations of D. Pelliccia and G. Campi)

Conclusions

Different kind of optics for microfocusing available with diffraction limit in the nanometer range.

Problems of power load and response to short pulses still open (mainly for ZP)