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Measurement of mechanical vibrations excited in aluminum resonators by 0.6 GeV electrons

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We present measurements of mechanical vibrations induced by 0.6 GeV electrons impinging on cylindrical and spherical aluminum resonators. To monitor the amplitude of the resonator's vibrational modes we used piezoelectric ceramic sensors calibrated by standard accelerometers. Calculations using the thermo-acoustic conversion model agree well with the experimental data, as demonstrated by specific variations of the excitation strengths with the absorbed energy, and with the traversing particles' track positions. For the first longitudinal mode of the cylindrical resonator we measured a conversion factor of 7.4 ± 1.4 nm/J, confirming the model value of 10 nm/J. Also, for the spherical resonator, we found the model values for the L=2 and L=1 mode amplitudes to be consistent with our measurement. We thus have confirmed the applicability of the model, and we note that calculations based on the model have shown that next generation resonant mass gravitational wave detectors can only be expected to reach their intended ultrahigh sensitivity if they are shielded by an appreciable amount of rock, where a veto detector can reduce the background of remaining impinging cosmic rays effectively. © 2000 American Institute of Physics. [S0034-6748(00)00803-0]

I. INTRODUCTION

A key issue for a resonant mass gravitational wave detector¹ of improved sensitivity with respect to existing detectors is the background due to impinging cosmic ray particles.^{2,3} The energy deposited in the detector's mass along a particle's track may excite the very vibrational modes that signal the passing of a gravitational wave. Computer simulations of such effects are based on the thermo-acoustic conversion model and earlier measurements of reso-

nant effects by Beron *et al.*⁴ and by Grassi Strini *et al.*⁵ According to the model, the energy deposited by a traversing particle heats the material around the particle track locally, which leads to mechanical tension and thereby excites acoustic vibrational modes.⁶ At a strain sensitivity of the order of 10^{-21} envisaged for a next generation gravitational-wave detector, computer simulations^{3,7} show that operation of the instrument at the surface of the Earth would be prohibited by the effect of the cosmic ray background. Since the applicability of the thermo-acoustic conversion model would thus yield an important constraint on the operating conditions of resonant mass gravitational wave detectors, Grassi Strini,

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TABLE	I.	Characteristics	of	our	setup.	
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Setup code name	BC	BU	SU
Resonator type	Bar	Bar	Sphere
Diameter	0.035 m	0.035 m	0.150 m
Length	0.2 m	0.2 m	-
Suspension	Plastic string	Plastic string	Brass rod
			$0.15 \text{ m} \times 0.002 \text{ m}$
Piezo sensors	1	2	2
Piezo hammer	0	1	1
Capacitor driver	1	0	0
Direct calibration	Yes	No	No
Beam energy	0.76 GeV	0.62 GeV	0.35 GeV
Beam peak current	3 mA	18 mA	19 mA
Electrons per burst	$\approx 10^9$	$\approx 5 \times 10^{10}$	$\approx 5 \times 10^{10}$
Mean absorbed energy per electron	0.02 GeV	0.02 GeV	0.1 GeV
Typical absorbed energy per burst	0.01 J	0.6 J	3.0 J

Strini, and Tagliaferri⁵ measured the mechanical vibrations in a bar resonator bombarded by 0.02 GeV protons and 5 $\times 10^{-4}$ GeV electrons. We extended that experiment by measuring the excitation patterns in more detail for a bar and a sphere excited by 0.6 GeV electrons. Even though we cannot think of a reason why the model, if applicable to the bar, would not hold for a sphere, we resorted to measuring with a sphere also. We exposed⁸ two aluminum 50ST alloy cylindrical bars and an aluminum alloy sphere, each equipped with piezoelectric ceramic sensors, to a beam of $\approx 0.6 \,\text{GeV}$ electrons used in single bunch mode with a pulse width of up to $\approx 2 \ \mu s$ and adjustable intensity of $10^9 - 10^{10}$ electrons. We recorded the signals from the piezo sensors, and Fourier analyzed their time series. Before and after the beam run we calibrated the sensor response of one of the bars for its first longitudinal vibrational mode at \approx 13 kHz to calibrated accelerometers.

II. EXPERIMENT SETUP AND METHOD

In the experiment we used three different setups in various runs: two bars and a sphere, summarized in Table I. With the uncalibrated BU bar we explored the feasibility of the measurement. Also, bar BU proved useful to indirectly determine the relative excitation amplitudes of higher longitudinal vibrational modes; see Sec. IV A. With bar BC calibrated at its first longitudinal vibrational mode, we measured its excitation amplitude in the beam directly. Finally, with the sphere we further explored the applicability of the model.

A. Electron beam

We used the Amsterdam linear electron accelerator MEA^{9,10} that delivers an electron beam with a pulse width of up to 2 μ s in its hand-triggered, single bunch mode. The amount of charge per beam pulse was varied, recorded by a calibrated digital oscilloscope, photographed and analyzed off line to determine the number of impinging electrons per burst.

B. Suspension and positioning

In both the BC and BU setups (see Fig. 1) the cylindrical aluminum bar was horizontally suspended in the middle, as indicated in Fig. 1, by a plastic string. The bar's cylinder axis was positioned 90° to the beam direction. The bar's suspension string was connected to a horizontally movable gliding construction, enabling us to handle the resonator by remote control and let the impinging electron beam hit it at different horizontal positions. The aluminum sphere SU (see Fig. 2) was suspended from its center by a brass rod. Either the bar's gliding construction or the sphere's suspension bar was attached to an aluminum tripod mounted inside a vacuum chamber,⁹ which was evacuated to about 10^{-5} mbar. By remote control, we rotated the tripod and moved it vertically to either let the beam pass the resonator completely or let it traverse the resonator. We let the beam traverse the sphere at different heights and different incident angles with respect to the piezo sensor positions on the sphere. We mark the beam heights E (Equator) and A (Africa) 0.022 m below the Equator. The E beam passed horizontally through the sphere's origin, remaining in the same vertical plane for the A beam.

C. Sensors and signal processing

In the BC setup we used a single piezo sensor $\approx 15 \times 3 \times 1 \text{ mm}^3$ and glued it over its full length 0.01 m off center on top of the bar. Bar BC was equipped with a capacitor plate 0.03 m in diameter ≈ 0.004 m from one of its end faces.

In setup BU one piezo sensor $\approx 3 \times 6 \times 0.3 \text{ mm}^3$ was fixed on one end face of the bar. A similar sensor of about the same dimensions was fixed in the same manner, oriented parallel to the cylinder's long axis at a position 35 mm away from the end face. In the third setup, SU (see Fig. 2) two piezo sensors $\approx 3 \times 6 \times 0.3 \text{ mm}^3$ were glued to the sphere's surface. One was situated at the equator, with respect to the vertical rotation axis, and the other one was at a relative displacement of 45° west longitude and 45° north latitude.



FIG. 1. Calibrated bar BC setup. The beam traverses the bar's front side perpendicular to the drawing plane.



FIG. 2. Spherical resonator setup, SU.

For the setup used, each sensor was connected to a charge amplifier $\approx 2 \times 10^{10}$ V/C in gain. The signals were sent through a Krohn-Hite 3202R low-pass 100 kHz pre-filter to a R9211C Advantest spectrum analyzer with internal 2 MHz pre-sampling and 125 kHz digital low-pass filtering. The oscillation signals were recorded for 64 ms periods at a 4 μ s sample rate. The beam pulse could be used as a delayed trigger for the Advantest analyzer. Using the memory option of the Advantest, the piezo signals were recorded from 0.3 ms onward *before* the arrival of the trigger. The data were stored on a disk and were Fourier analyzed off line.

D. Checks and stability

The data were taken at an ambient temperature of ≈ 23 °C. By exciting the resonator with the piezo hammer we could roughly check its overall performance. As will be discussed in Sec. III, setup BC was calibrated before and after the beam run. The instrument's stability was checked several times during the run by an electric driving signal on its capacitor endplate.

III. CALIBRATION OF BAR BC'S PIEZO CERAMIC SENSOR

A standard accelerometer mounted on the bar damped the vibrations too strongly to confidently measure their excitations in the electron beam. Therefore the response of the piezoelectric ceramic together with its amplifier was first calibrated against two 2.4 g Bruel & Kjaer 4375 accelerometers glued, one at a time, to bar BC's end face and connected to a 2635 charge amplifier. The resonator was excited through air by a nearby loudspeaker driven by the Advantest digitally tunable sine-wave generator. The output signals from both the piezoelectric ceramic amplifier and the accelerometer amplifier were fed into the Advantest. Stored time series were read by an Apple Mac 8100 AV, running LabView for on-line Fourier analysis, peak selection, amplitude and decay time determination. We took nine calibration runs, varying the charge amplifier's sensitivity setting, and dismounting and remounting either of the two accelerometers to the bar. For the lowest longitudinal vibrational mode



FIG. 3. Response at the lowest longitudinal acoustic frequency of bar BC's piezo (lower) and accelerometer (upper) Fourier amplitudes by constant driving of the amplitude loudspeaker. Left: On resonance, $f = 12\,950$ Hz. Right: Slightly off resonance, $f = 12\,850$ Hz.

we calculated the ratio of the Fourier peak signal amplitudes, R, from the piezoelectric ceramic and accelerometer.

With the calibrated bar BC positioned in the electron beam line we checked the stability of the piezoelectric ceramic's response intermittently with the beam runs by exciting the bar through its capacitor plate at one end face, electrically driving it at and around half of the bar's resonance frequency. We found the response to remain stable within a few percent.

After the beam runs we took additional calibration values in air with a newly acquired Bruel & Kjaer 0.5 g 4374S subminiature accelerometer and a Nexus 2692 AOS4 charge amplifier. In Fig. 3, typical frequency responses are shown when the bar is driven by a loudspeaker signal. The upper part gives the Fourier peak amplitude of the bar's 13 kHz resonance as measured with the accelerometer. The lower part gives the corresponding amplitude for the signal from the piezoelectric ceramic. The right-hand side of the picture shows the amplitudes to be smaller, as expected, when the bar is driver slightly off resonance. We calculate the decay time, τ , of the *k*th mode amplitude $A_k(t) = A_k(0) \cdot e^{-t/\tau}$ to be $\tau = 0.4$ s for this setup, which is equipped with the relatively light accelerometer.

Figure 4 shows the corresponding two signals when the bar is driven by the capacitor plate at 6.5 kHz, that is, at half the bar's resonance frequency. Here, the direct electric response of the piezoelectric ceramic's signal to the driving sine wave is present, clearly without a mechanical signal, which would have shown up in the accelerometer. The direct signal at 6.5 kHz remains constant. On the other hand, the bar's mechanical signals on and off its resonance frequency around 13 kHz show the expected amplitude change again, thereby demonstrating that around the bar's resonance, the



FIG. 4. Response of bar BC's piezo (lower) and accelerometer (upper) Fourier amplitudes by electrostatic capacitor plate driving. Left: On resonance, f = 6481.7 Hz. Right: Slightly off resonance, f = 6480.0 Hz. The piezo peak of constant amplitude at 6.5 kHz that arises from crosstalk is absent in the accelerometer, while the acoustic resonance is clearly seen at 13 kHz in both.

piezoelectric ceramic only responds to the mechanical signal, not to the electrical driving signal (see also the caption of Fig. 4).

We calculated the average value of $R_0 = V_{\text{piezo}}^{\text{Fourier}} / V_{\text{accel}}^{\text{Fourier}}$ and the error over all 29 measurements, finding for the calibration factor at f = 13 kHz,

$$\beta = R_0 S (2\pi f)^2 = (2.2 \pm 0.3) \text{V/nm}, \qquad (1)$$

where $S = 0.1 \text{ V/ms}^{-2}$ is the amplifier setting of the accelerometer.

IV. BEAM EXPERIMENTS

Sensor signals way above the noise level were observed for every beam pulse hitting the sphere or the bar. We ascertained that (a) the signals arose from mechanical vibrations in the resonator and (b) they were directly initiated by the effect of the beam on the resonator, and did not arise from an indirect effect of the beam on the piezo sensors. Our assertion is based on a combination of test results observed for both the bars and the sphere that will now be discussed.

First, when the beam passed underneath the resonator without hitting it, we observed no sensor signal above the noise. Second, as shown in Fig. 5, the sensors' delayed responses after the impact of the beam agreed with the sound velocity. Here the beam hit the sphere 5 mm above the sphere's south pole. The middle trace shows a beam pulse of $\approx 2 \ \mu$ s duration. The two other traces show both piezo sensors responding to a transient signal right from the start of the beam's arrival and begining to oscillate after some delay, depending on their distance from the beam. The distance of the equatorial sensor to the beam hitting the sphere at the



FIG. 5. 2 μ s electron bunch (middle trace) and the piezo sensors signals on the sphere (upper and lower traces).

south pole was 0.11 m, corresponding to an $\approx 22 \,\mu s$ travel time for a sound velocity of $\approx 5 \times 10^3$ m/s. The signal is indeed seen in the lowest trace that starts to oscillate at that delay time. The upper trace shows the signal from the second sensor situated on the northern hemisphere 0.14 m from the traversing beam, correspondingly starting to oscillate with a delay of $\approx 28 \,\mu s$ after impact of the beam. Third, after removing the piezo hammer from the resonator, we observed that the sensor signals did not change, which showed that the activation is not caused by the beam inducing triggering of the piezo hammer. Fourth, to simulate the electric effect of the beam pulse on the sensors, we coupled a direct current of 60 mA from a wave packet generator to the bar for a 2.5 μ s. Apart from the direct response of the piezo sensor during the input driving wave, no oscillatory signal was detected above the noise level. Finally, we measured the dependence of the amplitudes in several vibrational modes on the impact position of the beam, as will be described. We found the amplitudes to follow the patterns calculated with the thermoacoustic conversion model.

A. Results for the bar

In Fig. 6 a typical Fourier spectrum of bar BC is shown up to 55 kHz. The arrows point to identified vibrational modes.¹¹ From a fit of K and f_0 of the longitudinal frequencies $f_L = L \cdot f_0 (1 - L^2 K)^{12}$ of the modes for L = 1,...,4, we find $f_0 = 12\,933$, K = 0.0022, where f_0 is related to the sound velocity by $v_s = 2l \times f_0 = 5173$ m/s for our bar length of l = 0.2 m. For the Poisson ratio $\sigma = 2l \sqrt{K}/(\pi r)$, with r being the cylinder radius of the bar, from our fit we get σ = 0.338. The values agree well with Ref. 13 where σ = 0.33 and $v_s = 5000$ m/s for aluminum are reported. The root mean square error of the fit is 35 Hz, corresponding to the 30 Hz frequency resolution used in the Fourier analysis. Other peaks correspond to torsional and transverse modes.^{11,12}

The Fourier amplitudes A_k of $f(t) = \sum A_k e^{i\omega_k t}$ of the modes depend linearly (as shown for the 13 kHz, L=1 mode in Fig. 7) on the integrated charge in the beam pulse for a fixed beam position, and therefore also linearly on the energy deposited by the beam, which ranged in these runs from 0.06 to 0.8 J. The spread in the ratios of the amplitudes to the



FIG. 6. Typical Fourier spectrum of bar BC excited by the electron beam. Data were analyzed over 0.016 s, from 0.008 s onward after the beam passed. Identified vibrational modes are indicated.

beam charge shows that the Fourier amplitudes can be reproduced to within $\pm 10\%$.

The agreement of the model to within 10% with the measured data is shown in Fig. 8 which shows the measured Fourier amplitudes of bar BC at the piezo sensor and calculations according to Grassi Strini *et al.*^{5,14} as a function of the hit position along the cylinder's axis for the first four longitudinal modes. For each mode the average model value was scaled to the average measured value. The best fit was found with a shift of the hit positions along the bar, by an overall offset of $x_0 = -0.0075$ m, which corresponds to the crude way we aligned the bar with the beam line.

1. Lowest bar mode excitation amplitude

For the 13 kHz, L=1 mode we determine the absolute amplitude for a comparison with the model calculation of Refs. 5 and 14. First, we use the amplitude function $B_0(x)$ [see Eq. (9) of Refs. 5 and 14] by rewriting it in the form

$$B_0(x) = 2 \cdot \kappa_0 \cdot \Delta E / \pi$$

$$\times \cos(\pi x/l) \sin[\pi n/(2l)] / \pi n/(2l). \qquad (2)$$



FIG. 7. Correlation between the Fourier amplitude of the 12.6 kHz vibrational mode and the beam charge. Data points (*) and straight line fit (-).



FIG. 8. The measured, unnormalized Fourier amplitudes (+) and model calculations (-) as a function of the beam hit position along the cylinder axis for the four lowest longitudinal modes of bar BC.

with

$$\kappa_0 = \alpha \cdot l / (c_v \cdot M) = \alpha / (c_v \cdot \rho \mathcal{O}). \tag{3}$$

In this expressions x is the hit position along the cylinder axis, l the bar length, η the beam diameter, α the thermal linear expansion coefficient, ρ the density, c_v the specific heat, \mathcal{O} the cylindrical surface area of the bar, and ΔE the energy absorbed by the bar. From $B_0(x)$ we derive the functional form for the measured values of W_{sens} as

$$W_{\text{sens}}(x) = \frac{B_0(x)}{\Delta E} \beta D \frac{dE}{dQ},$$
(4)

where dE/dQ is the beam energy absorbed by the bar per unit of impinging beam charge, β the calibration factor discussed in Sec. III, and *D* the decay factor $e^{-t/\tau}$, since Eq. (2) applies at the excitation time and we have to correct the amplitude at the measuring time for the mode's decay, corresponding to its *Q* factor. Therefore,

$$W_{\text{sens}}(x) = \kappa_{\exp} 2/\pi \times \cos(\pi x/l) \sin[\pi \eta/(2l)]/\pi \eta/(2l), \quad (5)$$
with

with

$$\kappa_{\exp} = \beta D \kappa_0 \frac{dE}{dQ}.$$
 (6)

From fitting Eq. (5) to the measured values $W_{\text{sens}}(x)$ given in Table II with κ_{exp} as the free variable, we find our presently measured value for $\kappa_0^{\text{exp}} = \kappa_{\text{exp}}/(dE/dQ \cdot D \cdot \beta)$ which we compare to the model value in Eq. (3). Second, the decay time was measured by recording the sensor signals after a trigger delay of up to 1.6 s at a fixed beam hit position. An exponential fit $A(t) = A_0 \times e^{-t/\tau}$ to the mode amplitude gives $\tau = (0.36 \pm 0.01)$ s for the L = 1 mode. This corresponds to a Q value of ≈ 15000 , a value consistent with the room temperature measurement of aluminum in Ref. 15, and indicating a negligible influence of the suspension and piezoelectric

TABLE II. Excitation values W_{sens} , equaling the ratio of the Fourier amplitude measured and the beam pulse charge measured at each of the indicated hit positions on the bar for the 13 kHz, L=1 mode.

Hit position x (cm)	0	1	2	3	4	5	6	7	8	9	10
$W_{\text{sens}}(V/nC)$	0.185	0.216	0.167	0.180	0.225	0.152	0.152	0.157	0.112	0.089	0.057

ceramic sensor for this mode. From the measured value of τ and a mean delay time from the start of the beam pulse of 0.016 s, we calculate the decay factor to be D = 0.95. Third, as indicated in the second row of Table II we use the data for W_{sens} to fit the variable κ_{exp} in Eq. (5), where x is now the hit position given in row 1, l=0.2 m, and $\eta=0.002$ m. The value found in the fit is $\kappa_{exp} = (0.300 \pm 0.025)$ V/nC. Fourth, from a Monte Carlo simulation at the beam energy of 570 MeV used for these runs, we calculate the mean absorbed energy and the mean energy spread, which results from the fluctuating energy losses of the passing electrons and the energies of the secondaries escaping from the bar, as ΔE_{ρ} $=(19\pm2)$ MeV. The electron beam pulse thus deposits $dE/dQ = (0.019 \pm 0.002)$ J/nC in the bar. Using the measured calibration value at $f = 12986 \,\text{Hz}$ given in Eq. (1), β $=(2.2\pm0.3)$ V/nm, we arrive at

$$\kappa_0^{\exp} = (7.4 \pm 1.4) \text{ nm/J.}$$
 (7)

Finally, we calculate the model value of κ_0 from the material constants as being $\kappa_0 = 10$ nm/J, neglecting the much smaller error as arising from some uncertainty in the parameters. We conclude that $\kappa_0^{\exp}/\kappa_0 = (0.74 \pm 0.14)$, a result that is consistent with the validity of the model of Refs. 5 and 14.

The maximum excitation amplitude measured at beam position x=0 (see Fig. 8 for the 13 kHz, L=1 longitudinal mode) thus corresponds to (0.13 ± 0.02) nm.

2. Higher bar mode excitation amplitudes

Having determined the correspondence between the model calculation and the experiment's result for the first longitudinal vibrational mode amplitude, we return to some of the higher vibrational modes. To compare the modes we need to take the sensor position on the bar into account. We rewrite the displacement amplitude of Eq. (5) from Ref. 5 as a function of hit position x_h and sensor position x_s as

$$\Phi_{\text{odd}-L} = (2\kappa/L\pi)\sin(L\pi x_s/l)\cos(L\pi x_h/l),$$

$$\Phi_{\text{over}-L} = (2\kappa/L\pi)\cos(L\pi x_s/l)\sin(L\pi x_h/l). \tag{8}$$

where l is the bar length. We dropped the beam width correction term which would lead to a less than 0.1% correction even for L=4. We approximate the sensor response by the

local strain along bar BC's cylinder axis, that is, to the $d\Phi/dx_s$ of Eq. (8), arriving at a sensor response, S_L ,

$$S_{\text{odd}-L} = B_L \cos(L\pi x_h/l), B_{\text{odd}-L} = (2\epsilon\kappa/l)\cos(L\pi x_s/l),$$

$$S_{\text{even}-L} = B_L \sin(L\pi x_h/l), B_{\text{even}-L} = (2\epsilon\kappa/l)\sin(L\pi x_s/l),$$
(9)

where ϵ is a sensor response parameter. The x_s dependent term did not enter into the calculation of κ_0^{\exp} in Sec. IV A1, since the calibration was done at the same sensor position as the beam measurement. However, for a comparison between the modes, the dependence on the sensor position x_s has to be taken into account. Since the variables are strongly correlated, we first fitted for each mode the term B_L in the x_h dependent part of Eq. (9) to the measured value of W_{sens} for the mode, shifting the origin of x_h by 0.0075 m, as mentioned before. The results are given in the first row of Table III. Second, we corrected the amplitudes B_L^{meas} for the mode decay with a factor D, given in row 2, and corresponding to times $\tau_1 = 0.36 \text{ s}$, $\tau_2 = 0.10 \text{ s}$, $\tau_3 = 0.04 \text{ s}$, and $\tau_4 = 0.12 \text{ s}$, which leads to the values of B_L^{exp} in row 3. Finally, we multiplied with the factor $P_{odd-L} = 1/cos(L\pi x_s/l)$ and P_{even-L} = $1/\sin(L\pi x_s/l)$, where the bar length is l = 0.2 m. Since the sensor extends from 0.005 through 0.020 m from the center of the bar, we use the mean sensor position $x_s = 0.0125$ m. The resulting values of $\kappa' = 2\epsilon \kappa/l$, shown in the last row, should be independent of L. For L=2, 3, 4 they are rather closely scattered around a mean value of $\kappa' = 0.07$ which is, however, about half the L=1 value. This discrepancy might have originated from some resonances of the sensor itself, and we suspect the strong peak at 23 kHz, shown in Fig. 6, to be an indication of such resonances playing a role.

Since the amplitudes of the higher modes for bar BC do not comply with our expectations we turn, as a further check, to our uncalibrated measurements with bar BU. It had been equipped with a piezoelectric sensor at one end face where the longitudinal modes have maximum amplitude. The sensor had been mounted flatly with about half of its surface glued to the bar, and responding to the bar's surface acceleration, not its strain as at bar BC. We extract the κ_L values from our measurement analogously to those for bar BC, following again the model calculations of Grassi Strini *et al.*,⁵ using the L=1 mode as the reference. The results are given in Table IV.

TABLE III. Comparison of bar BC modes. The piezoelectric ceramic sensor responds to the bar's strain.

Description	Symbol	13 kHz, $L = 1$	25.6 kHz, $L=2$	38 kHz, L=3	50 kHz, $L = 4$
Amplitude	$B_L^{\rm meas}$	0.12 ± 0.01	$0.021 \!\pm\! 0.002$	0.033 ± 0.03	$0.052 \!\pm\! 0.005$
Decay correction	D	1.04 ± 0.001	1.17 ± 0.02	1.49 ± 0.06	1.14 ± 0.01
$B_L^{\text{meas}} \times D$	B_L^{exp}	0.12×0.01	0.025 ± 0.003	0.049 ± 0.005	$0.059 \!\pm\! 0.006$
Sensor position factor	P_L	1.02	2.61	1.20	1.41
$\epsilon \kappa 2/l = B_L^{\exp} \times P_L$ (a.u.)	κ'	0.12 ± 0.01	$0.065 \!\pm\! 0.007$	0.059 ± 0.006	0.083 ± 0.008

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TABLE IV. Comparison of bar BU modes. The piezoelectric ceramic sensor responds to the bar's acceleration. The value of κ_1^{meas} for the L=1, 13 kHz mode is used as the reference for the higher modes.

Description	Symbol	13 kHz, $L = 1$	25.6 kHz, $L = 2$	38 kHz, $L = 3$	50 kHz, $L = 4$
Relative amplitude	B ^{meas}	1	1.15	11.5	3.6
Relative decay correction	D	1	3±1	0.7 ± 0.2	1.3 ± 0.9
$(\omega_{L=1}/\omega_L)^2$	Ω	1	0.26	0.12	0.07
$B^{\text{meas}} \times D \times \Omega$	$\kappa_L^{\text{meas}}/\kappa_1^{\text{meas}}$	1	0.8 ± 0.3	0.9 ± 0.4	0.3 ± 0.3

After applying the decay correction factor D and the frequency normalization factor Ω , the results should be independent of L. The L=4 value is significantly low, which, again, might be due to some interfering resonance. The L=2 and L=3 values, however, do not significantly deviate from the L=1 value, thus confirming the model calculations for these higher modes too.

B. Results for the sphere

Our measurements on the sphere consisted of (a) hitting the sphere with the beam at one of two different heights in the vertically oriented plane through its suspension: at the equator (E) and at 0.022 m south of it (A); (b) rotating the sphere with its two fixed sensors up to 180° around the suspension axis at each beam height, and measuring it back and forth in steps of 30° several times to diminish the influence of temperature and beam fluctuations, and ending up on a 10° angular lattice. The Fourier amplitude spectrum of sensor 1, averaged over the angular positions, is shown in Fig. 9. The lowest spheroidal mode is most relevant for a *spherical* resonant mass gravitational wave detector, and we therefore focus on a few spheroidal modes. As expected, the lowest spheroidal L=2 mode is seen at 17.6 kHz, the lowest spheroidal L=1 mode at 24 kHz, and the lowest spheroidal L =0 mode at 37 kHz. Some other peaks are also indicated in Fig. 9, although not the toroidal modes, which we neglect completely. It should be noted that while the $L \neq 0$ amplitudes oscillate over the angles, the L=0 amplitude does not, leading to a relative enhancement of the latter in the angle-averaged Fig. 9.

The Fourier amplitudes, again, showed a linear dependence on the energy deposited. To determine the decay times at f = 17.6, 24 and 37 kHz (see Fig. 10) we took data with up to 4 s delay in the spectrum analyzer, and found $\tau \approx 1$, 0.4 and 0.1 s, respectively.

The angular distributions for the amplitude of the 37 kHz, L=0 mode at the two vertical beam positions, E and A, are shown in Fig. 11. The L=0 amplitude is independent of the angle, and since the amplitude is constant to within 20% we infer from the Fourier modulus' deviation from flatness a 20% variation of the beam intensity from one shot to another.

During our measurement with the sphere we were unable to use the beam pulse as a trigger, implying that the start time of data acquisition with respect to the beam pulse is unknown. In our further analysis we will therefore use only the Fourier modulus, and will not analyze the phases.

The absolute scale of the 37 kHz Fourier amplitude turned out to be ≈ 5 times larger than the model value for sensor 2 and ≈ 50 times larger for sensor 1. We assume this discrepancy to be based on some interference, possibly with



FIG. 9. Fourier amplitude spectrum of the sphere SU averaged over all measured angles for sensor 1 at beam height position E. The modes and frequencies calculated are indicated.



FIG. 10. Decay of sphere's vibrational modes as measured by delayed data taking of the spectrum analyzer. The curve shows the fit of $A(t)=A_0 \times e^{-t/\tau}$. Upper panel: 17.6 kHz, L=2, $\tau=1.1$ s. Middle panel: 24 kHz, L=1, $\tau=0.4$ s. Lower panel: 37 kHz, L=0, $\tau=0.1$ s.



FIG. 11. 37 kHz Fourier modulus angular distribution. The beam hits the sphere at E in the upper panel and at A in the lower panel.

a sensor resonance and a suspension bar mode, and we do not analyze the L=0 mode further. To unravel the angular distributions in general, we felt would put too much pressure on the results of our simple measurement for a couple of reasons. First, the Fourier amplitude A_L for any multipole order L in the sphere's case is actually a sum of M submodes. Although they would be degenerate for an ideal sphere, in practice some M modes might or might not turn out to be split beyond the frequency resolution of Δf = 30 Hz. Second, both sensors s_1 and s_2 should be taken as having unknown sensitivities, \mathbf{e}_{s_i} , in three orthogonal directions, with phase factors +1 or -1 for their orientation. Third, although each mode would start to be excited within the same subnanosecond time interval of the beam crossing, building up of each mode's resonance vibration may lead to a specific phase t_{M_1,b_k}^0 depending on the mode's spatial relation to the beam path.

1. The sphere's mode signatures

I

We now show that the calculated angular distributions have the signature of the *L* character of the measurement. Therefore, we write the Fourier modulus at different impinging beam positions b_k as a function of the angle ϕ as

$$A_{L,s_{j},b_{k}}(\phi) = \left| F_{L,s_{j},b_{k}} \epsilon_{s_{j}} \right| \\ \cdot \sum_{-M_{L}}^{+M_{L}} s_{L,M_{L},b_{k}} \mathbf{u}_{L,M_{L},s_{j},b_{k}}(\phi) e^{\omega_{L} t_{M_{L},b_{k}}^{0}} \right|,$$
(10)

where F_{L,s_j,b_k} is a frequency response function for each sensor that may depend also on the beam position. This normalization factor is expected to be of the order of 1, and is kept fixed at 1 for the L=2 distributions. It is used as a free



FIG. 12. Data points (+) and fit results (-) for the sphere's 24 kHz, L = 1 mode. Left column: Sensor 1; right colum: sensor 2. Upper row: Beam position E; lower row: beam position A. The *x* axes give the angle of sensor 1. Note that the *y* scales are different for the two sensors.

parameter for the L=1 distributions to compensate for the rather inaccurate knowledge of (a) the sensor positions on the sphere's surface, (b) the beam track location and (c) the electrons' and photons' shower development along the track, since the exact excitation strengths of the modes are quite sensitive to such data. As the first step in the fitting procedure we separately calculated the $s_{L,M_L,b_k} \mathbf{u}_{L,M_L,s_i,b_k}$, where s_{L,M_L,b_k} is the mode's strength from the beam excitation, given in detail in the Appendix. We inserted the calculated $s_{L,M_L,b_k} \mathbf{u}_{L,M_L,s_j,b_k}$ into a hierarchical fitting model to simultaneously fit¹⁶ the relevant parameters of Eq. (10) to the 17.6 kHz, L=2 Fourier modulus A_{L,s_i,b_k} for both sensors s_1 and s_2 at both the *E* and *A* beam positions. This fit led to a reduced $\chi^2 = 1.3$ at 59 degrees of freedom. Next, with fixed values for the sensor efficiencies $\epsilon_{s_{\pm}}$ thus established, we fitted the relevant parameters for the 24 kHz, L=1 Fourier peaks, including the L=1 sensor response factors F. At all stages the $|t_{M_{I},b_{i}}^{0}|$ of the phases were kept within the bounds of the period of mode L. With an uncertainty in the beam charge and in the Fourier peak amplitudes of $\approx 20\%$ each, the error amounts to $\approx 30\%$, and we took a minimum absolute error of 2×10^{-5} for sensor s_1 and 1×10^{-5} for sensor s_2 . In total we have 152 data points, while the total number of fitted parameters is 27, including a relative normalizing factor for the mean beam current at beam position A with respect to the mean current at beam position E. We found for the total fit a reduced $\chi^2 = 1.6$ at 125 degrees of freedom. The L=1 response factors remain within 1.1 and 0.2.

The results of the fits to the 17.6 and 24 kHz are given in Figs. 12 and 13 and Table V. Note the different vertical scales used for sensor 1 and sensor 2 in both pictures. Some of the parameters given in Table V are strongly correlated.

We conclude that the measured Fourier amplitude angular distributions are consistent with the model value for the L=2 and L=1 mode signatures.

2. The sphere's absolute displacement

Finally, to estimate the order of magnitude of the sphere's absolute displacement, we have to take an intermediate step by first normalizing bar BU to the calibrated results for bar BC and then use bar BU as a calibration for the sphere. With the sensors used on bar BU consisting of the



FIG. 13. Data points (+) and fit results (-) for the sphere's 17.6 kHz, L = 2 mode. Left column: Sensor 1; right colum: sensor 2. Upper row: Beam position E; lower row: beam position A. The *x* axes give the angle of sensor 1. Note that the *y* scales are different for the two sensors.

same sensor material and having been cut roughly to the same size, we assume them to be identical to the ones used on sphere SU. The amplifiers used are identical. The bar BU sensors, however, differ strongly from those of bar BC.

We arrive at an indirectly calibrated value for $\epsilon_{BU,2} = (3.4 \times 10^{-4} \pm 30\%) \text{ V/ms}^{-2}$ of sensor 2 on bar BU. The value of $\epsilon_{BU,1}$ for sensor 1 is about 10 times smaller. Then, for the sphere, the fitted value of ϵ_{s_j} given in Table V shows the largest value of sensor 2, $\epsilon_{\phi-SU} = (4 \times 10^{-4} \pm 10\%) \text{ V/ms}^{-2}$, leading to a ratio of $\approx (1.2 \pm 0.4)$ with ϵ_{BU} . Again, the values for sensor 1 are about 10 times smaller. The error of $\approx 33\%$ is the propagated statistical error only. The result seems reasonable. So the model calculation and our sphere measurement results are of the same order of magnitude on an absolute scale too.

From the maximum Fourier modulus, $V^{\text{max}} = 0.003 \pm 0.001 \text{ V}$, of the 17.6 kHz, L = 2 sphere mode measured in sensor 2 given in Fig. 13 and the absorbed energy of 3.1 J, we find the maximum sphere displacement to correspond to $(0.2 \pm 0.1) \text{ nm/J}$.

TABLE V. Results of a hierarchical fit to the data of the sphere at the 19 measuring angles of both sensors s_1 and s_2 of the Fourier modulus at 17.6 and 24 kHz at beam positions E and A.

Fit parameter	Fit result $\times 10^{-4} \text{ V/ms}^{-2}$	$\frac{\text{Error}}{\times 10^{-4} \text{ V/ms}^{-2}}$	
Sensor efficiency			
$\epsilon_{s_1} - r$	+0.10	0.02	
$\epsilon_{s_1} = \theta$	-0.5	0.2	
$\epsilon_{s_1} - \phi$	+0.4	0.1	
but less			
$\epsilon_{s_2} - r$	-0.5	0.2	
$\epsilon_{s_2} - \theta$	-0.5	0.2	
$\epsilon_{s_2} - \phi$	+4.0	0.3	
Intensity $\left(\frac{\text{beam }A}{\text{beam }E}\right)$	0.7	0.1	
Response factor			
$F_{L=1,s_1,\text{beam }A}$	0.3	0.1	
$F_{L=1,s_2,\text{beam }A}$	1.1	0.1	
$F_{L=1,s_1,\text{beam }E}$	0.2	0.1	
$F_{L=1,s_2,\text{beam }E}$	1.1	0.2	

V. DISCUSSION

Having confirmed the thermo-acoustic conversion model in the present experiment, we discuss some points about extrapolating these results to actual operation of a resonant gravitational wave detector. First, in our experiment many incident particles deposited their energy in the resonator, in contrast to a single muon hitting an actual detector. However, from this difference it seems unlikely that we will reach different conclusions, especially since in the process of depositing energy along its track, the muon will generate lots of secondary particles too. Also, we measured at room temperature while actual detectors would have to operate in the millikelvin range. An aluminum resonator, for instance, at such a temperature, would be superconducting, and it is as yet unclear how decoupling of the electron gas from the lattice would affect the process of acoustic excitation.

Therefore, we consider it of particular importance for the prospect of shielding a next generation resonant mass gravitational wave detector that an existing millikelvin detector like the Nautilus^{17,18} would succeed in measuring the impinging cosmic rays in correlation with the resonator mode. Such a result, as a test for the further applicability of the thermo-acoustic conversion model at operating temperature, would come the closest to the real situation envisaged for the new detectors.

Apart from such temperature effects, the applicability of the thermal acoustic conversion model^{4,5,14} is confirmed by the data and therefore cosmic rays should be anticipated to seriously disrupt, as calculated by the model, the possibility of detecting gravitational waves. It is beyond the scope of this article to go into detail.¹⁹ We want to point, however, to earlier calculations^{3,7,20} which, having used the model, clearly show, first, that a next generation spherical resonant mass gravitational wave detector of ultrahigh sensitivity will be significantly excited by cosmic rays. Second, the high impact rate of cosmic rays will prohibit gravitational wave detection at the Earth's surface with the sensitivity required.

Finally, shielding the instrument by an appreciable layer of rock as available in, for instance, the Gran Sasso laboratory, would suppress the cosmic ray background by a factor of $\geq 10^6$. Even then a vetoing system would be necessary and, with the radical reduction of the background rate thus established, it may indeed work effectively.

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APPENDIX: SPHERE EXCITATION MODEL CALCULATION

Our calculation of the (L,M)-mode excitation strengths is based on the source term of Eq. (5.10)/(11) of Ref. 14, $s = \Sigma/(\rho V) \times \int dz \nabla_{\perp} \cdot \mathbf{u}$, with $\Sigma = \gamma dE/dz$. Here, γ is the Grueneisen constant, ρV the sphere's mass, and dE/dz the absorbed energy per unit track length. The Fourier amplitudes, measuring the second time derivative of the mode amplitudes, are directly proportional to s, and the mode amplitudes follow from s/ω^2 , as in Eq. (5.18) of Ref. 14. However, the amount of energy absorbed per unit length in our case depends on the particle's position along the track. We therefore re-included the Σ term under the source term's integral by letting dE(z)/dz represent the electromagnetic cascade development of Ref. 21 as an approximation to the amount of energy absorbed per unit track length by the sphere at position z along the beam track,

$$s_{L,M_L} = \kappa \int_L \nabla_\perp \cdot \mathbf{u}_{L,M_L}(z) \frac{dE(z)}{dz} dz, \qquad (A1)$$

where z is measured from the beam's entrance point into the sphere. With E_{abs} being the total amount of energy absorbed by the sphere from the electron bunch, we write dE(z)/dz $=E_{abs} \times d[E(z)/E_{abs}]/dz$ and use the polynomial expansion $d[E(z)/E_{abs}]/dz = \sum_{i=0}^{3} c_i z^i$. Then $\int_{z_{min}}^{z_{max}} dz d(E(z)/E_{abs})/dz$ = 1. For the polynomial, measuring z in meters, we acquired the values $c_0 = 0.8332 \text{ m}^{-1}$, $c_1 = 226 \text{ m}^{-2}$, $c_2 = -1832 \text{ m}^{-3}$, and $c_3 = 4909 \text{ m}^{-4}$ from a fit to the form given in Ref. 21, with less than one percent deviation for our case of $0 \le z$ ≤ 0.15 m. The value for the energy absorbed by the sphere from a single electron, $E_{abs}^{e} = 123 \text{ MeV}$, we got from both our Monte Carlo simulation using GEANT²² and from EGS4.²³ At the 25 nC beam pulse charge measured this corresponds to a total E_{abs} = 3.1 J absorbed by the sphere. Then the value of $\kappa E_{abs} = \gamma E_{abs} / M = 1.00 \text{ m}^2/\text{s}^2$ for our case of $M = \rho V$ =4.95 kg and γ =1.6. Our sphere has a suspension hole which leads to a slight shift in the frequencies and the spatial distribution of the modes with respect to those of a sphere without a hole.²⁴ We approximated, however, our sphere's modes by the ideal hole-free sphere's eigenmode solutions $\mathbf{u}(z)$,²⁵ using the available computer code established in Ref. 26, and renormalizing to $\int \mathbf{u} \cdot \mathbf{u} dV = V$, as used in Ref. 14 from Eq. (5.6) onward. The source term s_{L,M_L} was calculated for each mode (L,M) by numerically integrating Eq. (A1). We checked that the surface term in the numerical procedure was negligible, as was assumed in the partial integration leading to the form of s used in Ref. 14. Each $\mathbf{u}(\phi)$ in Eq. (10) is the eigenmode solution, calculated for each sensor on the ϕ grid of the measured data, and each term s_{L,M_L,b_k} is the excitation factor s_{L,M_L} at the specific beam position b_k .

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