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Muon energy estimate through multiple scattering with the MACRO detector

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Abstract

Muon energy measurement represents an important issue for any experiment addressing neutrino-induced up-going muon studies. Since the neutrino oscillation probability depends on the neutrino energy, a measurement of the muon energy adds an important piece of information concerning the neutrino system. We show in this paper how the MACRO limited streamer tube system can be operated in drift mode by using the TDCs included in the QTPs, an electronics designed for magnetic monopole search. An improvement of the space resolution is obtained, through an analysis of the multiple scattering of muon tracks as they pass through our detector. This information can be used further to obtain an estimate of the energy of muons crossing the detector. Here we present the results of two dedicated tests, performed at CERN PS-T9 and SPS-X7 beam lines, to provide a full check of the electronics and to exploit the feasibility of such a multiple scattering analysis. We show that by using a neural network approach, we are able to reconstruct the muon energy for $E_\mu < 40$ GeV. The test beam data provide an absolute energy calibration, which allows us to apply this method to MACRO data.

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1. Introduction

The most recent studies of neutrino-induced up-going muons have been performed by two experiments: Superkamiokande [1], using a water Cherenkov detector, and MACRO [2], tagging neutrino events with a time-of-flight technique. Both experiments observed a flux deficit and a distortion of the up-going muon angular distribution with respect to the Monte Carlo expectation. The oscillation probability of neutrinos depends on the oscillation parameters (Δm^2, $\sin^2 2\theta$) and on the ratio L/E, where L is the distance between neutrino production and interaction point, while E is the neutrino energy. The energy of up-going neutrinos, interacting in the rock below the apparatus, is shared by the up-going muon and by the hadrons. Independent of the detector resolution, a precise measurement of the muon energy is prevented by the energy lost by the muon in the rock, while the hadrons are absorbed in the rock. Nevertheless, the residual muon energy can in principle be measured. In this paper we explore the possibility of performing such a measurement relying on muon multiple scattering (MS). The r.m.s. of the lateral displacement of the muon trajectory on a projected plane of material with depth X and radiation length X_0, can be written as

$$\sigma_{\text{proj}}^{\text{MS}} \simeq \frac{X}{\sqrt{3}} \frac{0.0136}{p\beta c} \sqrt{\frac{X}{X_0}}(1 + 0.038 \ln(X/X_0)) \qquad (1)$$

where p is in GeV/c and for MACRO, $X \simeq 25X_0/\cos\theta$, giving for vertical muons $\sigma_{\text{proj}}^{\text{MS}} \simeq 10\,\text{cm}/E(\text{GeV})$.

For a given amount of crossed material, the track deflection can be measured only when the particle displacement due to the MS is larger than the detector space resolution. The space point resolution of the tracking system of MACRO's $(3 \times 3)\,\text{cm}^2$ cross-section streamer tubes is of the order of 1 cm, and therefore provides a muon energy estimate through MS up to $\simeq 10$ GeV. Supposing $\Delta m^2 = \sim(10^{-3}\,\text{eV}^2)$ and $\sin^2 2\theta \simeq 1$, the neutrino-induced up-going muons, are not expected to experience neutrino oscillation at all energies. At the up-going muon median energy in MACRO, 11 GeV [3], the oscillation probability is still as high as 50% (Fig. 1), while it is just 10% for $E_\mu = 40$ GeV: an improvement of the space resolution offers the possibility of evaluating muon energy over an energy range sufficiently wide, to be sensitive to variations of the oscillation effect as a function of the muon (neutrino) energy.

In order to achieve this goal, we retrieve drift time information from the limited streamer tubes by using the TDC's implemented in the MACRO QTP electronic system [4].

In this paper we describe the use of this electronics to evaluate the MS effect along a muon track, showing the results obtained with two dedicated tests, performed at CERN PS-T9 and SPS-X7 beam lines, in October 2000 and August 2001, respectively. The application of the method to MACRO data is then presented.

2. The MACRO limited streamer tubes in drift mode

The MACRO streamer tube system [5] consists of about 5600 chambers; each chamber is made of 8 streamer tubes with cross-section $(3 \times 3)\,\text{cm}^2$ and

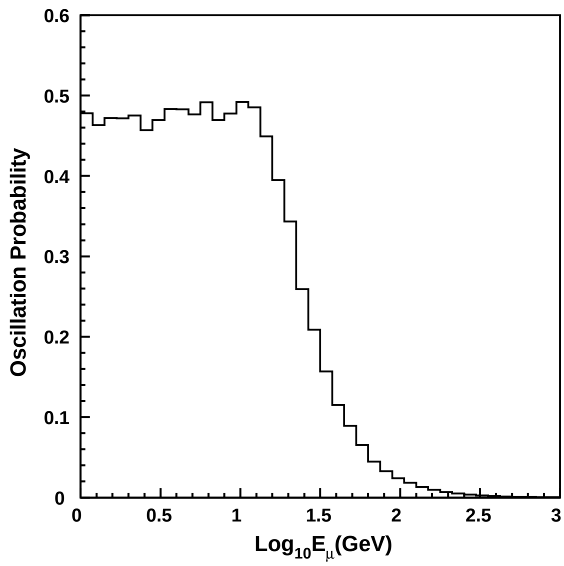

Fig. 1. Monte Carlo simulation: oscillation probability as a function of the energy of the muon entering in MACRO for $\Delta m^2 = 2.5 \times 10^{-3}\ \text{eV}^2$, $\sin^2 2\theta = 1$.

1200 cm length, for a total of about 50,000 wires. These tubes were built in "coverless" mode, i.e. the electric field of the inner four walls is not exactly the same. Despite this feature as well as the large cell dimension, the intrinsic space resolution of these chambers can be quite good, as demonstrated in Ref. [6], where using a MACRO streamer tube in drift mode, a resolution of $\sigma \simeq 250$ μm was obtained using standard LeCroy 2228A TDC (0.25 ns/bin). Such a resolution has to be considered as the ultimate resolution achievable with this device.

Although the MACRO streamer tube electronics does not contain a high resolution TDC system, information on streamer timing can be extracted using the QTP system [4]. This electronics, designed for our magnetic monopole search [7], consists of an ADC/TDC system with a 640 μs memory, during which the charge, the arrival time and the width of the streamer pulse of the particle crossing the cell are recorded. A slow particle in MACRO ($\beta \simeq 10^{-4}$) may take more than 500 μs to cross the detector. The QTP–TDC system allows us to distinguish randomly distributed background hits in this time window from a genuine slow particle, which, during the crossing time of the detector, are aligned in the space–time plane. The space–time plane is constructed by pairing the z-positions of the traversing particles with their time for crossing these planes. If we make a two-dimensional plot of these points, real tracks in space for particles moving with a constant velocity, will also appear as straight lines in the space–time plane.

For the magnetic monopole reconstruction optimization, a distributed clock of 6.7 MHz was chosen, resulting in an equivalent TDC bin width of $\Delta T \simeq 150$ ns. This clock frequency is quite coarse for drift time measurements in a single cell, given that the maximum drift time for MACRO streamer tubes, operated with a He (73%)/n-pentane (27%) mixture is $\simeq 600$ ns, but was chosen to match the transit time of a slow moving monopole through the detector.

The ultimate resolution that can be therefore obtained with such a system is $\sigma \simeq v_{\mathrm{drift}} \times \Delta T/\sqrt{12} \simeq 1.9$ mm, which is about an order of magnitude greater than the intrinsic precision of the streamer tube, operated in drift mode. Nevertheless, if such improved resolution could be achieved, it would be sufficient to estimate up-going muon energies up to 30–40 GeV.

In order to reduce the number of electronic channels, a single MACRO QTP channel, serves the OR of 4 chambers, for a total of 32 wires. Selecting only planes with a single fired tube, the association with the fired QTP channel is uniquely determined.

Given that our electronics was not designed for drift time measurements, the relative linearity was tested only for the much larger time scale of 500 μs rather than 600 ns. To avoid any systematic effects and to fully understand the capability of the QTP system in this context, we decided to test the electronics in a beam test at CERN PS-T9.

3. Streamer tube system performance in drift mode

To study the QTP–TDCs linearity, the drift velocity in He/n-pentane mixture and to develop the software used for muon tracking, we performed a test beam run in CERN PS-T9 beamline in October 2000.

For these tests, we reproduced a slice of the MACRO detector using 14 coverless streamer tube chambers, $(25 \times 3 \times 200)\,\mathrm{cm}^3$, filled with the standard MACRO gas mixture. The rock absorbers reproduced as much as possible those of MACRO. We built 7 iron boxes, $(40 \times 40 \times 32)\,\mathrm{cm}^3$, filled with rock excavated from the Gran Sasso tunnel ($\rho = 2.0\ \mathrm{g/cm}^3$). As in MACRO, each streamer tube chamber was equipped with a read-out card and the analog output of a chamber was sent to a QTP channel. The digital output, OR of each chamber signals (DIGOR) [5] was sent to a LeCroy 2228A TDC. Such double measurement of the drift time allowed us to make a comparison between QTP–TDCs and LeCroy TDCs on an event by event basis. The test beam layout is shown in Fig. 2. The trigger was provided by a fast coincidence of the scintillators S1, S2, S3. The last scintillator, following a 60 cm iron slab, suppresses the π, K contamination in the beam at high energies. The data acquisition was performed using LabView, running on a MacIntosh Quadra

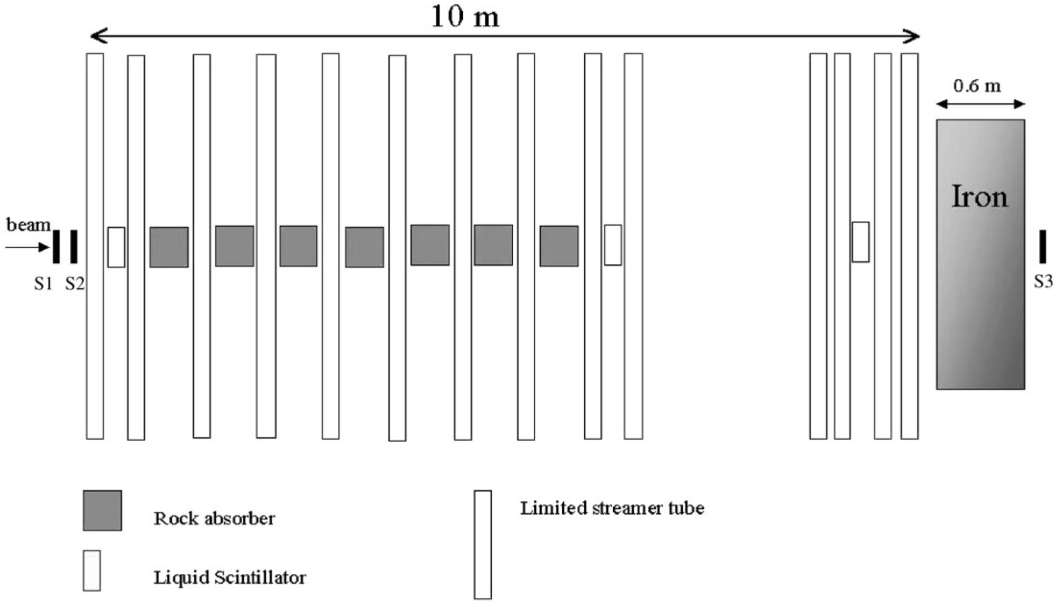

Fig. 2. Test beam layout at PS-T9: the trigger is provided by the fast coincidence of the scintillators S1, S2, S3.

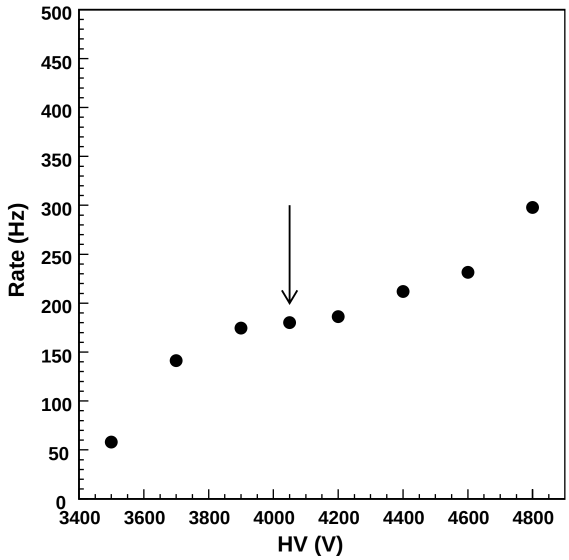

Fig. 3. Plateau of the streamer tubes: the arrow indicates the working point.

950. Fig. 3 shows the plateau curve of the streamer tubes used in the test beam. We operated these chambers at HV = 4050 V, where a full efficiency was reached. We collected 60 runs, with the beam stoppers closed, for a total of about 10^5 muons, with energy ranging from 2 to 12 GeV. Several runs were also taken with the rock absorbers removed, to study the QTP electronics and to allow for space resolution evaluation, without contributions of MS in the absorbers at these low muon energies.

First, we evaluated the QTP–TDCs linearity, by comparing its data with that recorded by the LeCroy TDCs. For each event, the time was measured twice with both the QTP–TDCs and the LeCroy TDCs. Fig. 4 shows the relationship between these two measurements for values of the QTP–TDC system (75, 225, 375, 525 ns), where we took the average of the LeCroy TDCs time distribution. The errors represent the width of the QTP–TDCs and the r.m.s. of the corresponding LeCroy-TDC time distributions.

Although the maximum drift time in our streamer tubes is about 600 ns, due to the non-homogeneity of the electric field in the streamer tube cell [6], the region between 500 ns $\leqslant T \leqslant$ 600 ns is not uniformly populated. We evaluated that this effect accounts for the $\simeq$10% observed shift-up of the QTP–TDCs, with respect to the expected average in that bin.

For $T \leqslant 450$ ns there is full consistency with LeCroy-TDC measurement. Considering the coarseness of QTP–TDC we conclude that the comparison if fully satisfactory. Therefore, we used the central value of each QTP–TDC bin (150 ns wide).

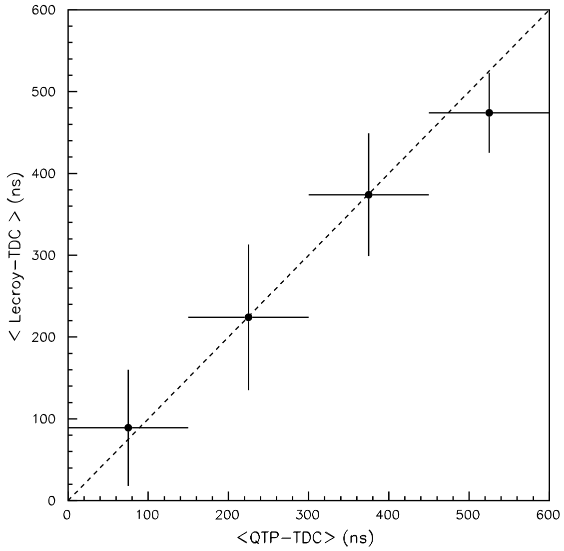

Fig. 4. Profile plot of LeCroy 2228A TDCs as a function of QTP–TDCs. The line is drawn to guide the eye.

We then studied the drift velocity in He/n-pentane mixture. Since in the test beam configuration N muons hit the detector at normal incidence:

$$\frac{dN}{dt} = \frac{dN}{dx}\frac{dx}{dt} = \frac{dN}{dx} v_{\mathrm{drift}} = K v_{\mathrm{drift}}. \tag{2}$$

The evaluation of v_{drift} can be therefore obtained fitting the LeCroy TDC spectrum distribution.

Fig. 5 shows the experimental results obtained, where we have superimposed the results of a GARFIELD [8] simulation for comparison. Such code performs a detailed simulation of electron drift and signal generation in gaseous wire detectors. The drift velocity as a function of electric field has been computed assuming the standard MACRO gas mixture by using the GARFIELD–MagBoltz [9] interface. The experimental data are in agreement with the simulation.

Once the TDC linearity has been checked and the v_{drift} has been measured, the test beam data can be used to measure the space resolution. Fig. 6 shows the residuals distribution for streamer tubes in drift mode using the LeCroy TDCs and the QTP–TDC system. Using the LeCroy TDC data,

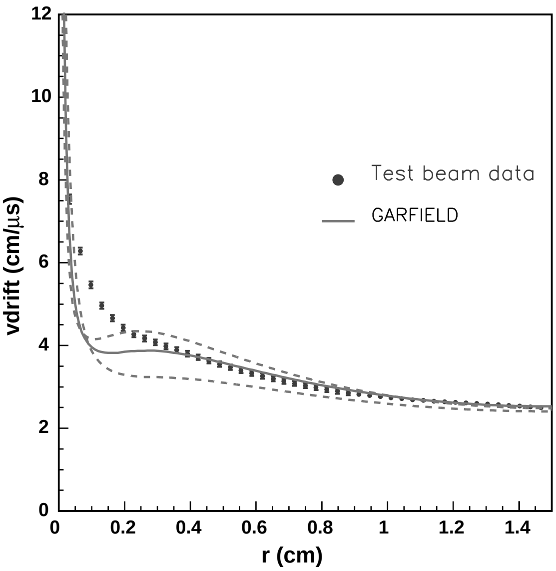

Fig. 5. Drift velocity as a function of the distance from the wire, measured at test beam and compared with the GARFIELD expectation. The dotted lines represent the effect of a 15% gas mixture variation.

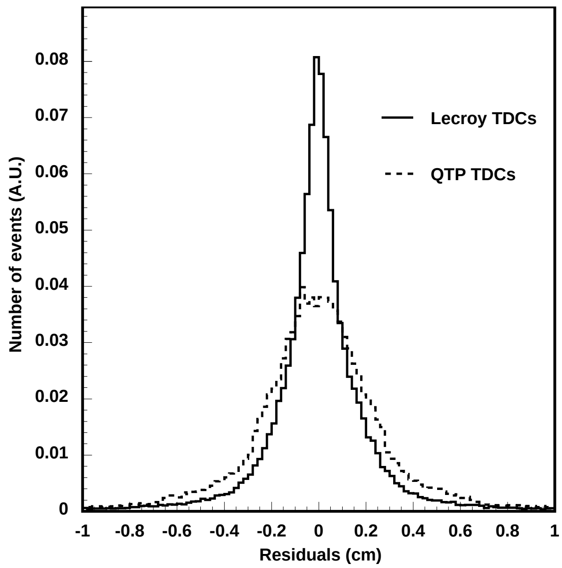

Fig. 6. Test beam results: residuals distribution obtained using LeCroy 2228A TDCs (continuous line) and QTP–TDCs (dotted line).

we find a resolution of 500 μm, while for the QTP–TDC data we obtained a resolution of $\sigma \simeq 2$ mm. This resolution limit is very close to that expected based on QTP–TDCs time resolution $\sigma = v_{\rm drift} \times 150\ {\rm ns}/\sqrt{12} \simeq 4\ {\rm cm}/\mu{\rm s} \times 150\ {\rm ns}/\sqrt{12} \simeq 1.9$ mm.

4. Study of the MACRO space resolution

To estimate the performance of the streamer tubes operated in drift mode in MACRO, we analysed a down-going muon sample, whose average energy is $\langle E_\mu \rangle \simeq 320$ GeV [10].

The analysis was performed by using the following steps:

(1) We considered the muon track reconstructed with the standard MACRO tracking (i.e. no QTP information is used at this stage);
(2) We selected those hits containing only a single fired tube;
(3) For each hit we looked at the corresponding QTP–TDC value in a time window of 2 μs. Given the background rate in the MACRO streamer tubes, $\simeq 40\ {\rm Hz/m^2}$, this corresponds to $\simeq 480$ Hz on 4 chambers (1 QTP channel), giving a probability $\simeq 10^{-3}$ for a spurious hit to mimic a genuine QTP–TDC count;
(4) After converting the TDC values to drift radii, by using the drift velocity measured in the test beam, a global fit of the track is performed.

The left–right ambiguity in drift tubes is usually resolved by choosing the track with the best chi squared, selected between the tracks obtained with a permutation of the left–right position of each hit. When operating in presence of MS, the choice of the best track to be selected is not uniquely determined. By using a Monte Carlo simulation we verified that the selection of the track with the smallest chi squared (i.e. with the smallest amount of MS) gives the best muon energy resolution.

As a first step, we used this procedure to perform an alignment of the detector database. The standard MACRO database was computed using the streamer tube data in digital mode, hence to take advantage of the improved space resolution achieved by this method, we first had to upgrade the precision of the detector database. To accomplish this we used 15×10^6 down-going muon tracks. Since the MACRO streamer tubes, 1200 cm long, are made of PVC, a flexible material, part of the misalignment may come from the deviation from a straight line along the main axis of each streamer tube (sagitta effect). We therefore divided the streamer tube length in six slices and computed the residuals in each slice separately. We generated a matrix of (14,2304,6) elements, where the first index runs over the number of horizontal planes, the second over the wire number and the last over the portion of the wire along its main axis. We adopted an iterative procedure, by adding at each step, for each element of the matrix, the mean value of the Gaussian of the residuals belonging to each portion of wire. As a result of this procedure, Fig. 7 shows the distribution of the track residuals for the MACRO streamer tube system in drift mode (black circles) and the MACRO simulation, GEANT based, (continuous line). The residuals of the down-going muons have a $\sigma = 3$ mm, in good agreement with the MACRO simulation. The

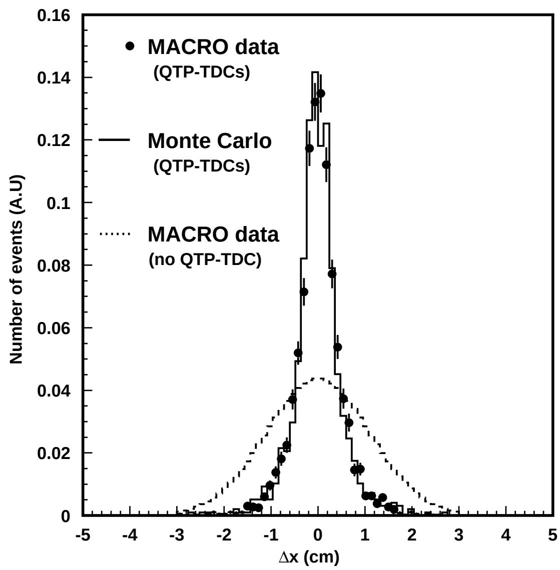

Fig. 7. MACRO data: residuals distribution obtained using the MACRO QTP–TDCs ($\sigma = 3$ mm) compared with the Monte Carlo, GEANT-based, expectation. The dotted line represents the residuals obtained using the MACRO streamer tube system in digital mode ($\sigma = 1$ cm).

continuous line shows the residuals distribution for the streamer tube system in digital mode ($\sigma =$ 1 cm), where we see that an improvement of the resolution by a factor $\simeq 3.5$ has been obtained.

For the MACRO data however, we expect the resolution to be worse than that measured in the PS-T9 test beam ($\sigma = 2$ mm) due to two effects. From our simulation, the most important contribution accounting for this difference comes from δ-rays and radiated photons produced in the rock absorbers. Both of these effects spoil the space resolution by producing streamers closer to the wire than those coming from the muon, resulting in smaller drift radii. Moreover, the MACRO down-going muons, despite an average energy of $\langle E_\mu \rangle \simeq 320$ GeV still suffer MS, mainly coming from the low energy tail of this distribution.

These hypotheses were tested during a second test beam, performed at SPS-X7 in August 2001, where high-energy muons with $15\ \mathrm{GeV} \leqslant E \leqslant 100\ \mathrm{GeV}$ were available, with the same setup used at PS-T9 (Fig. 8). The sigma of the residuals obtained with $E_\mu = 100$ GeV and rock absorbers inserted, was measured to be $\sigma =$ 3 mm, in good agreement with that obtained using the MACRO down-going muon data.

5. Muon energy estimate

A muon energy estimate can be performed in MACRO by measuring the amount of muon MS in the rock absorbers. The tests performed at CERN PS/SPS beamlines, allowed us to demonstrate this as well as offer the possibility of calibrating the MACRO system.

For each muon event we computed the following variables, sensitive to muon multiple scattering. The first three variables are just outputs from the track fitting procedure:

(1) the highest residual of the 14 measurements;
(2) the average of the residuals; and
(3) the standard deviation of the residuals.

For each track, we then considered the hit with the highest and lowest z-coordinate in the lower part of the detector (i.e. excluding the Attico hits), where z is the hit coordinate along the vertical axis. Then we selected a median hit, having the

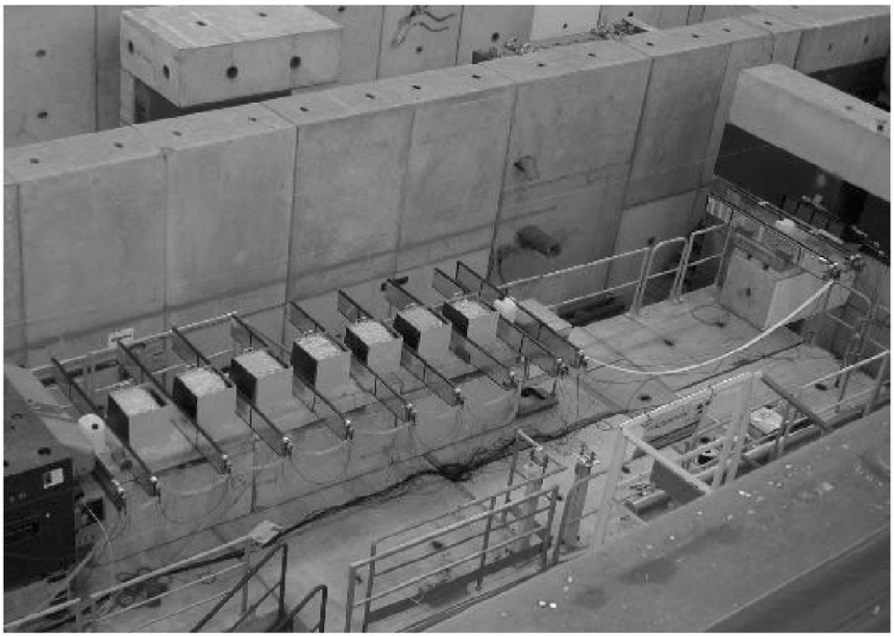

Fig. 8. Photo of the test beam performed at CERN SPS-X7.

maximum distance in height from the other two hits. From this we constructed the next two variables:

(4) the difference of the residuals of the highest hit and of the median hit; and
(5) the difference of the residuals of the lowest hit and of the median hit.

Lastly, we defined a "progressive fit" as the absolute value of the residual $d_i (i = 1, 14)$ as a function of the height of the streamer tube plane. For a high-energy muon, the average residual is roughly constant in the different planes, since the muon energy is almost constant while crossing the experimental setup. For instance a 20 GeV muon loses less than 5% of his energy after crossing the detector. In contrast, a low-energy muon loses a high fraction of its energy, by ionization, crossing the rock absorbers. As a result, the average residuals are higher for the last crossed planes. A linear fit of the absolute value of the residuals as a function of the streamer tube number, gives a small slope for high-energy muons, while the slope is much larger for low-energy muons. Guided by this analysis we introduce the following variables:

(6) the slope of the "progressive-fit"; and
(7) the intercept of the "progressive-fit".

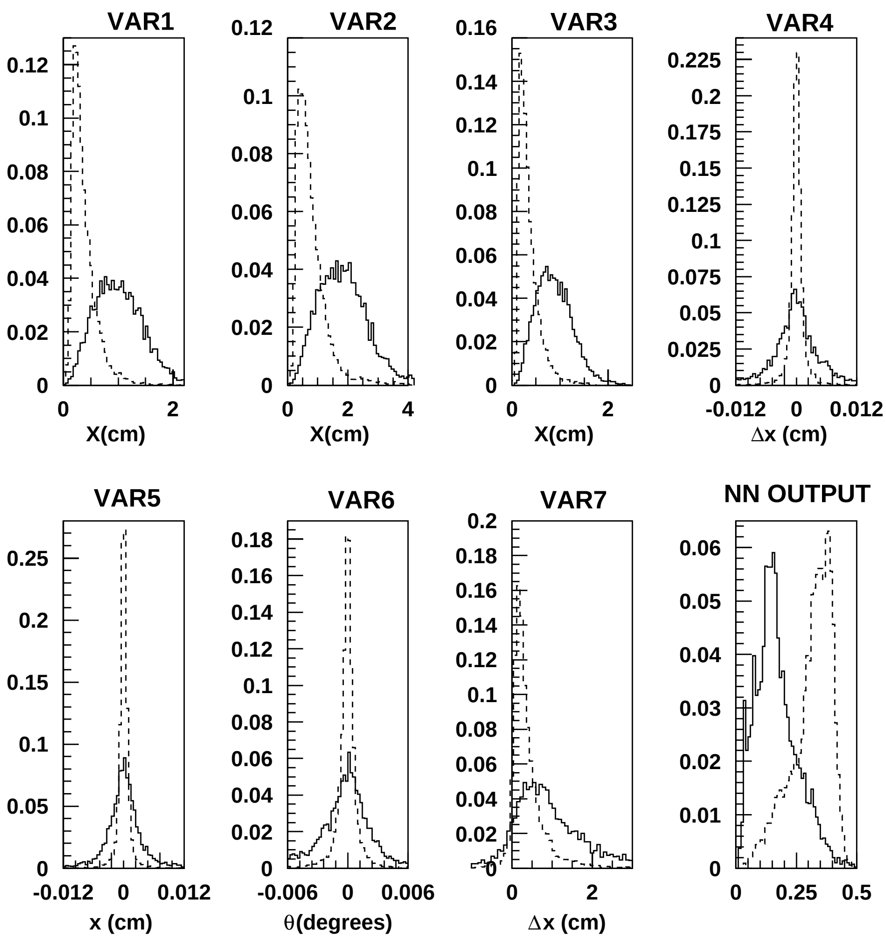

Fig. 9. Monte Carlo simulation: Distribution of the 7 input variables and of the neural network output(continuous line $E_\mu = 2$ GeV, dotted line $E_\mu = 100$ GeV).

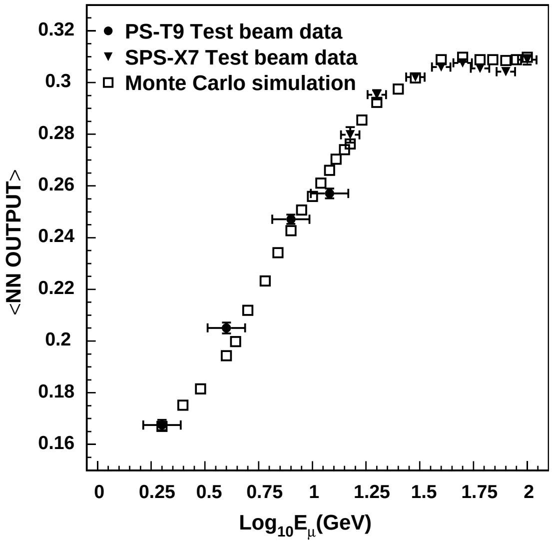

Fig. 10. Average neural network output as a function of the muon energy: empty squares (Monte Carlo), full circles (PS test beam data) and full triangles (SPS test beam data).

We followed a neural network approach (NN) in this analysis, choosing JETNET 3.0 [11], a standard package with a multilayer perceptron architecture and with back-propagation updating. The NN was configured with 7 input variables quoted above and 1 hidden layer, selecting the Manhattan upgrading function. Fig. 9 shows the distribution of the variables quoted above and of the NN output for muons with energy $E_\mu = 100$ GeV (continuous line) and for muons with energy $E_\mu = 2$ GeV (dotted line). Fig. 10 shows that the average NN output increases as a function of the muon energy up to $E_\mu \simeq 40$ GeV, saturating at higher energies.

The data collected during the PS test beam, provide an absolute energy calibration of the method, up to muon energy of 12 GeV. In order to check the NN output in the whole energy range of Fig. 10, we used the data collected at the CERN SPS-X7 beamline.

In Fig. 10 the test beam data and the Monte Carlo prediction are compared: empty squares represent the Monte Carlo expectation, black

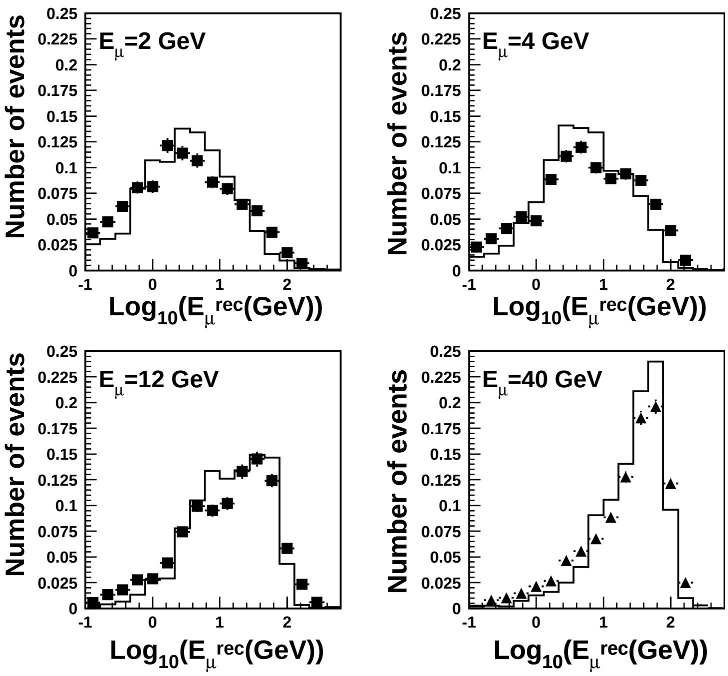

Fig. 11. Reconstructed energy distribution for 2,4,12,40 GeV muons. Monte Carlo: continuous line, test beam: full squares (PS-T9 data) and full triangles (SPS-X7 data).

Table 1
Reconstructed muon energy

E_μ(GeV)	2	3	5	10	40
Reconstructed energy (GeV)	$(2^{+6}_{-1.5})$	$(3^{+12}_{-2.5})$	(5^{+18}_{-4})	(10^{+30}_{-8})	(40^{+60}_{-21})

circles show the PS-T9 test beam points, while full triangles are the SPS-X7 test beam data. The NN output obtained with the test beam data is properly reproduced by the Monte Carlo simulation. The muon energy can be reconstructed by inverting the curve shown in Fig. 10. Fig. 11 and Table 1 show the reconstructed energy for $E_\mu = 2, 4, 12, 40$ GeV: data collected at PS-T9 test beam (full squares) and at SPS-X7 test beam (full triangles) are compared with the Monte Carlo expectation (continuous line), showing a reasonable agreement.

6. Conclusions

The use of the QTP–TDCs, offers the possibility of using the MACRO limited streamer tube system in drift mode. The test beam run performed at CERN PS-T9 confirmed such possibility. The QTP system allows us to improve the streamer tube system space resolution by a factor of $\simeq 3.5$, from $\sigma \simeq 1$ cm to 3 mm. These improvements were realized by using a neural network approach in order to obtain an energy estimate of muons crossing the detector. The average neural network output increases as a function of the muon energy up to $\simeq 40$ GeV. The comparison between Monte Carlo expectation and the test beam data shows a good agreement. This method offers the possibility to estimate the muon energy for neutrino-induced up-going muons in MACRO and thus to investigate the energy dependence of the neutrino oscillation signal.

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