Upper limit for a gravitational-wave stochastic background with the EXPLORER and NAUTILUS resonant detectors

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Abstract

We discuss the sensitivity of resonant-mass gravitational-wave detectors to a cosmic stochastic background of gravitational waves. We report the experimental upper limits given by the gravitational wave detectors EXPLORER and NAUTILUS.

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Among the possible gravitational waves (gw) signals, a cosmic stochastic background is one of the most interesting, as it might give information on the very early stages of the Universe and its formation. Several sources of stochastic background have been considered in the past years [1]. We recall the effect of the superposition of many continuous waves generated by the pulsars, the overlapping bursts due to gravitational collapses and to coalescence of binary systems.

Nucleosynthesis considerations put an upper limit on the ratio \( \Omega \) of the gw energy density to the critical density needed for a closed universe [1]. The upper limit is \( \Omega \leq 10^{-5} \). As well known the critical density is given by

\[
\rho_c = \frac{3H^2}{8\pi G} = 1.7 \times 10^{-5} \left( \frac{H}{100 \text{ km s}^{-1} \text{Mpc}^{-1}} \right)^2 \text{erg cm}^{-3}
\]

where \( H \) is the Hubble constant.

Recently a source based on string theory has been more deeply investigated [2,3]. The interesting feature of this theory, from the actual observer point of view, is that it might predict relic gw’s whose energy density increases, in a certain range, with the frequency \( f \) to the third power (remaining below the nucleosynthesis limit). In fact the previous models tend to predict gw’s in the frequency range below 1 Hz, lower than the operating frequency of the present detectors already in operation (resonant bars) or enter-
ing in operation in the next four to five years (long-
arm interferometers [4]). Only the newly proposed
space experiment LISA could explore, with good sen-
sitivity, a frequency range below 1 Hz, but such an
experiment, if approved, will fly after the year 2016.

The predictions of the new string cosmology models
depend on a number of parameters, such as the max-
imum frequency and the precise dependence of \( \Omega \) on
the frequency when it gets near the maximum value.
A measurement, even an upper limit, would help very
much in delineating the exact model. At this stage is
therefore very important to turn to the experiment.

The Rome group has operated the cryogenic reso-
nant antenna EXPLORER [5] since 1990 and the ul-
tracryogenic resonant antenna NAUTILUS [6] since
1995. Therefore it is worthwhile to study the recorded
data to look for useful experimental information on
relic gw.

A resonant gw detector consists of a carefully sus-
pended resonant mass, usually a cylindrical bar, whose
vibrational normal modes having the appropriate sym-
metry are excited by gw [1]. The mechanical oscilla-
tion of the resonant mass is transformed into an elec-
trical signal by a motion transducer and then amplified
by an electrical amplifier. Unavoidably, Brownian mo-
tion noise associated with dissipation in the resonant
mass and the transducer, and electronic noise from the
amplifier, limit the sensitivity of the detector.

The study of the problem of a stochastic background
detection has shown that the presently available reso-
nant detectors are suited for this type of measurement.
As a matter of fact it turns out, as shown below, that
the sensitivity of the resonant antennas to a stochastic
background (for the present detectors operating with
dcSQUID electronic amplifiers) depends essentially
on the quantity

\[
\frac{T}{MQ}
\]

(2)

where \( T \), \( M \) and \( Q \) are the detector thermodynamic
temperature, mass and quality factor. The bandwidth
of the apparatus enters to a minor extent, as it will be
shown later. Thus, what is the drawback of a resonant
detector, namely the small bandwidth, does not jeop-
dardize the measurement of the stochastic background.

The equation of motion for the bar end displacement \( \xi(t) \) is

\[
\ddot{\xi} + \frac{\Omega_0}{Q} \dot{\xi} + \omega_0^2 = \frac{F}{m}
\]

(3)

where \( F \) is the applied force, \( m \) the oscillator reduced
mass for a cylinder \( m = M/2 \), \( \omega_0 = 2\pi f_0 \) is the
angular resonance frequency and \( Q \) is the merit factor.

Expressing the effect of the Brownian and electronic
oise in term of displacement of the bar ends, we obtain [6] the noise power spectrum:

\[
S_\xi(f) = \frac{S_F}{(2\pi)^4 m^2} \times \left[ 1 + \Gamma \left( \frac{Q}{f_0} \right) \right] \left[ \frac{m^2}{Hz} \right]
\]

(4)

with

\[
S_F = \frac{2\Omega_0}{Q} m kT_e
\]

(5)

where \( T_e \) is the equivalent temperature that includes
the effect of the backaction from the electronic ampli-
der, and \( \Gamma \) is the spectral ratio between electronic and
Brownian noise [7] (usually \( \Gamma \ll 1 \)):

\[
\Gamma \approx \frac{T_n}{\beta Q T_e}
\]

(6)

where \( T_n \) is the amplifier noise temperature and \( \beta \) the cou-
pling parameter of the transducer to the bar \( (\beta \approx
10^{-2}-10^{-3}) \). The power spectrums are expressed in
two-sided form.

When a gravitational wave with amplitude \( h \) and
optimum polarization impinges perpendicularly to the
bar axis, the bar displacement corresponds [7] to the
action of a force

\[
f = \frac{2}{\pi^2} m L \dot{h}
\]

(7)

For a gw excitation with power spectrum \( S_h(f) \), the
spectrum of the corresponding bar end displacement is

\[
S_\xi(f) = \frac{4L^2 f^4 S_h}{\pi^4} \left( \frac{1}{f^2 - f_0^2} \right) \left[ \frac{m^2}{Hz} \right]
\]

(8)

We notice that the power spectrum of the bar dis-
placement for a constant spectrum of gw is similar to
that due to the action of the Brownian force. Therefore,
if only the Brownian noise were present, we would have an infinite bandwidth, in terms of signal to noise ratio (SNR):

By taking the ratio of the noise spectrum (4) and the signal spectrum (8) we obtain the signal to noise ratio (SNR)

$$\text{SNR}(f) = \frac{S_{\text{SNR}}(f)}{S_{\text{F}}(f)} = \frac{64L^2 f^4 l^2 m^2 S_h(f)}{S_F} \times \frac{1}{1 + \Gamma \left[ Q^2 \left( 1 - \left( \frac{f}{f_0} \right)^2 \right)^2 + \left( \frac{f}{f_0} \right)^2 \right]}$$ (9)

By equating to unity the above ratio we obtain the gw spectrum detectable with SNR = 1, that is the detector noise spectrum referred to the input:

$$S_h(f) = \frac{\pi}{8} \frac{k T_e}{M Q L^2} f_0^3 \times \left\{ 1 + \Gamma \left[ Q^2 \left( 1 - \left( \frac{f}{f_0} \right)^2 \right)^2 + \left( \frac{f}{f_0} \right)^2 \right] \right\}$$ (10)

At the resonance $f_0$ we have (being $\Gamma \ll 1$)

$$S_h(f_0) = \frac{\pi}{8} \frac{k T_e}{M Q L^2} \frac{1}{f_0^3}$$ (11)

We remark that the equivalent temperature $T_e$ reduces $T$ if the backaction from the electromechanical transducer can be neglected, as in the case of a dc-SQUID.

The above quantity $S_h(f)$ must be related to the quantity $\Omega(f)$ predicted by the theory, where

$$\Omega(f) = \frac{d \Omega}{d \ln(f)}$$ (12)

It turns out that [3]:

$$S_h(f) = \frac{3H^2}{4\pi^2} \frac{\Omega(f)}{f^3} = 8.0 \times 10^{-37} \frac{\Omega(f)}{\text{Hz}} \left( \frac{H}{100 \text{ km s}^{-1}\text{Mpc}^{-1}} \right)^2$$

$$= (@920 \text{ Hz}) \times \left( \frac{3.2 \times 10^{-23}}{\sqrt{\text{Hz}}} \frac{H}{100 \text{ km s}^{-1}\text{Mpc}^{-1}} \right) \sqrt{\Omega(f)}$$ (13)

The target sensitivity of ultracryogenic antennas like NAUTILUS [8] or AURIGA [9] with $f = 920$ Hz, $M = 3200$ kg, $T = 0.1$ K and $Q = 5 \times 10^6$ is $S_h(f) = (8.6 \times 10^{-23}/\sqrt{\text{Hz}})^2$. This is not sufficient to reach the limit imposed by the nucleosynthesis bound of $\Omega \leq 10^{-5}$, but it gives an upper limit.

We give now the results of measurements made with the antennas EXPLORER in the years 1991 (Fig. 1) and 1994 (Fig. 2) and NAUTILUS operating at $T = 1.3$ K in the year 1995 (Fig. 3). NAUTILUS is capable to operate below 0.1 K, but in 1995 we operated it at 1.3 K because we had some excess noise. We recall that both EXPLORER and NAUTILUS employ a resonant electromechanical transducer, thus showing two resonances which may have different sensitivity according to the noise on each one and to the tuning of the transducer to the bar.

With one detector only, the sensitivity does not depend on the length of the measuring time. Increasing the time of measurement would just reduce the error in the spectral determination, leaving practically unchanged the level of the spectrum. We notice that in 1994 we obtained at both the resonances (907 Hz and 923.32 Hz) a measurement $6 \times 10^{-22}/\sqrt{\text{Hz}}$. The upper limit from these measurements turns out to be still very high, about $\Omega = 300$.

At the frequency of 923.8 Hz we obtain from NAUTILUS $7 \times 10^{-22}/\sqrt{\text{Hz}}$.

Better sensitivity can be obtained by cross correlating the output of two antennas, because the local noises are uncorrelated and the sensitivity improves with a longer measuring time. It can be shown [10]...
that, in such a case, if the two identical antennas with respective spectral outputs $S_{1h}$ and $S_{2h}$ are close to each other, within a distance much smaller than the gw wavelength [11], the sensitivity is

$$\delta S_{gw}(\omega) = \frac{\sqrt{S_{1h}S_{2h}}}{\sqrt{t_m\Delta f}}$$

(14)

where $t_m$ is the measuring time and $\Delta f$ is the antenna bandwidth.

We see in the above formula the effect of the bandwidth, which enters as the 1/4 power for the usual sensitivity expressed in units $1/\sqrt{\text{Hz}}$. With the present resonant detectors at $T = 0.1$ K having $\Delta f = 1$ Hz

and for a measuring time of one year one can reach

$$\delta S_{gw}(f) = (1.1 \times 10^{-24}/\sqrt{\text{Hz}})^2$$

corresponding to $\Omega = 1.3 \times 10^{-3}$ at 920 Hz.

In order to reach the limit of $\Omega = 10^{-5}$, two resonant detectors cooled to 10 mK and with a ten times larger mass would be required. If such two resonant detectors operate at their quantum limit then the bandwidth may become as large as $\Delta f = 50$ Hz thus allowing to reach $\Omega = 5 \times 10^{-6}$ at $f = 920$ Hz.

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References