

# $V_{us}$ from kaon decays

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Recent measurements of kaon decays contributing to the determination of  $|V_{us}|$  are summarized, and up-to-date evaluations of  $|V_{us}|f_+(0)$  and  $|V_{us}|$  are presented.

## I. INTRODUCTION

At present, the first-row constraint  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$  (with  $|V_{ub}|^2$  negligible) offers the most precise test of CKM unitarity. Up until 2002 (and for the 2004 PDG evaluation [1]), the evaluation of  $|V_{us}|$  from older  $K_{\ell 3}$  data gave a  $2.3\sigma$  hint of unitarity violation in the first-row test. The 2003 measurement of  $\text{BR}(K_{e3}^+)$  by BNL E865 [2] gave a value for  $|V_{us}|$  consistent with unitarity. In the period 2004–2006, many new measurements of BRs, lifetimes, and form-factor slopes were announced by KLOE, KTeV, ISTRA+, and NA48. All of these new measurements are distinguished from the older measurements in that they are based on much higher statistics, and in that radiative corrections are applied consistently. Late-2005 evaluations of  $|V_{us}|$  by the CKM working group on the first row [3] and in the 2006 PDG review [4] include many, but not all, of these important developments. We present an up-to-date evaluation that includes preliminary results presented at this conference.

$|V_{us}|$  is evaluated from  $K_{\ell 3}$  data using the relation

$$\Gamma(K_{\ell 3}(\gamma)) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{\text{EW}} |V_{us}|^2 |f_+(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\text{EM}}), \quad (1)$$

where  $C_K$  is a Clebsch-Gordan coefficient;  $M_K$  is the appropriate kaon mass;  $S_{\text{EW}} = 1.0232$  is the universal short-distance electroweak correction;  $f_+(0) \equiv f_+^{K^0\pi^-}(0)$  is the value of the hadronic matrix element (form factor) for the  $K^0 \rightarrow \pi^-$  transition at zero momentum transfer to the leptonic system;  $I_{K\ell}$  is the phase-space integral of the normalized form factor, which depends on the values of one or more slope parameters  $\lambda$ ;  $\Delta_K^{SU(2)}$  and  $\Delta_{K\ell}^{\text{EM}}$  are  $SU(2)$ -breaking and long-distance electromagnetic corrections; and the subscripts  $K$  and  $\ell$  indicate dependence on the kaon charge and lepton flavor. Our main result is a current average value for  $|V_{us}|f_+(0)$ . We first review the recent data on  $K_{\ell 3}$  rates (BRs and lifetimes) and form-factor slopes.

## II. $K_L$ DECAY RATE DATA

Numerous measurements of the principal  $K_L$  BRs, or of various ratios of these BRs, have been published recently. For the purposes of evaluating  $|V_{us}|f_+(0)$ , these data can be used in a PDG-like fit to the  $K_L$  BRs and lifetime, so all such measurements are interesting.

KTeV has measured five ratios of the six main  $K_L$  BRs [5]. The six channels involved account for more than 99.9% of the  $K_L$  width and KTeV combines the five measured ratios to extract the six BRs. We use the five measured ratios in our analysis:  $\text{BR}(K_{\mu 3}/K_{e3}) = 0.6640(26)$ ,  $\text{BR}(\pi^+\pi^-/\pi^0/K_{e3}) = 0.3078(18)$ ,  $\text{BR}(\pi^+\pi^-/K_{e3}) = 0.004856(28)$ ,  $\text{BR}(3\pi^0/K_{e3}) = 0.4782(55)$ , and  $\text{BR}(2\pi^0/3\pi^0) = 0.004446(25)$ . The errors on these measurements are correlated; this is taken into account in our fit.

NA48 has measured the ratio of the BR for  $K_{e3}$  decays to the sum of BRs for all decays to two tracks, giving  $\text{BR}(K_{e3})/(1 - \text{BR}(3\pi^0)) = 0.4978(35)$  [6]. From a separate measurement of  $\text{BR}(K_L \rightarrow 3\pi^0)/\text{BR}(K_S \rightarrow 2\pi^0)$ , NA48 obtains  $\text{BR}(3\pi^0)/\tau_L = 3.795(58)$  MHz [7].

Using  $\phi \rightarrow K_L K_S$  decays in which the  $K_S$  decays to  $\pi^+\pi^-$ , providing normalization, KLOE has directly measured the BRs for the four main  $K_L$  decay channels [8]. The errors on the KLOE BR values are dominated by the uncertainty on the  $K_L$  lifetime  $\tau_L$ ; since the dependence of the geometrical efficiency on  $\tau_L$  is known, KLOE can solve for  $\tau_L$  by imposing  $\sum_x \text{BR}(K_L \rightarrow x) = 1$  (using previous averages for the minor BRs), thereby greatly reducing the uncertainties on the BR values obtained. Our fit makes use of the KLOE BR values before application of this constraint:  $\text{BR}(K_{e3}) = 0.4049(21)$ ,  $\text{BR}(K_{\mu 3}) = 0.2726(16)$ ,  $\text{BR}(K_{e3}) = 0.2018(24)$ , and  $\text{BR}(K_{e3}) = 0.1276(15)$ . The dependence of these values on  $\tau_L$  and the correlations between the errors [9] are taken into account.

KLOE has also measured  $\tau_L$  directly, by fitting the proper decay time distribution for  $K_L \rightarrow 3\pi^0$  events, for which the reconstruction efficiency is high and uniform over a fiducial volume of  $\sim 0.4\lambda_L$ . They obtain  $\tau_L = 50.92(30)$  ns [10].

There are also two recent measurements of  $\text{BR}(\pi^+\pi^-/K_{\ell 3})$ , in addition to the KTeV measurement of  $\text{BR}(\pi^+\pi^-/K_{e3})$  discussed above. KLOE obtains  $\text{BR}(\pi^+\pi^-/K_{\mu 3}) = 7.275(68) \times 10^{-3}$  [11], while NA48 obtains  $\text{BR}(\pi^+\pi^-/K_{e3}) = 4.826(27) \times 10^{-3}$

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TABLE I: Results of fit to  $K_L$  BRs and lifetime

Parameter	Value	$S$
$\text{BR}(K_{e3})$	0.40563(74)	1.1
$\text{BR}(K_{\mu 3})$	0.27047(71)	1.1
$\text{BR}(3\pi^0)$	0.19507(86)	1.2
$\text{BR}(\pi^+\pi^-\pi^0)$	0.12542(57)	1.1
$\text{BR}(\pi^+\pi^-)$	$1.9966(67) \times 10^{-3}$	1.1
$\text{BR}(2\pi^0)$	$8.644(42) \times 10^{-4}$	1.3
$\text{BR}(\gamma\gamma)$	$5.470(40) \times 10^{-4}$	1.1
$\tau_L$	51.173(200) ns	1.1

[12]. All measurements are fully inclusive of inner bremsstrahlung. The KLOE measurement is fully inclusive of the direct-emission (DE) component, DE contributes negligibly to the KTeV measurement, and a residual DE contribution of 0.19% has been subtracted from the NA48 value to obtain the number quoted above. For consistency, in our fit, a DE contribution of 1.52(7)% is added to the KTeV and NA48 values. Our fit result for  $\text{BR}(\pi^+\pi^-)$  is then understood to be DE inclusive.

In addition to the 14 recent measurements listed above, our fit for the seven largest  $K_L$  BRs and lifetime uses four of the remaining five inputs to the 2006 PDG fit and the constraint that the seven BRs sum to unity. The results are summarized in Table I.

Our fit gives  $\chi^2/\text{ndf} = 20.2/11$  (4.3%), while the 2006 PDG fit gives  $\chi^2/\text{ndf} = 14.8/10$  (14.0%). The differences between the output values from our fit and the 2006 PDG fit are minor. The poorer value of  $\chi^2/\text{ndf}$  for our fit can be traced to contrast between the KLOE value for  $\text{BR}(3\pi^0)$  and the other inputs involving  $\text{BR}(3\pi^0)$  and  $\text{BR}(\pi^0\pi^0)$ —in particular, the PDG ETAFIT value for  $\text{BR}(\pi^0\pi^0/\pi^+\pi^-)$ . The treatment of the correlated KTeV and KLOE measurements in the 2006 PDG fit gives rise to large scale factors for  $\text{BR}(K_{e3})$  and  $\text{BR}(3\pi^0)$ ; in our fit, the scale factors are more uniform. As a result, our value for  $\text{BR}(K_{e3})$  has a significantly smaller uncertainty than does the 2006 PDG value.

### III. $K_S$ DECAY RATE DATA

Two KLOE measurements provide the BR for the  $K_{e3}$  decay of the  $K_S$  with enough precision to be of interest for the determination of  $|V_{us}|$ . Using  $\phi \rightarrow K_L K_S$  decays in which the  $K_L$  is recognized by its interaction in the experiment's EM calorimeter, KLOE measures  $\text{BR}(K_{e3}/\pi^+\pi^-) = 10.19(13) \times 10^{-4}$  [13]. KLOE has also recently obtained  $\text{BR}(\pi^+\pi^-/\pi^0\pi^0) = 2.2459(54)$  [14], where this value is an average including the experiment's 2002 result. These measurements completely determine the main  $K_S$  BRs, and in particular give  $\text{BR}(K_{e3}) = 7.046(91) \times 10^{-4}$  [13]. Our evaluation of  $|V_{us}|f_+(0)$  uses this value, together with the lifetime value  $\tau_S = 0.08958(5)$  ns from the PDG fit to  $CP$  pa-

TABLE II: Results of fit to  $K^\pm$  BRs and lifetime

Parameter	Value	$S$
$\text{BR}(K_{\mu 2})$	63.442(145)%	1.3
$\text{BR}(\pi\pi^0)$	20.701(108)%	1.3
$\text{BR}(\pi\pi\pi)$	5.5921(305)%	1.0
$\text{BR}(K_{e3})$	5.121(38)%	1.6
$\text{BR}(K_{\mu 3})$	3.3855(203)%	1.2
$\text{BR}(\pi\pi^0\pi^0)$	1.7592(234)%	1.1
$\tau_\pm$	12.3840(213) ns	1.8

rameters, which is highly constrained by measurements from NA48 [15] and KTeV [16].

### IV. $K^\pm$ DECAY RATE DATA

There are several new results providing information on  $K_{\ell 3}^\pm$  rates. These results are mainly preliminary and have not been included in previous averages.

NA48/2 has recently submitted for publication measurements of the three ratios  $\text{BR}(K_{e3}/\pi\pi^0)$ ,  $\text{BR}(K_{\mu 3}/\pi\pi^0)$ , and  $\text{BR}(K_{\mu 3}/K_{e3})$  [17, 18]. These measurements are not independent; in our fit, we use the values  $\text{BR}(K_{e3}/\pi\pi^0) = 0.2496(10)$  and  $\text{BR}(K_{\mu 3}/\pi\pi^0) = 0.1637(7)$  and take their correlation into account.

ISTRA+ has also updated its preliminary value for  $\text{BR}(K_{e3}/\pi\pi^0)$ . They now quote  $\text{BR}(K_{e3}/\pi\pi^0) = 0.2449(16)$ , as reported at this conference [19].

KLOE has measured the absolute BRs for the  $K_{e3}$  and  $K_{\mu 3}$  decays [20]. In  $\phi \rightarrow K^+K^-$  events,  $K^+$  decays into  $\mu\nu$  or  $\pi\pi^0$  are used to tag a  $K^-$  beam, and vice versa. KLOE performs four separate measurements for each  $K_{\ell 3}$  BR, corresponding to the different combinations of kaon charge and tagging decay. The final averages are  $\text{BR}(K_{e3}) = 5.047(92)\%$  and  $\text{BR}(K_{\mu 3}) = 3.310(81)\%$ . Our fit takes into account the correlation between these values, as well as their dependence on the  $K^\pm$  lifetime [9].

The world average value for  $\tau_\pm$  is nominally quite precise; the 2006 PDG quotes  $\tau_\pm = 12.385(25)$  ns. However, the error is scaled by 2.1; the confidence level for the average is 0.2%. It is important to confirm the value of  $\tau_\pm$ . A preliminary measurement from KLOE,  $\tau_\pm = 12.367(78)$  ns [9], agrees with the PDG average, although at present the KLOE uncertainty is significantly larger.

Our fit for the six largest  $K^\pm$  BRs and lifetime makes use of the results cited above, plus the data used in the 2006 PDG fit, for a total of 30 measurements. The six BRs are constrained to sum to unity. The results are summarized in Table II.

The fit quality is poor, with  $\chi^2/\text{ndf} = 49/24$  (0.21%). However, when the five older measurements of  $\tau_\pm$  are replaced by their PDG average with scaled error,  $\chi^2/\text{ndf}$  improves to 31.3/20 (5.1%), with no significant changes in the results. Tension between the new measurements

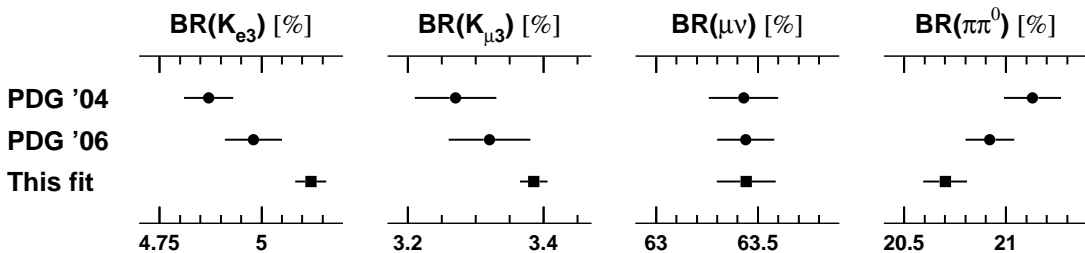


FIG. 1: Evolution of average values for main  $K^\pm$  BRs.

involving  $\text{BR}(K_{e3})$  and the older measurements is partly responsible for the poor fit quality (note the scale factor of 1.6 on  $\text{BR}(K_{e3})$  from our fit). Because of uncertainties concerning the treatment of radiative corrections, it would be desirable to eliminate some of the older measurements, as both we and PDG do for the  $K_L$  fit. However, for  $K^\pm$ , all of the new measurements involve the  $K_{\ell 3}$  BRs or the ratios  $\text{BR}(K_{\ell 3}/\pi\pi^0)$ . This leads to large correlations between the  $K_{\ell 3}$  BRs and  $\text{BR}(\pi\pi^0)$ , and makes the fit unstable when the older data are excluded. Both the significant evolution of the average values of the  $K_{\ell 3}$  BRs and the effect of the correlations with  $\text{BR}(\pi\pi^0)$  are evident in Fig. 1.

## V. FORM-FACTOR SLOPES

For  $K_{e3}$  decays, recent measurements of the quadratic slope parameters of the vector form factor ( $\lambda'_+, \lambda''_+$ ) are available from KTeV [21, 22], KLOE [23], ISTRA+ [19, 24], and NA48 [25]. For  $K_{\mu 3}$  decays, results of fits using the quadratic parameterization for the vector form factor ( $\lambda'_+, \lambda''_+$ ) and the linear parameterization for the scalar form factor ( $\lambda_0$ ) are available from KTeV [21], ISTRA+ [19, 26], and NA48 [18, 27]. ISTRA+ measures  $K^-$  decays; the other experiments measure  $K_L$  decays.

The  $K_{e3}$  data are summarized in Fig. 2. The uncertainties on the values of  $\lambda'_+$  and  $\lambda''_+$  reported by each experiment are highly correlated. This is an intrinsic property of the quadratic parameterization. The form-factor slopes represent a small modification of the kinematic density of the matrix element for  $K \rightarrow \pi$  transitions; in addition, sensitivity to  $\lambda''_+$  is poor because the kinematic density of the matrix element drops to zero at large  $t$ , where the form factor itself is maximal. Taking the correlations into account, we obtain a good fit to the data in Fig. 2:  $\lambda'_+ = 24.15(0.87) \times 10^{-3}$  and  $\lambda''_+ = 1.57(0.38) \times 10^{-3}$ , with  $\rho(\lambda'_+, \lambda''_+) = -0.941$  and  $\chi^2/\text{ndf} = 5.3/6$  (51%). The significance of the quadratic term is greater than  $4\sigma$ . The fit result is shown as the yellow ellipse in Fig. 2.

The  $K_{\mu 3}$  data are summarized in Fig. 3, which also shows the results for  $\lambda'_+$  and  $\lambda''_+$  from our  $K_{e3}$  fit, and the results of our combined fits to  $K_{e3}$  and  $K_{\mu 3}$  data. While the results for  $\lambda'_+$  and  $\lambda''_+$  from  $K_{\mu 3}$  decays are fairly consistent and reasonably agree with those from  $K_{e3}$  decays

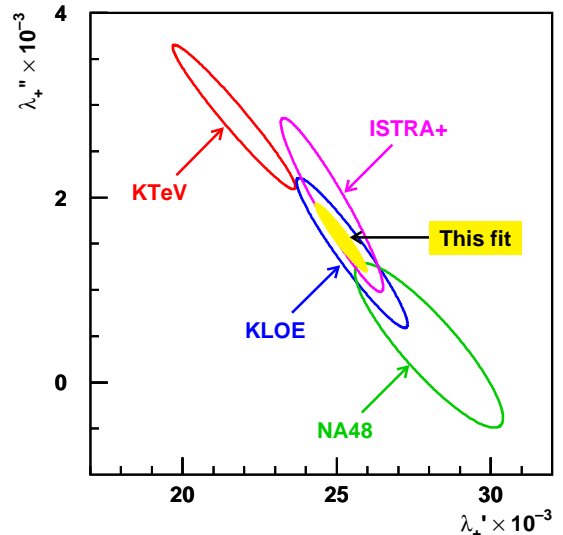
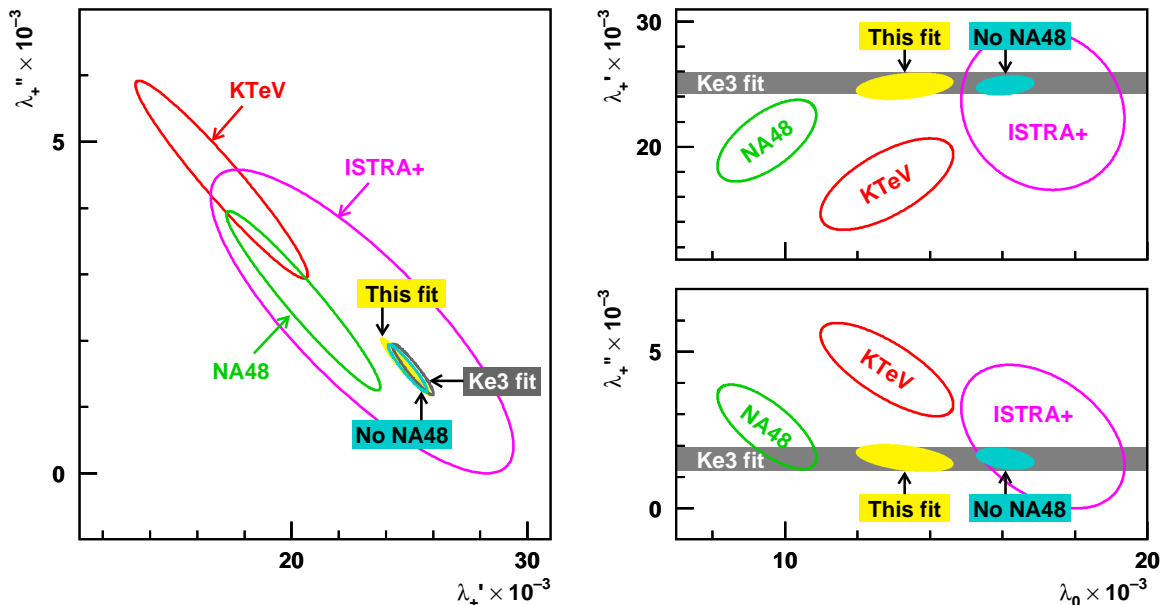


FIG. 2: Recent measurements of form-factor slopes for  $K_{e3}$  decays, with fit.

(Fig. 3, left panel), the results for  $\lambda_0$  show poor consistency (right panels). The most discrepant measurement is that of NA48. In both the  $\lambda'_+-\lambda_0$  and  $\lambda''_+-\lambda_0$  views, a locus of values within the  $1\sigma$  confidence contours from ISTRA+ and our  $K_{e3}$  fit lies along the major axis of the KTeV confidence ellipse. As a result, a fit to all data ( $K_{e3}$  and  $K_{\mu 3}$ ) excluding only the  $K_{\mu 3}$  results from NA48 gives  $\chi^2/\text{ndf} = 11.9/9$  (21.7%). The results of this fit are indicated by the cyan ellipses in Fig. 3. When the NA48  $K_{\mu 3}$  results are included,  $\chi^2/\text{ndf}$  increases to 52/12, for a probability of less than  $10^{-6}$ . The results are shown as the yellow ellipses in Fig. 3.

Since at the moment there is no a priori reason to exclude the NA48  $K_{\mu 3}$  results, we base our estimate of  $|V_{us}|f_+(0)$  on the fit to all data. However, we scale the errors on the slope parameters to reflect the inconsistency in the input data set. We obtain  $\lambda'_+ = 24.84(1.10) \times 10^{-3}$  ( $S = 1.4$ ),  $\lambda''_+ = 1.61(0.45) \times 10^{-3}$  ( $S = 1.3$ ), and  $\lambda_0 = 13.30(1.35) \times 10^{-3}$  ( $S = 2.1$ ), with  $\rho(\lambda'_+, \lambda''_+) = -0.944$ ,  $\rho(\lambda'_+, \lambda_0) = +0.314$ , and  $\rho(\lambda''_+, \lambda_0) = -0.420$ . From these results, we calculate the phase-space integrals for use with Eq. (1) to be  $I(K_{e3}^0) = 0.15452(29)$ ,  $I(K_{e3}^\pm) = 0.15887(30)$ ,  $I(K_{\mu 3}^0) = 0.10207(34)$ , and  $I(K_{\mu 3}^\pm) = 0.10501(35)$ .

FIG. 3: Recent measurements of form-factor slopes for  $K_{\ell 3}$  decays, with fits.TABLE III: Summary of  $SU(2)$ -breaking and EM corrections

Mode	$\Delta^{SU(2)}$	$\Delta^{\text{EM}}$
$K_{e3}^0$	0	+0.52(10)%
$K_{e3}^{\pm}$	+2.31(22)%	+0.03(10)%
$K_{\mu 3}^0$	0	+0.95(15)%
$K_{\mu 3}^{\pm}$	+2.31(22)%	+0.2(4)%

## VI. EVALUATION OF $|V_{us}|f_+(0)$

The strong  $SU(2)$ -breaking and EM corrections to  $f_+(0)$  used in our evaluation of  $|V_{us}|f_+(0)$  are summarized in Table III. For  $\Delta^{SU(2)}$ , we use the standard value from Ref. 28. All of the recent  $K_{\ell 3}$  BR measurements are fully inclusive of final-state radiation; the values we use for  $\Delta_{K\ell}^{\text{EM}}$  were calculated for the fully-inclusive rates. Specifically, for  $K_{e3}^0$  and  $K_{e3}^{\pm}$ , the values were obtained from the chiral perturbation theory calculation of Ref. 28, updated using the evaluations of the low-energy constants from Ref. 29. For  $K_{\mu 3}^0$ , the value is from Ref. 30, and was obtained using a generator implementing a hadronic-model calculation. For  $K_{\mu 3}^{\pm}$ , we do not know of any complete estimate. Our value is loosely based on Ref. 31, with a very generous error estimate. However, with this treatment,  $\Delta_{K\ell}^{\text{EM}}$  gives the largest single contribution to the uncertainty on  $|V_{us}|f_+(0)$  from this mode.

We use the results of the fits and averages described in Secs. II–V, together with the corrections in Table III, to evaluate the quantity  $|V_{us}|f_+(0)$  for each of the five  $K_{\ell 3}$  decay modes as listed in Table IV. The table also gives the fractional uncertainty for each determination of  $|V_{us}|f_+(0)$ , as well as the approximate breakdown of

TABLE IV: Values of  $|V_{us}|f_+(0)$  from data for different decay modes, with error breakdown

Mode	$ V_{us} f_+(0)$	% err	Approx contrib to % err			
			BR	$\tau$	$\Delta$	$I$
$K_L e3$	0.21639(55)	0.25	0.09	0.19	0.10	0.09
$K_L \mu 3$	0.21649(68)	0.31	0.10	0.18	0.15	0.17
$K_S e3$	0.21555(142)	0.66	0.65	0.03	0.10	0.09
$K^{\pm} e3$	0.21844(101)	0.46	0.38	0.11	0.24	0.09
$K^{\pm} \mu 3$	0.21809(125)	0.57	0.31	0.10	0.45	0.17

the contributions to the uncertainty from the various inputs (these contributions are understood to be added in quadrature). The source of the limiting uncertainty varies from mode to mode.

The average value of  $|V_{us}|f_+(0)$  from all modes, as obtained from a fit with correlations taken into account, is 0.21673(46). The fit gives  $\chi^2/\text{ndf} = 4.2/4$  (38%).

Although the value of  $\chi^2/\text{ndf}$  from the fit is satisfactory, one notes that the values of  $|V_{us}|f_+(0)$  for the two charged modes seem to be higher than those for the neutral modes. To quantify this, we perform the fit separately for charged and neutral modes, using the results of the overall fit to the form-factor slope data of Sec. V, and the  $SU(2)$ -breaking correction from Table III,  $\Delta_{\text{th}}^{SU(2)} = 2.31(22)\%$ , for the charged modes. We obtain  $|V_{us}|f_+(0) = 0.21635(50)$  for neutral modes and 0.21832(94) for charged modes, a  $1.9\sigma$  difference. If we take this as a suggestion that the  $SU(2)$ -breaking correction may be underestimated, and perform the fit leaving free the value of  $|V_{us}|f_+(0)$  for neutral modes and an empirical value of  $\Delta^{SU(2)}$ , we obtain  $\Delta_{\text{exp}}^{SU(2)} = 3.24(43)\%$ .

Comparison of the values of  $|V_{us}|f_+(0)$  for  $K_{e3}$  and

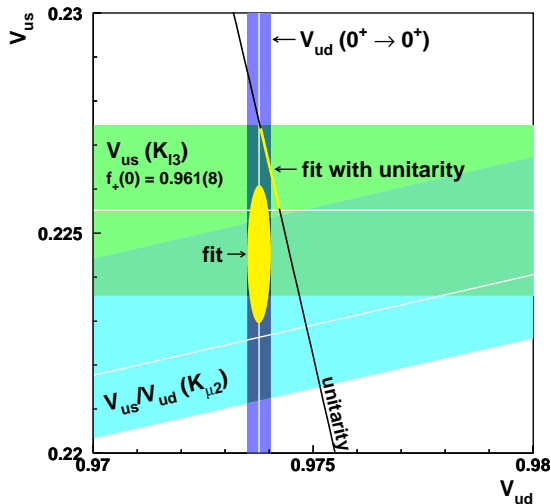


FIG. 4: Results of fits to  $|V_{ud}|$ ,  $|V_{us}|$ , and  $|V_{us}|/|V_{ud}|$ .

$K_{\mu 3}$  modes provides a test of lepton universality. Specifically,

$$r_{\mu e} = \frac{[|V_{us}| f_+(0)]_{\mu 3, \text{exp}}^2}{[|V_{us}| f_+(0)]_{e 3, \text{exp}}^2} = \frac{\Gamma_{\mu 3}}{\Gamma_{e 3}} \cdot \frac{I_{e 3} (1 + \delta_{e 3})}{I_{\mu 3} (1 + \delta_{\mu 3})}.$$

By comparison with Eq. (1),  $r_{\mu e}$  is equal to the ratio  $g_{\mu}^2/g_e^2$ , with  $g_l$  the coupling strength at the  $W \rightarrow \ell\nu$  vertex. In the Standard Model,  $r_{\mu e} = 1$ . We calculate  $r_{\mu e}$  separately for charged and neutral modes. The results are compatible; the average value is  $r_{\mu e} = 1.0003(53)$ . We note that the sensitivity of this test of lepton universality is beginning to approach that obtained in  $\pi \rightarrow \ell\nu$  decays:  $(r_{e\mu})_{\pi\ell 2} = 0.9966(30)$ , as reviewed in Ref. 32.

## VII. EVALUATION OF $|V_{us}|$

To test CKM unitarity, a value for  $f_+(0)$  is needed. In the chiral expansion,  $f_+(0) = 1 + f_{p^4} + f_{p^6} + \dots$ . A finite one-loop contribution gives rise to  $f_{p^4}$ ; its calculation [33, 34] involves essentially no uncertainty. A quark-model estimate of the contribution at  $\mathcal{O}(p^6)$  was

originally performed by Leutwyler and Roos [33]. More recently, a complete  $\mathcal{O}(p^6)$  calculation in chiral perturbation theory showed that meson-loop contributions are sizeable [35]. Analytic estimates of  $f_+(0)$  were surveyed at this conference [36], and lattice evaluations of  $f_+(0)$  are rapidly improving in precision (see, *e.g.*, Ref. 37). Until a definite consensus emerges on a new reference value for  $f_+(0)$ , however, we use the original estimate of Leutwyler and Roos,  $f_+(0) = 0.961(8)$ , which is supported by lattice calculations [37]. This gives  $|V_{us}| = 0.2255(19)$ .

Using the recent evaluation of  $|V_{ud}|$  from  $0^+ \rightarrow 0^+$  nuclear beta decays (Ref. 38; see also Ref. 39),  $|V_{ud}| = 0.97377(27)$ , one has  $|V_{ud}|^2 + |V_{us}|^2 - 1 = -0.0009(10)$ , a result compatible with unitarity.

Marciano [40] has observed that  $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$  can be precisely related to the product  $(|V_{us}|/|V_{ud}|)^2 (f_K/f_\pi)^2$ . The recent measurement  $\text{BR}(K^+ \rightarrow \mu^+\nu) = 0.6366(17)$  from KLOE [41], together with the preliminary lattice result  $f_K/f_\pi = 1.208(2)_{(-14)}^{(+7)}$  from the MILC Collaboration [42], gives  $|V_{us}|/|V_{ud}| = 0.2286_{(-15)}^{(+27)}$ . This ratio can be used in a fit together with the values of  $|V_{ud}|$  from Ref. 38 and  $|V_{us}|$  from  $K_{\ell 3}$  decays as above. Using the value for  $|V_{us}|$  obtained with  $f_+(0) = 0.961(8)$ , the fit gives  $|V_{ud}| = 0.97377(27)$  and  $|V_{us}| = 0.2245(16)$ , with  $\chi^2/\text{ndf} = 0.77/1$  (38%). The unitarity constraint can also be included, in which case the fit gives  $\chi^2/\text{ndf} = 3.10/2$  (21.2%, or  $1.2\sigma$ ). Both results are illustrated in Fig. 4.

In summary, from  $K_{\ell 3}$  data we obtain  $|V_{us}|f_+(0) = 0.21673(46)$ , where the uncertainty amounts to 0.21%. The dominant contribution to the uncertainty on  $|V_{us}|$  is from  $f_+(0)$ . Whether  $|V_{us}|$  is evaluated from  $K_{\ell 3}$  data alone or with the additional constraint from  $K_{\mu 2}$  decays, the first-row unitarity test is satisfied at about the  $1\sigma$  level.

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- [1] F. Gilman, K. Kleinknecht, and B. Renk, in *Review of Particle Properties*, Phys. Lett. B **592**, 130 (2004).
- [2] A. Sher et al., Phys. Rev. Lett. **91**, 261802 (2003).
- [3] E. Blucher et al. (2005), hep-ex/0512039.
- [4] E. Blucher and W. Marciano, in *Review of Particle Properties*, J. Phys. G **33**, 677 (2006).
- [5] T. Alexopoulos et al., Phys. Rev. D **70**, 092006 (2004).
- [6] NA48 Collab., A. Lai et al., Phys. Lett. B **602**, 41 (2004).
- [7] L. Litov, for the NA48 Collab., in *Proc. 32<sup>nd</sup> Int. Conf. on High-Energy Physics (ICHEP '04)* (Beijing, 2004), p. 817, hep-ex/0501048.
- [8] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **632**,

- 43 (2006).
- [9] T. Spadaro, these proceedings.
- [10] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **626**, 15 (2005).
- [11] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **638**, 140 (2006).
- [12] NA48 Collab., A. Lai et al., Phys. Lett. B **645**, 26 (2007).
- [13] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **636**, 173 (2006).
- [14] KLOE Collab., F. Ambrosino et al., Eur. Phys. J. C **48**, 767 (2006).
- [15] NA48 Collab., A. Lai et al., Phys. Lett. B **537**, 28 (2002).

- [16] KTeV Collab., A. Alavi-Harati et al., Phys. Rev. D **67**, 012005 (2003).
- [17] J. Batley et al. (2007), in press, hep-ex/0702015.
- [18] M. Veltri, these proceedings.
- [19] V. Romanovsky, these proceedings.
- [20] B. Sciascia, for the KLOE Collab., in *Proc. EPS Int. Europhysics Conf. on High-Energy Physics (HEP-EPS '05)* (Lisbon, 2005), hep-ex/0510028.
- [21] KTeV Collab., T. Alexopoulos et al., Phys. Rev. D **70**, 092007 (2004).
- [22] KTeV Collab., E. Abouzaid et al., Phys. Rev. D **74**, 097101 (2006).
- [23] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **636**, 166 (2006).
- [24] O. Yushchenko et al., Phys. Lett. B **589**, 111 (2004).
- [25] NA48 Collab., A. Lai et al., Phys. Lett. B **604**, 1 (2004).
- [26] O. Yushchenko et al., Phys. Lett. B **581**, 31 (2004).
- [27] NA48 Collab., A. Lai et al., Phys. Lett. B (2007), in press, hep-ex/0703002.
- [28] V. Cirigliano et al., Eur. Phys. J. C **23**, 121 (2002).
- [29] S. Descotes-Genon and B. Moussallam, Eur. Phys. J. C **42**, 403 (2005).
- [30] T. Andre (2004), hep-ph/0406006.
- [31] E. Ginsberg, Phys. Rev. D **1**, 229 (1970).
- [32] J. Erler and M. Ramsey-Musolf, Prog. Part. Nucl. Phys. **54**, 351 (2005).
- [33] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
- [34] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 517 (1985).
- [35] J. Bijnens and P. Talavera, Nucl. Phys. B **669**, 341 (2003).
- [36] J. Portolés, these proceedings.
- [37] J. Zanotti, these proceedings.
- [38] W. Marciano and A. Sirlin, Phys. Rev. Lett. **96**, 032002 (2006).
- [39] J. Hardy, these proceedings.
- [40] W. Marciano, Phys. Rev. Lett. **93**, 231803 (2004).
- [41] KLOE Collab., F. Ambrosino et al., Phys. Lett. B **632**, 76 (2006).
- [42] C. Bernard, in *Proc. 5<sup>th</sup> Int. Workshop on Chiral Dynamics (Chiral '06)* (Chapel Hill NC, USA, 2006), hep-lat/0611024.