

V_{us} from kaon decays

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for the FlaviaNet Working Group on Kaon Decays


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<http://www.Inf.infn.it/wg/vus>



Working Group on Precise SM Tests in K Decays



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News
Acknowledgements

Honorary chair: [Paolo Franzini \(LNF\)](#) **Coordinators:** [Mario Antonelli \(LNF\)](#) and [Gino Isidori \(LNF\)](#)

K_{l3} decays, V_{us} , and CKM unitarity

At present, most precise test of CKM unitarity is from 1st row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \equiv 1 - \Delta$$

$0^+ \rightarrow 0^+ \beta$ decays: $2|V_{ud}|\delta V_{ud} = 0.0005$
 $Kl3$ decays: $2|V_{us}|\delta V_{us} < 0.0010$

→ 2002
(2004 PDG)

Old $Kl3$ data give $\Delta = 0.0035(15)$
A 2.3σ hint of unitarity violation?

2003

BNL 865 measures $\text{BR}(K^+ \rightarrow \pi^0 e^+ \nu) = 5.13(10)\%$
Value for V_{us} consistent with unitarity

2004-2006

Many new measurements from KTeV, KLOE, ISTRA+, NA48

- **BRs, lifetimes, form-factor slopes**
- **Much higher statistics** than older measurements
- Importance of **radiative corrections**
- Proper reporting of **correlations** between measurements

**2005 CKM
WG1 report**

$|V_{us}|f_+(0) = 0.2173(8)$: unitarity to better than 1σ
Includes many (but not all) of these important developments

This talk

Update with all recent measurements (even if preliminary)

Determination of V_{us} using K_{l3} rates

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{Kl}(\{\lambda\}_{Kl}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{Kl}^{EM})$$

with $K \in \{K^+, K^0\}$; $l \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+ , 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t=0$)

$\Delta_{SU(2)}^K$ Form-factor correction for $SU(2)$ breaking

Δ_{EM}^{Kl} Form-factor correction for long-distance EM effects

Inputs from experiment:

$\Gamma(K_{l3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I^{Kl}(\{\lambda\}_{Kl})$ Integral of form-factor over phase space: λ s parameterize evolution in t

- K_{e3} : Only λ_+ (or λ_+', λ_+'')
- $K_{\mu3}$: Need λ_+ and λ_0

K_L branching ratios

KTeV
PRD 70 (2004)

5 ratios of main BRs from independent samples of 10^5 - 10^6 events collected with a single trigger

2-track ratios

$$\text{BR}(K_{\mu 3}/K_{e3}) = 0.6640(26)$$

$$\text{BR}(\pi^+\pi^-\pi^0/K_{e3}) = 0.3078(18)$$

$$\text{BR}(\pi^+\pi^-/K_{e3}) = 0.004856(28)$$

Neutral ratio

$$\text{BR}(2\pi^0/3\pi^0) = 0.004446(25)$$

Mixed ratio

$$\text{BR}(3\pi^0/K_{e3}) = 0.4782(55)$$

6 decays = 99.93% of K_L width
KTeV combines ratios to extract BRs

Our fit uses these BR ratios
Correlations available

NA48
PLB 602 (2004)

K_L beam only, 2-track sample, 80M events (6M signal)

$$\frac{\text{BR}(K_{e3})}{\text{BR}(2 \text{ track})} = 0.4978(35) \approx \frac{\text{BR}(K_{e3})}{1 - \text{BR}(3\pi^0)}$$

NA48
preliminary

From $\text{BR}(K_L \rightarrow 3\pi^0)/\text{BR}(K_S \rightarrow 2\pi^0)$

$$\text{BR}(3\pi^0) = 0.1966(34)$$

K_L branching ratios and lifetime

KLOE

PLB 632 (2006)

Absolute BRs: K_L decays tagged by $K_S \rightarrow \pi^+\pi^-$

Errors on absolute BRs dominated by error on τ_L

Dependence on τ_L of geometrical efficiency known

For KLOE results: Set $\sum_x \text{BR}(K_L \rightarrow x) = 1$ and solve for τ_L

For our fit: Use unconstrained BRs with dependence on τ_L

$$\text{BR}^{(0)}(Ke3) = 0.4049(21)$$

$$\text{BR}^{(0)}(K\mu3) = 0.2726(16)$$

$$\text{BR}^{(0)}(3\pi^0) = 0.2018(24)$$

$$\text{BR}^{(0)}(\pi^+\pi^-\pi^0) = 0.1276(15)$$

at $\tau_L^{(0)} = 51.54$ ns, with
 $d \text{BR}/\text{BR} = 0.67 d\tau_L/\tau_L$

Correlations available

KLOE

PLB 626 (2005)

Lifetime: Direct measurement with $K_L \rightarrow 3\pi^0$ events

High, uniform reconstruction efficiency over $0.4\lambda_L$

Independent of BR measurement

$$\tau_L = 50.92(30) \text{ ns}$$

$$\text{cf. Vosburgh '72: } \tau_L = 51.54(44) \text{ ns}$$

$$K_L \rightarrow \pi^+ \pi^-$$

New measurements of $K_L \rightarrow \pi^+ \pi^- (\gamma)$ also useful in global fit

KTeV

PRD 70 (2004)

$$\text{BR}(\pi^+ \pi^- / K e 3) = 4.856(29) \times 10^{-3}$$

1 of 5 ratios in K_L BR analysis

Contribution from direct emission (DE) negligible

KLOE

PLB 638 (2006)

$$\text{BR}(\pi^+ \pi^- / K \mu 3) = 7.275(68) \times 10^{-3}$$

Fully inclusive of DE component

NA48

hep-ex/0611052

$$\text{BR}(\pi^+ \pi^- / K e 3) = 4.826(27) \times 10^{-3}$$

Residual DE contribution of 0.19% subtracted

For consistency and to better satisfy $\Sigma \text{BR} = 1$ in global fit,
DE contribution of **1.52(7)%*** added to **KTeV** and **NA48** results

* From E731 '93, KTeV '01 and KTeV '06 $K_L \rightarrow \pi^+ \pi^- \gamma$ results

Fit to K_L BR and lifetime measurements

Availability of **comprehensive new K_L data set** with proper radiative corrections has **radically changed the PDG fit**

- **2004 fit used 50 measurements** - all but a handful pre-1990
- **2006 fit uses 17 measurements** - all but 2 post-2003

Compared to PDG 2006, our fit:

- Uses the KTeV BR ratios and KLOE BRs quoted before application of constraints
 - In each case, PDG uses the constrained results and removes the $3\pi^0$ from the fit
- Uses the NA48 preliminary BR($3\pi^0$) and new BR($\pi^+\pi^-$) measurements
- Implements consistent treatment of DE for $K_L \rightarrow \pi^+\pi^-$

Results of fit to K_L BRs, τ

18 input measurements:

5 **KTeV** ratios

NA48 BR($Ke3/2$ track)

NA48 BR($3\pi^0$) [prelim.]

4 **KLOE** BRs

with dependence on τ_L

KLOE, **NA48** BR($\pi^+\pi^-/Kl3$)

KLOE, **NA48** BR($\gamma\gamma/3\pi^0$)

PDG ETAFIT BR($2\pi^0/\pi^+\pi^-$)

KLOE τ_L from $3\pi^0$

Vosburgh '72 τ_L

Parameter	Value	S
BR($Ke3$)	0.40571(89)	1.4
BR($K\mu3$)	0.27055(81)	1.3
BR($3\pi^0$)	0.19447(103)	1.4
BR($\pi^+\pi^-\pi^0$)	0.12588(78)	1.5
BR($\pi^+\pi^-$)	$1.9860(74)\times 10^{-3}$	1.2
BR($2\pi^0$)	$8.603(54)\times 10^{-4}$	1.7
BR($\gamma\gamma$)	$5.453(43)\times 10^{-4}$	1.1
τ_L	51.148(211) ns	1.1

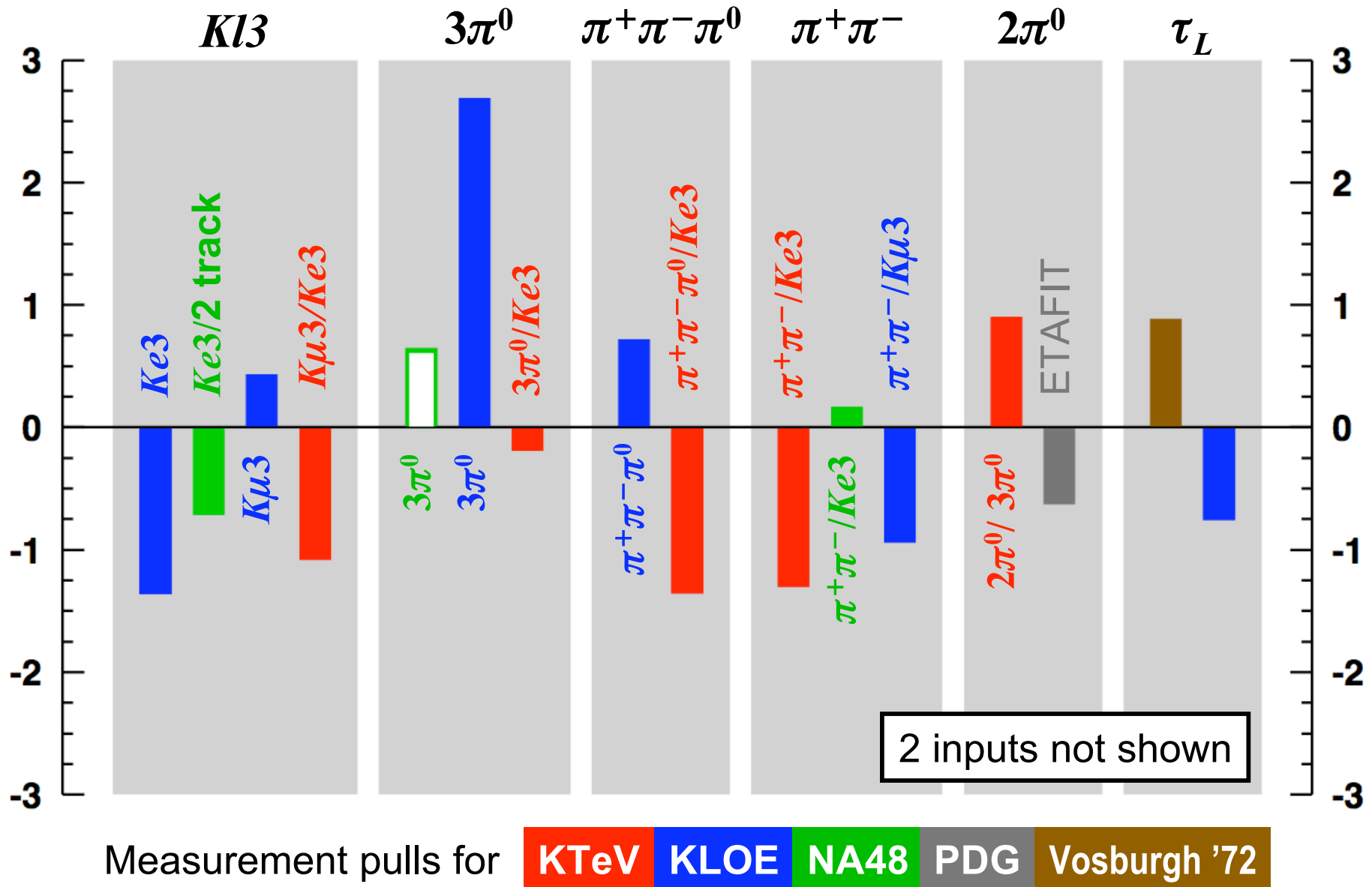
$\chi^2/\text{ndf} = 20.2/11$ (Prob = 4.3%)

compare PDG '06: 14.8/10 (14%)

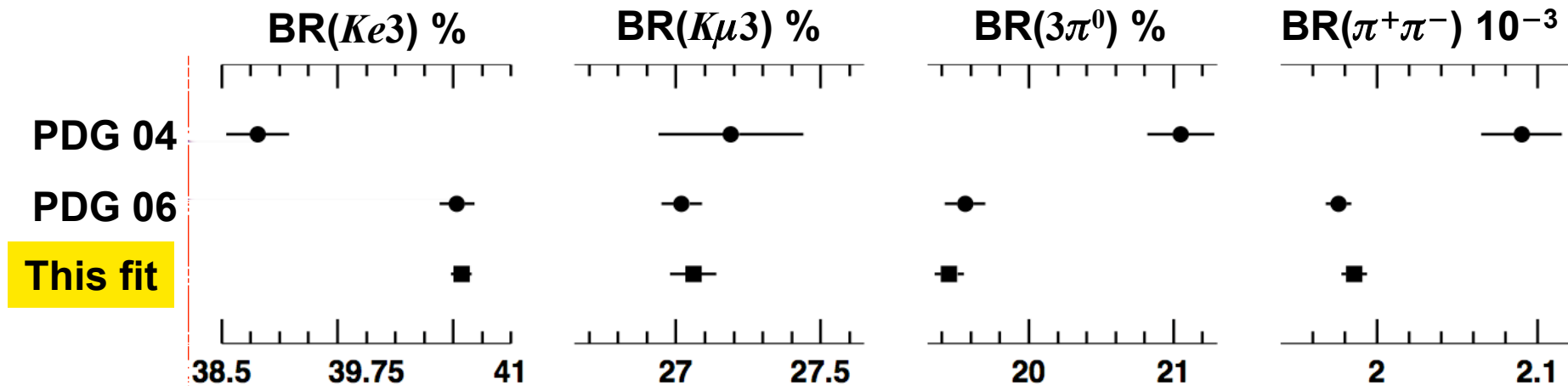
1 constraint: $\Sigma \text{BR} = 1$

- Without preliminary results & DE correction, our fit reproduces PDG
- Including ETAFIT pulls down $2\pi^0$ & $3\pi^0$ BRs, clashes with KLOE $3\pi^0$
- Even with scale factor, our fit reduces error on $Ke3$

Comparison: K_L BR fit vs. data



Evolution of K_L BRs



Differences between our fit and PDG '06 are minor

Since PDG '04 fit:

- BR($Ke3$) significantly increased
- BR($K\mu3$) changed little
- BR($3\pi^0$) significantly decreased

Proper radiative corrections
Especially for $Ke3$
Exclusion of NA31 measurements
Significantly reduces errors

$R_{\mu e}$ decreases from 0.701(9) to 0.6668(26)
Better agreement with lepton universality

BR($K_S \rightarrow \pi e \nu$) and K_S lifetime

KLOE

PLB 636 (2006)

Using tagged K_S beam

$$\text{BR}(K_S \rightarrow \pi e \nu) / \text{BR}(K_S \rightarrow \pi^+ \pi^-) = 10.19(13) \times 10^{-4}$$

KLOE

EPJC 48 (2006)

410 pb⁻¹, averaged with KLOE '02 result (17 pb⁻¹)

$$\text{BR}(K_S \rightarrow \pi^+ \pi^-) / \text{BR}(K_S \rightarrow \pi^0 \pi^0) = 2.2459(54)$$

These two measurements completely determine main K_S BRs

$$\text{BR}(K_S \rightarrow \pi e \nu) = 7.046(91) \times 10^{-4}$$

PDG

$$\tau_S = 0.08958(5) \text{ ns}$$

From fit to CP parameters, does not assume CPT

Dominated by **NA48 '02** and **KTeV '03** τ_S values

Preliminary results on K_{l3}^{\pm} BRs

NA48/2
preliminary

New results on $\text{BR}(K^{\pm}e3)/\text{BR}(\pi\pi^0)$ and $\text{BR}(K^{\pm}\mu3)/\text{BR}(\pi\pi^0)$!

Ratio for $K^{\pm}e3$ higher than previous preliminary value

Statistics dominated; main systematic from FF for acceptance

$$\text{BR}(K^{\pm}e3)/\text{BR}(\pi\pi^0) = 0.2496(9)(4)$$

$$\text{BR}(K^{\pm}\mu3)/\text{BR}(\pi\pi^0) = 0.1637(6)(3)$$

Correlation provided

ISTRA+
preliminary

Updated preliminary value

$$\text{BR}(K^{\pm}e3)/\text{BR}(\pi\pi^0) = 0.2449(4)(15)$$

KLOE
preliminary

Absolute $\text{BR}(K^{\pm}e3)$ and $\text{BR}(K^{\pm}\mu3)$ measurements

Separate measurements for each charge

Tagged by $K \rightarrow \mu\nu$ and $K \rightarrow \pi\pi^0$: 8 measurements total

$$\text{BR}^{(0)}(K^{\pm}e3) = 5.047(92)\%$$

$$\text{BR}^{(0)}(K^{\pm}\mu3) = 3.310(81)\%$$

at $\tau_{\pm}^{(0)} = 12.36$ ns, with $d \text{BR}/\text{BR} = -0.50 d\tau_{\pm}/\tau_{\pm}$

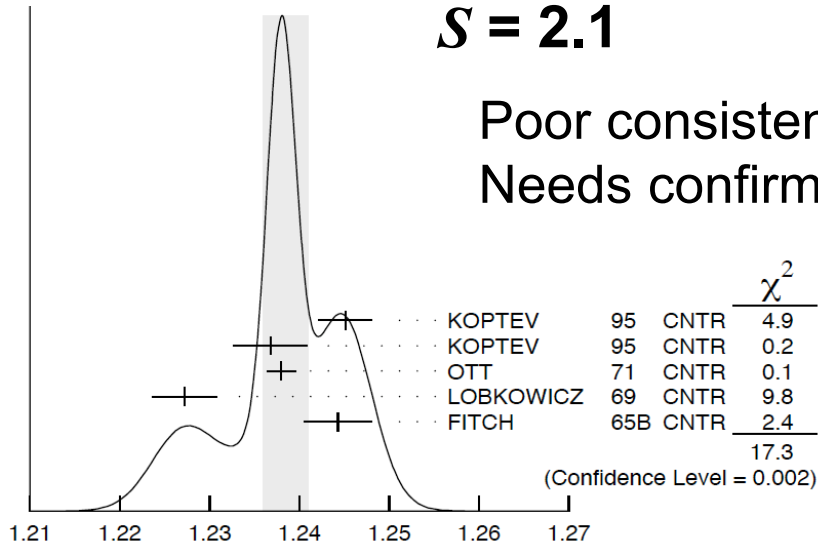
K^\pm lifetime

PDG
average

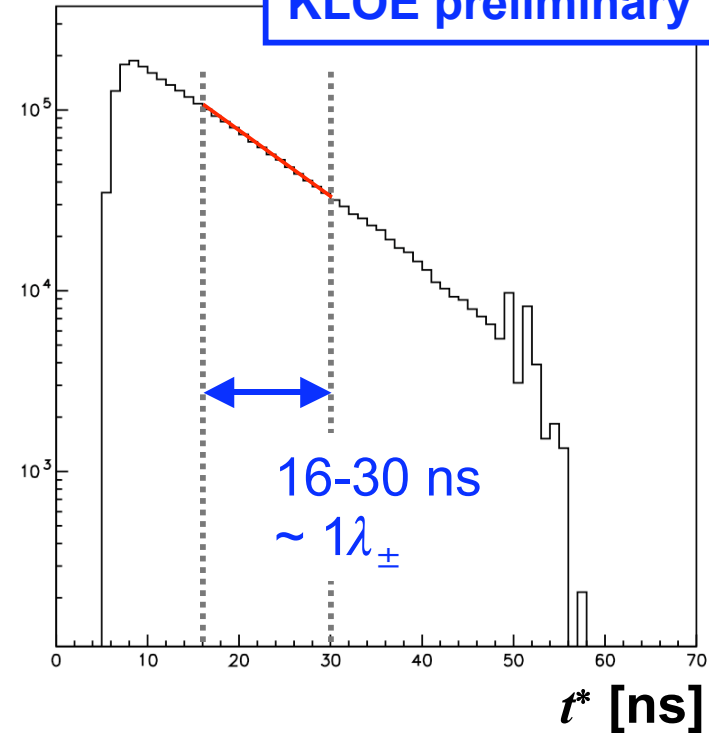
12.385(25) ns

$S = 2.1$

Poor consistency
Needs confirmation



KLOE preliminary



KLOE
preliminary

Fit to t^* distribution from decay length
Use all $K \rightarrow \mu\nu$ -tagged vertices in drift chamber

$\tau_\pm = 12.367(78) \text{ ns}$

Fit to K^\pm BR and lifetime measurements

Not possible to fit to only new K^\pm data (unlike for K_L)

Only $Kl3$ and $Kl3/\pi\pi^0$ have been measured recently

- $Kl3$ and $\pi\pi^0$ highly correlated in fit (-0.64 , -0.79 for $Ke3$, $K\mu3$)
- New measurement of $\pi\pi^0$ is crucial

For channels like $\pi\pi^0$ and $\pi^+\pi^+\pi^-$, fit rests heavily on Chiang '72

- No radiative corrections
- 6 BRs constrained by $\Sigma \text{BR} = 1$, correlations unavailable

Compared to PDG 2006, our fit:

- Uses preliminary results from KLOE, ISTRA+, and NA48/2
- Does not use $\text{BR}(\pi^0\pi^0e\nu)$ as a free parameter

Results of fit to K^\pm BRs, τ

30 input measurements:

5 older τ values in PDG

KLOE τ

KLOE BR($\mu\nu$)

KLOE $Ke3$, $K\mu3$ BRs

with dependence on τ

ISTRA+ BR($Ke3/\pi\pi^0$)

NA48/2 $Ke3/\pi\pi^0$, $K\mu3/\pi\pi^0$

E865 BR($Ke3/KDal$)

6 **Chiang '72** BRs

3 **old** BR($\pi\pi^0/\mu\nu$)

2 **old** BR($Ke3/2$ body)

3 $K\mu3/Ke3$ (2 **old**)

2 **old** + 1 **KLOE** results on 3π

1 **constraint: Σ BR = 1**

Parameter	Value	S
BR($\mu\nu$)	63.442(145)%	1.3
BR($\pi\pi^0$)	20.702(109)%	1.3
BR($\pi\pi\pi$)	5.5921(305)%	
BR($Ke3$)	5.120(38)%	1.6
BR($K\mu3$)	3.3853(205)%	1.2
BR($\pi\pi^0\pi^0$)	1.7592(234)%	1.1
τ_\pm	12.3840(213) ns	1.8

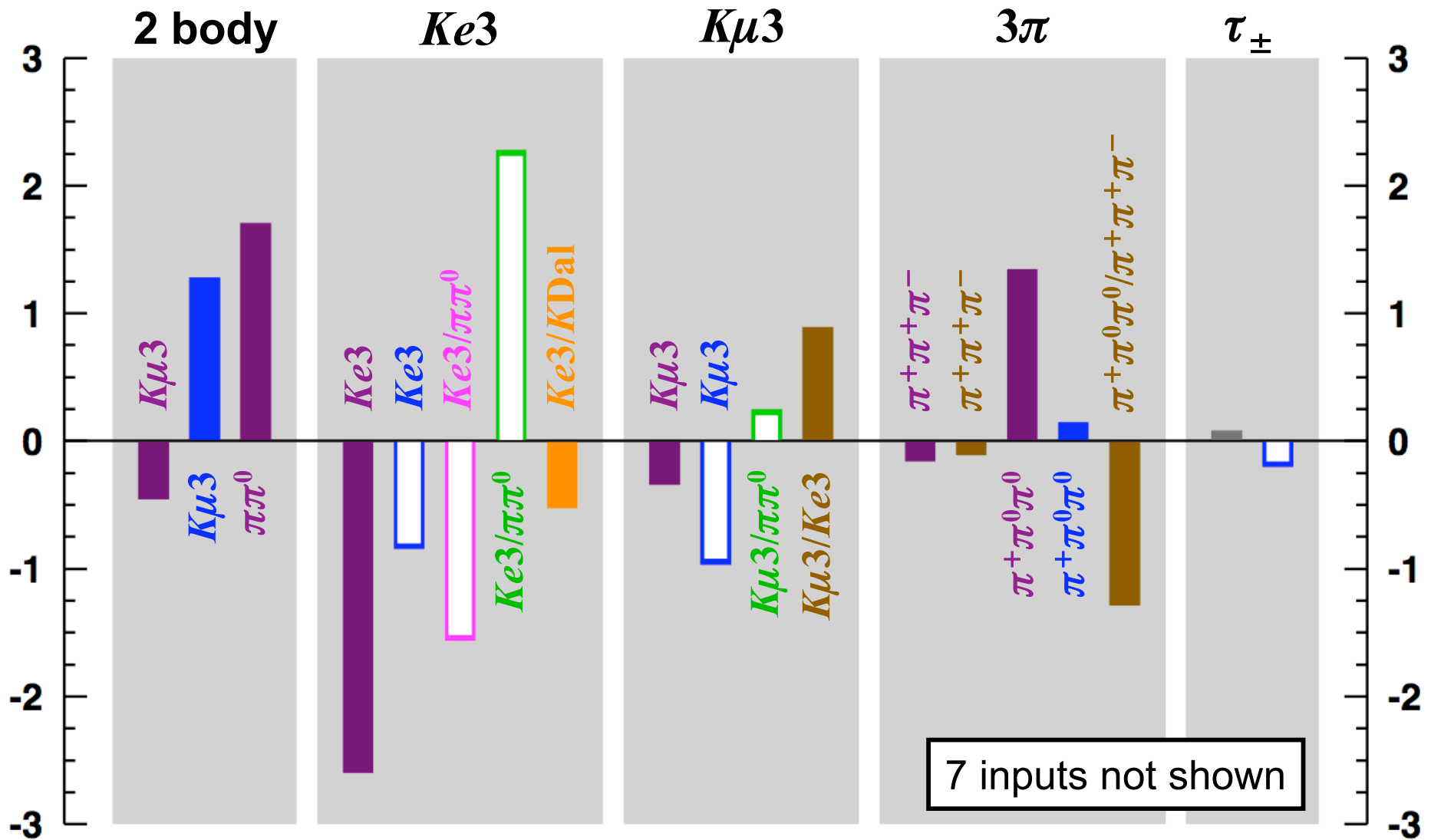
$\chi^2/ndf = 48.5/24$ (Prob = 0.22%)

compare PDG '06: 30.0/19 (5.2%)

Improves to $\chi^2/ndf = 31.2/20$ (5.3%)

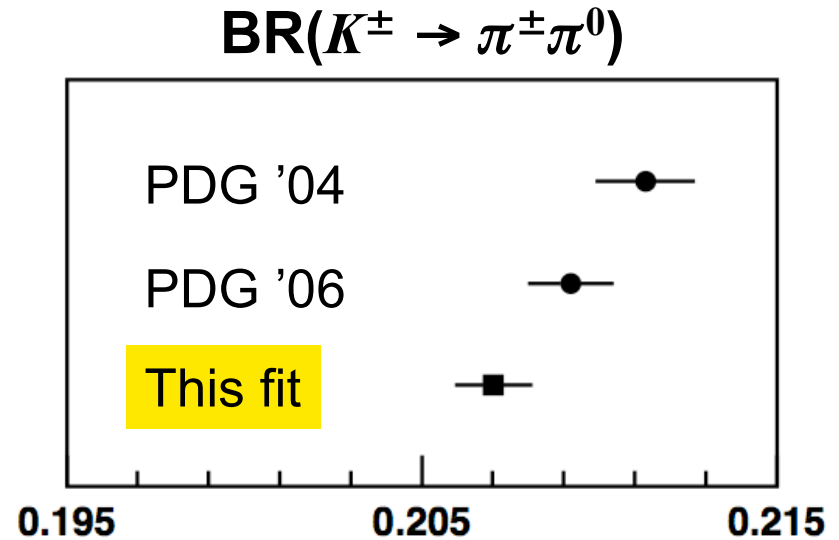
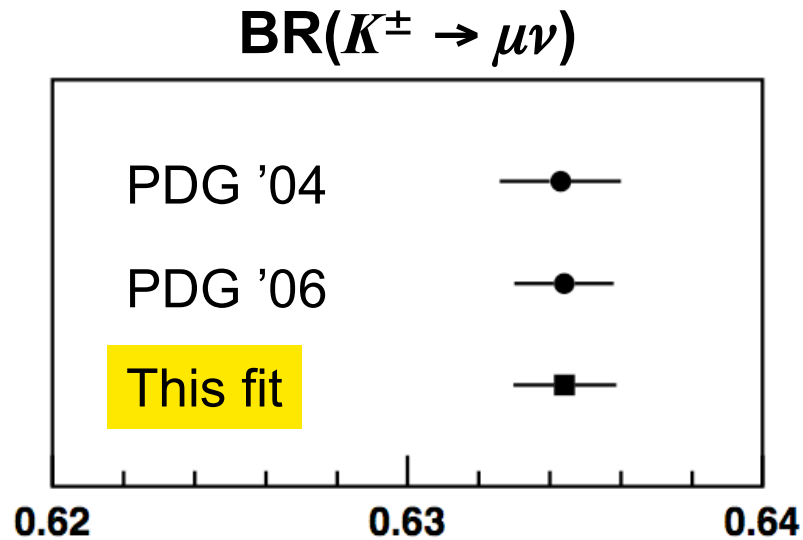
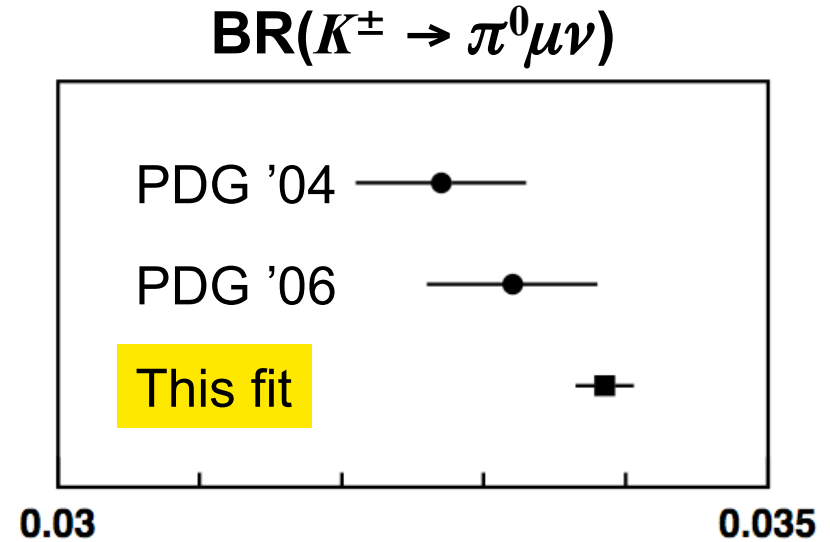
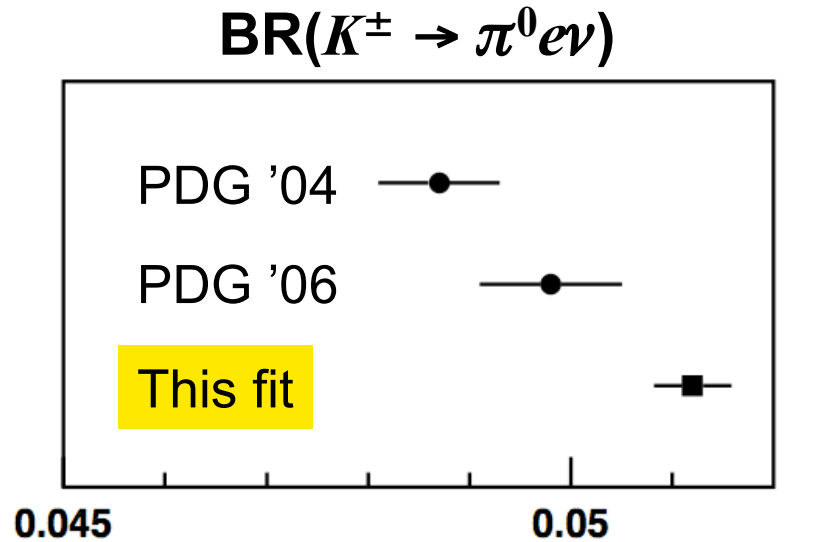
with no changes to central values or errors, if 5 older τ_+ measurements replaced by PDG avg (with $S = 2.1$)

Comparison: K^\pm BR fit vs. input data



Pulls from Chiang '72 KLOE ISTRA+ E865 NA48/2 others PDG avg

Evolution of K^\pm BRs



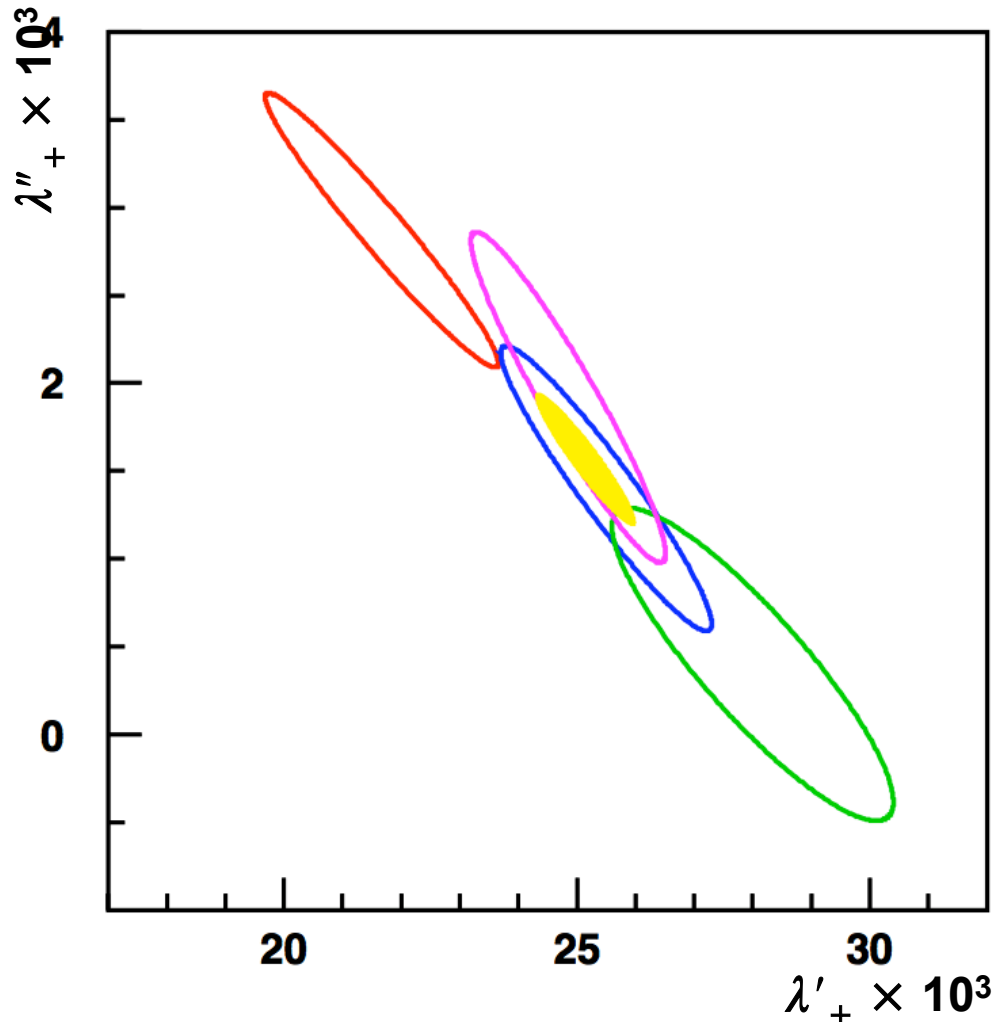
Current data on K_{l3} form-factor slopes

	Note	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
KTeV PRD 70 (2004)	$K_L e3$	21.7 ± 2.0	2.9 ± 0.8	
	$K_L \mu3$	17.0 ± 3.7	4.4 ± 1.5	12.8 ± 1.8
KLOE PLB 636 (2006)	$K_L e3$	25.5 ± 1.8	1.4 ± 0.8	
NA48 PLB 604 (2004)	$K_L e3$	28.0 ± 2.4	0.4 ± 0.9	
NA48 preliminary	$K_L \mu3$	16.8 ± 3.3	4.0 ± 1.4	9.1 ± 1.4
ISTRA+ PLB 581 (2004)	$K^- e3$	24.9 ± 1.7	1.9 ± 0.9	
ISTRA+ PLB 589 (2004)	$K^- \mu3$	23.0 ± 6.4	2.3 ± 2.3	17.1 ± 2.2

K_{e3} slopes: Quadratic fits

Slopes from

KTeV **KLOE** **ISTRA+** **NA48** **This fit**



Slope parameters $\times 10^3$

$$\lambda'_+ = 25.15 \pm 0.87$$

$$\lambda''_+ = 1.57 \pm 0.38$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 5.3/6 \text{ (51\%)}$$

Excellent compatibility
Significance of $\lambda''_+ > 4\sigma$

$$I(K^0 e3) = 0.15465(21)$$

$$I(K^+ e3) = 0.15901(22)$$

$K_{\mu 3}$ form-factor slopes

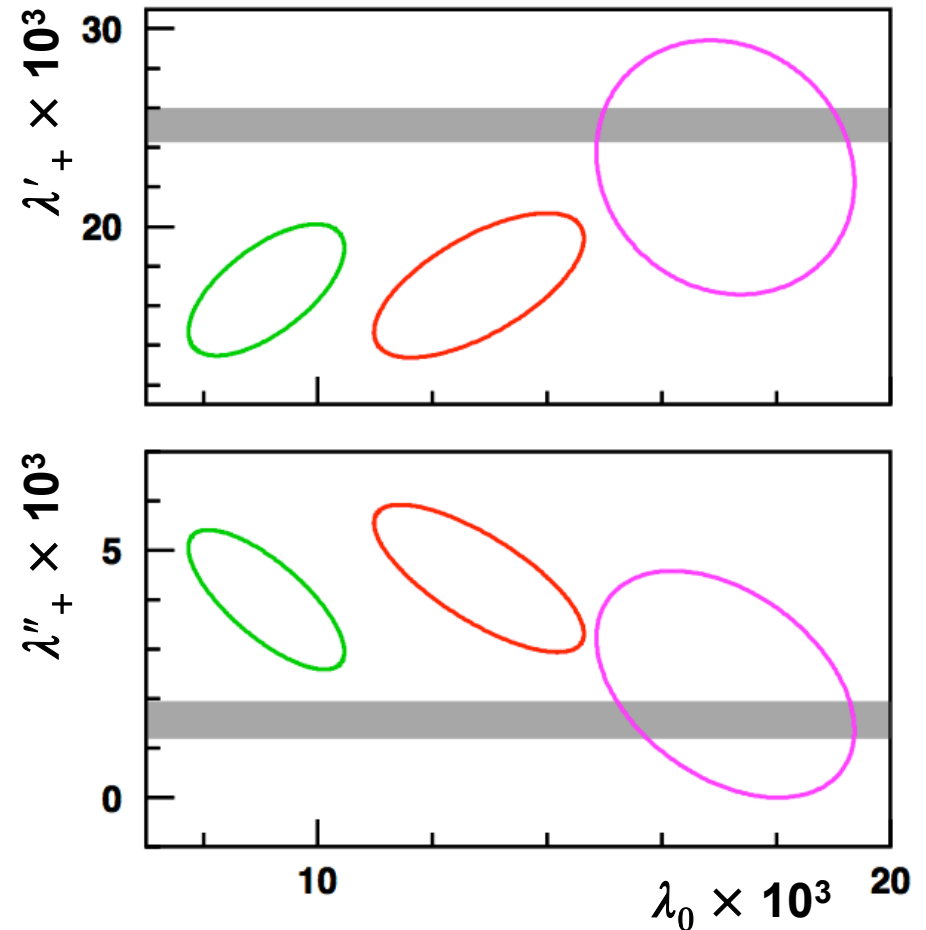
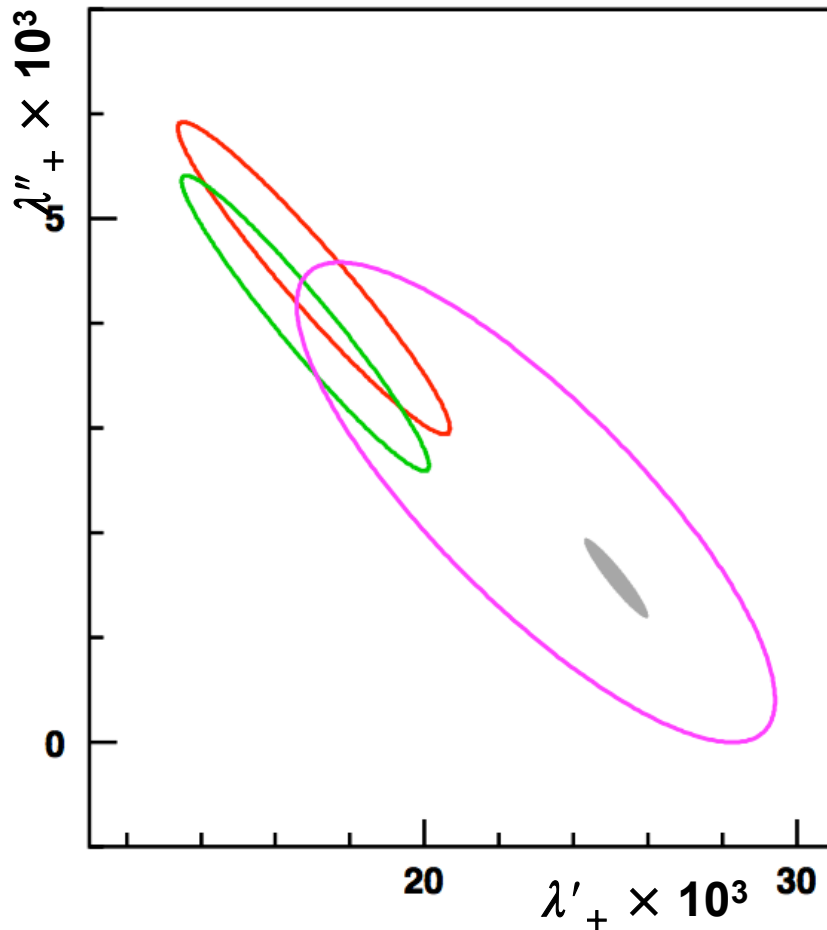
Slopes from

KTeV $K\mu 3$

ISTRA+ $K\mu 3$

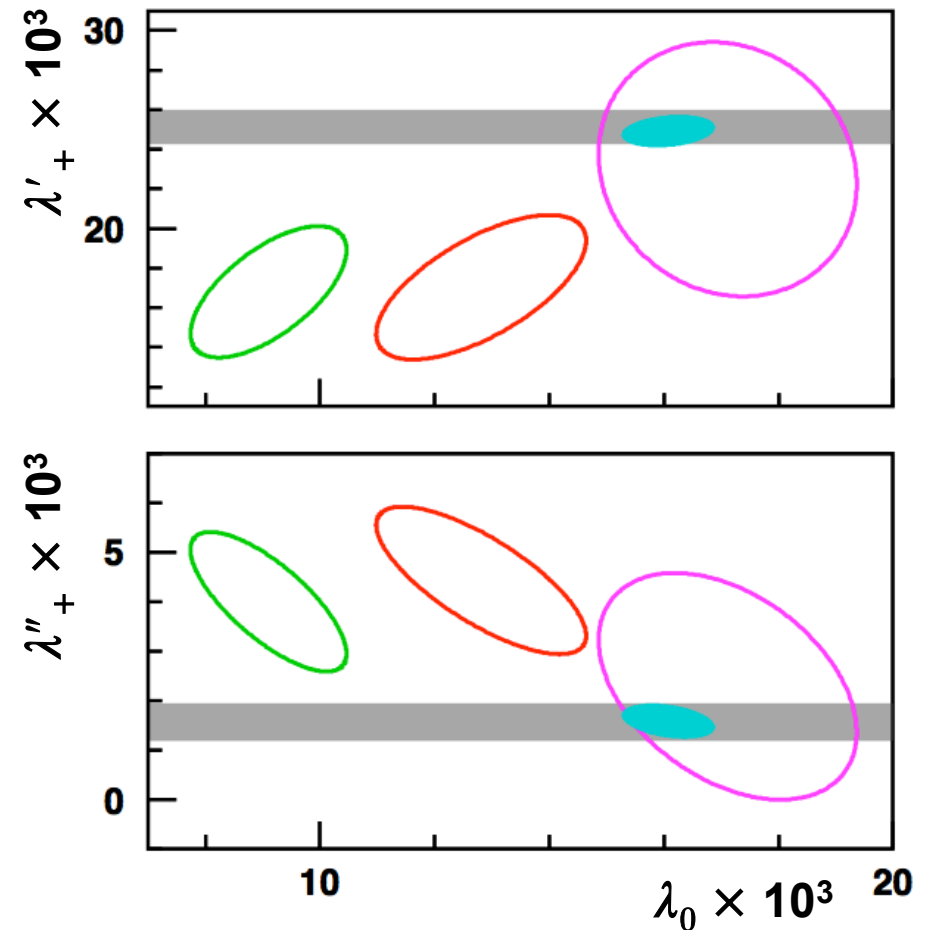
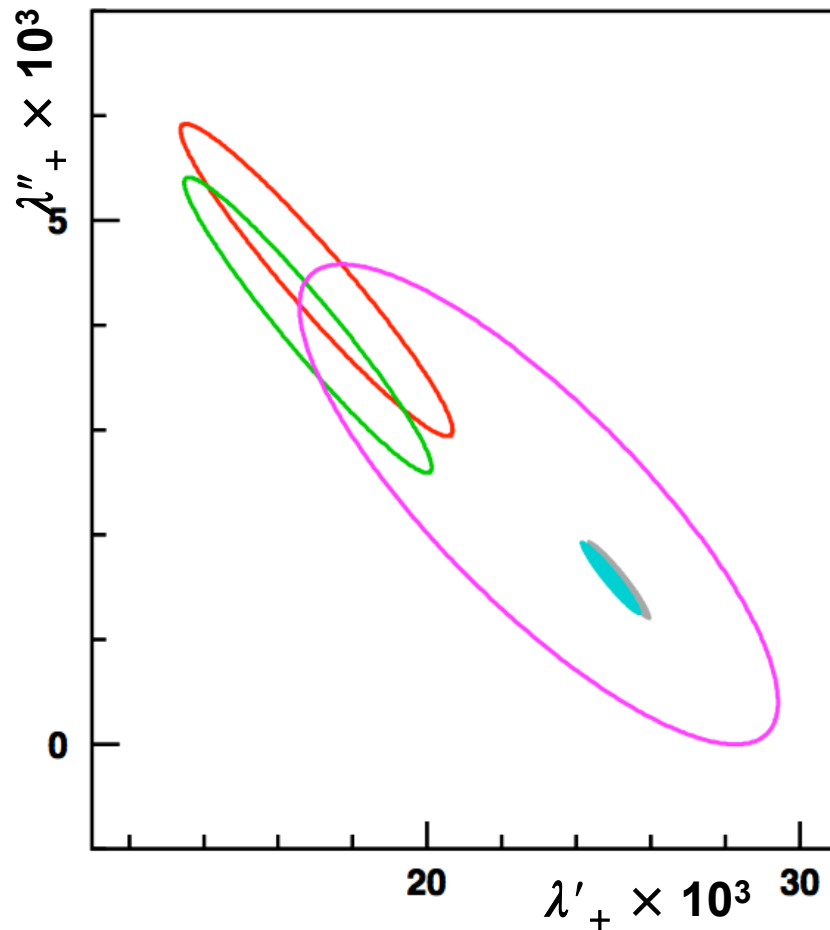
NA48 $K\mu 3$

Fit to $Ke 3$ data



Fit to K_{l3} form-factor slopes

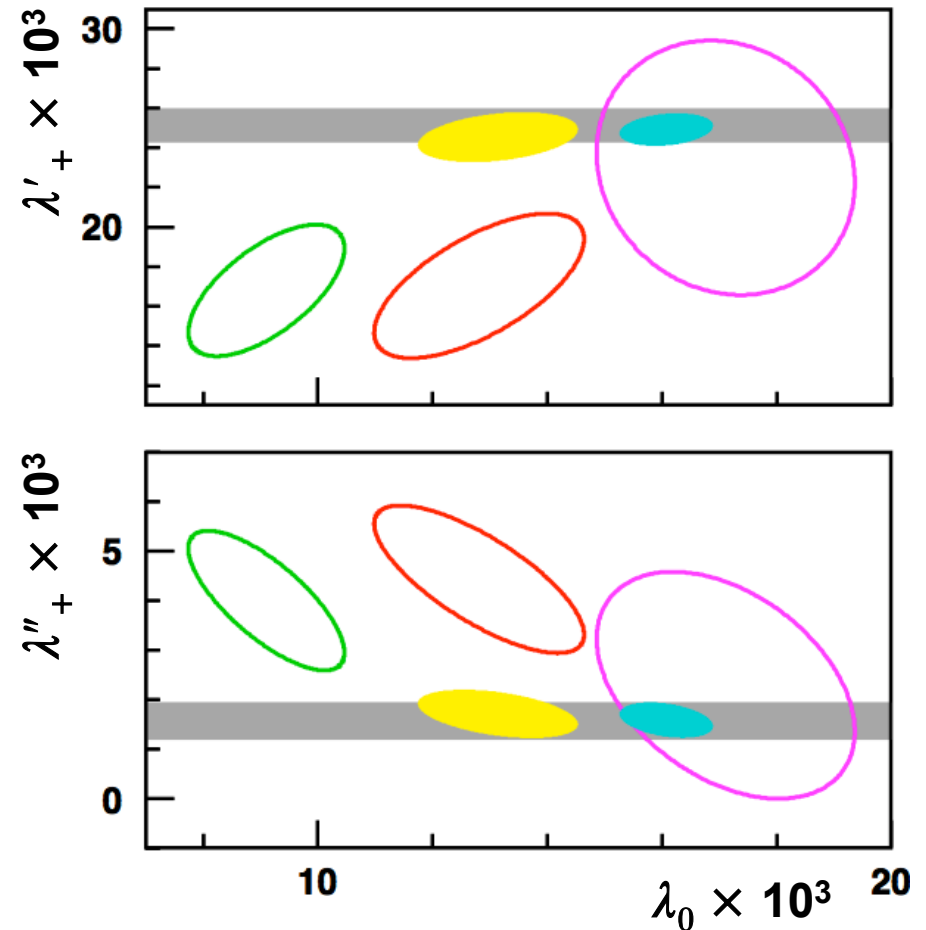
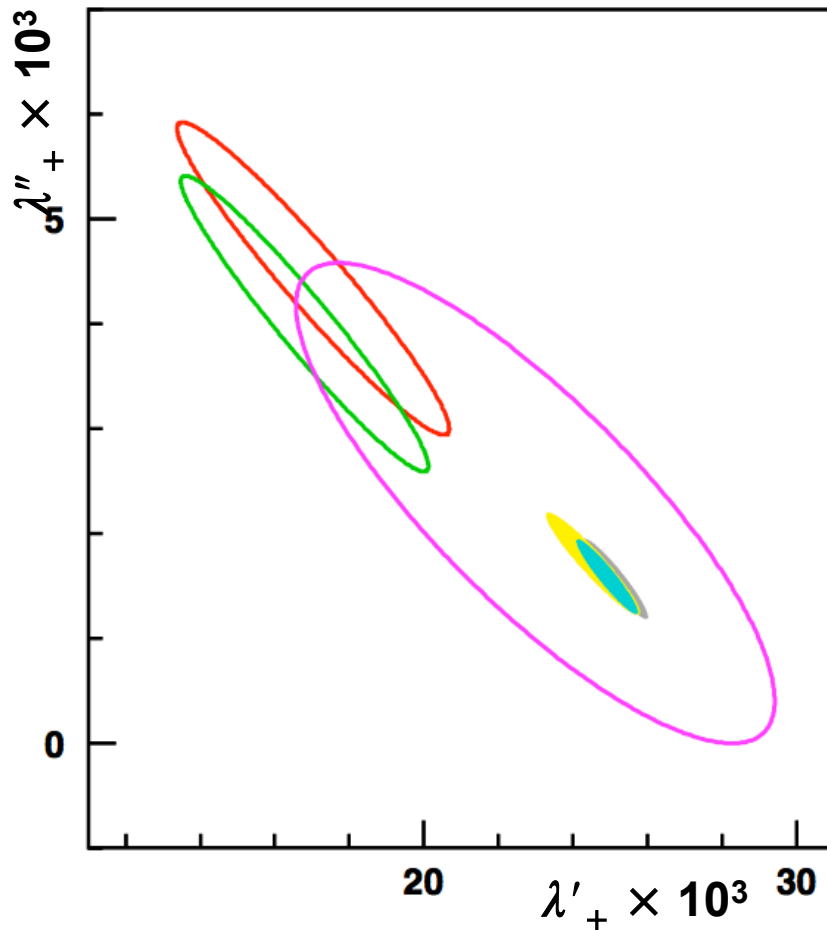
Slopes from **KTeV $K\mu 3$** **ISTRA+ $K\mu 3$** **NA48 $K\mu 3$** **Fit to $Ke 3$ data**



$Kl3$ fit, no NA48 $K\mu 3$: $\chi^2=11.9/9$ (21.7%)

Fit to K_{l3} form-factor slopes

Slopes from **KTeV $K\mu 3$** **ISTRA+ $K\mu 3$** **NA48 $K\mu 3$** **Fit to $Ke 3$ data**



$Kl3$ fit, no NA48 $K\mu 3$: $\chi^2=11.9/9$ (21.7%) **$Kl3$ fit, all data, $\chi^2=58/12$ (10^{-6})**

K_{l3} form-factor slopes: Fit results

Although compatibility poor, no *a priori* reason to exclude NA48 $K\mu 3$ data
 Inconsistency parameterized by scale factors for fit results

Slope parameters $\times 10^3$:

$$\lambda'_+ = 24.54 \pm 1.26 \quad S = 1.6$$

$$\lambda''_+ = 1.71 \pm 0.49 \quad S = 1.4$$

$$\lambda_0 = 13.14 \pm 1.40 \quad S = 2.2$$

$$\chi^2/\text{ndf} = 58/12 \quad (10^{-6})$$

Correlation coefficients:

	λ'_+	λ_0
λ'_+	-0.95	+0.31
λ''_+		-0.42

Integrals	
$I(K^0 e 3)$	0.15444(35)
$I(K^+ e 3)$	0.15879(35)
$I(K^0 \mu 3)$	0.10199(37)
$I(K^+ \mu 3)$	0.10493(38)

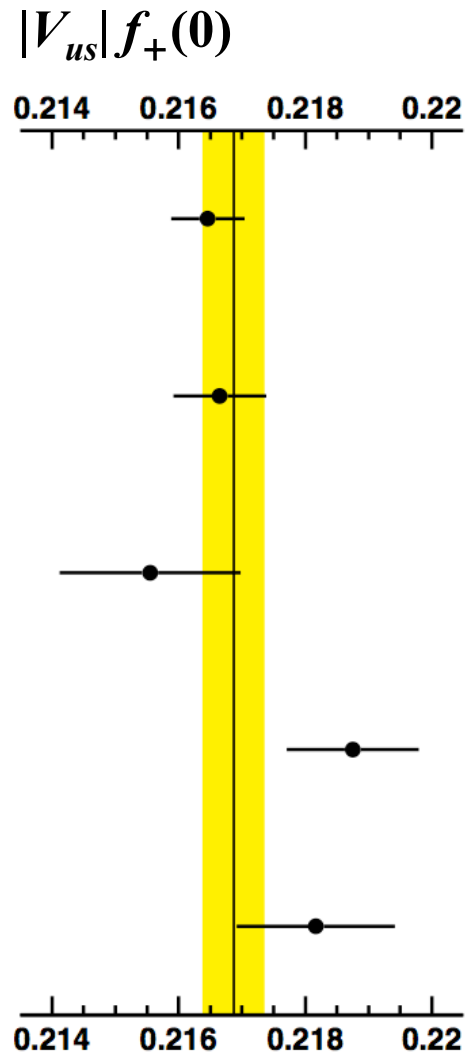
These results used to evaluate $|V_{us}| f_+(0)$ for all modes

$SU(2)$ and EM corrections

	$\Delta^{SU(2)}$	Δ^{EM}
$K^0 e3$	0	+0.55(10)%
$K^0 \mu3$	0	+0.95(15)%
$K^+ e3$	+2.31(22)%	-0.10(16)%
$K^+ \mu3$	+2.31(22)%	+0.20(40)%

- Δ^{EM} for full phase space - all measurements assumed fully inclusive
- $\Delta^{SU(2)}$ and $\Delta^{\text{EM}}(Ke3)$ are ChPT estimates from Cirigliano et al.
- $\Delta^{\text{EM}}(K^0\mu3)$ is hadronic-model estimate from Andre
 $\Delta^{\text{EM}}(K^0e3) = +0.65(15)$ from Andre agrees with value in table
- $\Delta^{\text{EM}}(K^+\mu3)$ a rough guess, with conservative error
 Currently the dominant contribution to error on $|V_{us}|f_+(0)$ from this mode

$|V_{us}| f_+(0)$ from K_{l3} data



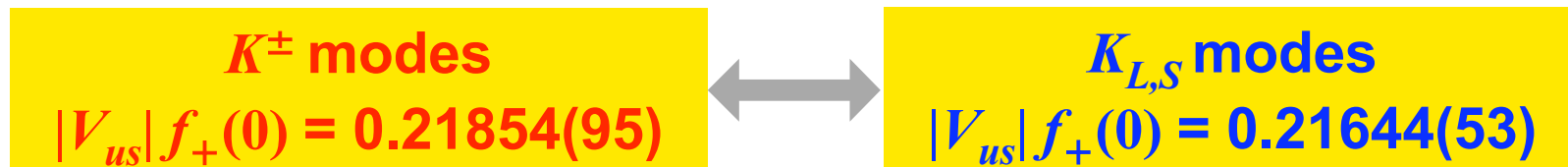
		% err	Approx. contrib. to % err from:			
			BR	τ	Δ	Int
$K_L e3$	$0.21646(59)$	0.27	0.09	0.19	0.10	0.11
$K_L \mu3$	$0.21665(71)$	0.33	0.12	0.19	0.15	0.18
$K_S e3$	$0.21555(143)$	0.66	0.65	0.02	0.10	0.11
$K^\pm e3$	$0.21875(104)$	0.47	0.37	0.07	0.27	0.11
$K^\pm \mu3$	$0.21817(125)$	0.57	0.30	0.07	0.45	0.18

Average: $|V_{us}| f_+(0) = 0.21686(49)$ $\chi^2/\text{ndf} = 5.0/4$ (29.0%)

$|V_{us}|f_+(0): K^\pm$ vs. $K_{L,S}$

Fit 5 modes with separate values of $|V_{us}|f_+(0)$ for K^\pm and $K_{L,S}$ modes

- Using results of overall fit to form-factor slopes
- With $SU(2)$ corrections for K^\pm modes [$\Delta^{SU(2)}_{\text{th}} = 2.31(22)\%$]



2.05 σ difference

$$\chi^2/\text{ndf} = 0.77/3 \text{ (86\%)} \quad \rho = 0.13$$

When fit performed without $SU(2)$ corrections for K^\pm modes, obtain an experimental value for $\Delta^{SU(2)}$

K^\pm modes, no $SU(2)$
 $|V_{us}|f_+(0) = 0.22359(85)$

$\Delta^{SU(2)}_{\text{exp}} = 3.30(43)\%$

K_{l3} data and lepton universality

For each state of kaon charge, we evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e 3}} \cdot \frac{I_{e 3} (1 + \delta_{e 3})}{I_{\mu 3} (1 + \delta_{\mu 3})} = \frac{[|V_{us}| f_+(0)]_{\mu 3, \text{obs}}^2}{[|V_{us}| f_+(0)]_{e 3, \text{obs}}^2} = \frac{(G_F^\mu)^2}{(G_F^e)^2}$$

K^\pm modes

$$r_{\mu e} = 0.9947(111)$$

Using 2004 BRs*

$$r_{\mu e} = 1.010(20)$$

$K_{L,S}$ modes

$$r_{\mu e} = 1.0022(62)$$

Using 2004 BRs*

$$r_{\mu e} = 1.052(15)$$

Average

(incl. $\rho = 0.11$)

$$r_{\mu e} = 1.0007(56)$$

Compare sensitivity from $\pi \rightarrow l\nu$ decays:

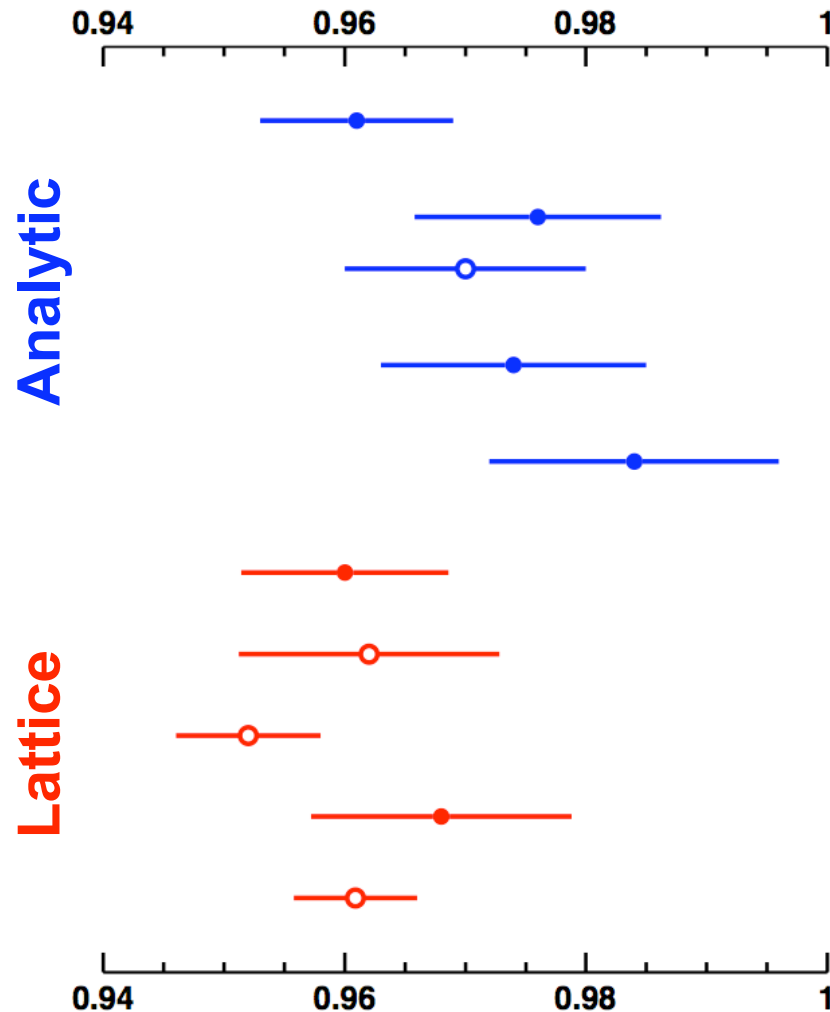
$$(r_{e\mu})_{\pi l 2} = 0.9966(30)$$

see Erler, Ramsey-Musolf '06

*Assuming current values for form-factor slopes

Evaluations of $f_+(0)$

$f_+(0)$



LR 84 quark model

BT 03 } ChPT + LR 84
Cir 05 }

JOP 04 ChPT + disp

C⁺ 05 ChPT + 1/N_c

SPQcdR 05 N_f = 0

FNAL/MILC/HPQCD 04 N_f = 2_{stag} + 1

JLQCD 05 N_f = 2

RBC 06 N_f = 2_{DW}

UKQCD/RBC 06 (revised) N_f = (2+1)_{DW}

Leutwyler & Roos estimate (LR 84) still widely used: $f_+(0) = \mathbf{0.961(8)}$

Lattice evaluations generally agree well with this value

$|V_{us}|$ from K_{l3} data and CKM unitarity

$$K_{l3} \text{ average: } |V_{us}| f_+(0) = 0.21686(49)$$

Leutwyler & Roos '84
 $f_+(0) = 0.961(8)$

Conventional choice for value of $f_+(0)$ until
a definitive evaluation becomes available

$$K_{l3} \text{ average: } |V_{us}| = 0.2257(20)$$

Marciano & Sirlin '06
 $|V_{ud}| = 0.97377(27)$

Average from $0^+ \rightarrow 0^+ \beta$ decays with recent
evaluation of EW radiative corrections

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0008(10)$$

Compatibility with unitarity -0.8σ

V_{ud} , V_{us} & $\text{BR}(K^\pm \rightarrow \mu^\pm \nu)$

Marciano '04

$$\frac{\Gamma(K^\pm \rightarrow \mu^\pm \nu(\gamma))}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\mu^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Uncertainty from SD virtual corrections \longrightarrow

MILC '06
preliminary

$$f_K/f_\pi = 1.208(2)^{(+7}_{-14)}$$

$$N_f = (2+1)_{\text{stag}}$$

Cancellation of lattice-scale uncertainties

KLOE
PLB 636 (2006)

$$\text{BR}(K^+ \rightarrow \mu^+ \nu(\gamma)) = 0.6366(17)$$

Uses $K^- \rightarrow \mu^- \nu$ to tag 2-body K decays

Counts $K^+ \rightarrow \mu^+ \nu$ from decay-momentum spectrum

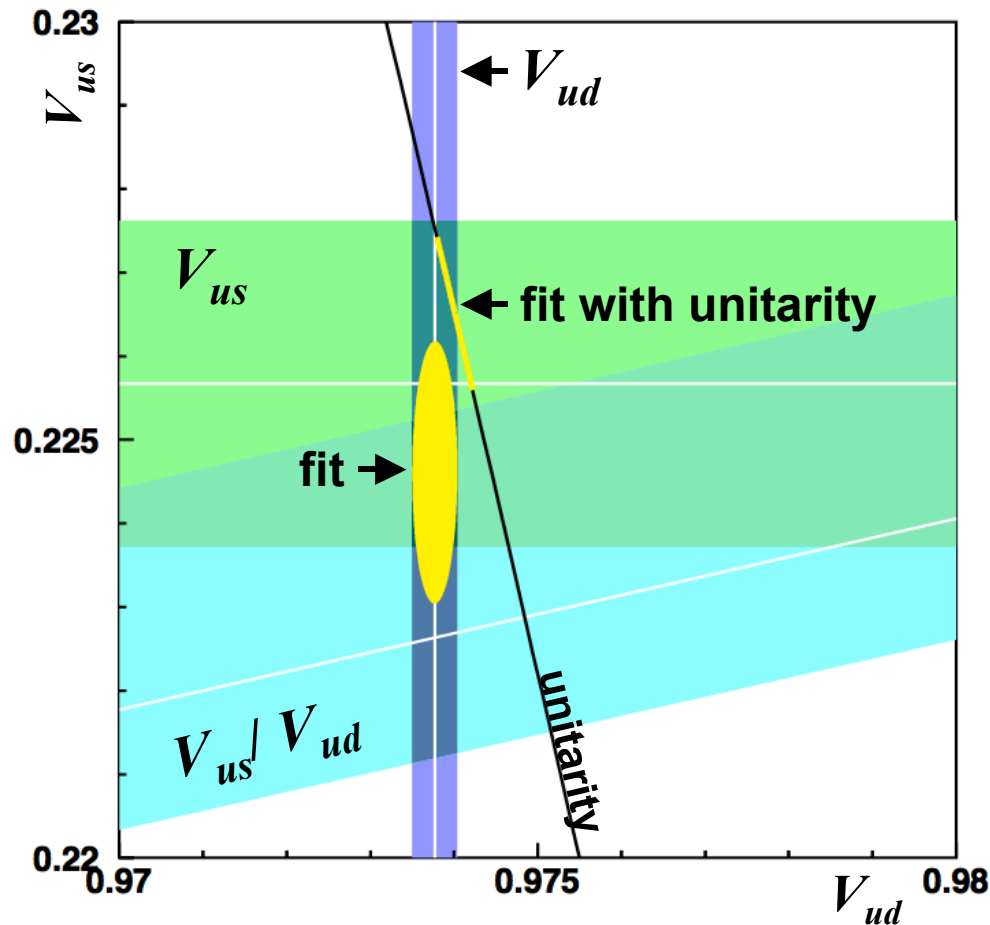
Use KLOE $\text{BR}(K^+ \rightarrow \mu^+ \nu(\gamma))$ instead of value from BR/lifetime fit:
Error slightly larger, but radiative contribution under better control

$$V_{us}/V_{ud} = 0.2286^{(+27}_{-15)}$$

V_{ud} , V_{us} & $\text{BR}(K^\pm \rightarrow \mu^\pm \nu)$

$f_+(0)$ from LR 84

$|V_{us}| = 0.2257(20)$ from $Kl3$



Fit results, no constraint:

$$\begin{aligned}
 V_{ud} &= 0.97377(27) \\
 V_{us} &= 0.2246(16) \\
 \chi^2/\text{ndf} &= 0.85/1 \quad (36\%)
 \end{aligned}$$

Fit results, unitarity constraint:

$$\begin{aligned}
 V_{ud} &= 0.97402(22) \\
 V_{us} &= 0.2265(9) \\
 \chi^2/\text{ndf} &= 3.04/2 \quad (22\%)
 \end{aligned}$$

Agreement with unitarity at 1.2σ

Summary and outlook

- Several new K^\pm measurements! None yet published
- BR/lifetime fits for both K_L and K^\pm have χ^2 probability $\sim 5\%$
For K^\pm , new measurements in normalization channels could help
- New NA48 $K\mu 3$ form-factor slopes disagree with other data
- Accuracy of $\Delta^{SU(2)}$, Δ^{EM} a significant issue for charged modes
Some evidence that $\Delta^{SU(2)}$ may be underestimated
- Experimental uncertainty on $|V_{us}|f_+(0)$ at 0.2% level
- Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$
- With $f_+(0) = 0.961(8)$, first-row unitarity test satisfied at $\sim 1\sigma$ level

$$K_B \text{ average: } |V_{us}|f_+(0) = 0.21686(49)$$

Additional information

Fit to K_L BRs, τ : Comparison to PDG

Parameter	This fit		PDG 2006	
	Value	S	Value	S
$Ke3$	0.40571(89)	1.4	0.4053(15)	2.1
$K\mu3$	0.27055(81)	1.3	0.2702(7)	
$3\pi^0$	0.19447(103)	1.4	0.1956(14)	1.9
$\pi^+\pi^-\pi^0$	0.12588(78)	1.5	0.1256(5)	
$\pi^+\pi^-$	$1.9860(74) \times 10^{-3}$	1.2	$1.976(8) \times 10^{-3}$	
$2\pi^0$	$8.603(54) \times 10^{-4}$	1.7	$8.69(4) \times 10^{-4}$	1.1
$\gamma\gamma$	$5.453(43) \times 10^{-4}$	1.1	$5.48(5) \times 10^{-4}$	1.2
τ	51.148(211) ns	1.1	51.14(21) ns	
18 measurements		17 measurements		
$\chi^2/\text{ndf} = 20.2/11$ (4.3%)		$\chi^2/\text{ndf} = 14.8/10$ (14.0%)		

Fit to K^\pm BRs, τ : Comparison to PDG

Parameter	This fit		PDG 2006	
	Value	S	Value	S
$\mu\nu$	63.442(145)%	1.3	63.44(14)%	1.2
$\pi\pi^0$	20.702(109)%	1.3	20.92(12)%	1.1
$\pi\pi\pi$	5.5921(305)%		5.590(31)%	1.1
$Ke3$	5.120(38)%	1.6	4.98(7)%	1.3
$K\mu3$	3.3853(205)%	1.2	3.32(6)%	1.2
$\pi\pi^0\pi^0$	1.7592(234)%	1.1	1.757(24)%	1.1
$\pi^0\pi^0e\nu$	Not in fit		$2.2(4) \times 10^{-5}$	
τ	12.3840(213) ns	1.8	12.385(24) ns	2.1
30 measurements		26 measurements		
$\chi^2/\text{ndf} = 48.5/24$ (0.22%)		$\chi^2/\text{ndf} = 30.0/19$ (5.2%)		

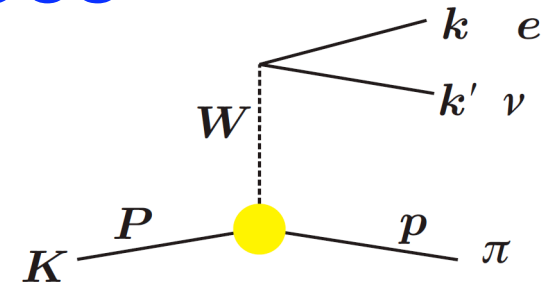
$K_{\mu 3}$ form-factor slopes

Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times [\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha]$$

$f_-(t)$ term only important for $K_{\mu 3}$.

For $K_{\mu 3}$, use $f_+(t)$ and $f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_{\pi^+}^2} f_-(t)$



For V_{us} , need integral over phase space of squared matrix element

Expand form factor:

Linear: $\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} [t/m_{\pi^+}^2]$

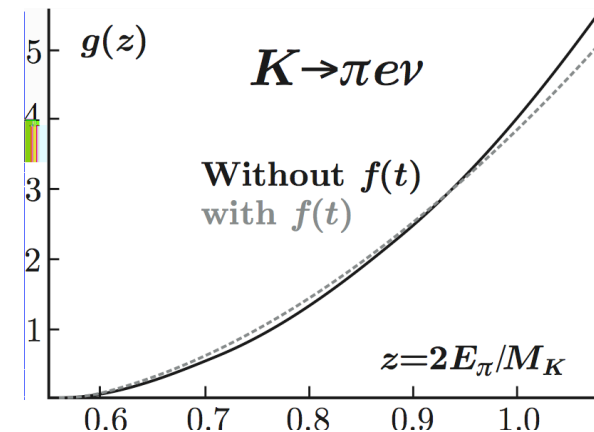
Quadratic: $\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} [t/m_{\pi^+}^2] + 1/2 \lambda''_{+,0} [t/m_{\pi^+}^2]^2$

Fits to t -distribution give poor sensitivity to quadratic terms

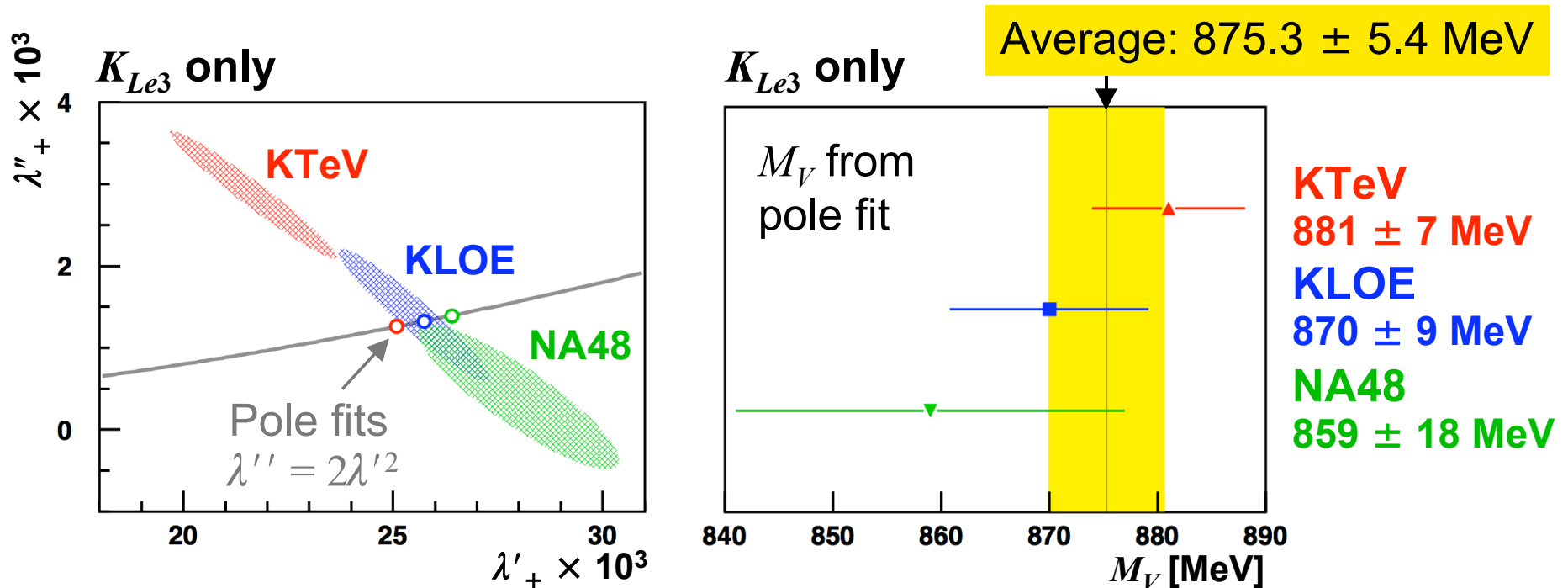
Polar fit:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t} \quad \lambda' = (m_{\pi^+}/M)^2$$

$$\lambda'' = 2\lambda'^2$$



K_{e3} slopes: Quadratic vs. pole fits



K_{Le3} data	$P(\chi^2)$ fit Quad	K_{Le3} integral Quad	$P(\chi^2)$ fit Pole	K_{Le3} integral Pole	Difference
KTeV	54%	0.15378(51)	43%	0.15449(25)	+0.46%
KLOE	92%	0.15472(42)	92%	0.15489(33)	+0.11%
K_{Le3} avg		0.15456(29)		0.15469(19)	+0.09%

Beyond quadratic and pole fits

Hill, PRD 74 (2006):

Power series expansion based on analyticity of form factors

Constraints from crossing symmetry, e.g., bounds on $f_+(t)$ from $\tau \rightarrow K\pi\nu$ data

Rigorous estimate of error from truncation of series expansion

KTeV

PRD 70 (2004)

Refit $Ke3$ data using Hill parameterization

$$I(K^0e3) = 0.15392(48)_{\text{exp}}(6)_{\text{th}}$$

Bernard et al., PLB 638 (2006):

Dispersion relation for $\ln f_0(t)$ subtracted at $t = 0$ and $t = m_K^2 - m_\pi^2$, giving:

$$f_0(t) = \exp \left[\frac{t}{m_K^2 - m_\pi^2} (\ln C - G(t)) \right] \quad G(t) \text{ evaluated using } K\pi \text{ scattering data and given as a polynomial}$$

1 fit parameter: $\ln C = \ln f_0(m_K^2 - m_\pi^2)$

Value of $\ln C$ a test for right-handed quark currents

NA48

preliminary

From new analysis of $K\mu3$ form-factor slopes

$$\ln C = 0.1533(138)$$

$|V_{us}|f_+(0): K_{L,S} \text{ vs. } K^\pm$

Using separate fit results for form-factor slopes:

K^\pm only, $e3$ and $\mu3$:

ISTRA+ $e3, \mu3$

$$\begin{aligned} \lambda'_+ &= 24.80 \pm 1.54 \\ \lambda''_+ &= 1.94 \pm 0.86 \\ \lambda_0 &= 16.76 \pm 1.20 \\ \chi^2/\text{ndf} &= 0.100/2 \text{ (95\%)} \\ I(Ke3) &= 0.159097(319) \\ I(K\mu3) &= 0.105949(298) \end{aligned}$$

K_L only, $e3$ and $\mu3$:

KTeV avg, KLOE $e3$, NA48 $e3, \mu3$

$$\begin{aligned} \lambda'_+ &= 25.26 \pm 1.96 \text{ (} S=1.9\text{)} \\ \lambda''_+ &= 2.20 \pm 0.72 \text{ (} S=1.7\text{)} \\ \lambda_0 &= 11.14 \pm 1.61 \text{ (} S=2.1\text{)} \\ \chi^2/\text{ndf} &= 31.2/7 \text{ (0.01\%)} \\ I(Ke3) &= 0.15413(55) \\ I(K\mu3) &= 0.10140(49) \end{aligned}$$

With $SU(2)$ corrections for K^\pm modes:

$$\begin{aligned} |V_{us}|f_+(0) &= \mathbf{0.21804(94)} \\ \chi^2/\text{ndf} &= 1.39/1 \text{ (23.9\%)} \end{aligned}$$

$$\begin{aligned} |V_{us}|f_+(0) &= \mathbf{0.21676(61)} \\ \chi^2/\text{ndf} &= 1.26/2 \text{ (53\%)} \end{aligned}$$

With NO $SU(2)$ corrections for K^\pm modes:

$$|V_{us}|f_+(0) = \mathbf{0.22308(84)}$$

$$\Delta^{SU(2)}_{\text{exp}} = \mathbf{2.92(48)\%}$$

Results without NA48 $K\mu 3$ slopes

Slope parameters $\times 10^3$
$\lambda'_+ = 24.92 \pm 0.83$
$\lambda''_+ = 1.59 \pm 0.36$
$\lambda_0 = 16.07 \pm 0.82$
$\chi^2/\text{ndf} = 11.9/9$ (21.7%)

Integrals	
$I(K^0 e 3)$	0.15455(21)
$I(K^+ e 3)$	0.15890(21)
$I(K^0 \mu 3)$	0.10268(21)
$I(K^+ \mu 3)$	0.10565(21)

$ V_{us} f_+(0)$	
$K_L e 3$	0.21638(56)
$K_L \mu 3$	0.21592(62)
$K_S e 3$	0.21547(142)
$K^+ e 3$	0.21868(102)
$K^- \mu 3$	0.21743(120)

$KL3$ average
 $|V_{us}|f_+(0) = 0.21655(43)$
 $\chi^2/\text{ndf} = 6.1/4$ (19.0%)

$$K^+ - K^0 \text{ diff.: } 2.1\sigma$$

$$\Delta^{SU(2)}_{\text{exp}} = 3.31(43)\% \quad r_{\mu e} = 0.9946(51)$$

$|V_{us}| = 0.2253(19)$
Unitarity test: -1.0σ