

KAON PHYSICS: RECENT EXPERIMENTAL PROGRESS

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Numerous recent measurements of kaon decays are described, with an emphasis on results offering constraints on chiral perturbation theory calculations. An up-to-date estimate of $|V_{us}|f_+(0)$ based on semileptonic kaon decay rates is presented.

Keywords: Kaon decays, chiral perturbation theory, CKM matrix, V_{us} .

1. Introduction

The last three years have been marked by very rapid progress in experimental studies of kaon decays. This review is an attempt to summarize those results that offer direct comparison for the predictions of chiral perturbation theory (ChPT), including in particular the determination of the $I = 0, 2$ $\pi\pi$ scattering lengths from $K \rightarrow 3\pi$ and K_{e4} data, recent results on a selection of rare and radiative kaon decays, and the determination of $|V_{us}|f_+(0)$ from $K_{\ell 3}$ rates.

2. Kaon Decays and the $\pi\pi$ Scattering Lengths

2.1. $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays

In the $M_{\pi^0\pi^0}^2$ distribution for 23×10^6 $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays from 2003 data, NA48/2 observes a cusp at $M_{\pi^0\pi^0}^2 = 4m_{\pi^+}^2$.¹ Cabibbo² has explained the cusp in terms of the interference between two amplitudes: the direct amplitude illustrated in Fig. 1a, and the rescattering amplitude illustrated in Fig. 1b. The latter amplitude is proportional to $a_0 - a_2$, the difference between the $I = 0, 2$ $\pi\pi$ scattering lengths; this quantity can thus be determined from fits to the $M_{\pi^0\pi^0}^2$ distribution. In the NA48/2 analysis, the fits are based on an extension of Cabibbo's original treatment, which includes contributions from the relevant two-loop diagrams.³ The theoretical

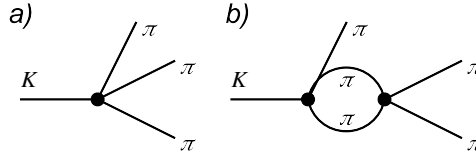


Fig. 1. $K \rightarrow 3\pi$ amplitudes used in the description of the cusp at $M_{\pi^0\pi^0}^2 = 4m_{\pi^+}^2$ in the treatment of Ref. 2.

contribution to the uncertainty on the determination of $a_0 - a_2$ is estimated to be 5%. The NA48/2 fits account for a possible contribution from pionium formation in the vicinity of the cusp, but the seven bins nearest the position of the cusp are excluded from the fit to avoid possible bias arising from the lack of radiative corrections in the model used. NA48/2 obtains a fit to the $M_{\pi^0\pi^0}^2$ distribution with $\chi^2/\text{ndf} = 146/139$ (32.5%) giving $(a_0 - a_2)m_{\pi^+} = 0.268(10)_{\text{st}}(4)_{\text{sy}}(13)_{\text{th}}$. This result compares favorably to the prediction of Colangelo *et al.*,⁴ based on a matching procedure between representations of the $\pi\pi$ scattering amplitude from $\mathcal{O}(p^6)$ ChPT calculations and from the Roy equations as constrained by $\pi\pi$ scattering data at higher energies: $(a_0 - a_2)m_{\pi^+} = 0.265(4)$. NA48/2 statistics will increase by a factor of five when all 2003–2004 data are analyzed, which should allow the experimental uncertainty on $a_0 - a_2$ to be reduced to a level comparable to the uncertainty on the predicted value. A precise comparison will require a reduction of the uncertainty arising from the theoretical description of the cusp. Recent work on a nonrelativistic effective field theory treatment by Colangelo *et al.*^{5,6} is promising in this regard. This scheme provides consistent power counting and allows electromagnetic corrections to be included in a standard way. NA48/2 is currently working on fits to the cusp using this treatment.⁷

2.2. K_{e4} decays

K_{e4} decays ($K \rightarrow \pi\pi e\nu$) also provide an opportunity to study the $\pi\pi$ interaction down to threshold. Fits to the kinematic distributions for K_{e4} decays provide sensitivity to the axial form factors F and G , and the vector form factor H . Recent work^{8,9} makes use of the parameterization of Ref. 10. In this parameterization, the form factors are expanded in partial waves, *e.g.*, $F = F_s e^{i\delta_0^0} + F_p e^{i\delta_1^1} \cos\theta_\pi$, where θ_π is the angle between the π^+ momentum in the $\pi\pi$ system and the momentum of the $\pi\pi$ system in the kaon rest frame, and $\delta_{L=0}^{I=0}$ and $\delta_{L=1}^{I=1}$ are the $\pi\pi$ phase shifts. The coefficients

F_s , F_p , etc., are expanded in powers of $q^2 = (M_{\pi\pi}^2 - 4m_\pi^2)/m_\pi^2$, so that, *e.g.*, $F_s = f_s + f'_s q^2 + f''_s q^4 + f_e(M_{e\nu}^2/4m_\pi^2) + \dots$. Fits to the kinematic distributions thus allow determination of the parameters f_s , f'_s , etc., which can be used to constrain ChPT couplings (F and G have been calculated to two loops in ChPT¹¹), as well as values for $\delta_0^0 - \delta_1^1$ in bins of $M_{\pi\pi}$.

The Roy equations can be used to relate the $\pi\pi$ phase shifts to a_0 and a_2 (see, *e.g.*, Ref. 12), but measurements of $\delta_0^0 - \delta_1^1$ provide very little constraint on the value of a_2 . However, consistency with the Roy equations and $\pi\pi$ scattering data above 0.8 GeV restricts possible values of a_0 and a_2 to a region in the a_0 - a_2 plane often called the universal band (UB). The UB constraint can be used to eliminate a_2 in fits to K_{e4} data.

The most recent published data on $K^\pm \rightarrow \pi^+\pi^-e\nu$ decays are from BNL E865.⁸ Using a sample of 388k $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decays, E865 measures $\text{BR} = 4.11(11) \times 10^{-5}$. For the kinematic analysis, the data are divided into 28800 bins (six in $M_{\pi\pi}^2$). For each bin in $M_{\pi\pi}^2$, the expansion parameters for the form factors F , G , and H are obtained, as well as values for $\delta_0^0 - \delta_1^1$. A separate fit for a_0 is performed without binning the data in $M_{\pi\pi}^2$. From this latter fit, with the application of the UB constraint, E865 obtains $a_0 m_{\pi^+} = 0.228(12)_{\text{st}}(4)_{\text{sy}}({}^{+12}_{-16})_{\text{th}}$, which compares well with the prediction from Ref. 4, $a_0 m_{\pi^+} = 0.220(5)$.

NA48/2 has recently obtained preliminary results on $K^\pm \rightarrow \pi^+\pi^-e\nu$ decays as well.^{7,9} The NA48/2 K_{e4}^\pm sample includes 235k K^+ and 135k K^- events. The data are divided into 15000 bins (ten in $M_{\pi\pi}$). Like E865, NA48/2 measures the expansion parameters for the form factors F , G , and H , as well as $\delta_0^0 - \delta_1^1$, in bins of $M_{\pi\pi}$. With the UB constraint, NA48/2 obtains $a_0 = 0.256(8)_{\text{st}}(7)_{\text{sy}}(18)_{\text{th}}$, which is in marginal ($\sim 1.7\sigma$) agreement with the prediction from Ref. 4. The data from the two experiments are compared in Fig. 2.

For either experiment, the experimental and theoretical contributions to the error on a_0 are 4–5% and 5–7%, respectively. In addition to the UB constraint, E865 uses the tighter constraint on a_0 vs. a_2 from Refs. 13 and 14 to obtain a value with a greatly reduced theoretical uncertainty: $a_0 m_{\pi^+} = 0.216(13)_{\text{st}}(4)_{\text{sy}}(2)_{\text{th}}$. NA48/2 also expects to obtain values with smaller theoretical uncertainties using alternate treatments.

NA48/2 has also obtained preliminary results in the channel $K^\pm \rightarrow \pi^0\pi^0e^\pm\nu$.⁹ In this channel, Bose symmetry considerations imply that only the $L = 0$ partial wave contributes. With 37k events from 2003 and 2004 data, NA48/2 has obtained values for the form-factor parameters f'_s/f_s and f''_s/f_s that are consistent with the results obtained for $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$.

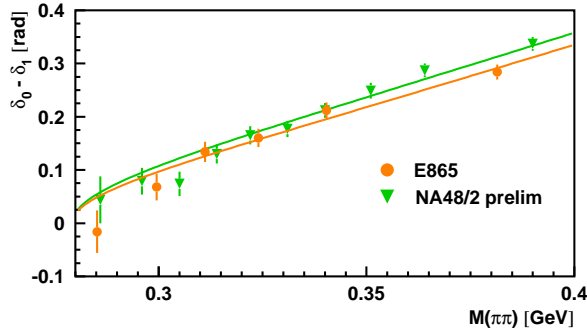


Fig. 2. Measurements of $\delta_0^0 - \delta_1^1$ from E865⁸ and NA48/2^{7,9} (NA48/2 data are preliminary). The solid curves illustrate the Roy equation solutions as parameterized in Ref. 12, with the values for a_0 quoted by each experiment. Adapted from a plot in Ref. 9.

3. Rare and Radiative Kaon Decays

3.1. Radiative $K_{\ell 3}$ decays

The amplitudes for radiative kaon decays can be divided into two components: an internal bremsstrahlung (IB) amplitude arising from radiation from external charged particles, and a direct-emission (DE), or structure-dependent (SD), amplitude arising from radiation from intermediate hadronic states. The components are isolated by analyzing the energy spectrum of the radiative photon; the study of the SD component can provide information about intermediate hadronic states in the decay.

For radiative $K_{\ell 3}$ decays, a convenient observable is

$$R_{\ell 3\gamma}(E_{\min}, \theta_{\min}) = \frac{\text{BR}(K_{\ell 3\gamma}, E_\gamma > E_{\min}, \theta_{\ell\gamma} > \theta_{\min})}{\text{BR}(K_{\ell 3(\gamma)})},$$

where E_{\min} and θ_{\min} are cuts on the energy of the radiated photon and on its angle of emission (in the kaon CM frame) with respect to the momentum of the lepton, and $\text{BR}(K_{\ell 3(\gamma)})$ signifies the inclusive branching ratio. These cuts are dictated by experimental necessity or by convention, bearing in mind that the IB amplitudes diverge for $E_\gamma \rightarrow 0$.

For $K_L \rightarrow \pi\mu\nu\gamma$, there is a recent measurement from KTeV:¹⁵ $R_{\mu 3\gamma}^0(E_\gamma > 30 \text{ MeV}) = 0.209(9)\%$. This is in good agreement with the earlier result from NA48:¹⁶ $R_{\mu 3\gamma}^0 = 0.208(26)\%$, as well as the predictions of Ref. 17. For $K_L \rightarrow \pi e\nu\gamma$, the situation is more complicated. Precise ChPT predictions are available—Gasser *et al.*¹⁸ have performed an $\mathcal{O}(p^6)$ ChPT calculation to obtain $R_{e 3\gamma}^0(E_\gamma > 30 \text{ MeV}, \theta_{e\gamma} > 20^\circ) = 0.96(1)\%$. On the experimental side, in 2001 KTeV published¹⁹ the re-

sult $R_{e3\gamma}^0 = 0.908(8)_{(-12)}^{(+13)}\%$; the data have recently been reanalyzed using more restrictive cuts that provide better control over systematic effects, but which reduce the statistics by a factor of three. The new KTeV result¹⁵ is $R_{e3\gamma}^0 = 0.916(17)\%$. Both results are at variance with the value obtained in Ref. 18. On the other hand, NA48²⁰ obtains $R_{e3\gamma}^0 = 0.964(8)_{(-9)}^{(+11)}\%$, which is in good agreement with Ref. 18. KLOE has announced a preliminary value²¹ based on 20% of its total data set: $R_{e3\gamma}^0 = 0.92(2)(2)\%$. As the errors on this result are reduced, KLOE will be able to comment on the apparent discrepancy between the KTeV and NA48 results.

For $K^\pm \rightarrow \pi^0 \mu^\pm \nu \gamma$ decays, experimental results have become available only recently. KEK E470 recently published²² the measurement $\text{BR}(K_{\mu 3\gamma}^\pm, E_\gamma > 30 \text{ MeV}, \theta_{\mu\gamma} > 20^\circ) = 2.4(5)(6) \times 10^{-5}$, obtained using stopped K^+ s. The ISTRA+ experiment at Protvino has used in-flight K^- decays to obtain measurements^{23,24} of $\text{BR}(K_{\mu 3\gamma}^\pm)$ for $5 < E_\gamma < 30 \text{ MeV}$ and $30 < E_\gamma < 60 \text{ MeV}$. All of these results are in good agreement with $\mathcal{O}(p^4)$ ChPT estimates.^{25,26} For $K^\pm \rightarrow \pi^0 e^\pm \nu \gamma$ decays, ISTRA+ has the preliminary result²⁴ $\text{BR}(K_{e 3\gamma}^\pm, E_\gamma > 30 \text{ MeV}, \theta_{e\gamma} > 20^\circ) = 3.05(2) \times 10^{-4}$, which is also in agreement with ChPT estimates.

3.2. Radiative $K \rightarrow \pi\pi$ decays

For the decay $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$, the IB component is suppressed by the $\Delta I = 1/2$ rule, leading to a relative enhancement of the DE component. The main contributions to the DE component are the M1 amplitude, arising from the chiral anomaly, and the E1 component. Both appear at $\mathcal{O}(p^4)$ in the chiral expansion. Interference between the M1 and E1 components vanishes in measurements inclusive with respect to the polarization of the radiative photon, but interference between the IB and E1 components is in principle observable. The decay is analyzed by looking at the variables T^* , the kinetic energy of the π^\pm in the K^\pm CM frame, and $W \equiv (p_K \cdot p_\gamma)(p_{\pi^+} \cdot p_\gamma)/(m_{K^+} m_{\pi^+})^2$. For a selected interval in T^* , chosen to reduce background from other K^\pm decays with π^0 s, the distribution in W is obtained. This can be written as the sum of a term from IB, a term from DE, and a term from IB/E1 interference. Fits to the W spectrum allow these components to be isolated.

Two K_{stop}^+ experiments have recently measured the DE BR for $55 < T^* < 90 \text{ MeV}$: the 2005 preliminary value from BNL E787²⁷ is $3.9(5)_{(-4)}^{(+3)} \times 10^{-6}$; the newly published value from KEK E470²⁸ is $3.8(8)(7) \times 10^{-6}$. Both measurements are based on samples of order 10k decays. No interference term is included in the fit in either case—both measurements are improve-

ments of previous analyses in which no evidence for an interference term was found. On the other hand, NA48/2, has a preliminary result based on 124k in-flight decays from the 2003 data⁷ obtained with an interference term included in the fit. The weights of the DE and interference terms are 3.35(35)(25)% and $-2.67(81)(73)\%$. The errors on these two values are highly correlated, but the interference term is observed with 3σ significance. The NA48/2 measurement is for $0 < T^* < 80$ MeV because of trigger considerations. For comparison with the K_{stop}^+ experiments, NA48/2 fits with no interference term and extrapolates to $55 < T^* < 90$ MeV, obtaining a DE fraction of 0.85(5)(2)% , which gives a DE BR of about 2.2×10^{-6} .

For the decay $K_L \rightarrow \pi^+\pi^-\gamma$, the IB and E1 amplitudes violate CP , leading to a relative enhancement of the M1 contribution. KTeV has recently published a new measurement in this channel²⁹ based on 112k events with $E_\gamma > 20$ MeV, updating their 2001 result with an increase in statistics of more than a factor of ten. In fits to the $(E_\gamma, \cos\theta_{\pi+\gamma})$ distribution, the form factor describing the M1 amplitude is based on a pole model with VMD photon couplings.³⁰ KTeV obtains the best measurement to date of the M1 form-factor parameters, as well as the 90% CL limit $|g_{E1}| < 0.21$. Setting $|g_{E1}|$ to zero, KTeV obtains the fraction of the radiation spectrum from M1 DE: $\text{DE}/(\text{DE} + \text{IB}) = 0.689(21)$ for $E_\gamma > 20$ MeV.

KTeV has also recently published a new measurement of the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$.³¹ In this decay, the polarization of the virtual photon is measured by the plane of the $\gamma^* \rightarrow e^+e^-$ conversion, so that the interference between the IB/E1 and M1 amplitudes can be observed. In addition, the process contributes in which the K_L emits a virtual photon and is transformed into a K_S , which then decays to $\pi^+\pi^-$; the amplitude for this process is proportional to $\langle r_K^2 \rangle$, the charge radius of the neutral kaon. With ~ 5200 events, KTeV obtains results for $\langle r_K^2 \rangle$ and the M1 form-factor parameters that agree with and improve upon the previous results from KTeV and NA48. The M1 form-factor parameter values also agree with those KTeV obtains for the $K_L \rightarrow \pi^+\pi^-\gamma$ channel, while the new KTeV limit on $|g_{E1}|/|g_{M1}|$ from $K_L \rightarrow \pi^+\pi^-e^+e^-$ is a stronger constraint on the size of the E1 amplitude than is obtained from $K_L \rightarrow \pi^+\pi^-\gamma$. The asymmetry A_ϕ about zero in the distribution of $\sin\phi\cos\phi$, where ϕ is the angle between the $\pi^+\pi^-$ and e^+e^- planes in the decay, parameterizes CP violation in the interference between IB and M1 amplitudes. The new KTeV measurement, $A_\phi = 0.136(14)(15)$, significantly improves on previous results from KTeV and NA48.

3.3. $K_S \rightarrow \gamma\gamma$

In ChPT calculations of the amplitude for $K_S \rightarrow \gamma\gamma$, since all particles involved are neutral, there are no tree-level contributions. Moreover, at $\mathcal{O}(p^4)$, only finite chiral-meson loops contribute. $BR(K_S \rightarrow \gamma\gamma)$ is predicted unambiguously at this level in terms of the couplings G_8 and G_{27} , giving 2.1×10^{-6} .³² The most precise published measurement of this BR is from NA48: $BR = 2.78(6)(4) \times 10^{-6}$.³³ This result would suggest the need for a significant $\mathcal{O}(p^6)$ correction in the ChPT calculation of the BR. The NA48 result may soon be confirmed by KLOE.²¹ While the number of $K_S \rightarrow \gamma\gamma$ events observed by KLOE is ~ 600 , as compared to the ~ 7500 observed by NA48, KLOE profits from the use of a tagged K_S beam and does not have to contend with irreducible background from $K_L \rightarrow \gamma\gamma$. The KLOE measurement is in progress; a total error of 5% is expected, which is sufficient to confirm the NA48 result.

4. Determination of $|V_{us}|f_+(0)$ from $K_{\ell 3}$ Decays

A precise test of CKM unitarity can be obtained from the first-row constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ (with $|V_{ub}|^2$ negligible). At present, the most precise value for $|V_{us}|$ is obtained from $K_{\ell 3}$ decay rates, via

$$\Gamma(K_{l3(\gamma)}) = N_K |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM}), \quad (1)$$

where the subscripts K and ℓ indicate dependence on the kaon charge state (K^\pm, K^0) and lepton flavor, and N_K is a well-determined constant. The value of the hadronic matrix element at zero momentum transfer, $f_+(t=0)$, differs from unity because of $SU(2)$ - and $SU(3)$ -breaking effects; conventionally, the value for $K^0 \rightarrow \pi^-$ decays is used in Eq. (1) and $SU(2)$ -breaking corrections are encoded in $\Delta_K^{SU(2)}$. $\Delta_{K\ell}^{EM}$ is the correction to the form factor for the effects of long-distance electromagnetic interactions. These theory inputs to Eq. (1), and especially the status of $f_+^{K^0\pi^-}(0)$, are discussed in the contribution to these proceedings by V. Cirigliano (see also Ref. 34). The inputs from experiment are $\Gamma(K_{l3(\gamma)})$, the radiation-inclusive decay rates, or in practice, BR and lifetime measurements; and $I_{K\ell}$, the phase-space integrals of the form factors, which are calculated from measurements of the form-factor slopes λ as discussed in Sec. 4.3.

In the 2002 PDG evaluation, $|V_{ud}|^2 + |V_{us}|^2 = 0.9965(15)$, a 2.3σ hint of CKM unitarity violation. The 2003 result from BNL E865,³⁵ $BR(K_{e3}^\pm) = 5.13(2)(10)\%$, is 2.7σ higher than the 2002 PDG average for this BR and seemed to resolve the problem. With respect to the older measurements

of $K_{\ell 3}$ decays, the E865 result made use of much higher statistics, and in addition was the first measurement with a well-defined treatment of the contribution from radiative decays. All newer measurements from KTeV, KLOE, ISTRA+, and NA48 share this feature.

4.1. K_L and K_S branching ratios and lifetimes

KTeV, NA48, and KLOE have recently published measurements of the BRs for the dominant K_L decay channels, including the $K_{\ell 3}$ decays.

KTeV has measured five ratios of the BRs for the principal K_L decays.³⁶ The six BRs involved account for 99.93% of the K_L width; the ratios are combined to determine the absolute BR values.

NA48 has measured the ratio of $\text{BR}(K_{e3})$ to the sum of the BRs for all decays to two tracks.³⁷ This is essentially $\text{BR}(K_{e3})/[1 - \text{BR}(3\pi^0)]$. NA48 normalizes using the average of the KTeV and NA31 measurements of $\text{BR}(3\pi^0)$; they also have a preliminary measurement of $\text{BR}(3\pi^0)$ normalized to $K_S \rightarrow \pi^0\pi^0$ decays that confirms this average.³⁸

KLOE has measured absolute BRs for the four dominant K_L modes using $\phi \rightarrow K_S K_L$ events in which a $K_S \rightarrow \pi^+\pi^-$ decay is used to tag the K_L decay, providing normalization.³⁹ The dominant contribution to the uncertainties on the absolute BRs comes from the uncertainty on τ_L , the K_L lifetime, which is needed to calculate the overall geometrical efficiency. Expressing the BRs as functions of τ_L and imposing the constraint $\sum \text{BR} = 1$ (with the 2004 PDG values used for the smaller K_L BRs), final values are obtained for the four BRs and for τ_L , with greatly reduced uncertainties. KLOE has also measured τ_L directly, using 10^7 $K_L \rightarrow 3\pi^0$ events,⁴⁰ for which the reconstruction efficiency is high and uniform inside the fiducial volume ($0.37\lambda_L$). The result, $\tau_L = 50.92(17)(25)$ ns, is consistent with the value obtained from the sum of the K_L BRs.

The results from the experiments are summarized in Table 1. Because of their interdependence for the purposes of normalization, they are best incorporated into a new evaluation of $|V_{us}|f_+(0)$ via a global fit akin to that performed by the PDG. The fit performed here uses the data in Table 1 in addition to four other measurements used in the 2006 PDG fit. The free parameters are the seven largest K_L BRs and τ_L ; the BRs in the fit are constrained to sum to unity. The principal difference between the fit performed here and the 2006 PDG fit is that here, the intermediate KTeV and KLOE values (*i.e.*, before applying constraints) are the inputs, and the complete error matrix is used to handle the correlations between the measurements from each experiment. (In the 2006 PDG fit, the final KTeV

Table 1. Recent K_L BR and lifetime measurements used in fit

KTeV ^a	BR($K_{\mu 3}/K_{e 3}$) = 0.6640(26)	BR($3\pi^0/K_{e 3}$) = 0.4782(55)
	BR($\pi^+\pi^-\pi^0/K_{e 3}$) = 0.3078(18)	BR($\pi^+\pi^-/K_{e 3}$) = 4.856(29) $\times 10^{-3}$
	BR($\pi^0\pi^0/3\pi^0$) = 4.446(25) $\times 10^{-3}$	
KLOE ^{a,b}	BR($K_{e 3}$) = 0.4049(21)	BR($K_{\mu 3}$) = 0.2726(16)
	BR($3\pi^0$) = 0.2018(23)	BR($\pi^+\pi^-\pi^0$) = 0.1276(15)
	$\tau_L = 50.92(30)$ ns	
NA48	BR($K_{e 3}/2$ track) = 0.4978(35)	BR($3\pi^0$) = 0.1966(34) ^c

Note: ^a In the fit, errors on these BRs are parameterized by the complete covariance matrix. ^b In the fit, these BR values are expressed as functions of τ_L as described in Ref. 39. ^c Preliminary.

and KLOE BR results were used and one measurement involving BR($3\pi^0$) was removed in each case.) Scale factors for the errors are calculated and used as per the PDG prescription. The fit has $\chi^2/\text{ndf} = 13.2/9$ (15.4%) and gives BR($K_{e 3}$) = 0.4047(11) ($S = 1.4$), BR($K_{\mu 3}$) = 0.2698(9) ($S = 1.3$), and $\tau_L = 51.11(21)$ ns ($S = 1.1$). These values are quite similar to those from the 2006 PDG fit.

KLOE also has recently published⁴¹ a measurement of BR($K_S \rightarrow \pi e \nu$) that is precise enough to contribute meaningfully to the evaluation of $|V_{us}|f_+(0)$. For this measurement, K_S decays are tagged by the observation of a K_L interaction in the KLOE calorimeter. The quantity directly measured is BR($\pi e \nu/\pi^+\pi^-$). Together with the recently published KLOE value BR($\pi^+\pi^-/\pi^0\pi^0$) = 2.2459(54), the constraint that the K_S BRs must sum to unity, and the assumption of universal lepton couplings, this completely determines the K_S BRs for $\pi^+\pi^-$, $\pi^0\pi^0$, $K_{e 3}$, and $K_{\mu 3}$ decays.⁴² In particular, BR($K_S \rightarrow \pi e \nu$) = 7.046(91) $\times 10^{-4}$. The KLOE measurement is performed separately for each lepton charge state, yielding the first result for the semileptonic charge asymmetry from K_S decays, $A_S = 15(96)(29) \times 10^{-4}$. This value has been used in tests of CPT symmetry and the $\Delta S = \Delta Q$ rule.⁴³

4.2. K^\pm branching ratios and lifetime

NA48/2, ISTRA+, and KLOE all have preliminary measurements of K^\pm BRs. These new measurements have a significant impact on the evaluation of $|V_{us}|f_+(0)$, as demonstrated by the updated fit performed here.

NA48/2 measures BR($K_{e 3}/\pi\pi^0$) and uses the 2004 PDG value for BR($\pi\pi^0$) to quote BR($K_{e 3}$) = 5.14(2)(6)%.³⁸ The fit performed here makes

use of the value $\text{BR}(K_{e3}/\pi\pi^0) = 0.2433(25)$, as well as of the NA48/2 result $\text{BR}(K_{\mu3}/K_{e3}) = 0.6749(35)(24)$.

ISTRA+ also measures $\text{BR}(K_{e3}/\pi\pi^0)$; they use the 2006 PDG value for $\text{BR}(\pi\pi^0)$ to quote $\text{BR}(K_{e3}) = 5.170(11)(57)\%$.²⁴ The fit performed here makes use of the value $\text{BR}(K_{e3}/\pi\pi^0) = 0.2471(23)$.

KLOE measurements the absolute K_{e3} and $K_{\mu3}$ BRs.⁴⁴ In $\phi \rightarrow K^+K^-$ decays, K^+ decays into $\mu\nu$ or $\pi\pi^0$ are used to tag a K^- beam, and vice versa. KLOE performs four separate measurements for each $K_{\ell3}$ BR, corresponding to the different combinations of kaon charge and tagging decay. The final averages are $\text{BR}(K_{e3}) = 5.047(46)(80)\%$ and $\text{BR}(K_{\mu3}) = 3.310(40)(70)\%$. The fit performed here takes into account the dependence of these BRs on the K^\pm lifetime.

The world average value for τ_\pm is nominally quite precise; the 2006 PDG quotes $\tau_\pm = 12.385(25)$ ns. However, the error is scaled by 2.1; the confidence level for the average is 0.2%. It is important to confirm the value of τ_\pm . KLOE has a preliminary measurement based on K^\pm decays tagged by $K^\mp \rightarrow \mu\nu$ and observed in a fiducial volume of $\sim 1\lambda_\pm$.⁴⁵ The result, $\tau_\pm = 12.336(44)(65)$ ns, agrees with the PDG average, although at present the KLOE uncertainty is significantly larger.

The fit performed here makes use of all preliminary results cited above, plus the data used in the 2006 PDG fit, for a total of 30 measurements. The free parameters are the six main K^\pm BRs and τ_\pm ; the BRs are constrained to sum to unity. The fit gives $\chi^2/\text{ndf} = 38/24$ (3.6%). The poor fit quality principally derives from the scatter in the five older measurements of τ_\pm ; when these are replaced with their PDG average with scaled error, $\tau_\pm = 12.385(25)$ ns, the fit gives $\chi^2/\text{ndf} = 20.5/20$ (42%), with no significant changes in the results. The results are $\text{BR}(K_{e3}) = 5.056(37)\%$ ($S = 1.3$), $\text{BR}(K_{\mu3}) = 3.399(29)\%$ ($S = 1.2$), and $\tau_\pm = 12.384(21)$ ns ($S = 1.8$). The significant evolution of the average values of the BRs for $K_{\ell3}^\pm$ decays and for the important normalization channels is evident in Fig. 3.

4.3. $K_{\ell3}$ form-factor slopes

Only the vector part of the weak current contributes to the hadronic matrix element for $K_{\ell3}$ decays:

$$\langle \pi | J_\alpha | K \rangle = f_+(t)(p_K + p_\pi)_\alpha + f_-(t)(p_K - p_\pi)_\alpha,$$

where $t = (p_K - p_\pi)^2$. When the squared matrix element is evaluated, a factor of m_ℓ^2/m_K^2 multiplies all terms containing $f_-(t)$. This form factor can be neglected for K_{e3} decays. For the description of $K_{\mu3}$ decays, it is

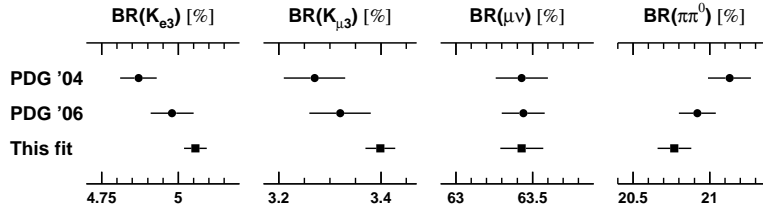


Fig. 3. Evolution of average values for main K^\pm BRs.

customary to use $f_+(t)$ and the scalar form factor $f_0(t) \equiv f_+(t) + [t/(m_K^2 - m_\pi^2)] f_-(t)$. The form factors are written as $f_{+,0}(t) = f_+(0) \tilde{f}_{+,0}(t)$, with $\tilde{f}_{+,0}(0) = 1$, and often expanded in powers of t as

$$\begin{aligned} \tilde{f}(t) &= 1 + \lambda \frac{t}{m_{\pi^+}^2} && \text{(linear),} \\ \tilde{f}(t) &= 1 + \lambda' \frac{t}{m_{\pi^+}^2} + \frac{1}{2} \lambda'' \left(\frac{t}{m_{\pi^+}^2} \right)^2 && \text{(quadratic).} \end{aligned} \quad (2)$$

The slopes λ are obtained from fits to the measured t distributions, but sensitivity to the quadratic terms is poor, in large part because the kinematic density of the matrix element drops to zero at large t , where the form factor itself is maximal. The vector form factor f_+ is dominated by the vector $K\pi$ resonances, *e.g.*, $K^*(892)$; this fact suggests the pole parameterization, $\tilde{f}_+(t) = M_V^2 / (M_V^2 - t)$. This one-parameter form generally fits experimental data better than does the linear parameterization; its expansion gives $\lambda' = (m_{\pi^+} / M_V)^2$; $\lambda'' = 2\lambda'^2$. A dispersive representation for $f_0(t)$ featuring a single experimental parameter has also been proposed.⁴⁶

KTeV, KLOE, and NA48 have measured the form-factor slopes in K_L decays; ISTRA+ has measured the slopes in K^- decays. KTeV and ISTRA+ have reported fit results for λ'_+ , λ''_+ , and λ_0 for both K_{e3} and $K_{\mu 3}$ decays; KLOE and NA48 have values for λ'_+ and λ''_+ from K_{e3} decays. These data are collected in Table 2. The experiments use different conventions in reporting the form-factor slopes; the data in the table have been adjusted for use with Eq. (2). Most experiments also quote various other combinations of linear, quadratic, and pole fit results. NA48 has preliminary results for λ_+ and λ_0 (linear fit) from $K_{\mu 3}$ decays.⁴⁷

To obtain reference values of the form-factor slopes for the phase-space integrals, the data in Table 2 have been averaged. Correlation coefficients are available for the KTeV and KLOE data; for NA48 and ISTRA+, their values have been inferred, in part by assuming the correlations to be intrinsic to the measurement and largely independent of the experimental

Table 2. Measurements of $K_{\ell 3}$ form-factor slopes

Experiment	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
KTeV $K_L e3\text{-}\mu 3$ avg. (Ref. 48)	20.6 ± 1.8	3.2 ± 0.7	13.7 ± 1.3
KLOE $K_L e3$ (Ref. 49)	25.5 ± 1.8	1.4 ± 0.8	
NA48 $K_L e3$ (Ref. 50)	28.0 ± 2.4	0.4 ± 0.9	
ISTRA+ $K^- e3$ (Ref. 51)	24.9 ± 1.7	1.9 ± 0.9	
ISTRA+ $K^- \mu 3$ (Ref. 52) ^a	23.0 ± 6.4	2.3 ± 2.3	17.1 ± 2.2

Note: ^a No systematic uncertainties are quoted for this fit.

details. In principle, this fact could be exploited to fix all correlations *a priori*.⁵³ The results are $\lambda'_+ = 24.72(84) \times 10^{-3}$, $\lambda''_+ = 1.67(36) \times 10^{-3}$, and $\lambda_0 = 15.72(97) \times 10^{-3}$, with $\rho(\lambda'_+, \lambda''_+) = -0.94$, $\rho(\lambda'_+, \lambda_0) = +0.30$, and $\rho(\lambda''_+, \lambda_0) = -0.40$. The fit gives $\chi^2/\text{ndf} = 11.6/9$ (23.9%).

KTeV, KLOE, and NA48 all quote values for M_V for K_{e3} decays. The average value is $M_V = 875.3 \pm 5.4$ MeV with $\chi^2/\text{ndf} = 1.83/2$ (40%). Using this or the average of the quadratic fit results stated above to calculate the phase-space integral for the $K_L e3$ form factor makes a 0.02% difference in the result. In the calculation of $|V_{us}|f_+(0)$, no additional error is assigned to account for differences obtained with quadratic and pole parameterizations for $\tilde{f}_+(t)$.

4.4. Discussion

Using the results of the fits discussed above for the BRs, lifetimes, and form-factor slopes, $|V_{us}|f_+(0)$ has been evaluated for each of the five decay modes using Eq. (1). The results are summarized in the left panel of Fig. 4. The most precise determination is from $K_L e3$ decays: $|V_{us}|f_+(0) = 0.2161(6)$. Indicatively, for K_L decays, the precision is limited by the uncertainties on the decay widths, and in particular, by the uncertainty on τ_L . For the $K_L \mu 3$ mode, the uncertainty on Δ^{EM} is also a significant issue. For both K^\pm decays, the uncertainties on the BR measurements are the limiting factor; for $K_{\mu 3}^\pm$, the uncertainty on Δ^{EM} is nearly as important. Although not a limiting factor at the moment, for K^\pm decays, the present $\sim 10\%$ uncertainty on $\Delta_{K^\pm}^{SU(2)}$ would ultimately prohibit obtaining the same level of precision as for K_L decays. The uncertainty on the phase-space integral is not currently a limiting factor for any mode.

A fit to all five values for $|V_{us}|f_+(0)$ taking all correlations into account has $\chi^2/\text{ndf} = 2.76/4$ (60%) and gives the average value $|V_{us}|f_+(0) = 0.2162(4)$. When $|V_{us}|f_+(0)$ is evaluated separately for K^0 and K^\pm decays

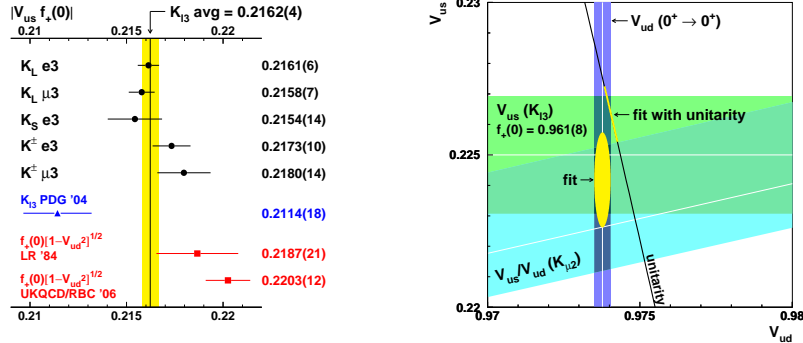


Fig. 4. Left: Determinations of $|V_{us}|f_+(0)$ for five $K_{\ell 3}$ decay channels, with average and comparison values. Right: Results of a fit to values for $|V_{ud}|$, $|V_{us}|$, and $|V_{us}|/|V_{ud}|$.

(making use of separate averages for the form-factor slopes for each case), the result for K^\pm decays is 1σ higher than that for K^0 decays. At present, the results obtained from all modes are consistent, though it is worth noting that all of the new $\text{BR}(K_{\ell 3}^\pm)$ results are still preliminary.

To test CKM unitarity, a value for $f_+(0)$ is needed. Conventionally, the original estimate of Leutwyler and Roos,⁵⁴ $f_+(0) = 0.961(8)$, is used; this gives $|V_{us}| = 0.2250(19)$. Using the most recent evaluation of $|V_{ud}|$ from $0^+ \rightarrow 0^+$ nuclear beta decays,⁵⁵ $|V_{ud}| = 0.97377(27)$, one has $|V_{ud}|^2 + |V_{us}|^2 - 1 = -0.0012(10)$, a result perfectly compatible with unitarity. As is evident from Fig. 4, this represents a significant evolution of the experimental picture since the 2004 PDG evaluation. However, lattice evaluations of $f_+(0)$ are rapidly improving in precision. For example, the UKQCD/RBC Collaboration has announced a preliminary result⁵⁶ from a lattice calculation with $2+1$ flavors of dynamical domain-wall quarks: $f_+(0) = 0.9680(16)$. This value implies $|V_{us}| = 0.2234(6)$ and $|V_{ud}|^2 + |V_{us}|^2 - 1 = -0.0019(6)$, a 3.2σ discrepancy with CKM unitarity.

Marciano⁵⁷ has observed that $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$ can be precisely related to the product $(|V_{us}|/|V_{ud}|)^2(f_K/f_\pi)^2$. The recent measurement $\text{BR}(K^+ \rightarrow \mu^+\nu) = 0.6366(9)(15)$ from KLOE,⁵⁸ together with the preliminary lattice result $f_K/f_\pi = 1.208(2)(\pm_{-14}^{+7})$ from the MILC Collaboration,⁵⁹ gives $|V_{us}|/|V_{ud}| = 0.2286(\pm_{-15}^{+27})$. This ratio can be used in a fit together with the values of $|V_{ud}|$ from Ref. 55 and $|V_{us}|$ from $K_{\ell 3}$ decays as above. Using the value for $|V_{us}|$ obtained with $f_+(0) = 0.961(8)$, the fit gives $|V_{ud}| = 0.97377(27)$ and $|V_{us}| = 0.2242(16)$, with $\chi^2/\text{ndf} = 0.52/1$ (47%). The unitarity constraint can also be included, in which case the fit gives $\chi^2/\text{ndf} = 3.43/2$ (18%). Both results are illustrated in Fig. 4, right. If in-

stead the newer lattice result for $f_+(0)$ is used to obtain $|V_{us}|$, the fit gives $|V_{ud}| = 0.97377(27)$ and $|V_{us}| = 0.2233(6)$, with $\chi^2/\text{ndf} = 0.074/1$ (79%). When the unitarity constraint is imposed, the fit gives $\chi^2/\text{ndf} = 10.5/2$, corresponding to a probability of 0.53%. These results reinforce the conclusion that the result of the first-row test of CKM unitarity depends mainly on the value and uncertainty assumed for $f_+(0)$. Confirmation of the new lattice result is a critical step towards a clearer understanding of the situation.

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