Electromagnetic contributions to $K_{\ell 3}$ decays

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Frascati, 19 May 2007

Semileptonic weak decays with electromagnetic contributions

dynamical degrees of freedom

pseudoscalar octet

$$\pi^\pm, \pi^0, K^\pm, K^0, \overline{K^0}, \eta$$

photon and light leptons

$$A_\mu \quad \ell, \nu_\ell (\ell = e, \mu)$$

Parameters of the effective Lagrangian

lowest order

$\mathcal{L}_{p^2}$	F, B, e, G_F
$\mathcal{L}_{e^2p^0}$	Z

next-to-leading order

$\mathcal{L}_{p^4}$	$L_1, L_2, \ldots L_{12}$	Gasser, Leutwyler
$\mathcal{L}_{e^2p^2}$	$K_1, K_2, \ldots K_{14}$	Urech
$\mathcal{L}_{\text{lept}}, \mathcal{L}_\gamma$	$X_1, X_2, \ldots X_8$	Knecht, N., Rupertsberger, Talavera

next-to-next-to-leading order

$\mathcal{L}_{p^6}$	$90 + 4$	Fearing, Scherer; Bijnens, Colangelo, Ecker

Present status of determination of low energy constants

$\mathcal{L}_{p^4}$ L_i^r determined from experimental input — Gasser, Leutwyler

$\mathcal{L}_{p^6}$ LECs relevant for $K_{\ell 3}$ determined — Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés

$\mathcal{L}_{e^2p^2}$ K_i determined — Baur, Urech; Ananthanarayan, Moussallam

$\mathcal{L}_{\rm lept}$ X_i determined — Descotes-Genon, Moussallam

$K_{\ell 3}$ decays

$$\Gamma(K_{\ell 3(\gamma)})_{\rm full} = \mathcal{N}_{\mathcal{K}}\, S_{\rm EW}\, |\tilde{f}_+^{K^0}(0)|^2\, I_{K_{\ell 3}}\, (1 + \delta_K^{\ell} + \delta_{\rm SU(2)})$$

$$\mathcal{N}_K = C_K^2 \frac{G_{\rm F}^2\, |V_{us}|^2 M_K^5}{128\pi^3}, \quad C_K = \begin{cases} 1 & \text{for } K_{e3}^0 \\ \frac{1}{\sqrt{2}} & \text{for } K_{e3}^+ \end{cases}$$

$$I_{K_{\ell 3}} = I_{K_{\ell 3}}(\lambda'_+, \lambda''_+, \lambda_0)$$

$$\delta_{\mathrm{SU}(2)} = 2\sqrt{3}\left(\varepsilon^{(2)} + \varepsilon_{\mathrm{S}}^{(4)} + \varepsilon_{\mathrm{EM}}^{(4)} + \ldots\right) = \begin{cases} 0 & \text{for } K_{\ell 3}^{0} \\ 0.048 \pm 0.005 & \text{for } K_{\ell 3}^{+} \end{cases}$$

Gasser, Leutwyler; Cirigliano, Knecht, N., Rupertsberger, Talavera

$\delta_{K^0}^{e} = 0.0114 \pm 0.0030$ (update)

$\delta_{K^+}^{e} = 0.0016 \pm 0.0030$ (update)

$\delta_{K^0}^{\mu} = 0.0159 \pm 0.0030$ (new, to be checked)

$\delta_{K^+}^{\mu} = -0.0024 \pm 0.0030$ (new, to be checked)

based on work by Cirigliano, Knecht, N., Pichl, Rupertsberger, Talavera

Determination of $V_{us}\tilde{f}^{K^0}$

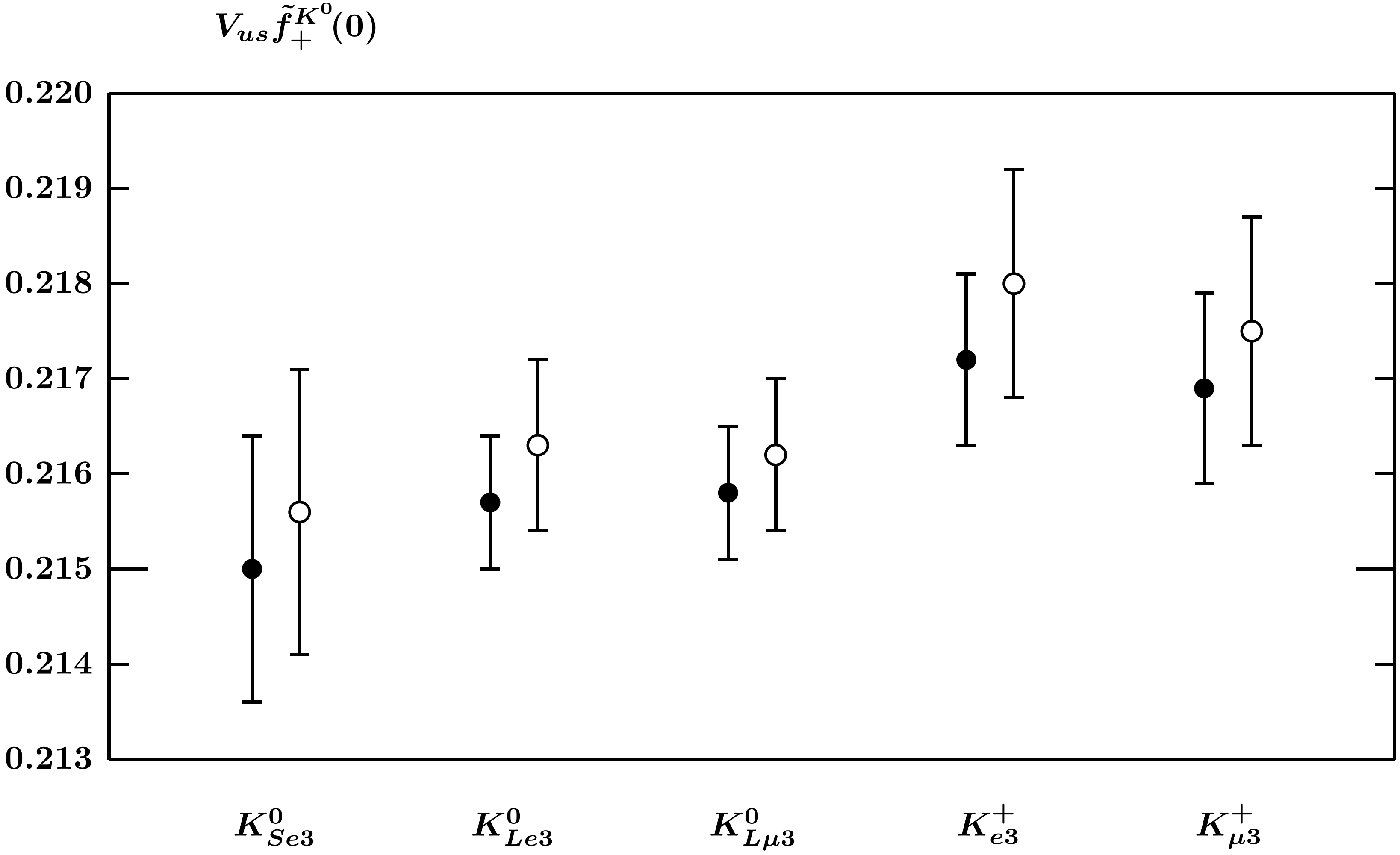
$\lambda'_+, \lambda''_+, \lambda_0$ from PDG 2006

$\mathrm{BR}(K^0_{L\ell 3})$ from PDG 2006

$\mathrm{BR}(K^0_{Se3})$ from KLOE

$\mathrm{BR}(K^+_{e3}) = (5.127 \pm 0.027)\%$ average of values from PDG 2006, NA48/2 (EPJC 50 (2007) 329), KLOE (prel., hep-ex/0701008v2)

$\mathrm{BR}(K^+_{\mu 3}) = (3.400 \pm 0.022)\%$ average of NA48/2 (EPJC 50 (2007) 329), KLOE (prel., hep-ex/0701008v2)



full circles: linear form factor fit
open circles: quadratic form factor fit

- good agreement between K^0_{Le3}, $K^0_{L\mu 3}$ and K^0_{Se3} data
- good agreement between K^+_{e3} and $K^+_{\mu 3}$ data
- small discrepancy between $K^0_{\ell 3}$ and $K^+_{\ell 3}$ data?
- isospin violation at $\mathcal{O}(p^6)$ larger than expected from naïve power counting?

average: $V_{us}\tilde{f}^{K^0}(0) = 0.2163 \pm 0.0005$ (quadratic fit)

Determination of V_{us} from $K_{\ell 3}$ data

need theoretical value for $\tilde{f}^{K^0}(0)$

Contribution at order p^4

$$
\begin{aligned}
\widetilde{f}_+^{K^0}(t) &= 1 \\
&+ \frac{1}{2} H_{K^+\pi^0}(t) + \frac{3}{2} H_{K^+\eta}(t) + H_{K^0\pi^-}(t) \\
&+ \sqrt{3}\,\varepsilon^{(2)} \left[H_{K\pi}(t) - H_{K\eta}(t) \right] + \ldots
\end{aligned}
$$

Gasser, Leutwyler

$$
\widetilde{f}_+^{K^0}(0) = 0.97689 \pm 0.00002
$$

contribution of order p^6

$$f_+(0)\Big|_{p^6} = f_+^{2-\mathrm{loops}}(0) + f_+^{\mathrm{L_i}\times\mathrm{loop}}(0) + f_+^{\mathrm{tree}}(0)\Big|_{p^6}$$

at $\mu = M_\rho$: $f_+^{2-\mathrm{loops}}(0) = 0.0113$, $f_+^{\mathrm{L_i}\times\mathrm{loop}}(0) = -0.0020 \pm 0.0005$

Bijnens, Talavera

$$f_+^{\mathrm{tree}}(0)\Big|_{p^6} = -0.002 \pm 0.008_{1/N_C} \pm 0.002_{M_S}$$

Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés

$$\rightarrow f_+(0)\Big|_{p^6} = 0.007 \pm 0.012$$

$$\rightarrow \tilde{f}_+^{K^0}(0) = 0.984 \pm 0.012$$

$$\tilde{f}_+^{K^0}(0) = 0.984 \pm 0.012 \rightarrow V_{us} = 0.2198 \pm 0.0027$$

comparison with $V_{us}^{\text{unit.}} = \sqrt{1 - V_{ud}^2}$

$V_{us}^{\text{unit.}} = 0.2275 \pm 0.0012$ (superallowed, PDG 2006)

$V_{us}^{\text{unit.}} = 0.2239 \pm 0.0083$ ($n\beta$, τ_n from PDG 2006)

$V_{us}^{\text{unit.}} = 0.2058 \pm 0.0090$ ($n\beta$, τ_n from Serebrov et al.)

$V_{us}^{\text{unit.}} = 0.2261 \pm 0.012$ ($\pi\beta$, PIBETA)

Leutwyler, Roos (1984) : $\tilde{f}_+^{K^0}(0) = 0.961 \pm 0.008 \rightarrow V_{us} = 0.2251 \pm 0.0020$

lattice results for $\tilde{f}_+^{K^0}(0)$:

$\tilde{f}_+^{K^0}(0) = 0.960 \pm 0.009$ Bećirević et al.

$\tilde{f}_+^{K^0}(0) = 0.968 \pm 0.011$ Dawson et al.

$\tilde{f}_+^{K^0}(0) = 0.961 \pm 0.005$ UKQCD/RBC Collab. (prel.)

Summary

- theoretical framework for treatment of all isospin-violating contributions in semileptonic decays $\surd$
- theoretical estimates of relevant LECs K_i^r, X_i^r $\surd$
- $K_{\ell 3}$ analysis at $\mathcal{O}(p^6, (m_d - m_u)p^2, e^2p^2)$ $\surd$
- experiments sensitive to isospin-breaking effects
- correct treatment of electromagnetic corrections mandatory
- small inconsistency between $K_{\ell 3}^+$ and $K_{\ell 3}^0$ data?
- remaining theoretical problem: $f_+(0)$