New ideas for $Kl3$-decays on the lattice

FLAVIANET MiniWorkshop
Frascati

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May 19, 2007

Collaboration

UKQCD

(Edinburgh - Southampton)

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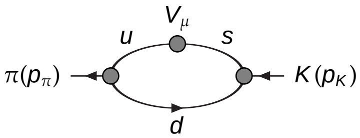
Outline

- Lattice calculation of $Kl3$ form factor
- Summary of systematics
- New idea: Use twisted boundary conditions [hep-lat/0703005]
- exploratory study

$Kl3$-form factor

- 

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle \qquad = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

- Currently our best lattice estimate

PRELIMINARY UKQCD-RBC (CKM 2006)

$$f_0(0) = f_+(0) = 0.9659(51)$$

more details see talk at Kaon 2007

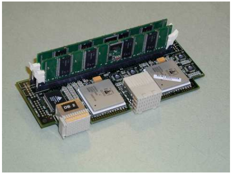
Matrix elements in lattice QCD

- Correlation functions in terms of **Euclidean** path integral

  $\langle O[\bar{\psi},\psi,A]\rangle = \frac{1}{Z}\int D\bar{\psi}D\psi DA\, O(\bar{\psi},\psi,A)\, e^{-S_G(U)-S_q(\bar{\psi},\psi,U)}$

  Ground state matrix elements for large Euclidean times
- discretisation – space time lattice as regulator – regulator π/a

  Statistical sampling of PI with QCDOC-computer by UKQCD/RBC

- from first principles:
  tune bare parameters (coupling and quark masses)
  - lattice spacing: $a^{-1} = \frac{m_\rho^{\mathrm{exp}}}{am_V}$
  - quark masses: $\frac{am_H}{am_V} = \frac{m_H^{\mathrm{exp}}}{m_V^{\mathrm{exp}}}$ $(H = \pi, K, D, \ldots)$

Fighting to control systematic errors

- statistical
- number of dynamical flavors: $N_f = 2, 2+1$
- quark mass

  extrapolation in the quark mass guided by effective theories (HQET, χPT)
- discretisation errors (cut-off effects)
  $\rightarrow$ systematic estimation $\rightarrow$ continuum extrapolation
- finite volume errors

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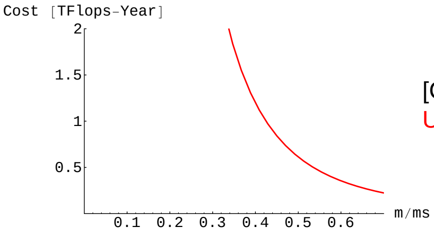

[Clark, Plot: Sachrajda]
Ukawa's Berlin Wall 2001

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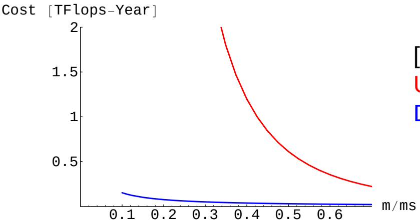

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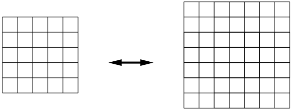
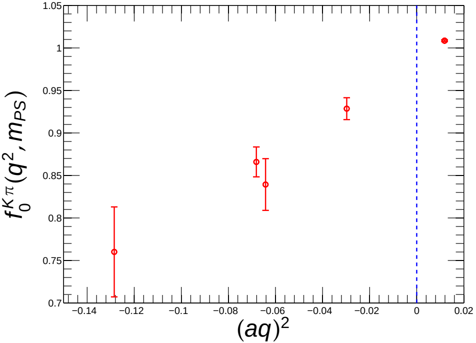
  lattice-box $\approx$ 2fm
  - negligible for $m_\pi L > 3-4$)
  - systematic estimation: 

[Becirevic et al.], [Okamoto et al.], [Tsutsui et al.], [Dawson et al.], [Antonio et al.]



- non-physical quark (meson) mass
- only discrete momenta

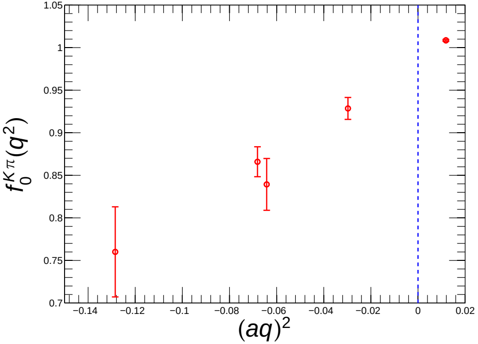
Another systematic effect: the lattice hadron momentum

- QCD in finite volume $\longrightarrow$ e.g. periodic boundary conditions
  $\rightarrow$ action single valued

fermion fields: $\tilde{\psi}(x + \hat{i}L) = \tilde{\psi}(x)$

quantised lattice momenta: $\vec{p}_{\rm lat} = \frac{2\pi}{L}\vec{n}$

Calculation in practice ('conventionally')



$$\langle K(0)|V_\mu|\pi(0)\rangle$$
$$\langle K(\tfrac{2\pi}{L})|V_\mu|\pi(0)\rangle$$
$$\langle K(0)|V_\mu|\pi(\tfrac{2\pi}{L})\rangle$$
$$\langle K(\sqrt{2}\tfrac{2\pi}{L})|V_\mu|\pi(0)\rangle$$
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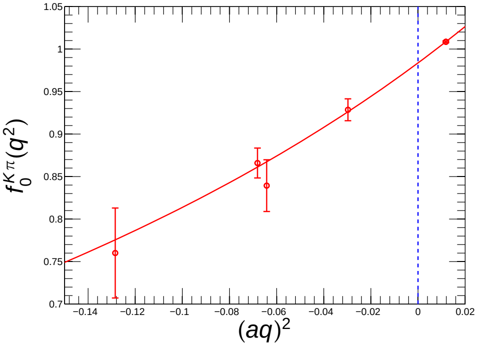
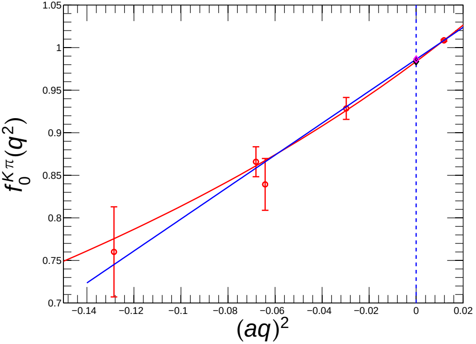


interpolate with phenomenologically motivated Ansatz, e.g.

$$f_+^{K\pi}(q^2) = \frac{f_+^{K\pi}(0)}{1 + cq^2}$$



or simply linear?

Twisted boundary conditions (I)

How to access momenta other then the Fourier modes:

- general ansatz for fermion bc's in finite volume [Bedaque 2004]:

$$q(x_i + L) = U_i q(x_i)$$

  action single-valued if U_i symmetry
- U_i diagonal for general diagonal mass matrix M
- q 3-flavour vector: $U_i \in U(3)$

$$U_i = \exp(i\Theta_i)$$

Twisted boundary conditions (II)

- Change of variables

$$\tilde{q}(x) = e^{-i\,\Theta/L x} q(x) \text{ where } \tilde{q}(x) \text{ periodic}$$

- Lagrangian

$$\mathcal{L} = \bar{\tilde{q}}(x)\left(\tilde{\not{D}} + M\right)\tilde{q}(x)$$

where $\tilde{D}_\mu = D_\mu + i\frac{\Theta_\mu}{L}$ $(\Theta_0 = 0)$

- Quark propagator

$$\tilde{S}(x, \theta) \equiv \langle \tilde{\psi}(x)\bar{\tilde{\psi}}(0)\rangle = \int \frac{dp_0}{2\pi}\frac{1}{L^3}\sum_{\vec{p}=\frac{2\pi}{L}\vec{n}} \frac{e^{ip\cdot x}}{i(\not{p} + \not{\theta}/L) + M}$$

momentum in denominator shifted by θ/L

Twisted boundary conditions (III)

- bc's are long-distance effect
- investigate FVE (in continuum) in effective theory
- amend Gasser-Leutwyler chiral Lagrangian [Sachrajda, Villadoro 2005] fundamental d.o.f:

$$\Sigma(x_i + L) = U_i \Sigma(x_i) U_i^\dagger$$

- as for the quark fields:

$$\tilde{\Sigma}(x) = e^{-i\Theta\cdot x/L} \Sigma(x) e^{i\Theta\cdot x/L}$$

- T(wisted)χPT-Lagrangian

$$\mathcal{L}_{\mathrm{T}\chi\mathrm{PT}} \frac{f^2}{8} \langle \tilde{D}^\mu \tilde{\Sigma}^\dagger \tilde{D}_\mu \tilde{\Sigma} \rangle - \frac{f^2}{8} \langle \tilde{\Sigma} \chi^\dagger + \chi \tilde{\Sigma}^\dagger \rangle$$

where

$$\tilde{D}_\mu = D_\mu + i[B_\mu, \tilde{\Sigma}], \qquad B_i = \Theta/L, \qquad B_0 = 0$$

"Standard chiral Lagrangian coupled to a vector field"

- twisting for charged mesons:

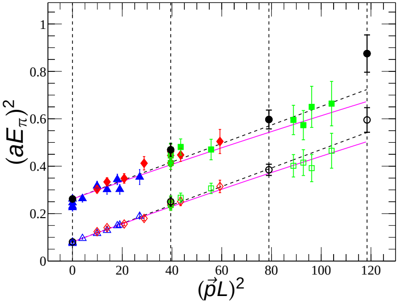
$$[B_i, \pi^{\pm}] = \pm \frac{\theta_{u,i} - \theta_{d,i}}{L} \pi^{\pm}$$

- twisting for neutral mesons:

$$[B_i, \pi^0] = 0 \rightarrow \text{no shift}$$

- $E_{\pi^{\pm}} = \sqrt{m^2_{\pi^{\pm}} + (\vec{p}_{\mathrm{lat}} - \frac{1}{L}(\vec{\theta}_u - \vec{\theta}_d))^2}$

  hadron momentum shifted by the sum of quark shifts



[Flynn et al. 2005]

- twists determine momentum transfer $\langle \pi(p_\pi)|V_4(0)|K(p_K)\rangle$

  $$q^2 = (p_K - p_\pi)^2 = \left\{[E_K(\vec{p}_K) - E_\pi(\vec{p}_\pi)]^2 - \left[(\vec{p}_{\mathrm{FT},K} + \vec{\theta}_K/L) - (\vec{p}_{\mathrm{FT},\pi} + \vec{\theta}_\pi/L)\right]^2\right\}$$

  [Guadagnoli et al. 2006], [Boyle et al. 2007]

- how to extract $f_+^{K\pi}(0)$ [Boyle et al. 2007]:
  1) twist quarks such that $q^2 = 0$ in

     $$\langle \pi(p_\pi)|V_4(0)|K(p_K)\rangle = f_+^{K\pi}(0)(E_K + E_\pi) + f_-^{K\pi}(0)(E_K - E_\pi)$$

  2) subtract $f_-^{K\pi}(0)$ by linearly combining

     $$1)\quad \langle \pi(0)|V_4|K(\vec{\theta}_K)\rangle : \quad |\vec{\theta}_K| = L\sqrt{\left(\frac{m_K^2+m_\pi^2}{2m_\pi}\right)^2 - m_K^2} \quad \text{and} \quad \vec{\theta}_\pi = \vec{0}$$

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$K l3$ form factor and twisted boundary conditions

- twists determine momentum transfer $\langle \pi(p_\pi)|V_4(0)|K(p_K)\rangle$

$$q^2 = (p_K - p_\pi)^2 = \left\{ [E_K(\vec{p}_K) - E_\pi(\vec{p}_\pi)]^2 - \left[(\vec{p}_{\mathrm{FT},K} + \vec{\theta}_K/L) - (\vec{p}_{\mathrm{FT},\pi} + \vec{\theta}_\pi/L) \right]^2 \right\}$$

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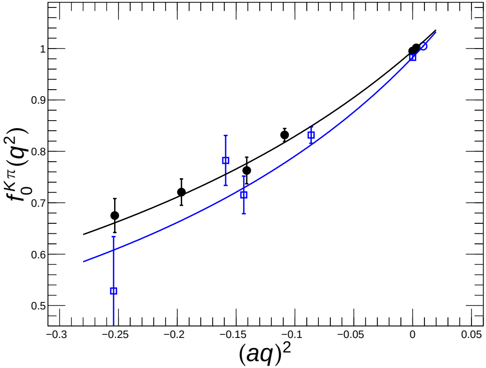
Exploratory study

- DWF with DBW2 gauge action (UKQCD/RBC, $\beta = 0.72$)
- $16^3 \times 32 \times 8$ lattice
- $a^{-1} \approx 1.6 - 1.7$GeV, $L \approx 2$fm
- unitary points $am_l = 0.02$ and 0.01 ($m_\pi \approx 0.6$GeV and 0.48GeV)

Results - Kaon form factor

Conventional approach for $Kl3$

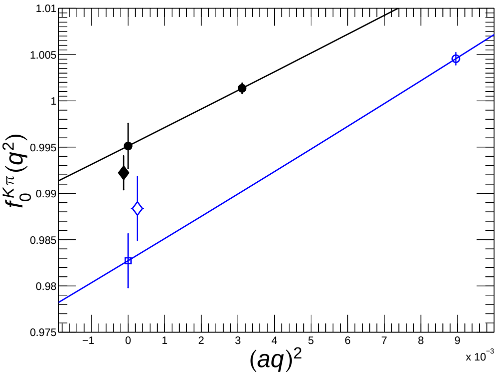


$am = 0.02$ ($m_\pi = 0.6$GeV)

$am = 0.01$ ($m_\pi = 0.48$GeV)

Results - Kaon form factor

Conventional and new approach for $Kl3$

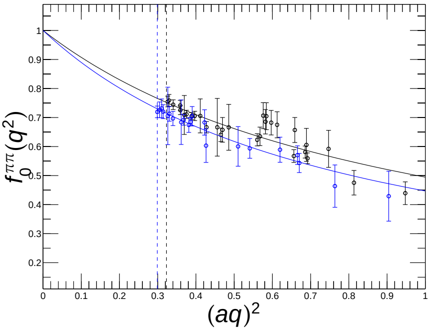


- procedure works
- compatible with pole dominance interpolation
- competitive errors

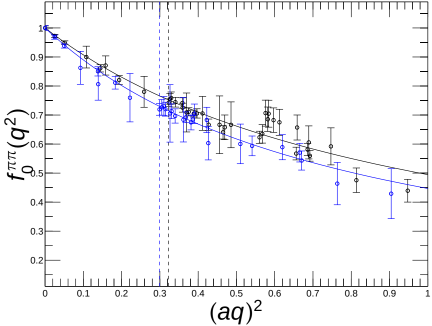
Summary and outlook

- new method allows to control one more systematic
- will now be applied to check current large scale simulations (cf. talk at Kaon 2007)
- many other applications: Pion/Kaon form factors, Baryons, distribution amplitudes

Conventional approach $f_{\pi\pi}(q^2)$

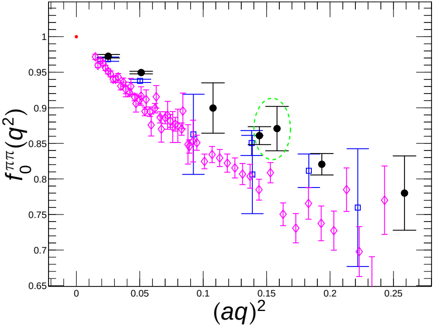


New approach $f_{\pi\pi}(q^2)$



A brief aside the pion form factor

New approach $f_{\pi\pi}(q^2)$



Experiment (magenta): NA7

lattice study: hep-lat/0703005