$K_{\ell 3}$ AT TWO LOOPS IN CHIRAL PERTURBATION THEORY

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Motivation
- Chiral Perturbation Theory
- $K_{\ell 3}$
- What is new when including isospin breaking?
- First *very preliminary* results with isospin breaking at two loops

Motivation

Basically two: Test ChPT and determine V_{us}
From Moulson, hep-ex/0703013, Flavianet working group result

Mode	$\Delta(SU(2))$	$\Delta(EM)$	Decay	$V_{us}f_+(0)$
K^0_{e3}	0	+0.52%	K_{Le3}	0.21639(55)
$K^0_{\mu 3}$	0	+0.95%	$K_{L\mu 3}$	0.21649(68)
$K^\pm_{e3}$	2.31%	+0.52%	$K^\pm_{e3}$	0.21844(101)
$K^\pm_{\mu 3}$	2.31%	+0.95%	$K^\pm_{\mu 3}$	0.21809(125)

About 1% discrepancy: is this real or a fluke

The 2.31% is from on loop ChPT

Actual value of $f_+(0)$: nothing new

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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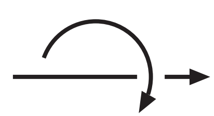
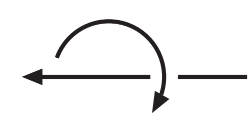
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via



$v < c$, $m_q \neq 0 \Longrightarrow$

$v = c$, $m_q = 0 \not\Longrightarrow$



Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

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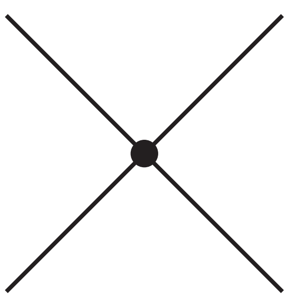
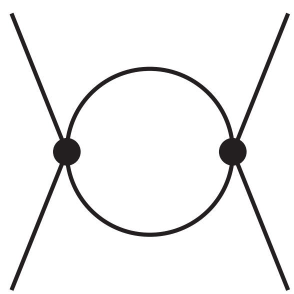
Power counting in momenta:

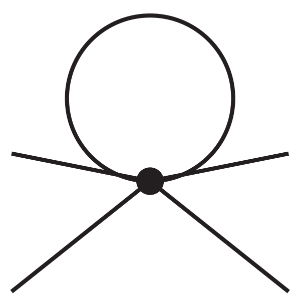


p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$



$(p^2)\,(1/p^2)\,p^4 = p^4$

$\int d^4p$ $\quad p^4$

Chiral Perturbation Theory

For lectures, review articles: see
http://www.thep.lu.se/~bijnens/chpt.html

Review paper on Two-Loops: JB, LU TP 06-16
hep-ph/0604043

Two Loop: Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

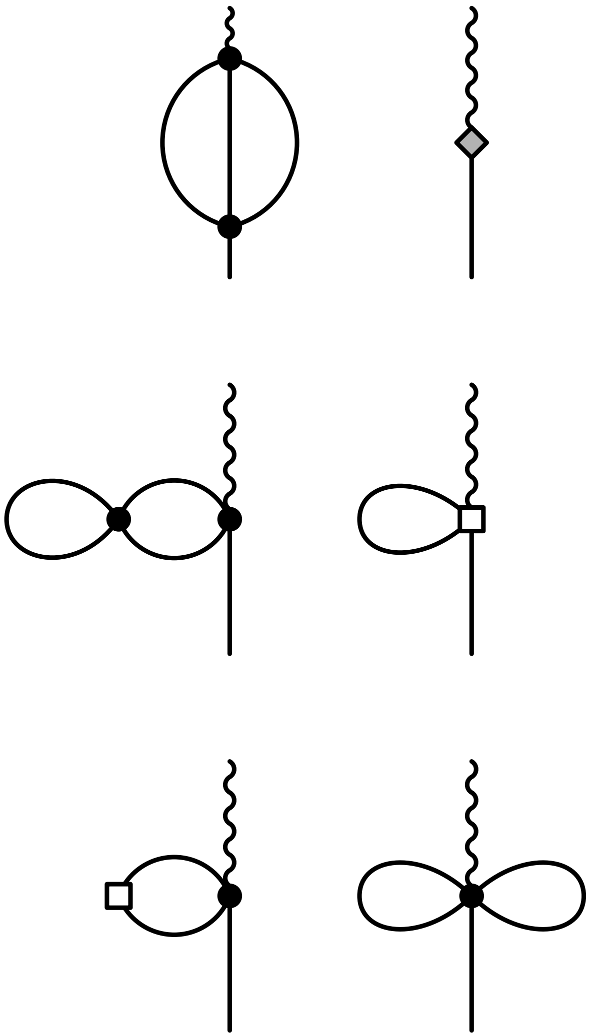
p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

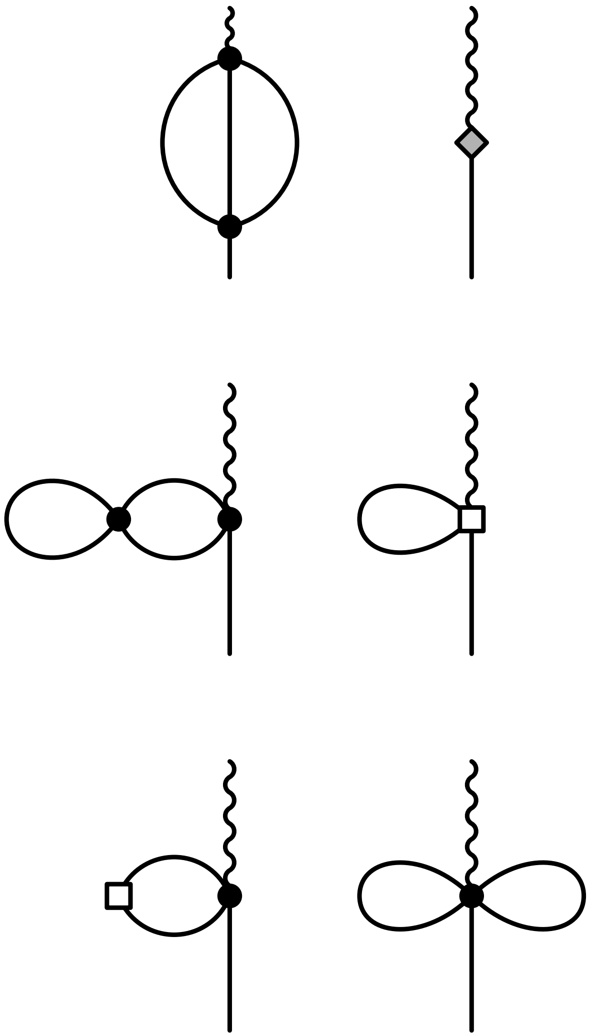
  $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$

Long Expressions



$\Longrightarrow$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} = &\ \pi_{16}\, L_0^r\, \left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \ -\ 2\,\pi_{16}\,L_1^r\,\chi_{13}^2\\
- &\ \pi_{16}\,L_2^r\,\left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\,L_3^r\,\left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right]\\
+ &\ \pi_{16}^2\,\left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \ -\ 48\,L_4^rL_5^r\,\bar\chi_1\chi_{13} \ -\ 72\,L_4^{r2}\,\bar\chi_1^2\\
- &\ 8\,L_5^{r2}\,\chi_{13}^2 \ +\ \bar A(\chi_p)\,\pi_{16}\,\left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\,R^p_{q\eta} + 1/16\,\bar\chi_1\,R^c_p - 1/48\,R^p_{q\eta}\,\chi_p - 1/16\,R^p_{q\eta}\,\chi_q\right.\\
+ &\ \left.1/48\,R^\eta_{pp}\,\chi_\eta + 1/16\,R^c_p\,\chi_{13}\right] \ +\ \bar A(\chi_p)\,L_0^r\,\left[8/3\,R^p_{q\eta}\,\chi_p + 2/3\,R^c_p\,\chi_p + 2/3\,R^d_p\right] \ +\ \bar A(\chi_p)\,L_3^r\,\left[2/3\,R^p_{q\eta}\,\chi_p\right.\\
+ &\ \left.5/3\,R^c_p\,\chi_p + 5/3\,R^d_p\right] \ +\ \bar A(\chi_p)\,L_4^r\,\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0} - 2\,\bar\chi_1\,R^p_{q\eta} + 3\,\bar\chi_1\,R^c_p\right] \ +\ \bar A(\chi_p)\,L_5^r\,\left[-2/3\,\bar\chi^{pp}_{\eta\eta1} - R^p_{q\eta}\,\chi_p\right.\\
+ &\ \left.1/3\,R^p_{q\eta}\,\chi_q + 1/2\,R^c_p\,\chi_p - 1/6\,R^c_p\,\chi_q\right] \ +\ \bar A(\chi_p)^2\,\left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\,R^p_{q\eta}R^c_p + 1/288\,(R^c_p)^2\right]\\
+ &\ \bar A(\chi_p)\bar A(\chi_{ps})\,\left[-1/36\,R^p_{q\eta} - 5/72\,R^p_{s\eta} + 7/144\,R^c_p\right] \ -\ \bar A(\chi_p)\bar A(\chi_{qs})\,\left[1/36\,R^p_{q\eta} + 1/24\,R^p_{s\eta} + 1/48\,R^c_p\right]\\
+ &\ \bar A(\chi_p)\bar A(\chi_\eta)\,\left[-1/72\,R^p_{q\eta}R^v_{\eta13} + 1/144\,R^c_pR^v_{\eta13}\right] \ +\ 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \ +\ 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\,R^\eta_{pp}\\
+ &\ \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\,\left[1/4\,\chi_p - 1/18\,R^p_{q\eta}R^c_p\,\chi_p - 1/72\,R^p_{q\eta}R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\,R^c_pR^d_p\right]\\
+ &\ \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\,\left[1/18\,R^\eta_{pp}R^c_p\,\chi_p - 1/18\,R^\eta_{13}R^c_p\,\chi_p\right] \ +\ \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\,\left[-1/72\,R^p_{q\eta}R^d_q + 1/144\,R^c_pR^d_q\right]\\
- &\ 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\,R^p_{s\eta}\,\chi_{ps} \ -\ 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\,R^q_{p\eta}R^c_p\,\chi_p\\
+ &\ 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^c_pR^d_p\,\chi_p \ +\ \bar A(\chi_p;\varepsilon)\,\pi_{16}\,\left[1/8\,\bar\chi_1R^p_{q\eta} - 1/16\bar\chi_1\,R^c_p - 1/16\,R^c_p\,\chi_p - 1/16\,R^d_p\right]\\
+ &\ \bar A(\chi_{ps})\,\pi_{16}\,\left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \ -\ 2\,\bar A(\chi_{ps})\,L_0^r\,\chi_{ps} \ -\ 5\,\bar A(\chi_{ps})\,L_3^r\,\chi_{ps} \ -\ 3\,\bar A(\chi_{ps})\,L_4^r\,\bar\chi_1\\
+ &\ \bar A(\chi_{ps})\,L_5^r\,\chi_{13} \ +\ \bar A(\chi_{ps})\bar A(\chi_\eta)\,\left[7/144\,R^\eta_{pp} - 5/72\,R^\eta_{ps} - 1/48\,R^\eta_{qq} + 5/72\,R^\eta_{qs} - 1/36\,R^\eta_{13}\right]\\
+ &\ \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\,\left[1/24\,R^p_{s\eta}\,\chi_p - 5/24\,R^p_{s\eta}\,\chi_{ps}\right] \ +\ \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,R^\eta_{ps}R^z_{qp\eta}\,\chi_p\right.\\
- &\ \left.1/9\,R^\eta_{ps}R^z_{qp\eta}\,\chi_{ps}\right] \ -\ 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\,R^d_q \ +\ 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\,R^q_{s\eta}\,\chi_s\\
+ &\ 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\,R^q_{s\eta} \ +\ 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\,\left[\chi_s + \bar\chi_1\right] \ -\ 1/8\,\bar A(\chi_{p4})^2 \ -\ 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6})\\
+ &\ 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \ -\ 1/32\,\bar A(\chi_{p6})^2 \ +\ \bar A(\chi_\eta)\,\pi_{16}\,\left[1/16\,\bar\chi_1\,R^v_{\eta13} - 1/48\,R^v_{\eta13}\,\chi_\eta + 1/16\,R^v_{\eta13}\,\chi_{13}\right]\\
+ &\ \bar A(\chi_\eta)\,L_0^r\,\left[4R^\eta_{13}\,\chi_\eta + 2/3\,R^v_{\eta13}\,\chi_\eta\right] \ -\ 8\,\bar A(\chi_\eta)\,L_1^r\,\chi_\eta \ -\ 2\,\bar A(\chi_\eta)\,L_2^r\,\chi_\eta \ +\ \bar A(\chi_\eta)\,L_3^r\,\left[4R^\eta_{13}\,\chi_\eta + 5/3\,R^v_{\eta13}\chi_\eta\right]\\
+ &\ \bar A(\chi_\eta)\,L_4^r\,\left[4\,\chi_\eta + \bar\chi_1\,R^v_{\eta13}\right] \ -\ \bar A(\chi_\eta)\,L_5^r\,\left[1/6\,R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\,R^v_{\eta13}\,\chi_\eta\right] \ +\ 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta13})^2\\
+ &\ 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\,R^v_{\eta13} \ +\ \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\,\left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta1} - 1/18\,R^p_{q\eta}R^\eta_{pp}\,\chi_p + 1/18\,R^\eta_{pp}R^c_p\,\chi_p\right.\\
+ &\ \left.1/144\,R^d_pR^v_{\eta13}\right] \ +\ \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta1} + 1/18\,(R^\eta_{pp})^2R^z_{qp\eta}\,\chi_p\right]\\
- &\ 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\,R^\eta_{ps}\,\chi_{ps} \ -\ \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\,\left[1/216\,R^v_{\eta13}\,\chi_4 + 1/27\,R^v_{\eta13}\,\chi_6\right]\\
- &\ 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\,R^1_{\eta\eta}R^3_{\eta\eta}\,\chi_\eta \ +\ 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^\eta_{pp}R^d_p\,\chi_p \ +\ \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\,\left[1/8\,\chi_\eta\right.\\
- &\ \left.1/16\,\bar\chi_1\,R^v_{\eta13} - 1/8\,R^\eta_{13}\,\chi_\eta - 1/16\,R^v_{\eta13}\,\chi_\eta\right] \ +\ \bar A(\chi_1)\bar A(\chi_3)\,\left[-1/72\,R^p_{q\eta}R^c_q + 1/36\,R^1_{3\eta}R^3_{1\eta} + 1/144\,R^c_1R^c_3\right]\\
- &\ 4\,\bar A(\chi_{13})\,L_1^r\,\chi_{13} \ -\ 10\,\bar A(\chi_{13})\,L_2^r\,\chi_{13} \ +\ 1/8\,\bar A(\chi_{13})^2 \ -\ 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k)\\
+ &\ 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \ +\ 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \ +\ 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \ -\ 24\,\bar A(\chi_4)\,L_1^r\,\chi_4 \ -\ 6\,\bar A(\chi_4)\,L_2^r\,\chi_4\\
+ &\ 12\,\bar A(\chi_4)\,L_4^r\,\chi_4 \ +\ 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \ +\ 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\,\left[R^p_{4\eta}R^\eta_{p4}\,\chi_4 - R^p_{4\eta}R^\eta_{q4}\,\chi_4\right]\\
- &\ 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_4 \ -\ 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\,R^1_{4\eta}R^3_{4\eta}\,\chi_4 \ +\ 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4\\
- &\ 32\,\bar A(\chi_{46})\,L_1^r\,\chi_{46} \ -\ 8\,\bar A(\chi_{46})\,L_2^r\,\chi_{46} \ +\ 16\,\bar A(\chi_{46})\,L_4^r\,\chi_{46} \ +\ \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\,\left[1/9\,\chi_{46} + 1/12\,R^\eta_{pp}\,\chi_p\right.\\
+ &\ \left.1/36\,R^\eta_{pp}\,\chi_4 + 1/9\,R^\eta_{p4}\,\chi_6\right] \ +\ \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,R^\eta_{pp}\,\chi_4 - 1/9\,R^\eta_{p4}\,\chi_6 + 1/9\,R^\eta_{q4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right]\\
- &\ 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\,\left[R^\eta_{pp} - R^\eta_{13}\right] \ +\ 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta13}\,\chi_{46} \ -\ \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\,\left[2/9\,\chi_{46}\right.\\
+ &\ \left.1/9\,R^\eta_{p4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right] \ -\ 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\,R^\eta_{13} \ +\ 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46}\\
+ &\ \bar B(\chi_p,\chi_p;0)\,\pi_{16}\,\left[1/16\,\bar\chi_1\,R^d_p + 1/96\,R^d_p\,\chi_p + 1/32\,R^d_p\,\chi_q\right] \ +\ 2/3\,\bar B(\chi_p,\chi_p;0)\,L_0^r\,R^d_p\,\chi_p\\
+ &\ 5/3\,\bar B(\chi_p,\chi_p;0)\,L_3^r\,R^d_p\,\chi_p \ +\ \bar B(\chi_p,\chi_p;0)\,L_4^r\,\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta0}\chi_p - 4\,\bar\chi_1R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1R^c_p\,\chi_p + 3\,\bar\chi_1R^d_p\right]\\
+ &\ \bar B(\chi_p,\chi_p;0)\,L_5^r\,\left[-2/3\bar\chi^{pp}_{\eta\eta1}\chi_p - 4/3\,R^p_{q\eta}\,\chi_p^2 + 4/3\,R^c_p\,\chi_p^2 + 1/2\,R^d_p\,\chi_p - 1/6\,R^d_p\,\chi_q\right]\\
+ &\ \bar B(\chi_p,\chi_p;0)\,L_6^r\,\left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta1} + 8\,\bar\chi_1R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1R^c_p\,\chi_p\right] \ +\ 4\,\bar B(\chi_p,\chi_p;0)\,L_7^r\,(R^d_p)^2\\
+ &\ \bar B(\chi_p,\chi_p;0)\,L_8^r\,\left[4/3\,\bar\chi^{pp}_{\eta\eta2} + 8/3\,R^p_{q\eta}\,\chi_p^2 - 8/3\,R^c_p\,\chi_p^2\right] \ +\ \bar B(\chi_p,\chi_p;0)^2\,\left[-1/18\,R^p_{q\eta}R^d_p\,\chi_p + 1/18\,R^c_pR^d_p\,\chi_p\right.\\
+ &\ \left.1/288\,(R^d_p)^2\right] \ +\ 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\,\left[R^\eta_{pp}R^d_p\,\chi_p - R^\eta_{13}R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

Usual ChPT two-loop: A list

Two-Loop Two-Flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006

Two-Loops Three flavours

- $\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$ — Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ — Maltman
- $\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ — Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} — Moussallam $\boxed{L_4^r, L_6^r}$

Usual ChPT two-loop: A list

- $\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ Amorós, JB, Talavera
- $K_{\ell 4}, \langle\overline{q}q\rangle$ Amorós, JB, Talavera L_1^r, L_2^r, L_3^r
- $F_M, m_M, \langle\overline{q}q\rangle\ (m_u \neq m_d)$ Amorós, JB, Talavera $L_{5,7,8}^r, m_u/m_d$
- $F_{V\pi}, F_{VK+}, F_{VK^0}$ Post, Schilcher; JB, Talavera L_9^r
- $K_{\ell 3}$ Post, Schilcher; JB, Talavera V_{us}
- $F_{S\pi}, F_{SK}$ (includes σ-terms) JB, Dhonte L_4^r, L_6^r
- $K, \pi \to \ell\nu\gamma$ Geng, Ho, Wu L_{10}^r
- $\pi\pi$ JB,Dhonte,Talavera
- πK JB,Dhonte,Talavera

$K_{\ell 3}$

- H. Leutwyler and M. Roos, Z.Phys.C25:91,1984.
- J. Gasser and H. Leutwyler, Nucl.Phys.B250:517-538,1985.
- J. Bijnens and P. Talavera, hep-ph/0303103, Nucl. Phys. B669 (2003) 341-362
- V. Cirigliano et al., hep-ph/0110153, Eur.Phys.J.C23:121-133,2002.
- J. Bijnens and K. Ghorbani, to be published.

$K_{\ell 3}$ Definitions

$$K_{\ell 3}^{+}: \quad K^{+}(p) \rightarrow \pi^{0}(p')\ell^{+}(p_\ell)\nu_\ell(p_\nu)$$

$$K_{\ell 3}^{0}: \quad K^{0}(p) \rightarrow \pi^{-}(p')\ell^{+}(p_\ell)\nu_\ell(p_\nu)$$

$$K_{\ell 3}^{+}: \quad T = \frac{G_F}{\sqrt{2}} V_{us}^{\star} \ell^{\mu} F_{\mu}{}^{+}(p', p)$$

$$\ell^{\mu} = \bar{u}(p_\nu)\gamma^{\mu}(1-\gamma_5)v(p_\ell)$$

$$F_{\mu}{}^{+}(p', p) = < \pi^{0}(p') \mid V_{\mu}^{4-i5}(0) \mid K^{+}(p) >$$

$$= \frac{1}{\sqrt{2}}[(p'+p)_{\mu} f_{+}^{K^{+}\pi^{0}}(t) + (p-p')_{\mu} f_{-}^{K^{+}\pi^{0}}(t)]$$

Isospin: $f_{+}^{K^{0}\pi^{-}}(t) = f_{+}^{K^{+}\pi^{0}}(t) = f_{+}(t)$

$f_{-}^{K^{0}\pi^{-}}(t) = f_{-}^{K^{+}\pi^{0}}(t) = f_{-}(t)$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leutwyler-Roos: see later
  - Radiative corrections: Cirigliano et al
- Know experimentally $f_+(0)$

V_{us}

PDG2002:

$|V_{ud}| = 0.9734 \pm 0.0008 \qquad |V_{us}| = 0.2196 \pm 0.0026$

$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) =$
0.9957 ± 0.0019

PDG2006:

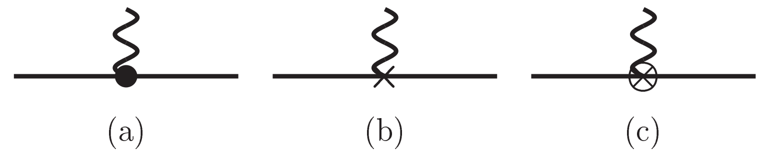
$|V_{ud}| = 0.97377 \pm 0.00027 \qquad |V_{us}| = 0.2257 \pm 0.0021$

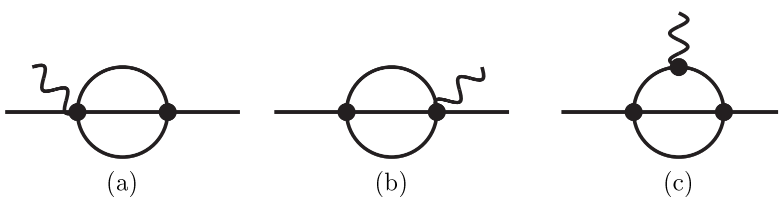
$|V_{ud}|^2 + |V_{us}|^2 = (0.94823 \pm 0.00054) + (0.05094 \pm 0.00095) =$
0.99917 ± 0.00110

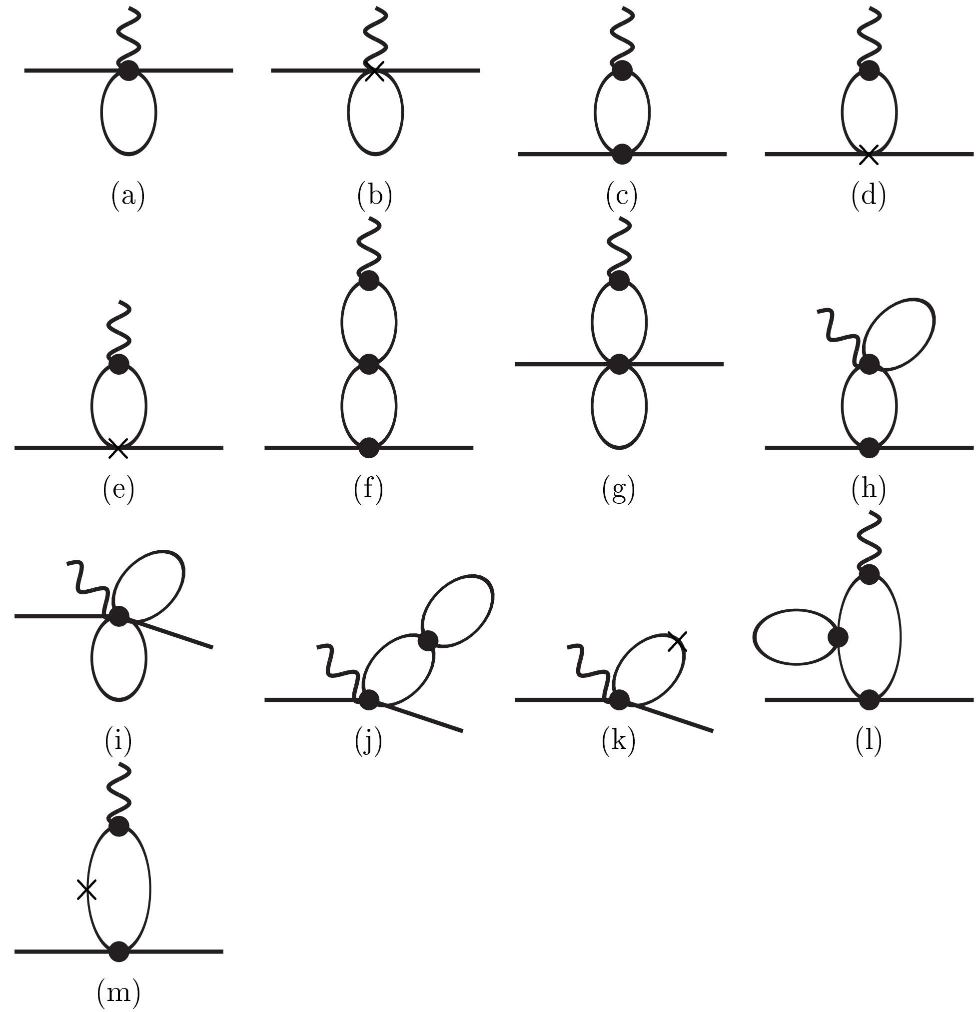
Problems:

- Ignores $\Delta(0) = 0.0113$ from pure two-loop
- Comparison between experiments: we'll hear more next week

$K_{\ell 3}$ Diagrams







● : p^2 vertex

× : p^4 vertex

⊗ : p^6 vertex

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

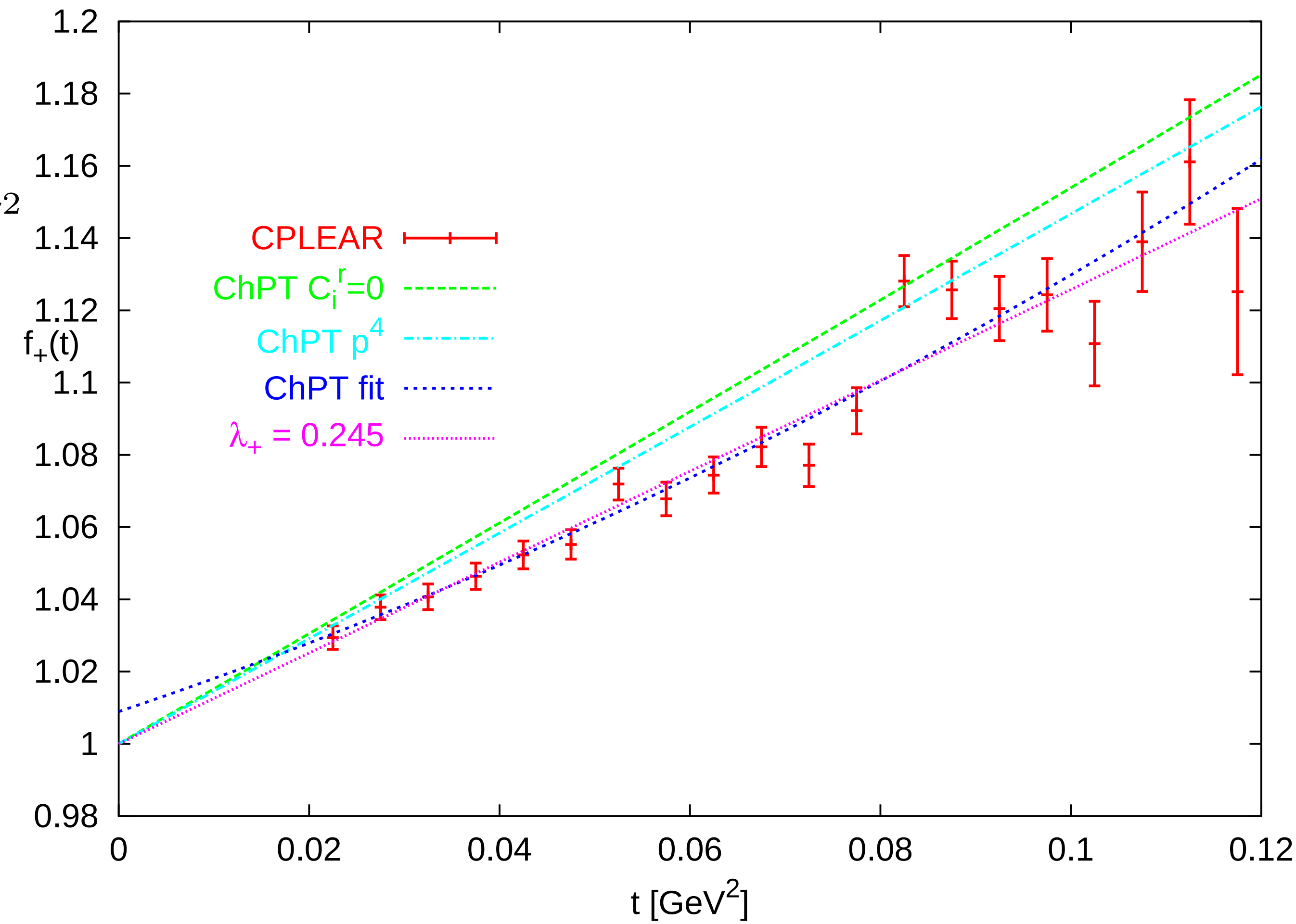
ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-(4.7 \pm 0.5)\ 10^{-5}$ GeV^2

($c_+ = 3.2$ GeV^{-4})

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \;=\;& 1-\frac{8}{F_\pi^4}\left(C_{12}^r+C_{34}^r\right)\left(m_K^2-m_\pi^2\right)^2 \\
&+8\frac{t}{F_\pi^4}\left(2C_{12}^r+C_{34}^r\right)\left(m_K^2+m_\pi^2\right)+\frac{t}{m_K^2-m_\pi^2}\left(F_K/F_\pi-1\right) \\
&-\frac{8}{F_\pi^4}t^2C_{12}^r+\overline{\Delta}(t)+\Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$
\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.
$$

Experiment

Form Factor comparison

KTeV [PRD 70(2004)]

K^0_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 4σ level

$K^0_{\mu 3}$ quadraqtic fit: $\lambda_0 = (13.72 \pm 1.31)\ 10^{-3}$

Slopes consistent for K^0_{e3} and $K^0_{\mu 3}$

ISTRA+ [PLB 581(2004), PLB589(2004)]

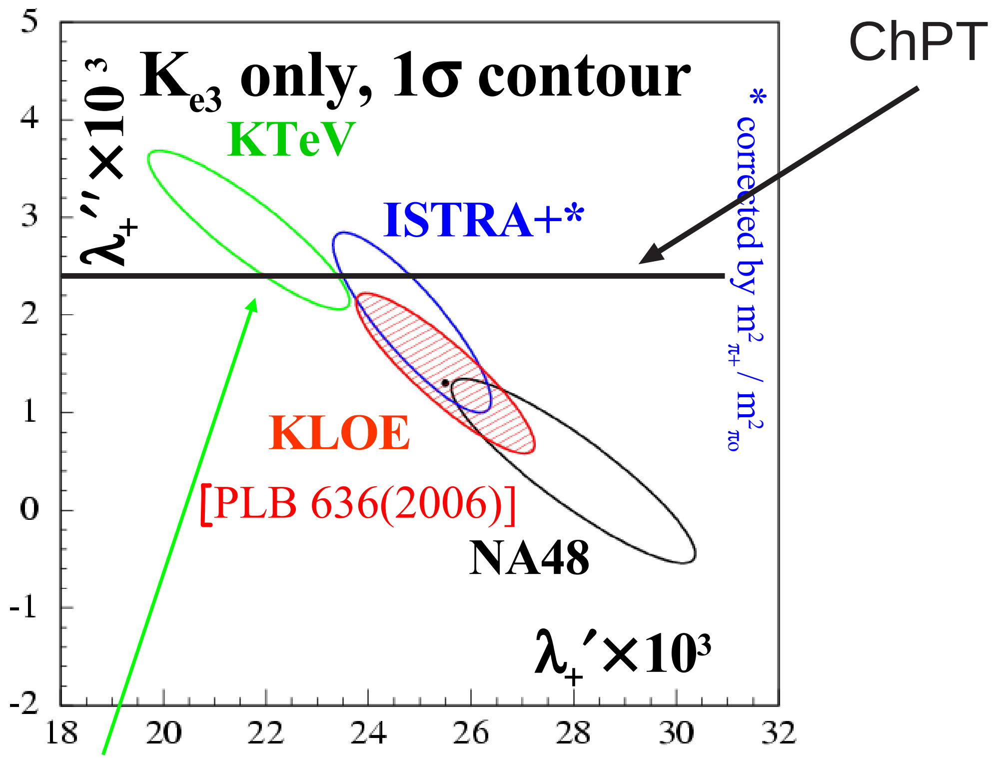
K^-_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 2σ level

$K^-_{\mu 3}$ quadratic fit: $\lambda_0 = (17.11 \pm 2.31)\ 10^{-3}$

NA48 [PLB 604(2004), HEP2005 289]

K^0_{e3}: No evidence for quadratic term

$K^0_{\mu 3}$ linear fit: $\lambda_0 = (12.0 \pm 1.7)\ 10^{-3}$



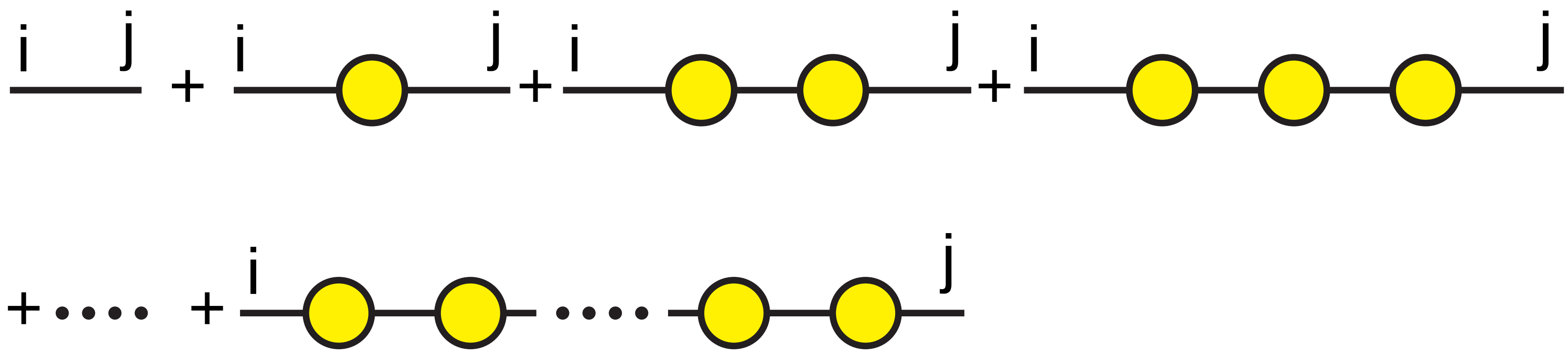
$\lambda''_+ \to -1\%$ phase space integral

$\to +0.5\%$ for V_{us}

A WG on Precise SM tests in K decays M.Antonelli LNF-INFN – FLAVIANET meeting Barcelona

New for isospin breaking at two loops

Two-point functions:



The diagrams for $\langle 0 | \phi_i \, \phi_j | 0 \rangle \equiv G_{ij}$

Problem: Mixing

Take LSZ into account properly

Amoros, JB, Talavera, 2001

Isospin Breaking

$$G = iP + iP(i\Pi iP) + iP(i\Pi iP)^2 + \ldots = iP\left(1 + \Pi P\right)^{-1}\,.$$

No mixing: Π, P and G all diagonal

$$\det(G^{-1}) = \det\left(\left(1 + \Pi P\right)P^{-1}\right) = \det\left(P^{-1} + \Pi\right) = 0\,.$$

Zeros give particle masses, residues can also be obtained.

Solve perturbatively for our case of π^0-η mixing.

Lowest order: $\pi^0 \equiv \cos(\epsilon)\,\pi_3^0 + \sin(\epsilon)\,\eta_8$

lowest order mixing angle ϵ: $\tan(2\epsilon) = \frac{\sqrt{3}}{2}\frac{m_d - m_u}{m_s - \hat{m}}$,

Isospin Breaking

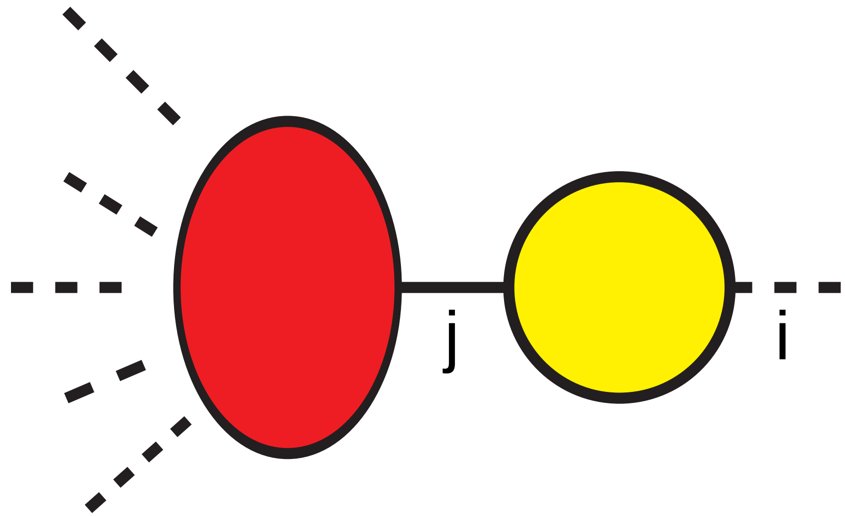
Define lowest order diagonal basis $P_{ij} = \dfrac{\delta_{ij}}{p^2 - m_{i0}^2}$

Π starts at next-to-leading order

$$
\begin{aligned}
m^2_{phys} &= m_0^2 + (m^2)^{(4)} + (m^2)^{(6)} + \ldots \\
m_0^2 &= m_{30}^2 \,, \\
(m^2)^{(4)} &= -\Pi_{33}^{(4)}(m_{30}^2, m_{i0}^2, F_0) \,, \\
(m^2)^{(6)} &= -\Pi_{33}^{(4)}(m_{30}^2, m_{i0}^2, F_0) - (m^2)^{(4)} \left. \frac{\partial}{\partial p^2} \Pi_{33}^{(4)}(p^2, m_{i0}^2, F_0) \right|_{p^2=r} \\
&\quad + \frac{1}{m_{30}^2 - m_{80}^2} \left(\Pi_{38}^{(4)}(m_{30}^2, m_{i0}^2, F_0) \right)^2
\end{aligned}
$$

Matrix element

Matrix element:



$$\mathcal{A}_{i_1\ldots i_n} = \left(\frac{(-i)^n}{\sqrt{Z_{i_1}\ldots Z_{i_n}}}\right)\prod_{i=1}^{n}\lim_{k_i^2\to m_i^2}(k_i^2-m_i^2)\,G_{i_1\ldots i_n}(k_1,\ldots,k_n)$$

Isospin Breaking

$G^{-1} = -i\mathcal{P}$, $\mathcal{P} = P^{-1} + \Pi$

$$Z_3 = \frac{1}{\left.\frac{\partial}{\partial p^2}\left(\det \mathcal{P}(p^2)\right)\right|_{p^2=m_\pi^2}} \mathcal{P}_{88}(m_\pi^2)\,,$$

Leading to $\mathcal{A}_3 = \dfrac{1}{\sqrt{\mathcal{P}_{88}\frac{\partial}{\partial p^2}\left(\mathcal{P}_{33}\,\mathcal{P}_{88} - \mathcal{P}_{38}^2\right)}}\left\{\mathcal{P}_{88}\mathcal{G}_3 - \mathcal{P}_{38}\mathcal{G}_8\right\}\,.$

Now expand in chiral orders and . . .

Isospin Breaking

$$
\begin{aligned}
\mathcal{A}_3 \;=\;& \mathcal{G}_3^{(2)} + \left\{\mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2}\mathcal{G}_8^{(2)}\right\} \\
&+\left[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8}\left(Z_{33}^{(4)}\right)^2 \mathcal{G}_3^{(2)}\right. \\
&+\frac{Z_{38}^{(4)}\Pi_{38}^{(4)}}{\Delta m^2}\mathcal{G}_3^{(2)} - \frac{1}{2}\left(\frac{\Pi_{38}^{(4)}}{\Delta m^2}\right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2}\mathcal{G}_8^{(4)} \\
&\left.-\frac{\Pi_{38}^{(6)}}{\Delta m^2}\mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)}\,\Pi_{88}^{(4)}}{\Delta m^2}\mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)}\,\frac{\Pi_{38}^{(4)}}{\Delta m^2}\mathcal{G}_8^{(2)}\right]
\end{aligned}
$$

Isospin Breaking

$$
\begin{aligned}
\mathcal{A}_3 \;=\;& \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
&+ \Bigg[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left(Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \\
&+ \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left(\frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
&- \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \Bigg]
\end{aligned}
$$

Compute all diagrams, produce numerical programs, . . .

Numerical results for $f_+(0)$

VERY PRELIMINARY

Decay	p^2	p^4	pure 2-loop	L_i^r at p^6	C_i^r	total
			Iso conserving calculation			
K^0	1	-0.02266	0.01130	0.00332	???	0.99196
K^+	1	-0.02276	0.01104	0.00320	???	0.99154
			$m_u/m_d = 0.45$			
K^0	1	-0.02310	0.01131	0.00325	???	0.99146
K^+	1.02465	-0.01741	0.00379	0.00648	???	1.01751
ratio	1.02465	1.0311				1.0262
			$m_u/m_d = 0.58$			
K^0	1	-0.02299	0.01124	0.00325	???	0.99150
K^+	1.01702	-0.01897	0.00657	0.00551	???	1.01013
ratio	1.0170	1.0215				1.0188

Some Comments

- K^0 only again on $C^r_{12} + C^r_{34}$
  Ademollo-Gatto + Callan-Treiman as in isoconserving case
  but $(m_s - m_u)^2$
- Not checked yet whether C^r_i in K^+ decay can b determined
- p^6 lowers the isospin breaking
- p^6 fit has $m_u/m_d = 0.45$ and not 0.58
- $0.58 \to 0.52$ from p^6 and $0.52 \to 0.45$ from violation of Dashen's theorem
- Very preliminary numerics: decrease discrepancy by about 0.5%.