WHAT CAN BE LEARNED COMPARING K_{l3}^{+} AND K_{l3}^{0}

Jan Stern

Institut de Physique Nucléaire, Orsay

Collaborateurs : V. Bernard (LPT-Strasbourg)
M. Oertel (LUTH-Meudon)
E. Passemar (IPN-Orsay)

Different sources of isospin breaking

- $m_d - m_u \neq 0$ integral part of QCD / ChPT
- local electromagnetic contributions (Effective theory description, LECs C, K, X)
- long distance (non local) electromagnetic contributions

After factorization of the last type of elmg contributions:
4 form form factors:

$f_+^K(t)$, $f_S^K(t) = f_+^K(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$.

Here $K = K^+$ for $K^+ \to \pi^0$ decay and
$K = K^0$ for $K^0 \to \pi^-$ decay

$f^K_{+,S}(t) = f^K_+(0) F^K_{+,S}(t)$ where $F^K_{+,S}(0) = 1$

$f^K_+(0)\mathcal{V}^{us}_{eff} \ldots \rightarrow$ Decay rates

$F^K_{+,S}(t)\ldots \rightarrow$ form factor shapes, Dalitz distributions, needed to calculate phase space integrals

Two types of $m_d - m_u$ effects

- "normal size" $\frac{m_d - m_u}{4\pi F_\pi} \ldots$
  few $\times 10^{-3}$ affect both K^0 and K^+.
- enhanced by small denominator $\frac{m_d - m_u}{m_s}$
  $\eta - \pi^0$ mixing , $\mathcal{O}(1)$ in ChPT,
  $\sim 10^{=2}$, affects $K^+ \rightarrow \pi^0$ decay rate.

ChPT suggests:
isospin asymmetry in form factor **SHAPES** is negligible.

$F_+^{K^0}(t) = F_+^{K^+}(t)$ and $F_S^{K^0}(t) = F_S^{K^+}(t)$
implying the equality of corresponding slopes curvatures etc

Defining further $r^{+0} = \frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} = 1 + \Delta_{SU(2)}$
the whole experimental information is stored in

- $f_+^{K^0}(0)\mathcal{V}_{eff}^{us}$
- $\Delta_{SU(2)}$
- form factor shapes $F_{+,S}(t)$ defining in turn phase space integrals

This is the standardly used framework. In addition it is customary to use **the input value** $\Delta_{SU(2)} = 0.023 \pm 0.003$
This allows to put apples and pears together and make an average.

Theoretical questions behind $\Delta_{SU(2)}$

The dominant contribution comes from $\eta - \pi^0$ mixing given by $\frac{<\eta|\bar{q}\mathcal{M}q|\pi^0>}{M_\eta^2 - M_\pi^2}$,

If the three flavor condensate $< \bar{q}q > |m_s = m_u = m_d = 0$ is the leading order parameter (c.f. Cirigliano et al)

$$\Delta_{SU(2)} = \sqrt{3}[\epsilon^{(2)} + \epsilon_S^{(4)} + \epsilon_{EM}^{(4)}] - \frac{\alpha}{4\pi} \ln \frac{M_K^2}{M_\pi^2} - 16\pi\alpha X_1$$

dominated by the quark mass ratio

$$\epsilon^{(2)} = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - m}$$

The remaining terms are small and known : $-16\pi\alpha X_1 = 0.001$, (Descotes-Genon, Moussallam - 2004)

The quark mass ratio $R = \frac{m_s - m}{m_d - m_u}$

Fundamental parameter of QCD. Assuming (once more) **the dominance of the three flavor $\bar{q}q$ condensate** , ChPT relates R to :

- Kaon mass difference in the absence of electromagnetism:
  $\frac{1}{2}R(r+1) = \frac{M_K^2(M_K^2 - M_\pi^2)}{M_\pi^2(M_{K^0}^2 - M_{K^+}^2)}[1 + \mathcal{O}(m_s^2)] \equiv Q^2$
  where $r = m_s/m$. The electromagnetic contribution to the Kaon mass difference is hard to estimate precisely.
- $\eta \to 3\pi$ partial width $\sim 1/R^2$. Sensitive to higher orders in the expansion in m_s . Dispersive analysis possible but **crucial experimental inputs are not available**.

Summary of this situation by Leutwyler in 1996 : $R = 40.8 \pm 3.2$

- Increase of $\Gamma(\eta \to 2\gamma)$ - normalization channel in the last eddition of PDG $\to$ decrease of R.
- Errors quoted by Leutwyler are likely underestimated.

If the three flavor condensate is suppressed due to important fluctuations of $\bar{s}s$ vacuum pairs - the above predictions could be substantially modified , increasing the expected value of $\Delta_{SU(2)}$.

The question is closely related to ongoing tests of ChPT prediction for low-energy $\pi\pi$ scattering phases (NA48/2 K_{e4} decay analysis) and is currently under investigation (Descotes-Genon, JS)
$\boldsymbol{\Delta_{SU(2)}}$ has to be measured

I_K	Inputs (τ_K, BRs)	$\mid f_+(0)V_{us}\mid_e^{K^0}$	$\mid f_+(0)V_{us}\mid_e^{K^+}$	$\Delta SU(2)_e$
$\lambda'_+ = 0.02484(110), \lambda''_+ = 0.00161(45)$	Moulson [Nag]	0.21639(55)	0.22348(90)	0.0328(49)
$\rho(\lambda'_+, \lambda''_+) = -0.944, \rho(\lambda'_+, \lambda_0) = 0.314$	NA48		0.22567(87)	0.0429(48)
$\rho(\lambda''_+, \lambda_0) = -0.420$	ISTRA		0.22354(101)	0.0330(53)
Moulson [Nagoya]				

New connection : $\Gamma(\pi^0 \to 2\gamma)$

Leading contribution to the amplitude : ABJ anomaly $\mathcal{O}(p^4)$

Corrections $\mathcal{O}(p^6, e^2p^4, (m_d - m_u)p^4)$ determined by Ananthanarayan and Moussallam (2002): They are of percent level and **they are dominated by $\eta - \pi^0$ mixing ... new access to $\Delta_{SU(2)}$**

$A(\pi^0 \to 2\gamma) = \frac{\alpha}{\pi F_\pi}[1 + 1.24\Delta_{SU(2)} + \ldots]$

calculated corrections of 0.001 level.

This is likely the most accurate way how to measure F_π independently of EW couplings of quarks to W:

$F_\pi = [92.16 \pm 0.11](1 + 2\epsilon_{NS})$ MeV

where ϵ_{NS} is a yet undetermined small parameter arising from the chiral CKM mixing (talk by Micaela Oertel).

The result of the PRIMEX measurement of $\Gamma(\pi^0 \to 2\gamma)$ to better than 1 percent is expected by the summer.

SUMMARY

Three crucial tests of ChPT are still pending

- $\pi\pi$ scattering lengths and phases
- Callan-Treiman Theorem
- $\Delta_{SU(2)}$

The prediction $\Delta_{SU(2)} = 0.023 \pm 0.002$ has never been tested. Sanctifying it as the TRUTH using it as an input in other analysis, is the best way how **to never test it**.