Frascati, May 2007

Improved estimates of the rare K decay matrix-elements from $K_{\ell 3}$ decays

Christopher Smith

b UNIVERSITÄT BERN

In collaboration with
Federico Mescia,
[hep-ph] 0705.2025

- Outline

A- Introduction

Present status for rare K decays

Goals and strategy to improve

B- Vector form-factors beyond LO

FCNC form-factors at $\mathcal{O}(p^4)$ *and ratios of form-factors*

C- Numerical study

Slopes and phase-space integrals

f(0) and branching ratios

D- Conclusion

Introduction

- Why rare K decays are so interesting?

$$K_L \to \pi^0 \nu\bar{\nu},\ K^+ \to \pi^+ \nu\bar{\nu},\ K_L \to \pi^0 e^+ e^-,\ K_L \to \pi^0 \mu^+ \mu^-$$

- "*Would-be forbidden*" modes in the SM → *New Physics can be dominant*
  - Helicity-suppression: $Br(K_L \to e^+ e^-)^{\exp} = 9^{+6}_{-4} \times 10^{-12}$
  - Lepton Flavor Violation: $Br(K_L \to \mu^\pm e^\mp)^{\exp} < 4.7 \times 10^{-12}$
  - Flavor Changing Neutral Currents

    ↳ *GIM mechanism*: probe the SM at the quantum level (loop).

- *CP-violating FCNC*: Additional suppression in the SM $\left(\operatorname{Im}\lambda_t = \operatorname{Im}(V_{td}V_{ts}^*) \sim 10^{-4}\right)$

  Heaviest SM particle (top quark) gives the largest contribution

  → Well-controlled perturbative regime.

- *Semi-leptonic decays*: hadronic effects under excellent control (compare with ε'/ε) (FCNC and CC matrix elements are related).

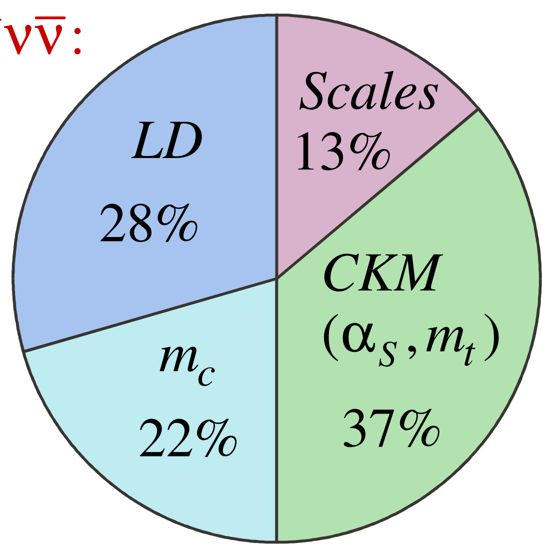
- *The only theoretically clean window on the $\Delta S = 1$ sector*

  → Essential input for the "inverse problem" in the LHC era.

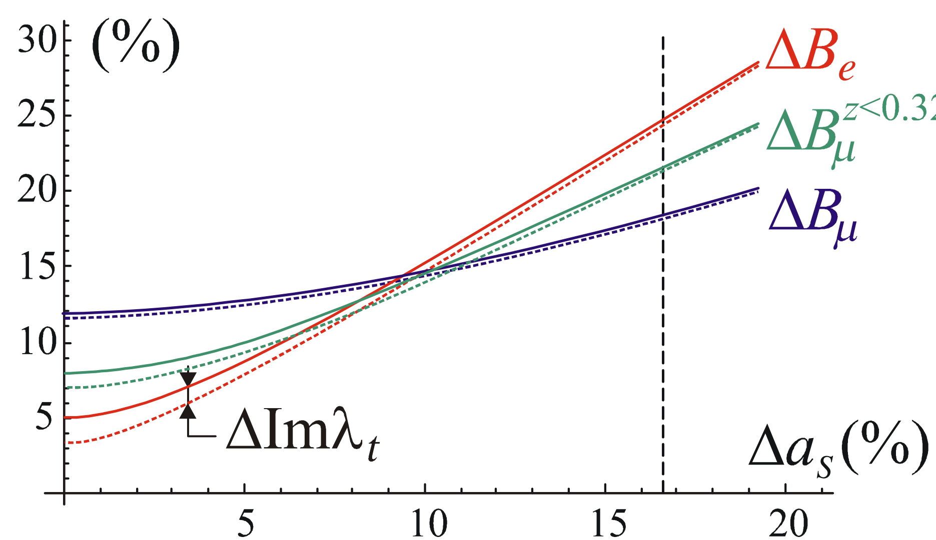
- Summary of current status in the SM

	V,A	K^0-$\bar{K}^0$	2^{++}	0^{++}	SM $(\times 10^{-11})$	Experiment
$K_L \to \pi^0 \nu\bar{\nu}$	100%	(≈ 1%)	–	–	$2.81^{+0.56}_{-0.56}$	$< 2.1 \cdot 10^{-7}$ E391a
$K_L \to \pi^0 e^+ e^-$	40%	60%	(<3%)	–	$3.54^{+0.98}_{-0.85}$	$< 2.8 \cdot 10^{-10}$ KTeV
$K_L \to \pi^0 \mu^+ \mu^-$	30%	35%	–	35%	$1.41^{+0.28}_{-0.26}$	$< 3.8 \cdot 10^{-10}$ KTeV
$K^+ \to \pi^+ \nu\bar{\nu}$	100%	–	–	–	$8.0^{+1.1}_{-1.1}$	$14.7^{+13.0}_{-8.9} \cdot 10^{-11}$ E787 E949

Theory errors for $K^+ \to \pi^+ \nu\bar{\nu}$:



Theory errors for $K_L \to \pi^0 \ell^+ \ell^-$:



Haisch, FlavLHC ('06)
Buras, Gorbahn, Haisch, Nierste ('05, '06)

- Precision physics with the $K \to \pi\nu\bar{\nu}$ decays

$$H_{eff}(\bar{s}d \to \bar{\nu}\nu) = \frac{G_F \alpha}{\sqrt{2}} \frac{1}{2\pi \sin^2\theta_W} y_\nu \, (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

Matrix-elements from $K_{\ell 3}$

$$\Gamma(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_\nu^+ \frac{|y_\nu|^2}{|V_{us}|^{10}}, \quad \Gamma(K_L \to \pi^0\nu\bar{\nu}) = \kappa_\nu^L \frac{(\mathrm{Im}\, y_\nu)^2}{|V_{us}|^{10}}$$

$$\kappa_\nu^+ = (0.504 \pm 0.017)\cdot 10^{-10}, \quad \kappa_\nu^L = (2.20 \pm 0.07)\cdot 10^{-10}$$

Marciano, Parsa ('96) updated by Buras, Gorbahn, Haisch, Nierste ('05)

Wilson coefficient: $y_\nu = (\mathrm{Re}\,\lambda_t + i\,\mathrm{Im}\,\lambda_t)\, X(x_t) + |V_{us}|^4\, \mathrm{Re}\,\lambda_c (P_c + \delta P_{u,c})$

Dimension six *t-quark*: $X(x_t) \overset{NLO}{=} 1.464 \pm 0.041$ — *Buchalla, Buras ('93)*

Dimension six *c-quark*: $P_c \overset{NNLO}{=} \lambda^4 (0.37 \pm 0.04)$ — *Buras, Gorbahn, Haisch, Nierste ('05)*

c-quark dim-8 operators, *u-quark* LD contributions: $\delta P_{u,c} = \lambda^4 (0.04 \pm 0.02)$ — *Isidori, Mescia, C.S. ('05)*

• Rare K decay matrix-elements and $K_{\ell 3}$

CC and FCNC operator matrix-elements related by isospin-symmetry.

For all these $K \to \pi$ transitions:

$$\langle \pi^j(p') | \bar{q}\lambda^a\gamma^\mu q | K^i(p) \rangle = C_{ij} \left(f_+^{ij}(q^2)(p+p')^\mu + f_-^{ij}(q^2)(p-p')^\mu \right)$$

$$\lambda^{FCNC} \sim \lambda_{6\pm i7} : \bar{s}\gamma^\mu d, \;\; \lambda^{CC} \sim \lambda_{4\pm i5} : \bar{s}\gamma^\mu u$$

At lowest order: $f_+^{ij}(q^2) = 1, f_-^{ij}(q^2) = 0$ (conserved vector current).

Extracting *FCNC* form-factors from *CC* data: no need to study SU(3)-breaking.

*Our goal is to improve on the current LO analysis**: both the S.D. theoretical predictions for rare K decays, and $K_{\ell 3}$ data have significantly improved.

More specifically, we want:

**Marciano,Parsa ('96)*

- To study *isospin-breaking (IB) at NLO*, and partially NNLO,
- To include *long-distance QED corrections,*
- To use the *latest $K_{\ell 3}$ data,*
- To perform a detailed *error study.*

Parametrization in terms of the *form-factor at the origin* and its derivatives (*slopes*):

$$\Gamma(K^i \to \pi^j \ell^+ \nu_\ell(\gamma)) = C_{ij}^2 \frac{G_F^2 S_{EW} M_{K^i}^5}{192\pi^3} \left| V_{us} \times f_+^{K^i \pi^j}(0) \right|^2 \mathcal{I}_\ell^{ij} (1 + 2\Delta_{\ell,EM}^{ij})$$

C_{ij}: Clebsch-Gordan coefficients.

G_F fixed from μ-decay, S_{EW} the leading SD correction. *Sirlin ('82)*

Long-distance QED corrections moved to $\Delta_{\ell,EM}^{ij}$. *Cirigliano,Knecht,Neufeld, Rupertsberger,Talavera('02) Cirigliano,Neufeld,Pichl('04)*

$\mathcal{I}_\ell^{ij}$: Phase-space integrals, functions of the slopes.

$$B(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \kappa_\nu^+ (1 + \Delta_{EM}) \frac{|y_\nu|^2}{|V_{us}|^{10}}, \quad B(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_\nu^L \frac{(\mathrm{Im}\, y_\nu)^2}{|V_{us}|^{10}}$$

$$\kappa_\nu^{+,L} = \tau_{+,L} \frac{G_F^2 M_{K^{+,L}}^5 \alpha^2}{256\pi^5 \sin^4\theta_W} |V_{us}|^8 \left| V_{us} \times f_+^{K^{+,0}\pi^{+,0}}(0) \right|^2 \mathcal{I}_\nu^{+,0}$$

Our goal is thus: to study IB for the slopes and for the form-factors at the origin.

FCNC form-factors beyond LO

• Vector form-factors for the $K \to \pi$ transitions

1. Form-factors at $\mathcal{O}(p^4\varepsilon^{(2)}, p^2\alpha)$

- Loops with $\mathcal{O}(p^2\varepsilon^{(2)})$ and $\mathcal{O}(p^0\alpha)$ vertices (i.e. $Z_{em} e^2 F_\pi^4 \langle U^\dagger Q U Q \rangle$)

Ecker,Gasser,Pich,de Rafael('89)

- $\mathcal{O}(p^4\varepsilon^{(2)})$ and $\mathcal{O}(p^2\alpha)$ counterterms (the L_i and K_i)

Gasser,Leutwyler ('85) / Urech('95),Neufeld,Rupertsberger('96)

- No leptonic counterterms (only external currents),
  - → included in Δ_{EM} for $K_{\ell 3}$,
  - → included in the Wilson coefficients y_ν for rare decays.

Knecht,Neufeld, Rupertsberger,Talavera('89)

$$f_+^{K^0\pi^0} = 1 + 3H^{\eta}_{K^0} + H^{\pi^0}_{K^0} + 2H^{\pi^+}_{K^+} - \sqrt{3}\varepsilon^{(2)}(1 + H^{\eta}_{K^0} + 3H^{\pi^0}_{K^0} + 2H^{\pi^+}_{K^+}) - \sqrt{3}\varepsilon^{(4)}$$

$$f_+^{K^+\pi^0} = 1 + 3H^{\eta}_{K^+} + H^{\pi^0}_{K^+} + 2H^{\pi^+}_{K^0} + \sqrt{3}\varepsilon^{(2)}(1 + H^{\eta}_{K^+} + 3H^{\pi^0}_{K^+} + 2H^{\pi^+}_{K^0}) + \sqrt{3}\varepsilon^{(4)} - \frac{\alpha}{2\pi}\delta^{K^+}_{EM}$$

$$f_+^{K^0\pi^+} = 1 + 3H^{\eta}_{K^+} + H^{\pi^0}_{K^+} + 2H^{\pi^+}_{K^0} + 2\sqrt{3}\varepsilon^{(2)}(H^{\pi^0}_{K^+} - H^{\eta}_{K^+}) - \frac{\alpha}{2\pi}\delta^{\pi^+}_{EM}$$

$$f_+^{K^+\pi^+} = 1 + 3H^{\eta}_{K^0} + H^{\pi^0}_{K^0} + 2H^{\pi^+}_{K^+} - 2\sqrt{3}\varepsilon^{(2)}(H^{\pi^0}_{K^0} - H^{\eta}_{K^0}) - \frac{\alpha}{2\pi}J_{EM}(q^2)$$

- QED corrections moved into the Δ_{EM}'s (including QED-induced $\pi^0 - \eta$ mixing), But physical masses kept in the two-point loop functions H_i^j.

Cirigliano,Knecht,Neufeld,Rupertsberger,Talavera('02), Cirigliano,Neufeld,Pichl('04)

Vector F.F.

The $\pi^0-\eta$ *mixing parameters* are *Gasser,Leutwyler ('85)/Leutwyler('96)/Cirigliano et al. ('02)*

$$\varepsilon^{(2)} = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}} = 0.01061 \pm 0.00083, \quad \varepsilon^{(4)} = \varepsilon^{(2)}(0.27 \pm 0.09),$$

At *zero momentum-transfer*:

$$f_+^{K^0\pi^0}(0) = 0.9775 - \sqrt{3}(0.963\varepsilon^{(2)} + \varepsilon^{(4)})$$
$$f_+^{K^+\pi^0}(0) = 0.9773 + \sqrt{3}(0.963\varepsilon^{(2)} + \varepsilon^{(4)})$$
$$f_+^{K^0\pi^+}(0) = 0.9773 - 0.0250\varepsilon^{(2)}$$
$$f_+^{K^+\pi^+}(0) = 0.9775 + 0.0257\varepsilon^{(2)}$$

Varying the η-mass between theory and physical → global shift by -0.0004.

Though very precise, the SU(3)-breaking at NNLO has to be added,
and this generates a "large" 1% uncertainty: *Bijnens,Talavera('03)*

$$f_{p^6}(0) = \delta_{loops} + \delta_{CT} = -0.001 \pm 0.010, \quad \delta_{CT} = -\frac{8(M_K^2 - M_\pi^2)^2}{F_\pi^4}(C_{12}^r + C_{34}^r)$$

$$\delta_{CT}(LR) \approx -0.016$$ *Leutwyler,Roos('84)/…*

CKM unitarity → $\delta_{CT}(lattice) \approx -0.031$ *Becirevic et al.('05),…*

To avoid this, we consider only *ratios of $K \to \pi$ form-factors.*

2. The ratio r

Spectator quarks play no role at NLO, and only $\pi^0 - \eta$ mixing is relevant for:

$$r_{0+} = \frac{f_+^{K^+\pi^0}(q^2)}{f_+^{K^0\pi^+}(q^2)} = \frac{f_+^{K^+\pi^+}(q^2)}{f_+^{K^0\pi^0}(q^2)} = 1 + \sqrt{3}(\varepsilon^{(2)} + \varepsilon^{(4)})$$

(→ CC / FCNC slopes are equal two-by-two)

More interestingly, $r = \frac{f_+^{K^+\pi^0}(q^2) f_+^{K^0\pi^0}(q^2)}{f_+^{K^+\pi^+}(q^2) f_+^{K^0\pi^+}(q^2)} = 1 + \mathcal{O}((\varepsilon^{(2)})^2)$

$\mathcal{O}(p^6\varepsilon^{(2)})$ should induce corrections for r_{0+}, but these effects could well continue to cancel in the double ratio r. Indeed, local contributions do cancel out:

$$r_{0+} = 1 + \sqrt{3}(\varepsilon^{(2)} + \varepsilon^{(4)} + \varepsilon^{(6)}) + \frac{16(M_K^2 - M_\pi^2)^2}{\sqrt{3}F_\pi^4}\varepsilon^{(2)}(4C_{12} - 6C_{35}) + \mathcal{O}(q^2, p^4\alpha)$$

But: $r = 1 - \frac{64\pi\alpha Z_{em}}{F_\pi^2}(2(M_K^2 - M_\pi^2)C_{12} - q^2 C_{90}) + \mathcal{O}((\varepsilon^{(2)})^2)$

Therefore: $r = 1.000 \pm 0.0002$ to account for residual *non-local* $\mathcal{O}(p^6\varepsilon^{(2)})$ contributions and $\mathcal{O}((\varepsilon^{(2)})^2, p^4\alpha)$ corrections.

3. The ratio r_K

The second ratio needed to extract FCNC form-factors is

$$r_K = \frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = (1.00027 \pm 0.00008) + (0.051 \pm 0.01)\varepsilon^{(2)}$$

At $\mathcal{O}(p^4\varepsilon^{(2)})$ spectator-quark effects negligible. Large corrections at $\mathcal{O}(p^6\varepsilon^{(2)})$?

$$\delta r_K = -\frac{8}{\sqrt{3}}\varepsilon^{(2)}\delta_{CT} - 64\pi\alpha Z_{em}\frac{M_K^2 - M_\pi^2}{F_\pi^2}C_{12}$$

Same combination of counterterms as for $f_{p^6}(0)$, but *suppressed by* $\varepsilon^{(2)}$.

Conservatively taking $\delta_{CT} = -0.016 \pm 0.016$ (uncertainties for the CT + loops):

$$r_K = (1.00027 \pm 0.00008) + (0.12 \pm 0.07)\varepsilon^{(2)} = \boxed{1.0015 \pm 0.0007}$$

Numerical study

1. Slopes:

ISTRA, KLOE, KTeV and NA48:

$$\begin{cases} \lambda'_+ = (24.84 \pm 1.10) \cdot 10^{-3} \\ \lambda''_+ = (1.61 \pm 0.45) \cdot 10^{-3} \\ \lambda_0 = (13.30 \pm 1.35) \cdot 10^{-3} \end{cases}$$

Moulson ('07)

(+ correlation)

Slopes arise at $\mathcal{O}(p^4)$, where they are dominated by L_9 (large SU(3)-breaking at $\mathcal{O}(p^6)$ expected). As a result, one finds a ridiculously small error for the ratio:

$$\frac{\lambda_+^{FCNC}}{\lambda_+^{CC}} = 0.9986 \pm 0.0002$$

What about SU(2)-breaking at $\mathcal{O}(p^6 \varepsilon^{(2)})$? $\frac{\lambda_+^{FCNC}}{\lambda_+^{CC}} = \frac{M^2(K^{*+})}{M^2(K^{*0})} = 0.990\ (\pm 0.005)$

$$\mathcal{I}_\nu^+ = 0.15268 \pm 0.00028_{\exp} \pm 0.00007_{th}$$

$$\mathcal{I}_\nu^L = \mathcal{I}_e^{V,A} = 0.16042 \pm 0.00030_{\exp} \pm 0.00008_{th}$$

$$\mathcal{I}_\mu^V = 0.03765 \pm 0.00010_{\exp} \pm 0.00003_{th}$$

Dominated by the exp. error on λ'_+

$$\mathcal{I}_\mu^A = 0.08619 \pm 0.00078_{\exp} \pm 0.00010_{th}$$

Dominated by the exp. error on λ_0

2. Form-factor at the origin

ISTRA, KLOE, KTeV and NA48: $$\begin{cases} K^0_{\ell 3}: \left|V_{us}\times f_+^{K^0\pi^+}(0)\right|_{\exp} = 0.21635(41) \\ K^+_{\ell 3}: \left|V_{us}\times f_+^{K^+\pi^0}(0)\right|_{\exp} = 0.22343(72) \end{cases}$$

Moulson ('07)

The most precise strategy:

$$\left|V_{us}\times f_+^{K^0\pi^0}(0)\right| = r\, r_K \frac{\left|V_{us}\times f_+^{K^0\pi^+}(0)\right|^2_{\exp}}{\left|V_{us}\times f_+^{K^+\pi^0}(0)\right|_{\exp}} = 0.2098(10)$$

$$\left|V_{us}\times f_+^{K^+\pi^+}(0)\right| = r_K \left|V_{us}\times f_+^{K^0\pi^+}(0)\right|_{\exp} = 0.2167(4)$$

Errors dominated by experiment:

Uncertainties due to residual $\mathcal{O}(p^6\varepsilon^{(2)})$ effects, i.e. errors on r and r_K, account for only 9%(3%) and 28%(0%) of the final errors, respectively.

Discussion:

For $K^0\pi^0$, there is a complication, because r_{0+} seems too large experimentally:

$$(r_{0+})_{\exp} = \frac{\left|V_{us} \times f_+^{K^+\pi^0}(0)\right|_{\exp}}{\left|V_{us} \times f_+^{K^0\pi^+}(0)\right|_{\exp}} = 1.0328 \pm 0.0039$$

$$(r_{0+})_{th} = 1 + \sqrt{3}(\varepsilon^{(2)} + \varepsilon^{(4)}) = 1.0238(35)_{linear} \quad (or\,(22)_{quad.})$$

This casts some doubts on theory (higher-orders), Δ_{EM} (th/exp) or experiment.
Note: experimentally, the K^+ data relies on older measurements.

Waiting for new measurements, we also present the final estimates for:

$$\left|V_{us} \times f_+^{K^0\pi^0}(0)\right|_{th} = \frac{r\, r_K}{(r_{0+})_{th}} \left|V_{us} \times f_+^{K^0\pi^+}(0)\right|_{\exp} = 0.2119 \pm 0.0004_{\exp} \pm 0.0008_{th}$$

$$\left|V_{us} \times f_+^{K^0\pi^0}(0)\right|_{KLOE} = r\, r_K \frac{\left|V_{us} \times f_+^{K^0\pi^+}(0)\right|^2_{\exp}}{\left|V_{us} \times f_+^{K^+\pi^0}(0)\right|^{KLOE}_{\exp}} = 0.2116 \pm 0.0018$$

Based on KLOE *preliminary*: $B(K^+_{e3}) = 5.047(92)\%, B(K^+_{\mu 3}) = 3.310(81)\%$.

3. Final results for the κ coefficients

Avoid propagating the K_L lifetime error by using (or the BR, but remove first Δ_{EM}!)

$$\left(\tau_L \left|V_{us} \times f_+^{K^0\pi^+}(0)\right|^2\right)_{\exp} = 0.23960(65)\ 10^{-8}$$

	$(r_{0+})_{th}$	$(r_{0+})_{KLOE}$	$(r_{0+})_{exp}$	*Exp.* τ_+	*Exp.* $f(0)$	*Exp.* $\mathcal{I}$	*Th.* r_K	*Th.* r	Future?
κ_ν^+	0.5168(25)	0.5168(25)	0.5168(25)	19	43	21	17	-	±0.0023
κ_ν^L	2.229(17)	2.229(36)	2.190(18)	-	77	12	9	2	±0.013
$\kappa_e^{V,A}$	0.7825(61)	0.7825(127)	0.7691(64)	-	77	12	9	2	±0.0046
κ_μ^V	0.1837(15)	0.1837(30)	0.1805(16)	-	73	16	8	2	±0.0011
κ_μ^A	0.4204(50)	0.4204(78)	0.4132(51)	-	54	38	6	2	±0.0031

$$\frac{\kappa_\nu^+}{\kappa_\nu^L} = \frac{1}{r^2}\frac{M_{K^+}^5}{M_{K^0}^5}\frac{\mathcal{I}_\nu^+}{\mathcal{I}_\nu^0}\frac{\left(\tau_+ \left|V_{us} \times f_+^{K^+\pi^-}(0)\right|^2\right)_{\exp}}{\left(\tau_L \left|V_{us} \times f_+^{K^0\pi^+}(0)\right|^2\right)_{\exp}} = 0.2359 \pm 0.0017$$

Future? (± 0.0008)

Future? = Errors on λ'_+, λ_0 and $\left|V_{us} \times f_+^{K^+\pi^0}(0)\right|$ (from $K_{\ell 3}^+$) reduced by 50%.

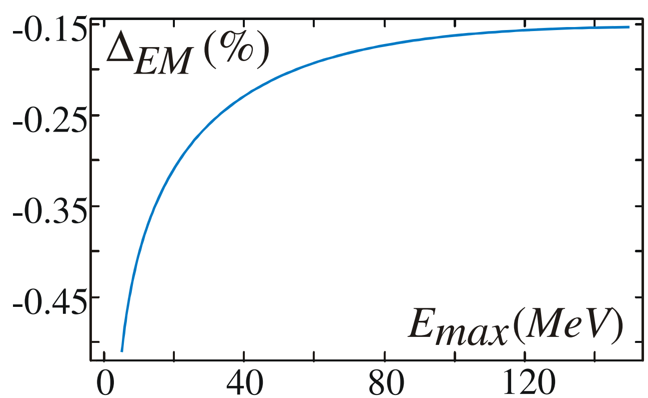
4. QED correction

Virtual photons: $B(K^+ \to \pi^+ \nu\bar{\nu})_{QED} = -\kappa_\nu^+ \frac{|y_\nu|^2}{|V_{us}|^{10}} \frac{1}{\mathcal{I}_\nu^+} \frac{\alpha}{\pi} \int_0^{(1-r_\pi)^2} dz \lambda_\pi^3 J_{EM}(z, r_\pi)$

$$f_+^{K^+\pi^+} = 1 + 3H_{K^0}^{\eta} + H_{K^0}^{\pi^0} + 2H_{K^+}^{\pi^+} - 2\sqrt{3}\varepsilon^{(2)}(H_{K^0}^{\pi^0} - H_{K^0}^{\eta}) - \frac{\alpha}{2\pi} J_{EM}(q^2)$$

Bremsstrahlung: $B(K^+ \to \pi^+ \nu\bar{\nu}\gamma) = \kappa_\nu^+ \frac{|y_\nu|^2}{|V_{us}|^{10}} \frac{1}{\mathcal{I}_\nu^+} \frac{\alpha}{\pi} \int_0^{(1-r_\pi)^2} dz \lambda_\pi^3 J_{BR}(z, r_\pi, E_{\max})$

Combination: $\Delta_{EM}(E_{\max}) = \frac{1}{\mathcal{I}_\nu^+} \frac{\alpha}{\pi} \int_0^{(1-r_\pi)^2} dz \lambda_\pi^3 \left(J_{BR}(z, r_\pi, E_{\max}) - J_{EM}(z, r_\pi) \right)$



Conclusion

Conclusion

Higher-order corrections can be controlled for two very clean *ratios of FCNC and CC form-factors*.

At present, a remaining source of uncertainty is due to the observed *isospin-breaking in $K_{\ell 3}$ data*, a priori *too large* to be accounted for theoretically. This should be resolved by *eagerly awaited new data*.

For now, using the theoretical IB, we get the improvements:

LO: $\kappa_\nu^+ = (0.504 \pm 0.017) \cdot 10^{-10}$ $\kappa_\nu^L = (2.20 \pm 0.07) \cdot 10^{-10}$

NLO: $\kappa_\nu^+ = (0.5168 \pm 0.0025) \cdot 10^{-10}$ $\kappa_\nu^L = (2.229 \pm 0.017) \cdot 10^{-10}$

Giving, for the $K \to \pi \nu \bar{\nu}$ branching ratios (for $E_{\max}$ about 20MeV):

$$B(K^+ \to \pi^+ \nu \bar{\nu}(\gamma)) = (7.82 \pm 0.82) \cdot 10^{-11}$$

Errors: $X_t : 20\%,\ P_{u,c} : 31\%,\ \kappa_\nu^+ : 3\%,\ CKM : 46\%$

$$B(K_L \to \pi^0 \nu \bar{\nu}) = (2.49 \pm 0.39) \cdot 10^{-11}$$

Errors: $X_t : 25\%,\ \kappa_\nu^L : 4\%,\ CKM : 71\%$

Hadronic uncertainties are exceptionally small, below the percent level! These modes will therefore allow us to *test the SM with a very high precision*.