$K^+ \to \pi^+\pi^0\gamma$: theoretical issues

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Old work with Gino, Gao, Portoles and New results based on work L. Cappiello and on preliminary work with Gino

- Introduction /CHPT
- Strong and weak counterterms, VMD
- $K^+ \to \pi^+\pi^0\gamma$ counterterm combination/kinematics and form factor
- Conclusions

Vector Meson Dominance in the strong sector

Ecker,Gasser,de Rafael,Pich

i	$L_i^r(M_\rho)$	V	A	Total	Total$^{c)}$
1	0.4 ± 0.3	0.6	0	0.6	0.9
2	1.4 ± 0.3	1.2	0	1.2	1.8
3	-3.5 ± 1.1	-3.6	0	-3.0	-4.9
4	-0.3 ± 0.5	0	0	0.0	0.0
5	1.4 ± 0.5	0	0	1.4	1.4
6	-0.2 ± 0.3	0	0	0.0	0.0
7	-0.4 ± 0.2	0	0	-0.3	-0.3
8	0.9 ± 0.3	0	0	0.9	0.9
9	6.9 ± 0.7	$6.9^{a)}$	0	6.9	7.3
10	-5.5 ± 0.7	-10.0	4.0	-6.0	-5.5

$^{c)}$ uses QCD "inspired" relations

$$F_V = 2G_V = \sqrt{2} f_\pi, \quad F_A = f_\pi \quad M_A = \sqrt{2} M_V$$

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

Theoretical motivations of success of VMD

- VMD improves the matching with QCD Ecker,Gasser,Leutwyler,Pich,de Rafael

$$F_V^{\pi^\pm}(t) \approx \frac{M_\rho^2}{M_\rho^2 - t},$$

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2).$$

- Picture consolidated by in the chiral quark model χQM Georgi-Manohar;Espriu,de Rafael,Taron
This is a dynamical interpretation (in CHPT) of the success of the non-relativstic quark model. In this model

$$L_i^{\chi \text{QM}} \sim L_i^{VMD}$$

- Also Large N

- Strong consolidated picture: Must work also in the WEAK sector in some way: **WHICH WAY?**

- Many CT's (37) Ecker,Kambor, Wyler; G.D. Portoles

$$\mathcal{L}^{(4)}_{|\Delta S|=1} = G_8 F^2 \sum_{i=1}^{37} N_i W_i$$

- There are **tests**
- $K^+ \longrightarrow \pi^+\pi^0\gamma$ and $K_S \longrightarrow \pi^+\pi^-\gamma$ same CT combination
- $K \longrightarrow \pi l^+ l^-$, $K^+ \longrightarrow \pi^+\pi^0\gamma$, $K^+ \longrightarrow \pi^+\gamma\gamma$ (observed or close to observation) and others probe the the same CT's $\Longrightarrow$ **CHPT tests**
- We need desperately some dynamical info and also some theory prejudice
- VMD must work and it is easy testable: form factor

Weak interactions in CHPT and VMD

- We know is tough: see $\Delta I = 1/2$-rule
- Actually even in the strong sector: $V^{\mu\nu}$ better than V^{μ}
- $\mathcal{L}^{(4)}_{\Delta S=1}$: $V^{\mu\nu}$ and V^{μ} similar results $(+)$ but important differences

Is there any chance that VMD works in CHPT weak?

- Evidence that VMD form factor describes better the photon energy distribution in $K_L \longrightarrow \pi^+\pi^-\gamma$
- size and sign of a_V in $K_L \longrightarrow \pi^0\gamma\gamma$ were postdicted in VMD
- the large slope in $K^+ \longrightarrow \pi^+l^+l^-$ maybe explained by the large size of VMD
- It is not a lot ...We need more infos

i	W_i	Vectors	Axials
14	$i\langle \Delta \{ f_+^{\mu\nu}, u_\mu u_\nu \} \rangle$	$\frac{1}{2} f_V^2 \eta_V$	
15	$i\langle \Delta u_\mu f_+^{\mu\nu} u_\nu \rangle$	$f_V^2 \eta_V$	
16	$i\langle \Delta \{ f_-^{\mu\nu}, u_\mu u_\nu \} \rangle$		$\frac{1}{2} f_A^2 \eta_A$
17	$i\langle \Delta u_\mu f_-^{\mu\nu} u_\nu \rangle$		$f_A^2 \eta_A$
18	$\langle \Delta (f_{+\,\mu\nu}^2 - f_{-\,\mu\nu}^2) \rangle$	$-\frac{1}{4} f_V^2 \eta_V$	$\frac{3}{4} f_A^2 \eta_A$

η_V **Weak Vector coupling**

η_A **Weak Axial coupling**

we have used $f_V = 2\ g_V$
This approx. too drastic
(when V-contributions cancel)

$K^+ \longrightarrow \pi^+ \pi^0 \gamma$
$N_{14} - N_{15} - N_{16} - N_{17}$

$K^+ \longrightarrow \pi^+ \gamma\gamma$
$N_{14} - N_{15} - 2N_{18}$
size and sign from Axials
$f_V \stackrel{?}{=} 2\ g_V$

Counterterm combination	Processes	VMD weak coupling
$N_{14}^r - N_{15}^r$	$K^+ \to \pi^+\gamma^*$ $K^+ \to \pi^+\pi^0\gamma^*$	$-0.020\,\eta_V + 0.004\,\eta_A$
$2N_{14}^r + N_{15}^r$	$K_S \to \pi^0\gamma^*$	$0.08\,\eta_V$
$N_{14} - N_{15} - 2N_{18}$	$K^+ \to \pi^+\gamma\gamma$ $K^+ \to \pi^+\pi^0\gamma\gamma$	$-0.01\,\eta_A$
$N_{14} - N_{15} - N_{16} - N_{17}$	$K^+ \to \pi^+\pi^0\gamma$ $K_S \to \pi^+\pi^-\gamma$	$-\,0.010\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$	$K_L \to \pi^+\pi^-\gamma^*$	$-0.004\,\eta_V + 0.018\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$	$K_S \to \pi^+\pi^-\gamma^*$	$0.05\,\eta_V - 0.04\,\eta_A$
$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$	$K^+ \to \pi^+\pi^0\gamma^*$	$0.12\,\eta_V + 0.01\,\eta_A$
$N_{29} + N_{31}$	$K_L \to \pi^+\pi^-\gamma$	$0.005\,\eta_V + 0.003\,\eta_A$
$3N_{29} - N_{30}$	$K^+ \to \pi^+\pi^0\gamma$	$-0.005\,\eta_V - 0.003\,\eta_A$

Observation hidden by other effects: different analysis maybe useful (Kaon charge radius)
NA48 has a good chance

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} + \text{c}$ $\qquad$ E_D, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.44 \pm 0.07)10^{-5}$ PDG	$M1, E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

CP asymmetry

- In the asymmetry in the slope, $\dfrac{\partial^2\Gamma^{\pm}}{\partial T_c^* \partial W^2}$ select a favourable kin. region (large W^2)

- This asymm., Ω, in extensions of SM $\sim \mathcal{O}(10^{-4})$ Colangelo et al.

- SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

- Assuming the expts. are almost seeing the CP conserving $E1$ Statistics seems tough but previous limit (Smith eta l. 76) weak

- Similar analysis for CPV in K_L: but time interf. required

$K_L \longrightarrow \pi^+\pi^-\gamma$

M1 transtions clearly measured KTeV (00) with large slope

form factor measured

$$\mathcal{F} = 1 + \frac{\mathrm{a}}{1 - \frac{m_k^2}{m_\rho^2} + \frac{2m_K E_\gamma^*}{m_\rho^2}}$$

E_γ^* photon energy

KTeV:

- $\mathrm{a}{=}{-}1.243 \pm 0.057$
-

	linear slope	quadratic slope	$\mathcal{F}$
χ^2/DOF	$43.2/27$	$37.6/26$	$38.8/27$

$\Rightarrow$ Large VMD: $\rho-$pole

p^4 CT's to $K_L \to \pi^+\pi^-\gamma$

$$a_1 = 8\,\pi^2\,N_{28} \quad , \quad a_2 = 32\,\pi^2\,N_{29} \;,\; a_3 = \frac{16}{3}\,\pi^2\,N_{30} \quad , \quad a_4 = 16\,\pi^2\,N_{31}$$

$$\mathcal{L}^{(4)}_{\Delta S=1} \quad \stackrel{K_L\to\pi^+\pi^-\gamma}{\Longrightarrow} \quad M1 \sim a_2 + 2a_4 + \quad h.o.$$

$$\Downarrow$$

Large VMD in the a_i. Not automatic in all spin-1 formulations

[G.D. Portoles, G.D. Gao, Cappiello G.D]

Consistent with M1 in $K^+ \to \pi^+\pi^0\gamma$ and predictive for spectrum (work for NA48/2)

$K^+ \to \pi^+\pi^0\gamma$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

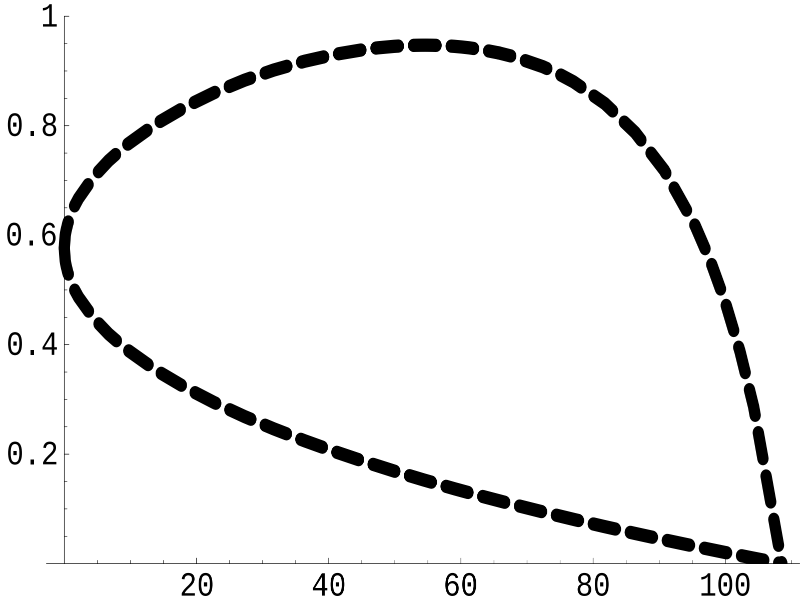
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

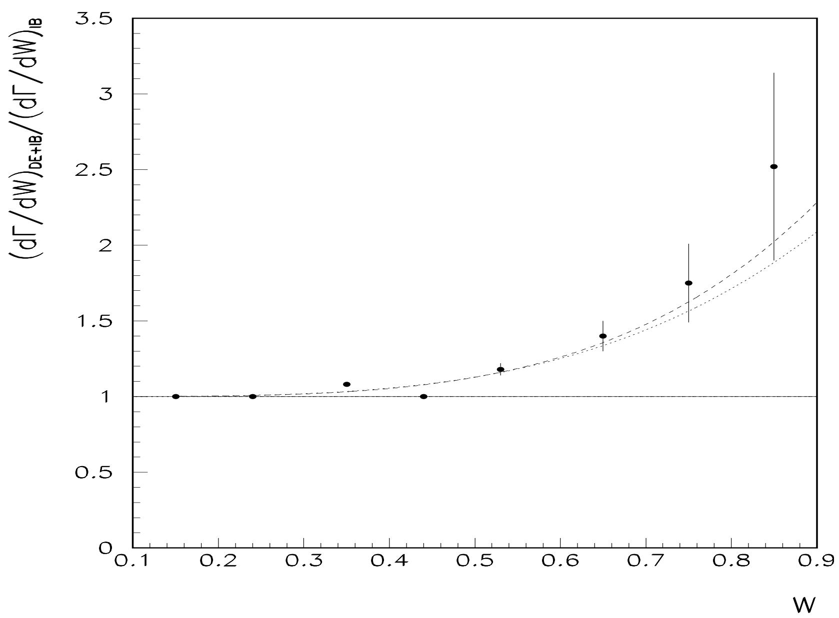
$$W^2 = (q\cdot p_K)(q\cdot p_+)/(m_\pi^2 m_K^2)$$

$$A = A(K^+ \to \pi^+\pi^0)$$

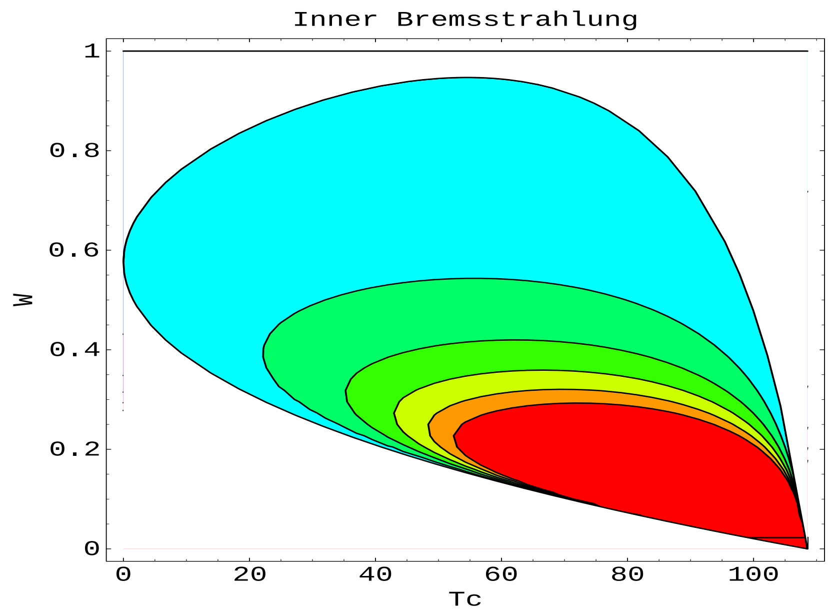
$K^+ \to \pi^+\pi^0\gamma\ W - T_c$ **Dalitz plot**

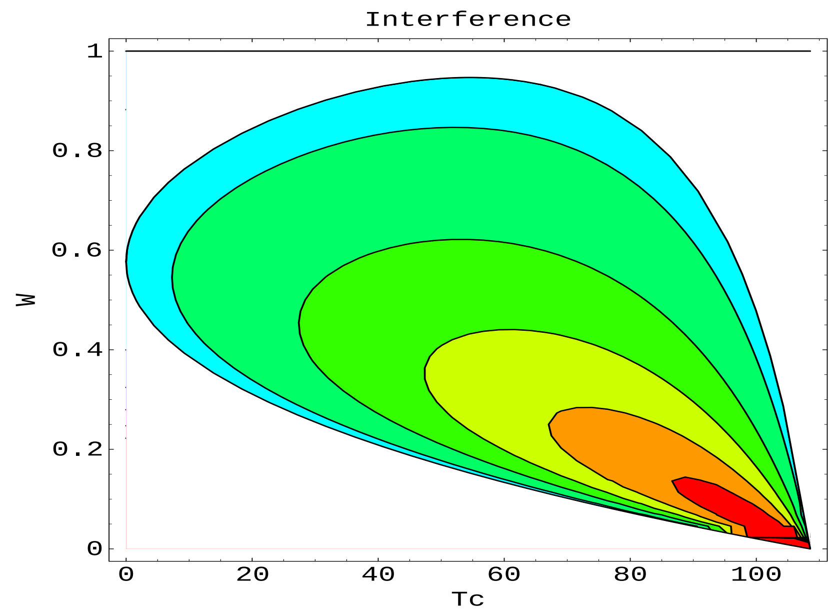
Integrating over T_c deviations from IB measured





$K^+ \to \pi^+\pi^0\gamma$

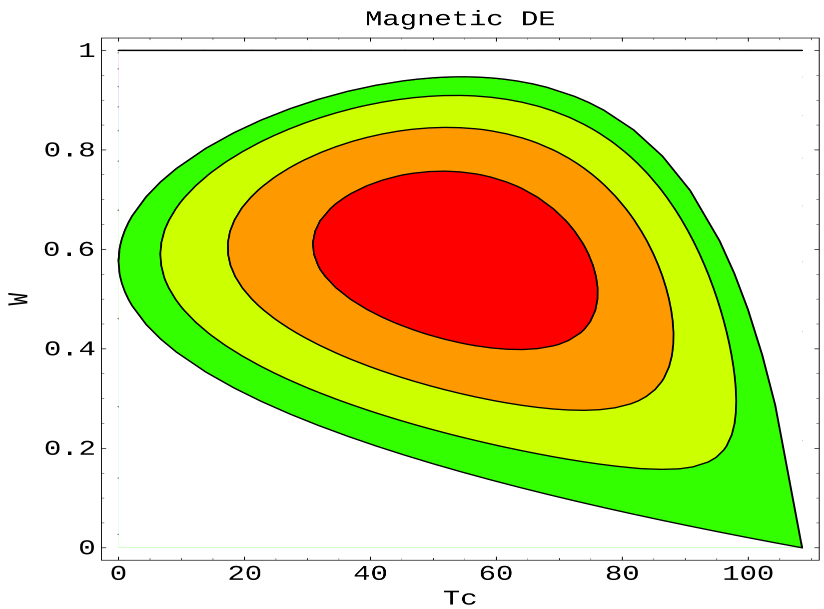




- E787 has measured M1 and Re$\left(\frac{E1}{E_{IB}}\right) \sim (-0.4 \pm 1.6)\%$

$$\downarrow$$

- E1 dominated by CT $\Rightarrow$ E787 constrains models $(k_f < 1)$ NA48/2$\Longrightarrow k_f = -0.4$



M1 $\sim$ (-2 + $3a_2 - 6a_3$)

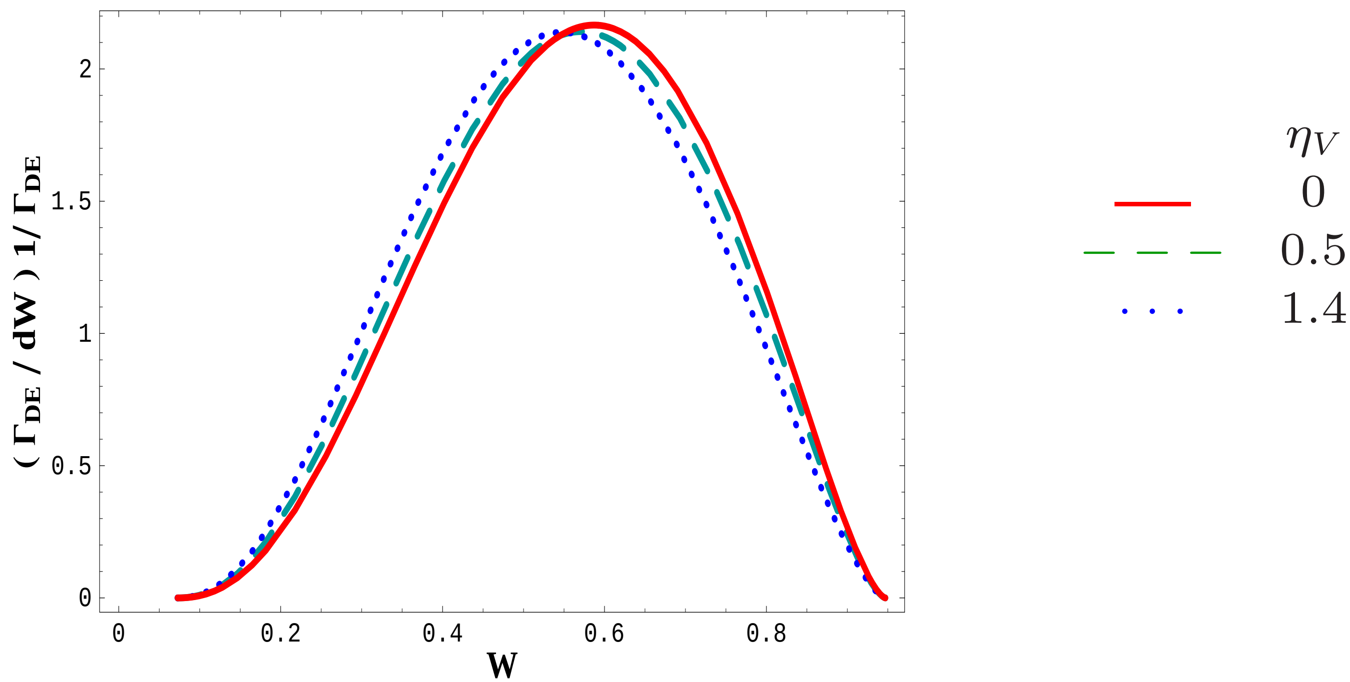
Exp. $\sim$ -2.5

Possible scenario a_i small?

$\overset{\text{new expts.}}{\Longrightarrow} -1$

Cheng Bijnens, Ecker, Pich

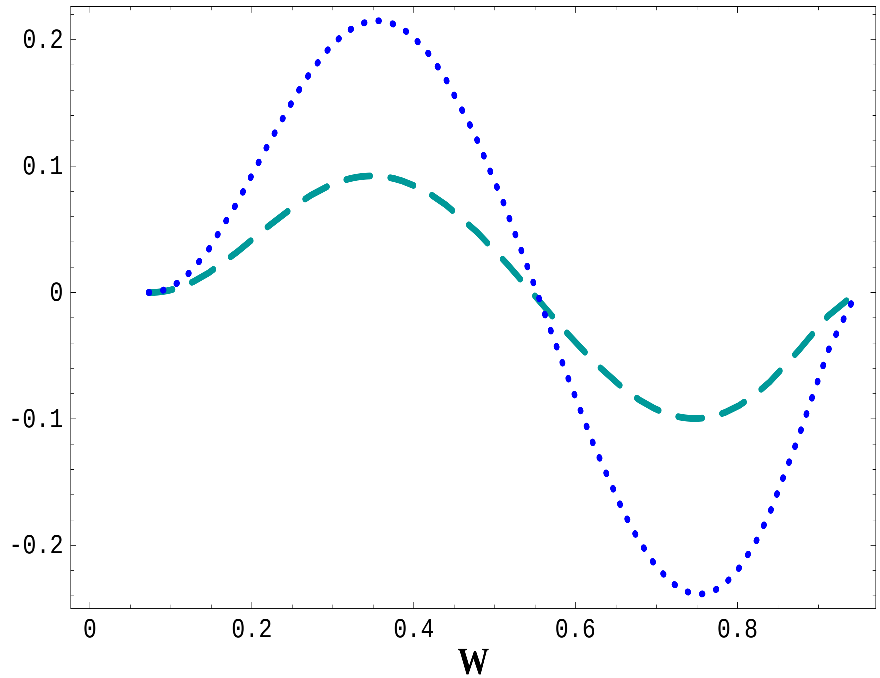
We have instead insisted, motivated by $K_L \to \pi^+\pi^-\gamma$ on the VMD $\Longrightarrow$ form factor



- $\frac{d\Gamma}{dW}$ may be crucial to study well the form factor $W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$

- to establish VMD

Is this the origin of the interference?



Subtracting a constant DE amplitude from a DE amplitude with form factor $\eta_V = 0.5$ (slashed line) and $\eta_V = 1.4$ (dotted line)

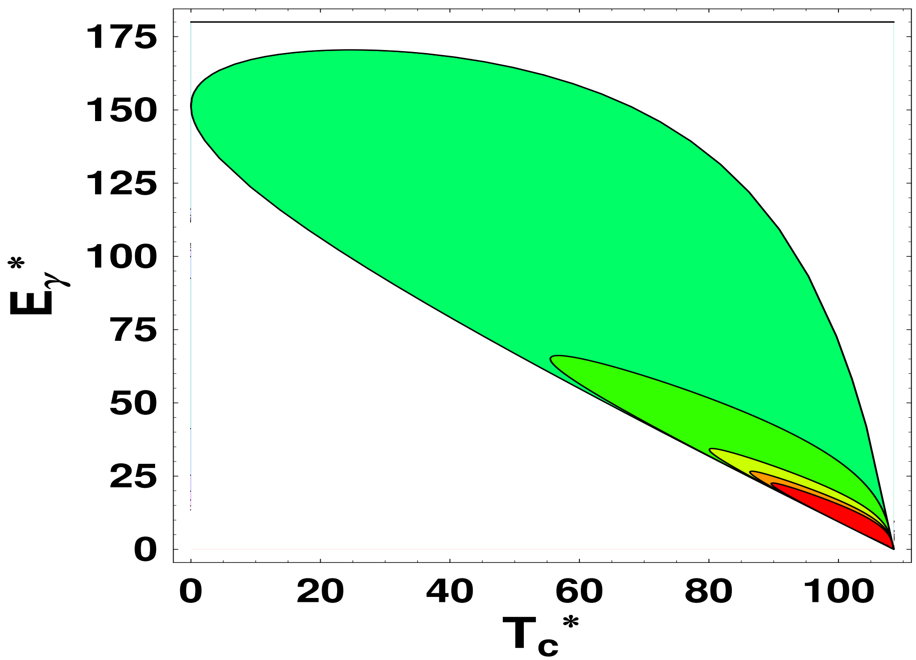
Different kin. variables

$$\frac{\partial^2\Gamma}{\partial T_c^* \partial E_\gamma^*} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^* \partial E_\gamma^*}\left[1 \quad + 2Re\left(\frac{E_{DE}}{e\mathcal{A}}\right)\left(\frac{m_K}{2} - E_0 - \frac{\delta\mu^2}{2m_K}\right)\frac{E_\gamma^*}{m_K} + \left(\left|\frac{E_{DE}}{e\mathcal{A}}\right|^2 + \left|\frac{M_{DE}}{e\mathcal{A}}\right|^2\right)\left(\frac{m_K}{2} - E_0 - \frac{\delta\mu^2}{2m_K}\right)^2 \frac{E_\gamma^{*2}}{m_K^2}\right],$$

where E_0 is the π^0-energy,

$$\delta\mu^2 = m_{\pi+}^2 - m_{\pi 0}^2$$

Christ Dalitz plot



Correlations

REFS.	$B(K^+ \to \pi^+\pi^0\gamma)_{\rm DE}$ $T_c^* \in [55, 90]$
BNL E787	$(4.7\pm0.9)\times10^{-6}$
PDG 06	$(4.4\pm0.7)\times10^{-6}$
KEK-E470	$(3.8\pm0.8 \pm 0.7)\times10^{-6}$
NA48/2	$(2.22 \pm 0.13 \pm 0.05)\times10^{-6}$

NA48/2 analysis	$T_c^* \in [0, 80]$ MeV
$B(K^+ \to \pi^+\pi^0\gamma)^{\rm INT}$	$(-4.91 \pm 2.00)\times10^{-6}$
$B(K^+ \to \pi^+\pi^0\gamma)^{\rm DE}$	$(6.16 \pm 0.79)\times10^{-6}$

Conclusions

- We need all the available info to understand the chiral dynamics N_i's
- Negative interf. for $K^+ \to \pi^+\pi^0\gamma$ not excluded
- Picture to be consolidated

WZW-VMD plot