From $K_{\ell 3}$ data to $K \to \pi\nu\bar{\nu}$ decay rates

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OVERVIEW - Master formula for $K \to \pi\nu\bar{\nu}$

In the Standard Model, the long- and short-distance part are conventionally separated as

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\,(\gamma)\right) = \kappa_\nu^+\,(1+\Delta_{EM})\,|y_\nu|^2\,,\quad \mathcal{B}\left(K_L \to \pi^0\nu\bar{\nu}\right) = \kappa_\nu^L\,(\mathrm{Im}\,y_\nu)^2 \tag{1}$$

$$\begin{aligned} \mathrm{LD}: \kappa_\nu^{+,L} &= \frac{G_F^2 M_{K^{+,0}}^5 \alpha\,(M_Z)^2}{256\pi^5 \sin^4\theta_W}\,\tau_{+,L}|V_{us}\times f_+^{K^{+,0}\pi^{+,0}}(0)\,|^2 \mathcal{I}_\nu^{+,0}\,,\\ \mathrm{SD}: y_\nu &= \frac{1}{|V_{us}|}\left[(\mathrm{Re}\lambda_t + i\mathrm{Im}\lambda_t)\,X_t + (0.2248)^4\mathrm{Re}\lambda_c P_{u,c}\right]\,, \end{aligned} \tag{2}$$

with $\lambda_q = V_{qs}^* V_{qd}$, $X_t = 1.464 \pm 0.041$ [1], $P_{u,c} = 0.41 \pm 0.04$ for $m_c\,(m_c) = 1.30 \pm 0.05$ [1, 2]. We also set $\alpha_{\overline{MS}}(M_Z)^{-1} = 127.9$, $(\sin^2\theta_W)_{\overline{MS}} = 0.231$ [1], since their errors are assumed to be part of that on X_t, together with scheme dependences [3]. The long-distance QED correction, with a soft-photon energy cut-off at about 20 MeV, is $\Delta_{EM} = (-0.30 \pm 0.05)\%$ [4].

As summarized in this note (see Ref. [4] for more details), experimental data on $K_{\ell 3}$ branching ratios and form-factor slopes permit to estimate the κ coefficients very precisely

$$\kappa_\nu^+ = 0.7867(43)\cdot 10^{-5}\,,\quad \kappa_\nu^L = 3.3624(264)\cdot 10^{-5},\quad r_\kappa \equiv \frac{\kappa_\nu^+}{\kappa_\nu^L} = 0.2340(13)\,, \tag{3}$$

or, with the normalization of Ref. [4], $\kappa_\nu^+ \times (0.2248)^8 = 0.5131(28)\cdot 10^{-10}$ and $\kappa_\nu^L \times (0.2248)^8 = 2.1929(172)\cdot 10^{-10}$. The errors are reduced by a factor of ~ 7 and ~ 4, respectively, compared to previous estimates [1, 5].

Using these values, together with the CKM parameters of Ref. [6] (compatible with Ref. [7]):

$$\lambda_t = (-3.23(19) + i1.402(94))\cdot 10^{-4},\ \mathrm{Re}\,\lambda_c = -0.22029(73),\ |V_{us}| = 0.22653(77)\,, \tag{4}$$

we get, neglecting correlations between these CKM inputs:

$$\begin{aligned} &\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = 2.76(40)\cdot 10^{-11} && (69_{\mathrm{CKM}},\ 27_{X_t},\ 4_{\kappa_\nu^L})\%\,,\\ &\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}\,(\gamma)) = 8.22(84)\cdot 10^{-11} && (47_{\mathrm{CKM}},\ 20_{X_t},\ 30_{P_{u,c}},\ 3_{\kappa_\nu^+},\ 0.3_{\Delta_{EM}})\%\,,\\ &\frac{\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}\,(\gamma))}{\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})} = 2.98\,(46) && (71_{\mathrm{CKM}},\ 6_{X_t},\ 21_{P_{u,c}},\ 2_{r_\kappa},\ 0.2_{\Delta_{EM}})\%\,. \end{aligned} \tag{5}$$

As the error budget shows, the errors on CKM matrix elements largely dominates, while those from LD κ coefficients are negligible. In other words, the intrinsic theoretical errors on $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}\,(\gamma))$ and their ratio are of 5.6%, 7.1% and 6.3%, respectively.

INPUT 1 - Theoretical ratios

The good theoretical control over LD effects in $K \to \pi\nu\bar{\nu}$ stems from the excellent theoretical control over two ratios of $K_{\ell 3}$ and $FCNC$ form-factors in Chiral Perturbation Theory

$$r \equiv \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^+}(0)}\frac{f_+^{K^0\pi^0}(0)}{f_+^{K^+\pi^+}(0)} = 0.99994(43)\,,\ r_K \equiv \frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = 1.0012(10)\,. \tag{6}$$

These predictions include the $\mathcal{O}(p^4, e^2p^2)$ corrections from Ref. [4], and the non-local $\mathcal{O}(p^6, e^0)$ corrections from Ref. [8]. The local C_i contribution vanishes for r [4], while for r_K, we take the estimate of Ref. [4](with a conservative error). The dependence on the light-quark mass ratio is mild and we use $\varepsilon^{(2)} = 0.0143$ [8].

For the FCNC form-factor slopes, we simply rescale those of $K_{\ell 3}$ to account for the relocation of the vector meson pole from $m_{K^{*+}(892)}$ to $m_{K^{*0}(892)}$, and adding a conservative error [4]:

$$r_\lambda \equiv \frac{\lambda_+^{\prime FCNC}}{\lambda_+^{\prime CC}} = 0.990(5) \approx \frac{m^2_{K^{*+}(892)}}{m^2_{K^{*0}(892)}} \; . \tag{7}$$

INPUT 2 - $K_{\ell 3}$ data

The input data for the $K_{\ell 3}$ form-factor slopes is [9]

$$\left\{ \begin{array}{l} \lambda'_+ = (24.920 \pm 1.105) \cdot 10^{-3} \; , \\ \lambda''_+ = (1.612 \pm 0.447) \cdot 10^{-3} \; , \\ \lambda_0 = (13.438 \pm 1.190) \cdot 10^{-3} \; , \end{array} \right. \quad \rho = \begin{pmatrix} 1 & -0.944 & 0.328 \\ & 1 & -0.439 \\ & & 1 \end{pmatrix} \; . \tag{8}$$

For $K_{\ell 3}$ branching ratios and K lifetimes, the input values are [9]

$$\begin{array}{lll} \mathcal{B}(K^L_{e3}) = 40.563\,(74)\,\% \; , & \mathcal{B}(K^L_{\mu 3}) = 27.047\,(71)\,\% \; , & \tau_L = 51.173\,(200) \text{ ns}, \\ \mathcal{B}(K^+_{e3}) = 5.0780\,(258)\,\% \; , & \mathcal{B}(K^+_{\mu 3}) = 3.3650\,(271)\,\% \; , & \tau_+ = 12.3840\,(193)\,\text{ns}, \\ \mathcal{B}(K^S_{e3}) = 0.07046\,(91)\,\% \; , & - & \tau_S = 0.08958\,(5)\,\text{ns}. \end{array} \tag{9}$$

STEP 1 - From slopes to $K \to \pi\nu\bar{\nu}$ phase-space integrals

The $K \to \pi\nu\bar{\nu}$ phase-space integrals, accounting for the rescaling Eq.(7), can be immediately computed from Eq.(8) as

$$\mathcal{I}^i_\nu(r_\lambda, \lambda'_+, \lambda''_+) = \int_0^{(1-r_{\pi i})^2} dz \; \lambda^{3/2} \left(1 + r_\lambda \lambda'_+ \frac{z m^2_{K^i}}{m^2_{\pi^\pm}} + r^2_\lambda \lambda''_+ \frac{z^2 m^4_{K^i}}{2 m^4_{\pi^\pm}}\right)^2 \quad (i = +, 0), \tag{10}$$

with $\lambda = r^4_{\pi i} - 2\,(1+z)\,r^2_{\pi i} + (1-z)^2$ and $r_{\pi i} \equiv m_{\pi^i}/m_{K^i}$, giving

$$\left\{ \begin{array}{l} \mathcal{I}^+_{e3} = 0.158924\,(297) \\ \mathcal{I}^+_{\mu 3} = 0.105077\,(319) \\ \mathcal{I}^0_{e3} = 0.154572\,(289) \\ \mathcal{I}^0_{\mu 3} = 0.102128\,(310) \\ \mathcal{I}^+_\nu = 0.152712\,(294) \\ \mathcal{I}^0_\nu = 0.160448\,(311) \end{array} \right. \; , \quad \rho_{\mathcal{I}} = \begin{pmatrix} 1 & 0.6188 & 1 & 0.6200 & 0.9674 & 0.9646 \\ & 1 & 0.6192 & 1 & 0.6043 & 0.5915 \\ & & 1 & 0.6203 & 0.9675 & 0.9645 \\ & & & 1 & 0.6054 & 0.5925 \\ & & & & 1 & 0.99939 \\ & & & & & 1 \end{pmatrix} \; . \tag{11}$$

STEP 2 - From $K_{\ell 3}$ rates to $K \to \pi\nu\bar{\nu}$ matrix elements and κ coefficients

Given that $K \to \pi\nu\bar{\nu}$ and K_{e3} phase-space integrals are correlated at above 95%, we proceed from the BR of Eq.(9) by isolating the following quantities:

$$F_0 \equiv \tau_L |V_{us} \times f_+^{K^0\pi^+}(0)|^2 \mathcal{I}^0_{e3}, \quad F_+ \equiv \tau_+ |V_{us} \times f_+^{K^+\pi^0}(0)|^2 \mathcal{I}^+_{e3} \; . \tag{12}$$

They are found separately averaging neutral and charged $K_{\ell 3}$ modes

$$F_0 = \frac{192\pi^3}{G_F^2 S_{EW} M_{K^0}^5}\left(\frac{\mathcal{B}(K_{e3}^L)}{1+2\delta_{em}^{0,e}};\frac{\mathcal{B}(K_{\mu 3}^L)}{1+2\delta_{em}^{0,\mu}}\frac{\mathcal{I}_{e3}^0}{\mathcal{I}_{\mu 3}^0};\frac{\mathcal{B}(K_{e3}^S)}{1+2\delta_{em}^{0,e}}\frac{\tau_L}{\tau_S}\right)_{av.} = 0.37038(101)\cdot 10^{-9}, \quad (13)$$

$$F_+ = \frac{384\pi^3}{G_F^2 S_{EW} M_{K^+}^5}\left(\frac{\mathcal{B}(K_{e3}^+)}{1+2\delta_{em}^{+,e}};\frac{\mathcal{B}(K_{\mu 3}^+)}{1+2\delta_{em}^{+,\mu}}\frac{\mathcal{I}_{e3}^+}{\mathcal{I}_{\mu 3}^+}\right)_{av.} = 0.97425(479)\cdot 10^{-10}, \quad (14)$$

with $S_{EW} = 1.0232$ and $\delta_{em}^{i,\ell}$ the long-distance QED corrections from Ref. [9]. From F_0, we directly get the κ_ν^+ coefficient as

$$\kappa_\nu^+ = \frac{G_F^2 M_{K^+}^5 \alpha\left(M_Z\right)^2}{256\pi^5 \sin^4\theta_W}\frac{\tau_+}{\tau_L}(r_K)^2 F_0 \frac{\mathcal{I}_\nu^+}{\mathcal{I}_{e3}^0} = 0.7867(43)\cdot 10^{-5}\ , \quad (15)$$

while for κ_ν^L, we first compute the very precise ratio $r_\kappa = \kappa_\nu^+/\kappa_\nu^L$ and find

$$r_\kappa = \frac{1}{r^2}\frac{M_{K^+}^5}{M_{K^0}^5}\frac{F_+}{F_0}\frac{\mathcal{I}_\nu^+\mathcal{I}_{e3}^0}{\mathcal{I}_\nu^0\mathcal{I}_{e3}^+} = 0.2340(13) \quad \rightarrow \kappa_\nu^L = \frac{\kappa_\nu^+}{r_\kappa} = 3.3624(264)\cdot 10^{-5}\ . \quad (16)$$

References

[1] A.J. Buras, M. Gorbahn, U. Haisch and U. Nierste, Phys. Rev. Lett. **95** (2005) 261805; JHEP **0611** (2006) 002.

[2] G. Isidori, F. Mescia and C. Smith, Nucl. Phys. **B718** (2005) 319.

[3] G. Buchalla and A. J. Buras, Phys. Rev. D **57** (1998) 216.

[4] F. Mescia and C. Smith, Phys. Rev. **D76** (2007) 034017.

[5] W.J. Marciano, Z. Parsa, Phys. Rev. **D53** (1996) R1.

[6] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. **C41** (2005) 1, and 2007 updated results presented at EPS07 & LP07.

[7] M. Bona *et al.* [UTfit Collaboration], JHEP **0610** (2006) 081.

[8] J. Bijnens and K. Ghorbani, arXiv:0711.0148 [hep-ph].

[9] FlaviaNet web pages: http://www.lnf.infn.it/wg/vus/ .