Virtual Hadronic and Heavy-Fermion $\mathcal{O}(\alpha^2)$ Corrections to Bhabha Scattering

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Outline

The Luminosity Monitor

Luminosity of a collider depends on the machine and the beam: all is complicated (not mentioning errors estimation)

$$\frac{dN}{dt} = \mathcal{L}(t), \quad N = \sigma \int dt \mathcal{L}(t) = \sigma \mathcal{L} \qquad (1)$$

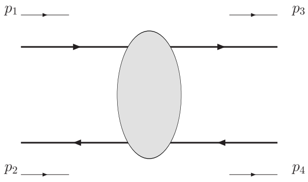
better is to choose a well known process:

$$\mathcal{L} = \frac{1}{\sigma_{Bhabha}} N_{Bhabha}$$

then we may determine any cross section in (1)

Kinematical Regions for Bhabha

Two regions where the Bhabha-scattering cross section is large and QED dominated



$$s = (p_1+p_2)^2 = 4E^2 > 4m_e^2, \quad t = (p_1-p_3)^2 = -4\,(E^2-m_e^2)\sin^2\frac{\theta}{2} < 0$$

- $\sqrt{s} \sim 10^2$ GeV $\Rightarrow$ small θ
- SABS $\Rightarrow \mathcal{L}$ at LEP, ILC

  $\sim$ a few degrees

- $\sqrt{s} \sim$ 1-10 GeV $\Rightarrow$ large θ
- LABS $\Rightarrow \mathcal{L}$ at KLOE

  $\theta \sim 55^0 \; - \; 125^0$

Experimental Precision on $\mathcal{L}$

LABS $\rightarrow 10^{-3}$

- KLOE $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\mathrm{exp}} = 0.3\%$
  DAΦNE-VEPP-2M $\Rightarrow$ 0.1 %

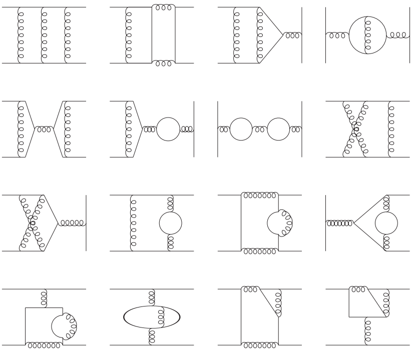
SABS $\rightarrow 10^{-4}$

- LEP $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\mathrm{exp}} = 0.03\%$
  ILC $\Rightarrow \frac{\delta\mathcal{L}}{\mathcal{L}}|_{\mathrm{exp}} = 0.02(1)\%$ Giga-Z

Status around 2004, from S. Jadach, hep-ph:0306083

Type of correction/error	LEPEWWG	hep-ph:9905235	update
Technical precision	–	– (0.03%)	**0.03%**
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027% (0.013%)	**0.013%**
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015% (0.006%)	**0.006%**
Vacuum polarization	0.04%	0.040%	**0.025%**
Light pairs	0.03%	0.010%	**0.010%**
Z-exchange	0.015%	0.015%	**0.015%**
Total	0.11%	0.054% (0.055%)	**0.045%**

NNLO QED Corrections [Bern, Dixon, Ghinculov '00]



from Bern,Dixon,Ghinculov, arXiv:hep-ph/0010075

For practical applications

$$\sigma = \int \mathrm{d}\Phi\, 2\,\mathrm{Re}\left[\mathcal{M}^{\text{2loop}} \cdot \mathcal{M}^{\text{tree}*}\right]$$

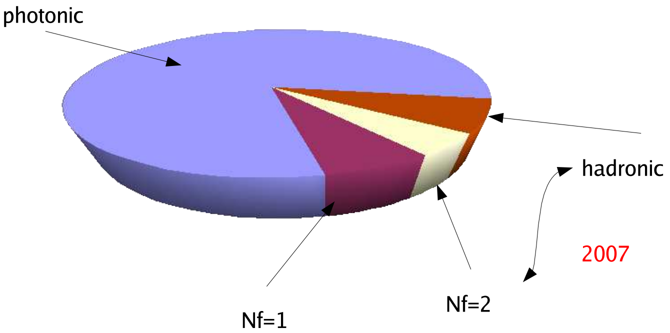
Simplification $m_e = 0$

- + 2-scale problem s, t
- + simple result $\ln(x)$, $\mathrm{Li}_n(x)$
  $x = -t/s = \sin^2(\theta/2)$
- \- bad for MCs, $m_e \neq 0$

$\Rightarrow$ Recompute with $m_e \neq 0$

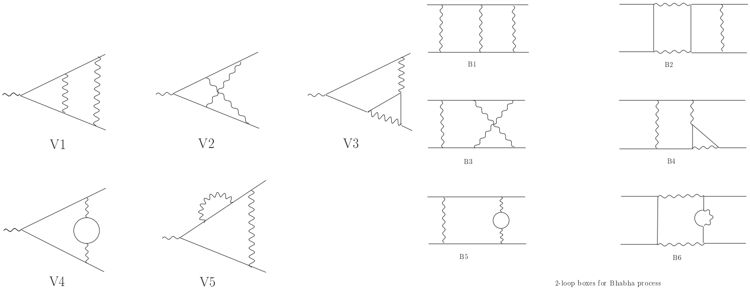
$\Rightarrow$ Massive from massless

First look, progress during last 2 years



Remarkable: photonic, $N_f = 1, N_f = 2$ NNLO corrections doubly (triply) cross-checked

NNLO photonic and fermionic $N_f = 1, 2$ topologies



2-loop boxes for Bhabha process

- SE loop insertions (without photonic line) are so called fermionic diagrams, rest represents photonic.
- Closed fermionic loop can be muon, tau, top or light quarks.
- In general, box B5 is a 4-scale problem: $m_e, m_f, s, t(u)$.

Photonic NNLO massive QED Corrections

Photonic corrections (no fermion loop)

$$\frac{d\sigma_2}{d\sigma_0} = \underbrace{A \ln^2\left(\frac{s}{m_e^2}\right) + B \ln\left(\frac{s}{m_e^2}\right)}_{\text{collinear}} + \underbrace{C}_{\text{non collinear}}$$

- [Arbuzov,Kuraev,Shaikhatdenov '98] [Glover,Tausk,van der Bij '01] $\Rightarrow$ A, B
- [Penin '05] $\Rightarrow$ $C = C_0 + \mathcal{O}(m_e^2/s)$ from massless result

- Result of A. Penin confirmed by independent calculation

[Becher-Melnikov '07]

Heavy Fermions with IR Matching excluding $\mathcal{O}(m_e^2/s)$

[Penin '05]: photonic corrections follow from a change in the regularization scheme

$$\mathcal{M}(m=0, \underbrace{1/\epsilon}_{\text{IR and collinear}}) \Rightarrow \mathcal{M}(\underbrace{\ln(s/m_e^2)}_{\text{collinear}}, \underbrace{\ln(\lambda^2/m_e^2)}_{\text{IR}}) + \delta\mathcal{M}$$

[Mitov-Moch '06]: for $2 \to n$ QED/QCD scattering process

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \mathcal{M}(m=0) \quad \mathcal{Z}_j = \frac{F_D(Q^2, m \neq 0)}{F_D(Q^2, m=0)}$$

[Becher-Melnikov '07]: in QED assuming $m_e^2 << m_f^2 << s, t, u$ heavy fermions can be included

$$\mathcal{M}(m \neq 0) = \prod_j \mathcal{Z}_j^{1/2} \underbrace{\mathcal{S}(m_f)}_{\text{process dependent}} \mathcal{M}(m=0)$$

$N_f = 1$ NNLO massive QED Corrections

Fermionic corrections: electrons in SE loops

- [Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij '05]
  $\Rightarrow$ electron loops with full m_e dependence

recalculated in 2007

- [Actis, JG, Czakon, Riemann '07]
  $\Rightarrow$ electron loops with full m_e dependence

and in small electron limit:

- [Becher-Melnikov '07]
  $\Rightarrow$ electron loops $m_e^2 << s, t$

Evaluation of the MIs Electron-Loop Case

MIs evaluated using the method of differential equations

[Kotikov '91] [Remiddi '97] [Caffo,Czyz,Laporta,Remiddi '98]

Three-scale problem: s, t, m_e

$$x = \frac{\sqrt{s} - \sqrt{s - 4m_e^2}}{\sqrt{s} + \sqrt{s - 4m_e^2}} \qquad y = \frac{\sqrt{4m_e^2 - t} - \sqrt{-t}}{\sqrt{4m_e^2 - t} + \sqrt{-t}}$$

MIs: HPLs [Remiddi,Vermaseren '99] and 2dHPLs [Gehrmann,Remiddi '00]

Full agreement with the existing result

[Bonciani,Ferroglia,Mastrolia,Remiddi,van der Bij '05]

[Actis.,Czakon,JG,Riemann '07]

$N_f = 2$ NNLO leptonic massive QED Corrections

First results:

- [Becher-Melnikov '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$
- [Actis, JG, Czakon, Riemann '07]

  $\Rightarrow m_e^2 << m_f^2 << s, t, u$

[Bonciani, Ferroglia, Penin '07]

-

  $\Rightarrow$ here: $m_e^2 << m_f^2, s, t, u$

Finally, yet another approach (dispersion relations)

- [Actis, JG, Czakon, Riemann '08]

  $\Rightarrow m_e^2 << m_f^2, s, t, u$

I: Evaluation of the MIs Heavy Fermions

Four-scale problem: s, t, m_e, m_f → new heavy-fermion scale

- Exploit the hierarchy of scales $m_e^2 << m_f^2 << s, t, u$
- Evaluate the MIs neglecting $\mathcal{O}(m_e^2/m_f^2)$, $\mathcal{O}(m_e^2/s)$, $\mathcal{O}(m_f^2/s)$

Mellin-Barnes method [Smirnov '99] [Tausk '99] efficient

- $$\mathcal{I}^{2L} \sim \underbrace{\int dz \left(\frac{m_e^2}{m_f^2}\right)^{z+\epsilon} \frac{\prod \Gamma_i(z,\epsilon)}{\prod \Gamma_j(z,\epsilon)}}_{\text{integral}} \Rightarrow \underbrace{\sum_k c_k \left(\frac{m_e^2}{m_f^2}\right)^k}_{\text{sum}}$$
- MB representations by AMBRE [Kajda, JG, Riemann '07]
- analytical continuation in ϵ with `MB` [Czakon '05]
- sums with `XSummer` [Moch,Uwer '05]

Example: box MIs, $L_m(x) = \ln(-m^2/x)$ and $R = m^2/M^2$

$$
\begin{aligned}
\texttt{B[x,y]} \;=\; & \frac{m^{-4\epsilon}}{x}\Big\{\frac{1}{\epsilon^2}L_m(x) + \frac{1}{\epsilon}\Big(-\zeta_2 + 2L_m(x) + \frac{1}{2}L_m^2(x) + L_m(x)L_m(y)\Big) \\
- \; & 2\zeta_2 - 2\zeta_3 + 4L_m(x) + L_m^2(x) + \frac{1}{3}L_m^3(x) - 4\zeta_2 L_m(y) \\
+ \; & 2L_m(x)L_m(y) + L_m(x)L_m^2(y) - \frac{1}{6}L_m^3(y) \\
- \; & \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
- \; & \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) + \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\texttt{Bd[x,y]} \;=\; & \frac{m^{-4\epsilon}}{xy}\Big\{\frac{1}{\epsilon}\Big[-L_m(x)L_m(y) + L_m(x)L(R)\Big] - 2\zeta_3 + \zeta_2 L_m(x) + 4\zeta_2 L_m(y) \\
- \; & 2L_m(x)L_m^2(y) + \frac{1}{6}L_m^3(y) - 2\zeta_2 L(R) + 2L_m(x)L_m(y)L(R) - \frac{1}{6}L^3(R) \\
+ \; & \Big(3\zeta_2 + \frac{1}{2}L_m^2(x) - L_m(x)L_m(y) + \frac{1}{2}L_m^2(y)\Big)\ln\Big(1+\frac{y}{x}\Big) \\
+ \; & \Big(L_m(x) - L_m(y)\Big)\mathrm{Li}_2\Big(-\frac{y}{x}\Big) - \mathrm{Li}_3\Big(-\frac{y}{x}\Big)\Big\}.
\end{aligned}
$$

Checks on the Computation

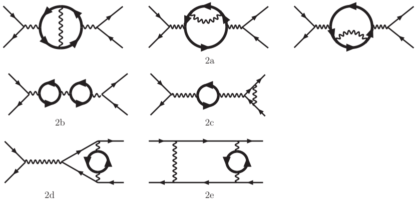
$\Rightarrow$ UV/IR/collinear structure of the result

- UV divergencies cancel after inserting counterterms
- IR divergencies cancel after including single-photon emission
- $\mathcal{O}(m_e^{-n})$ from IBPIs cancel $\Rightarrow$ collinear divergencies $\ln\left(s/m_e^2\right)$

$\Rightarrow$ Dirac FF agrees with [Burgers '85] [Kniehl,Krawczyk,Kühn,Stuart '88]

$\Rightarrow$ Expansions of MIs checked with sector decomposition [Binoth,Heinrich '00]

Structure of the result



SE

reducable diagrams

irreducable diagrams

After combining the 2-loop terms with the loop-by-loop terms and with soft real corrections:

$$\frac{d\sigma^{\rm NNLO}}{d\Omega} + \frac{d\sigma_{\gamma}^{\rm NLO}}{d\Omega} = \frac{d\sigma^{\rm NNLO,e}}{d\Omega} + \sum_{f\neq e} Q_f^2 \frac{d\sigma^{\rm NNLO,f^2}}{d\Omega} + \sum_{f\neq e} Q_f^4 \frac{d\sigma^{\rm NNLO,f^4}}{d\Omega} + \sum_{f_1,f_2\neq e} Q_{f_1}^2 Q_{f_2}^2 \frac{d\sigma^{\rm NNLO,2f}}{d\Omega}.$$

Irreducable terms

$$\frac{d\sigma^{\mathrm{NNLO,f^2}}}{d\Omega} = \frac{\alpha^2}{s}\left\{\sigma_1^{\mathrm{NNLO,f^2}} + \sigma_2^{\mathrm{NNLO,f^2}} \ln\left(\frac{2\omega}{\sqrt{s}}\right)\right\}$$

$$\sigma_1^{\mathrm{NNLO,f^2}} = \frac{\left(1-x+x^2\right)^2}{3x^2}\left\{-\frac{1}{3}\left[\ln^3\left(\frac{s}{m_e^2}\right) + \ln^3\left(\frac{m_e^2}{m_f^2}\right)\right]\right.$$
$$+ \ln^2\left(\frac{s}{m_e^2}\right)\left[\frac{55}{6} - \ln\left(\frac{m_e^2}{m_f^2}\right) + \ln\left(-1+1/x\right)\right]$$
$$+ \cdots\cdots\cdots$$

Structure of the Result

$$\frac{d\sigma_2}{d\sigma_0} = \left(\frac{\alpha}{\pi}\right)^2 \left[A \ln\left(\frac{2\omega}{\sqrt{s}}\right) + B \ln\left(\frac{s}{m_e^2}\right) + C\right]$$

$$C = C_3 \ln^3\left(\frac{s}{m_f^2}\right) + C_2 \ln^2\left(\frac{s}{m_f^2}\right) + C_1 \ln\left(\frac{s}{m_f^2}\right) + C_0$$

$$C_3 = -\frac{1}{9} \qquad C_2 = \frac{19}{18} + \frac{1}{3}\ln\left(\frac{1-x}{x}\right) \qquad x = -\frac{t}{s} = \sin^2\left(\frac{\theta}{2}\right)$$

$C_0 \Big\} \Rightarrow$

$$\begin{aligned}
- & \frac{1}{324(1-x+x^2)^2}\Big\{-4795+9590x-14385x^2+9590x^3-4795x^4-1404\zeta_2+1674x\,\zeta_2 \\
+ & 2484x^2\zeta_2-4158x^3\zeta_2+2808x^4\zeta_2+432\,\zeta_3-864x\zeta_3+1296x^2\zeta_3-864x^3\zeta_3+432x^4\zeta_3 \\
- & 672\,L_{-x}+966xL_{-x}-2016x^2L_{-x}+966x^3\,L_{-x}-672x^4L_{-x}+648\zeta_2L_{-x}-2160x\zeta_2\,L_{-x} \\
+ & 3456x^2\zeta_2L_{-x}-2160x^3\zeta_2\,L_{-x}+648x^4\zeta_2L_{-x}-360L_{-x}^2+1044x\,L_{-x}^2-1458x^2L_{-x}^2 \\
+ & 1044x^3\,L_{-x}^2-360x^4L_{-x}^2+108L_{-x}^3-216xL_{-x}^3+252x^2L_{-x}^3-144\,x^3L_{-x}^3+36x^4L_{-x}^3 \\
+ & 3534L_{+x}-5259xL_{+x}+6309x^2L_{+x}-2775x^3L_{+x}+672x^4\,L_{+x}+648\zeta_2L_{+x}-1944x\zeta_2\,L_{+x} \\
+ & 702x^2\zeta_2L_{+x}+864x^3\zeta_2L_{+x}-648x^4\,\zeta_2L_{+x}+1080L_{-x}L_{+x}-2106x\,L_{-x}L_{+x}+1998x^2L_{-x}L_{+x} \\
- & 540x^3\,L_{-x}L_{+x}-135x^2L_{-x}^2L_{+x}+162\,x^3L_{-x}^2L_{+x}-108x^4L_{-x}^2L_{+x}-990\,L_{+x}^2+1656xL_{+x}^2 \\
- & 1512x^2L_{+x}^2+144x^3\,L_{+x}^2+360x^4L_{+x}^2-324L_{-x}L_{+x}^2+540x\,L_{-x}L_{+x}^2-216x^2L_{-x}\,L_{+x}^2 \\
- & 108x^3L_{-x}L_{+x}^2+108x^4L_{-x}L_{+x}^2+36\,L_{+x}^3-279xL_{+x}^3+198x^2L_{+x}^3+18x^3\,L_{+x}^3-36x^4L_{+x}^3 \\
- & 36\left(1-x+x^2\right)\left[-20\,\left(1-x+x^2\right)+3x(-1+2x)L_{-x}-6(-2+x)\,L_{+x}\right]\mathrm{Li}_2(1-x) \\
+ & 72\left(1-x+x^2\right)\left[-10\left(1-x+x^2\right)+3\left(3-2\,x+x^2\right)L_{+x}\right]\mathrm{Li}_2(x)+\Big[216\,L_{-x}-324xL_{-x} \\
+ & 324x^2\,L_{-x}-108x^3\,L_{-x}-216L_{+x}\,+324xL_{+x}\,-324x^2L_{+x}+108x^3\,L_{+x}\Big]\mathrm{Li}_2\left(\frac{x}{x-1}\right) \\
+ & \left[-108x\,+324x^2-324x^3+216x^4\right]\mathrm{Li}_3(1-x)+\left[-216+432x\,-648x^2+432x^3-216x^4\right]\times \\
\times & \mathrm{Li}_3(x)+\left[216\,-324x\,+324x^2-108x^3\right]\mathrm{Li}_3\left(\frac{x}{x-1}\right)\Big\}
\end{aligned}$$

$$\begin{aligned}
- & \frac{1}{108(1-x+x^2)^2}\Big[818-1636x+2454x^2-1636x^3+818x^4+216\zeta_2-432x\zeta_2-162x^2\zeta_2 \\
+ & 540x^3\zeta_2-432x^4\zeta_2+120L_{-x}-186xL_{-x}+360x^2L_{-x}-186x^3L_{-x}+120x^4L_{-x}+72L_{-x}^2 \\
- & 198xL_{-x}^2+270x^2L_{-x}^2-198x^3L_{-x}^2+72x^4L_{-x}^2-444L_{+x}+672xL_{+x}-846x^2L_{+x} \\
+ & 402x^3L_{+x}-120x^4L_{+x}-216L_{-x}L_{+x}+432xL_{-x}L_{+x}-378x^2L_{-x}L_{+x}+108x^3L_{-x}L_{+x} \\
+ & 36L_{+x}^2-153xL_{+x}^2+135x^2L_{+x}^2+9x^3L_{+x}^2-72x^4L_{+x}^2-144\left(1-x+x^2\right)^2\mathrm{Li}_2(1-x) \\
+ & 144\left(1-x+x^2\right)^2\mathrm{Li}_2(x)\Big]
\end{aligned}$$

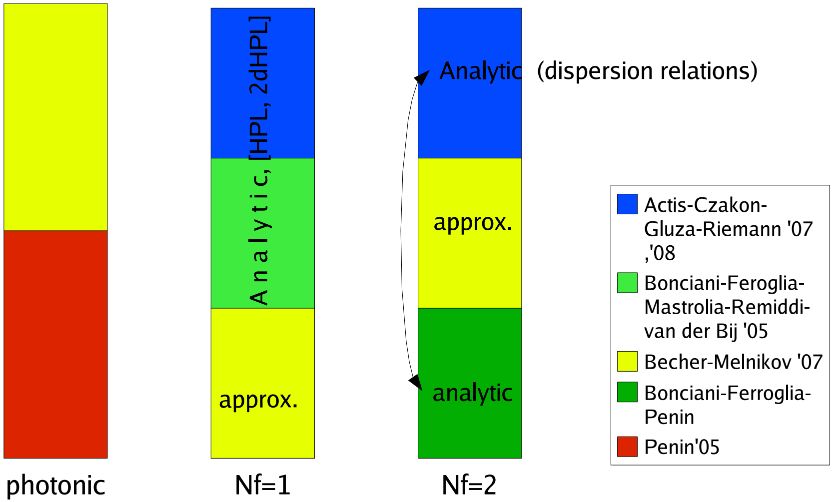
$\Uparrow$

$\overbrace{}$ C_1

[S.A.,Czakon,Gluza,Riemann '07]

[Becher,Melnikov '07]

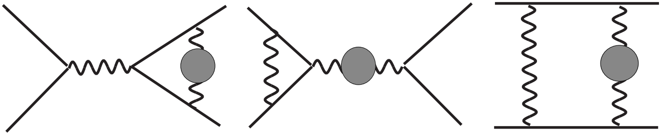
Present situation, virtual NNLO QED



Hadronic contributions

1. Actis, Czakon, JG, Riemann,
   arXiv:0711.3847v2, Physical Review Letters, 100, 2008,

   long paper: arXiv:0807.4691, submitted to PRD
2. Johann H. Kühn, Sandro Uccirati, arXiv:0807.1284



(a) (b) (c)

Beyond $m_f^2 << s$

- KLOE $\sqrt{s} = 1$ GeV $< m_\tau$
- ILC $\sqrt{s} = 500$ GeV $\Rightarrow m_t < \sqrt{-t}, \sqrt{-u}$
  $40^0 < \theta < 140^0 \neq$ region for luminosity

$\Rightarrow$ Kinematical regions where expansions don't work

$\Rightarrow$ General method for including hadronic effects

Dispersion Relations

- Obtain fermionic corrections inserting Π_R in $\Delta_\gamma^{\mu\nu}$

$$\frac{g_{\mu\nu}}{q^2 + i\,\delta} \;\to\; \frac{g_{\mu\alpha}}{q^2 + i\,\delta}\,\left(q^2\,g^{\alpha\beta} - q^\alpha\,q^\beta\right)\,\Pi_R(q^2)\,\frac{g_{\beta\nu}}{q^2 + i\,\delta}$$

- Represent Π_R through a dispersion integral

$$\Pi_R(q^2) = -\frac{q^2}{\pi}\int_{4M^2}^{\infty}\frac{dz}{z}\,\frac{\operatorname{Im}\Pi(z)}{q^2 - z + i\,\delta}$$

Leptons, top (perturbative)

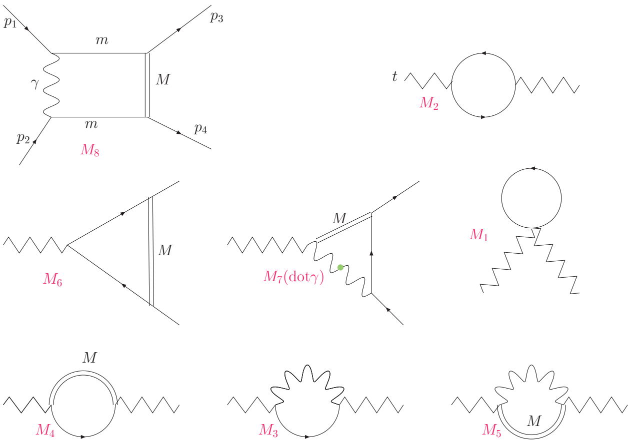
$$\operatorname{Im}\Pi_f(z) = -\left(\frac{\alpha}{\pi}\right)F_\epsilon\left(\frac{m_e^2}{m_f^2}\right)^{\epsilon} Q_f^2\,C_f\times$$
$$\times\,\theta\left(z - 4\,m_f^2\right)\,\frac{\pi}{3}\left\{\frac{\beta_f(z)}{2}\left[3 - \beta_f^2(z)\right]\right\}$$

Light quarks (non pert.)

Unitarity $\Rightarrow \operatorname{Im}\Pi_{\text{had}}(z) = -\frac{\alpha}{3}R_{\text{had}}(z)$

$$R_{\text{had}}(z) = \frac{\sigma(\{e^+e^- \to \gamma^\star \to \text{hadrons}\};z)}{(4\pi\alpha^2)/(3z)}$$

Effectively we must calculate following MIs



► again expanded from MB representations

Box Master, M_8

$$\int \frac{d^D k}{(k^2 - z)\left[(k+p_3)^2 - m_e^2\right](k+p_3-p_1)^2\left[(k+p_3-p_1-p_2)^2 - m_e^2\right]}$$

$$= \frac{1}{s\,(t-z)}\left\{\frac{1}{\epsilon}\left[\ln\left(-\frac{m_e^2}{t}\right) + \ln\left(-\frac{z}{s}\right) - \ln\left(-\frac{z}{t}\right)\right] - 2\,\zeta_2\right.$$

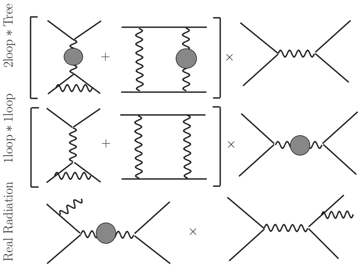
$$+ \ln\left(-\frac{m_e^2}{t}\right)\left[\frac{1}{2}\ln\left(-\frac{m_e^2}{t}\right) + \ln\left(-\frac{z}{s}\right) + \ln\left(-\frac{z}{t}\right) - 2\ln\left(1 - \frac{z}{t}\right)\right]$$

$$- \frac{3}{2}\ln^2\left(-\frac{z}{t}\right) + \ln\left(-\frac{z}{s}\right)\ln\left(-\frac{z}{t}\right)$$

$$\left. - 2\ln\left(1 - \frac{z}{t}\right)\left[\ln\left(-\frac{z}{s}\right) - \ln\left(-\frac{z}{t}\right)\right] - \mathrm{Li}_2\left(1 + \frac{z}{s}\right)\right\} + \mathcal{O}(m_e^2).$$

IR finite results with boxes

The 4 direct and 4 crossed fermionic 2-loop box diagrams have to be combined with other diagrams for an IR-finite contribution:



- assembling all diagrams, the terms in $\ln(s/m_e^2)$ drop out and the total contribution of fermionic box diagrams is free of collinear divergencies
- a sensible, infrared safe cross-section contains the complete sum of all the single IR-divergent diagrams, or no one of them

Numbers from Bonciani, Ferroglia, Penin, ’08

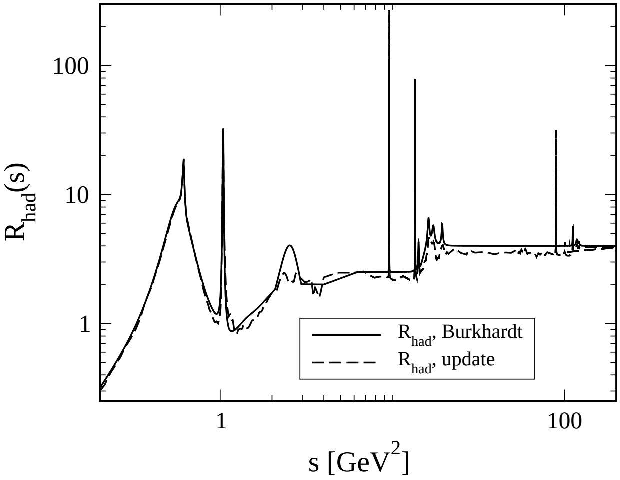
Hadronic $\rightarrow$ leptonic, $m_\pi \rightarrow m_f$, plus change of kernels

$\sqrt{s} = 1$ GeV

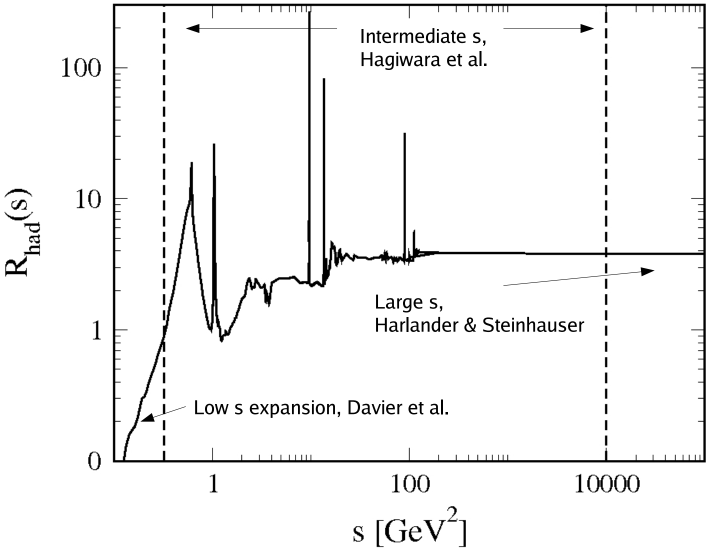
θ	e (10^{-4})	μ (10^{-4})	c (10^{-4})	τ (10^{-4})	b (10^{-4})
50°	17.341004	1.7972877	0.0622677	0.0264013	0.0010328
60°	18.407836	2.2267654	0.0861876	0.0367058	0.0014184
70°	19.438718	2.6504950	0.1086126	0.0465329	0.0018907
80°	20.465455	3.0655973	0.1253094	0.0540991	0.0022442
90°	21.463240	3.4581845	0.1321857	0.0576348	0.0024428
100°	22.366427	3.8070041	0.1268594	0.0560581	0.0024304
110°	23.099679	4.0922189	0.1098317	0.0495028	0.0022024
120°	23.605216	4.3030725	0.0843311	0.0392810	0.0018086
130°	23.847394	4.4392717	0.0549436	0.0273145	0.0013297

agreement (in all digits for μ)
and for $m_\tau = 1.7$ GeV [but not 1.777 GeV] :-))

Hadronic contributions



Hadronic contributions



Hadronic contributions

in contrast to PRL paper, narrow resonances are implemented replacing the rapidly varying cross section ratio with the parametrization

$$R_{\rm res}(z) = \frac{9\pi}{\alpha^2} M_{\rm res} \Gamma^{e^+e^-}_{\rm res}\, \delta(z - M^2_{\rm res}).$$

For the numerical evaluation of the contribution due to the narrow resonances, we use the values listed in the Burkhardt's routine

Hadronic contributions

resonance	M_{res} [GeV]	$\Gamma_{\text{res}}^{e^+e^-}$ [keV]
$\omega(782)$	0.7826	0.66
$\phi(1020)$	1.0195	1.31
$J/\psi(1S)$	3.0969	4.7
$\psi(2S)$	3.6860	2.1
$\psi(3770)$	3.7699	0.26
$\psi(4040)$	4.0300	0.75
$\psi(4160)$	4.1590	0.77
$\psi(4415)$	4.4150	0.47
$\Upsilon(1S)$	9.4600	1.22
$\Upsilon(2S)$	10.0234	0.54
$\Upsilon(3S)$	10.3555	0.40
$\Upsilon(4S)$	10.577	0.24
$\Upsilon(10860)$	10.865	0.31
$\Upsilon(11020)$	11.019	0.13

Table: taken directly from Burkhardt routines.

Agreement with KU, 0807.1284, 2008 for IPAR=0

θ [°] | $\sqrt{s}$ [GeV]	$\theta = 20$ | 1	$\theta = 20$ | 10	$\theta = 3$ | M_Z	$\theta = 3$ | 500
vertices [μ+τ+hadr.]	-0.001086	-0.00022513	-0.007982	-0.00129296
vertices [e]	-0.102787	-0.00325449	-0.092546	-0.00574577
soft pairs e^+e^-	0.130264	0.00403772	0.112763	0.00685890
rest: e	0.235562	0.00497834	0.135650	0.00672652
μ	0.009518	0.00135040	0.040792	0.00287809
	– 0.017214	0.00134282	0.040688	0.00287795
τ	0.000074	0.00005385	0.002706	0.00087639
	$\times$	$\times$	– 0.009610	0.00083969
hadr.	0.008642	0.00269490	0.087618	0.00810781

Table: dispersion-based approach (first line) and the one the analytical expansion (second line), neglecting $\mathcal{O}(m_f^2/x)$, where $x = s, |t|, |u|$. When $m_f^2 > x$, the entry is suppressed.

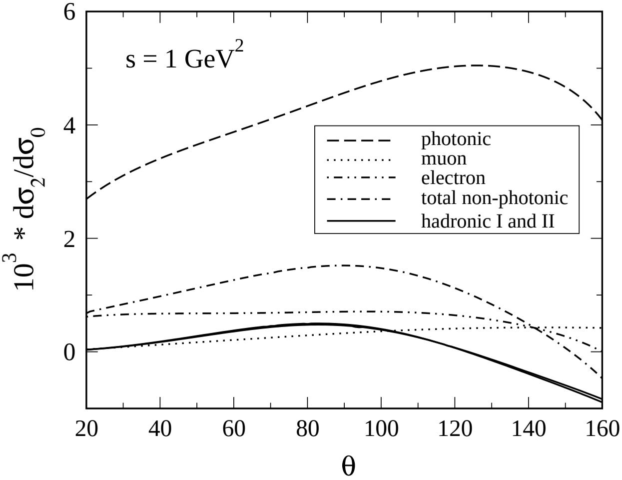
soft pairs: Arbuzov, Kuraev, Merenkov, Trentadue, 1995

Agreement with KU, 0807.1284, 2008 for IPAR=0

$\sqrt{s}$ [GeV]	1	10	M_Z	500
vertices [μ+τ+hadr.]	-16.351	-2.0437	-0.125208	-0.0104275
vertices [e]	-477.620	-12.3010	-0.298589	-0.0155751
soft pairs e^+e^-	648.275	16.0690	0.376531	0.0191990
rest: e	807.476	14.5277	0.270575	0.0119285
μ	160.197	6.0819	0.147046	0.0072579
	152.890	6.0809	0.147046	0.0072579
τ	2.383	1.3335	0.075268	0.0045713
	$\times$	1.0739	0.075214	0.0045712
hadr.	232.674	16.0670	0.469944	0.0246035

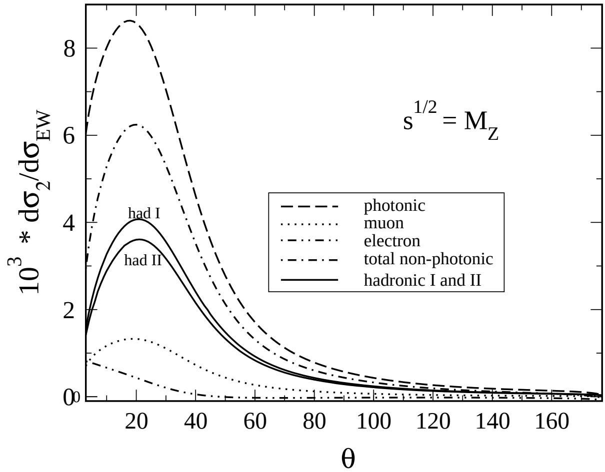
Table: Numerical values for the differential cross section in nanobarns at a scattering angle $\theta = 90^\circ$, in units of 10^{-4}.

PRD paper



s = 1 GeV^2
photonic
muon
electron
total non-photonic
hadronic I and II
10^3 * dσ_2/dσ_0
θ
6
4
2
0
20
40
60
80
100
120
140
160

PRD paper



$s^{1/2} = M_Z$
photonic
muon
electron
total non-photonic
hadronic I and II
had I
had II
$10^3 * d\sigma_2/d\sigma_{EW}$
0
2
4
6
8
20
40
60
80
100
120
140
160
θ

Summary

- NNLO QED Bhabha corrections have been extensively studied by different groups and they reach the level of several permille in the regions needed for measuring the luminosity
- the results have to be compared and interfaced with the available MC generators

谢谢