Radiative corrections for the $e^+e^- \to e^+e^-$***,*** $\mu^+\mu^-$***,*** $\pi^+\pi^-$***,*** $\gamma\gamma$ ***processes***

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Large Angle Bhabha Scattering

A.Arbuzov, E.Kuraev *et al.*, JHEP 97 10(1997) 001
Eur. Phys. J. C **46**, 689 (2006)

- The first order $\dfrac{\alpha}{\pi}$ is taken into account exactly.
- All orders of large leading logarithms $\left(\dfrac{\alpha}{\pi}L\right)^n$ are calculated by means of the Structure Function method, where $L = \ln\dfrac{s}{m_e^2}$ is large logarithm.

Estimated accuracy $\leq$ 0.2%

Primary integral for $e^+e^- \to e^+e^-\gamma$

$$\frac{\mathrm{d}\sigma^{e^+e^-(\gamma)}}{\mathrm{d}\Omega_-} = \int\limits_{z_1}^{1}\mathrm{d}z_1\int\limits_{z_2}^{1}\mathrm{d}z_2\,\mathcal{D}(z_1)\mathcal{D}(z_2)\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-}\left(1+\frac{\alpha}{\pi}K_{SV}\right)\Theta$$

$$\times\int\limits_{y_{\mathrm{th}}}^{Y_1}\frac{\mathrm{d}y_1}{Y_1}\int\limits_{y_{\mathrm{th}}}^{Y_2}\frac{\mathrm{d}y_2}{Y_2}\,\mathcal{D}(\frac{y_1}{Y_1})\mathcal{D}(\frac{y_2}{Y_2})\;-\;\mathrm{Compensators}$$

$$-\;\frac{\mathrm{d}\tilde{\sigma}_0}{\mathrm{d}\Omega_-}\frac{8\alpha}{\pi}\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\Delta\varepsilon}{\varepsilon}\;+\;\frac{\alpha^3}{2\pi^2 s}\int\limits_{\substack{k^0>\Delta\varepsilon\\ \pi-\theta_0>\theta>\theta_0}}\frac{WT}{4}\,\Theta\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-}$$

$\mathcal{D}(z)$ – structure function giving probability for electron to have an energy $E_e = z\times E_{beam}$ without angular dependency.

Shifted Born cross-section

In quasi-real electron approximation considering process

$$e^-(z_1 p_-) \; + \; e^+(z_2 p_+) \; \longrightarrow \; e^-(\tilde{p}_-) \; + \; e^+(\tilde{p}_+)$$

$$\begin{aligned}\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-} &= \frac{4\alpha^2}{sa^2}\left\{\frac{1}{|1-\Pi(\tilde{t})|^2}\left[\frac{a^2+z_2^2(1+c)^2}{2z_1^2(1-c)^2} = \mathbf{\frac{5}{2}}\right]\right. \\ &+ \frac{1}{|1-\Pi(\tilde{s})|^2}\left[\frac{z_1^2(1-c)^2+z_2^2(1+c)^2}{2a^2} = \mathbf{\frac{1}{4}}\right] \\ &- \left. \mathrm{Re}\,\frac{1}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{s}))^*}\left[\frac{z_2^2(1+c)^2}{az_1(1-c)} = \mathbf{\frac{1}{2}}\right]\right\}\end{aligned}$$

where $a = z_1 + z_2 + (z_1 - z_2)c$.

Red numbers for $\mathrm{d}\tilde{\sigma}_0(1,1)$ at 90 degree.

Calculating integral

To merge one photon and $\mathcal{D}$-function we have to introduce auxiliary parameters:

- $\omega/\varepsilon < \Delta$ – soft and virtual region, hard photon $\omega/\varepsilon > \Delta$
- $\dfrac{1}{\gamma} \ll \theta_0 \sim \dfrac{1}{\sqrt{\gamma}} \ll 1$ – collinear region

And we also need to introduce compensator to subtract first order nonleading part integrating outside collinear region from $\mathcal{D}$-function because we already have this part in one photon.

Singularities isolation

Primary integral is taken by Monte Carlo method $\Rightarrow$ isolate singularities to increase generator efficiency.

Infrared divergence $$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_\gamma} \sim \frac{1}{\omega(E-\omega)}$$

Collinear singularity $$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_\gamma} \sim \frac{1}{1-\beta_e^2\cos^2\theta_\gamma}$$

t-channel scattering $$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_-} \sim \frac{1}{(1-\cos\theta_-)^2}$$

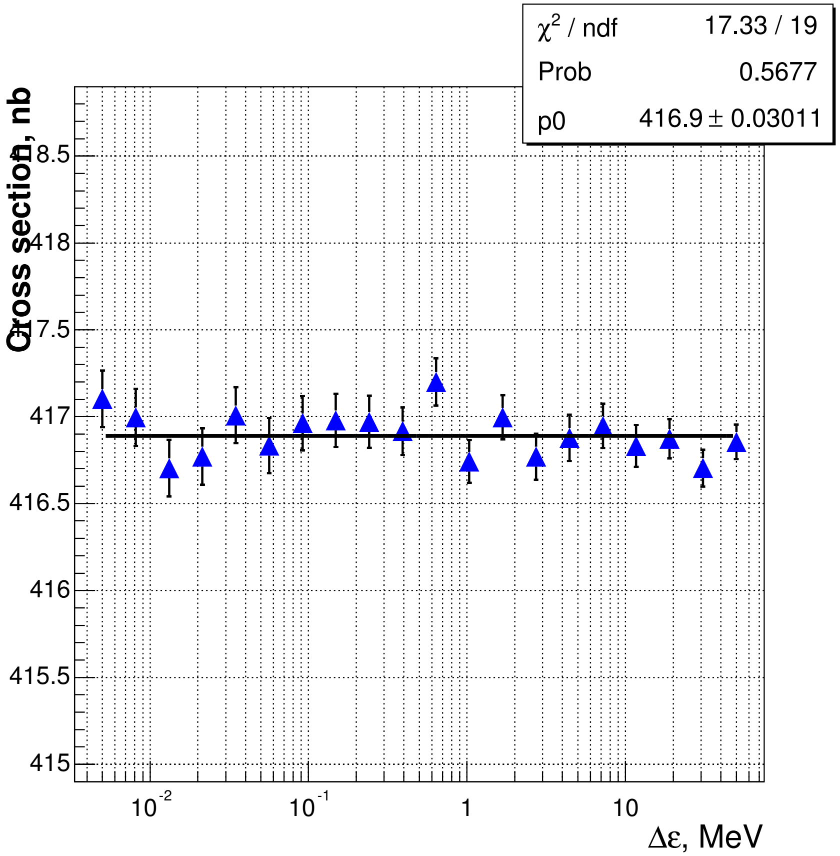
Selection criteria

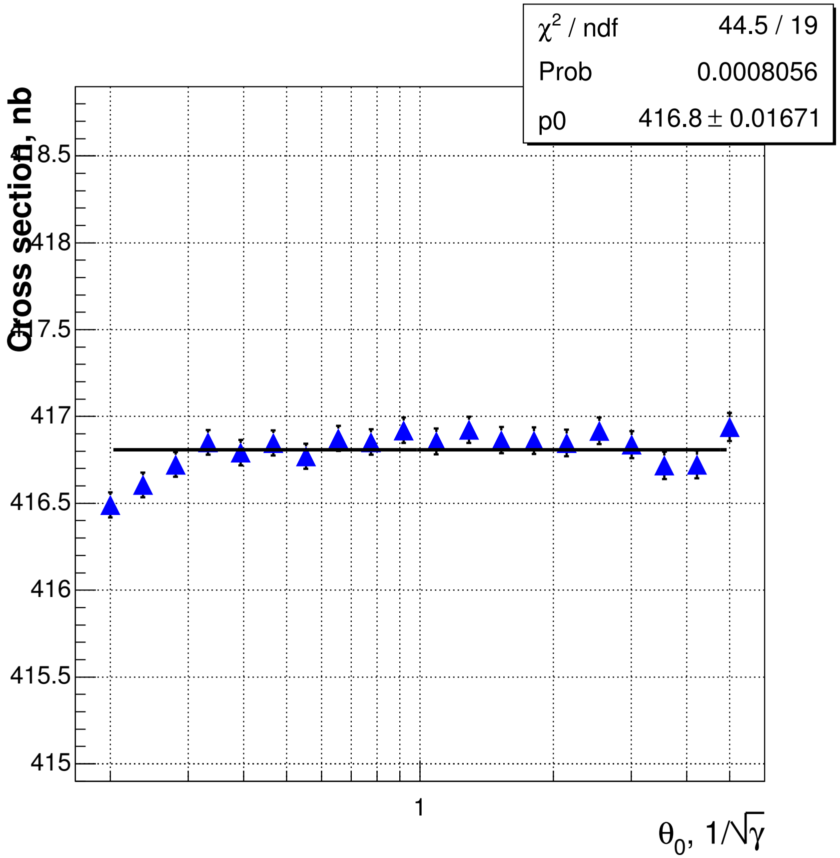
- $|\Delta\theta| < 0.25$ rad, where $\Delta\theta = \theta_+ + \theta_- - \pi$
- $|\Delta\phi| < 0.15$ rad, where $\Delta\phi = |\phi_+ - \phi_-| - \pi$
- $1.1 < \theta_{\mathrm{average}} < \pi - 1.1$, where $\theta_{\mathrm{average}} = (\theta_+ - \theta_- + \pi)/2$
- $P^{\pm}_{\mathrm{tran}} > 90$ MeV/c

This selections are used in the pictures below unless otherwise pointed

Dependence on auxiliary parameters

$$e^+e^- \to e^+e^-\gamma$$

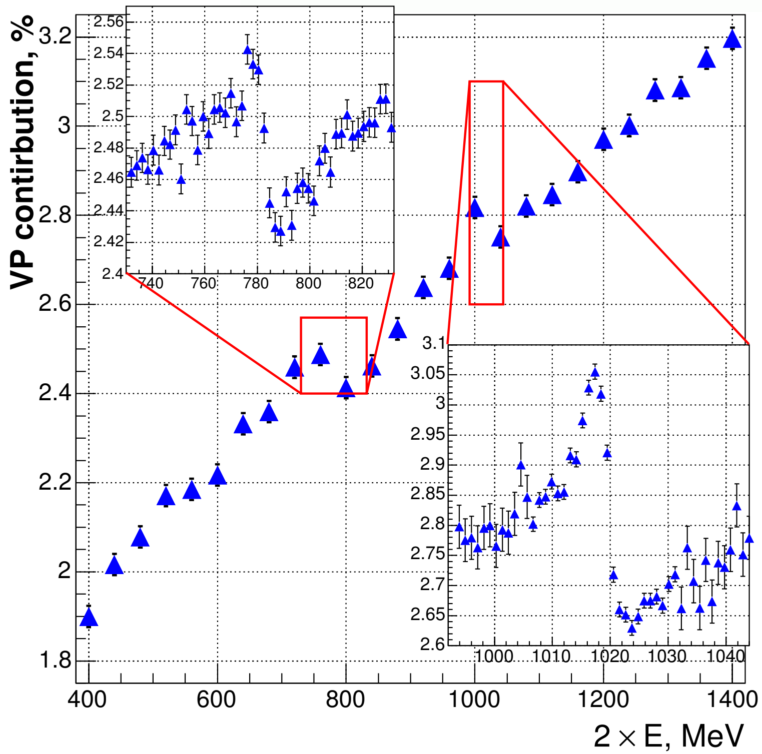




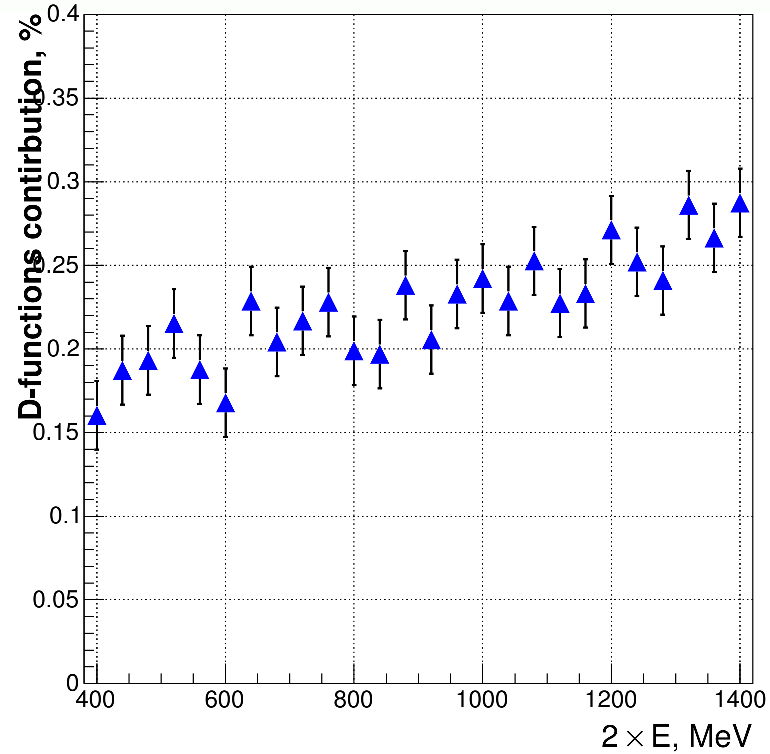
$\Delta\varepsilon$

θ_0 in $1/\sqrt{\gamma}$ units

Contributions to the $e^+e^- \to e^+e^-\gamma$ *cross section*

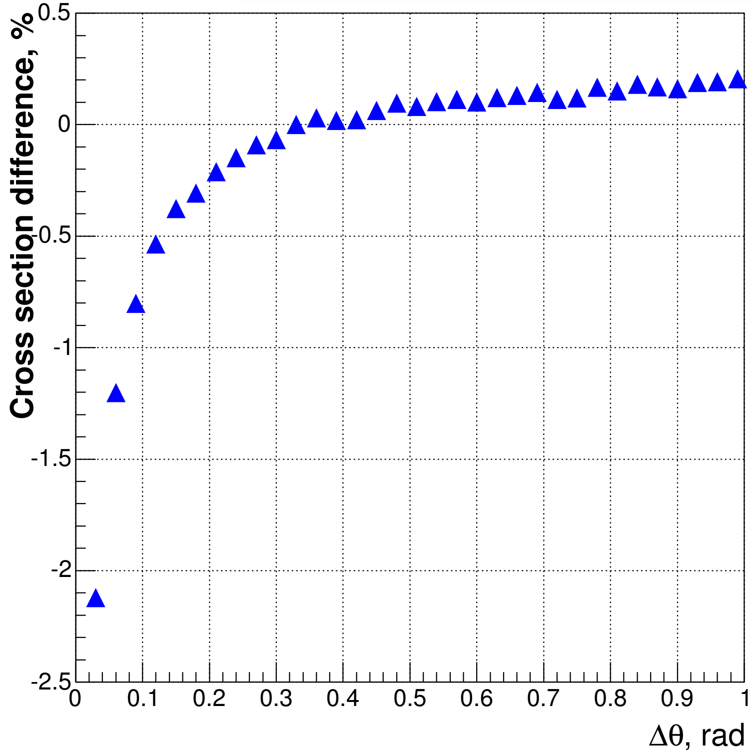


Vacuum polarization contribution

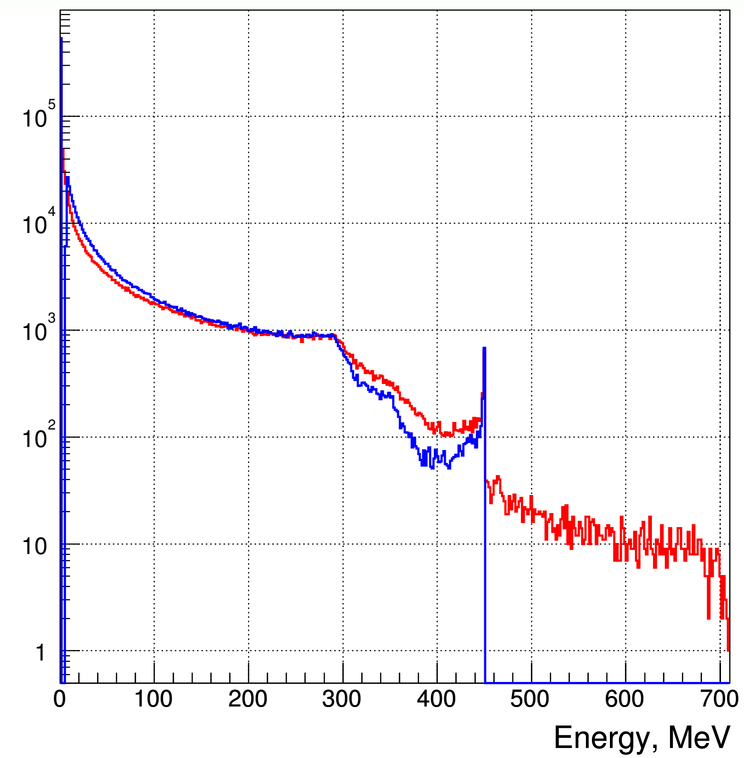


High order leading logarithm contribution in collinear region

Comparison with one photon



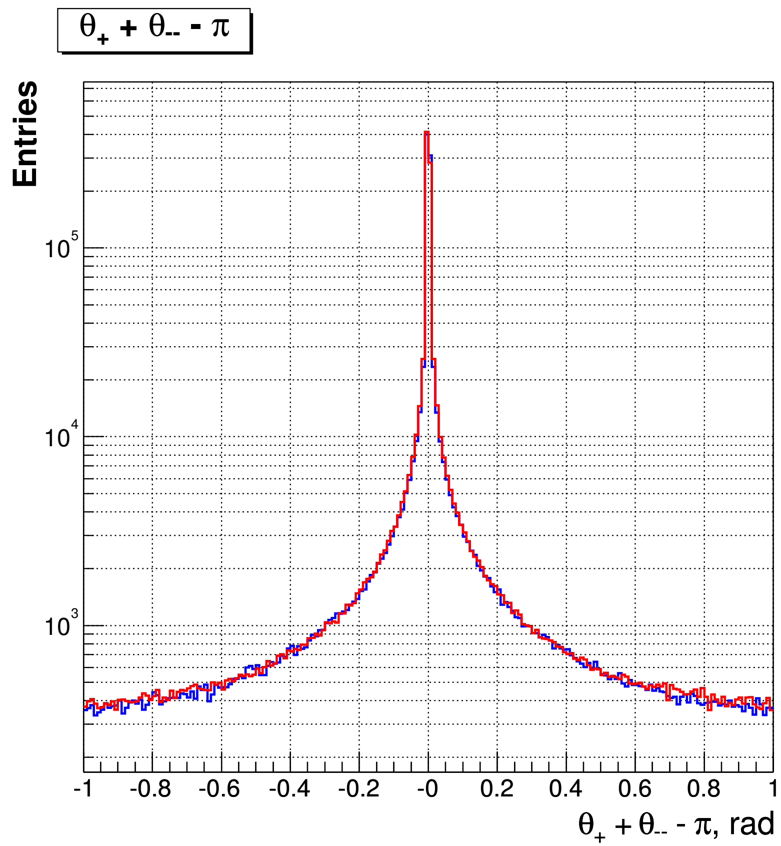
Cross section difference for one photon emission and high order contributions depend on $\Delta\theta$

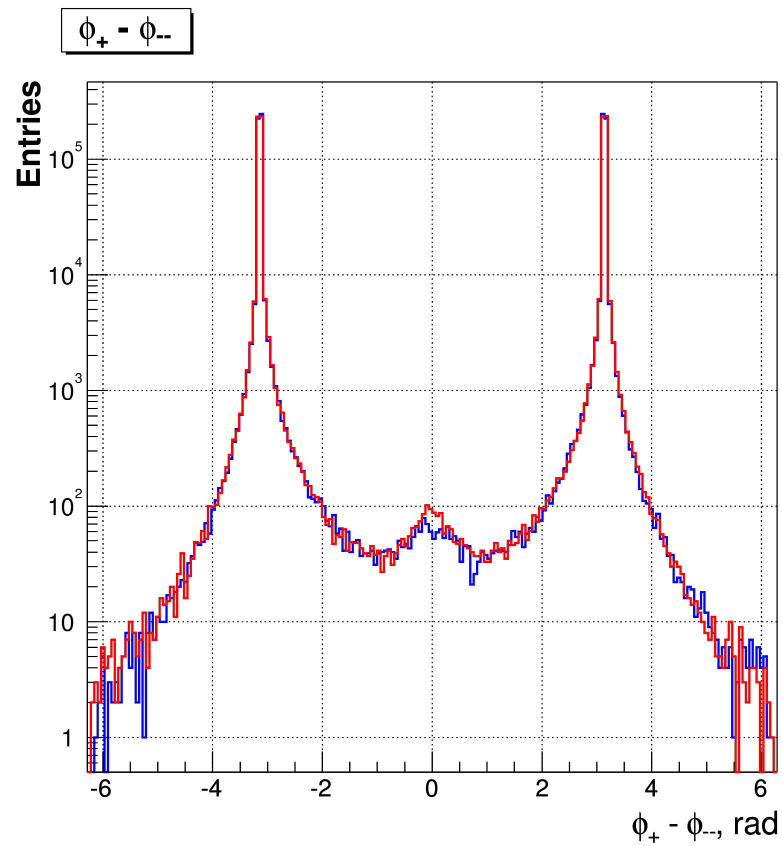


Spectra of emitted energy, $|\Delta\theta| < 1$ rad, $|\Delta\phi| < \pi$ rad

Comparison with BHWIDE

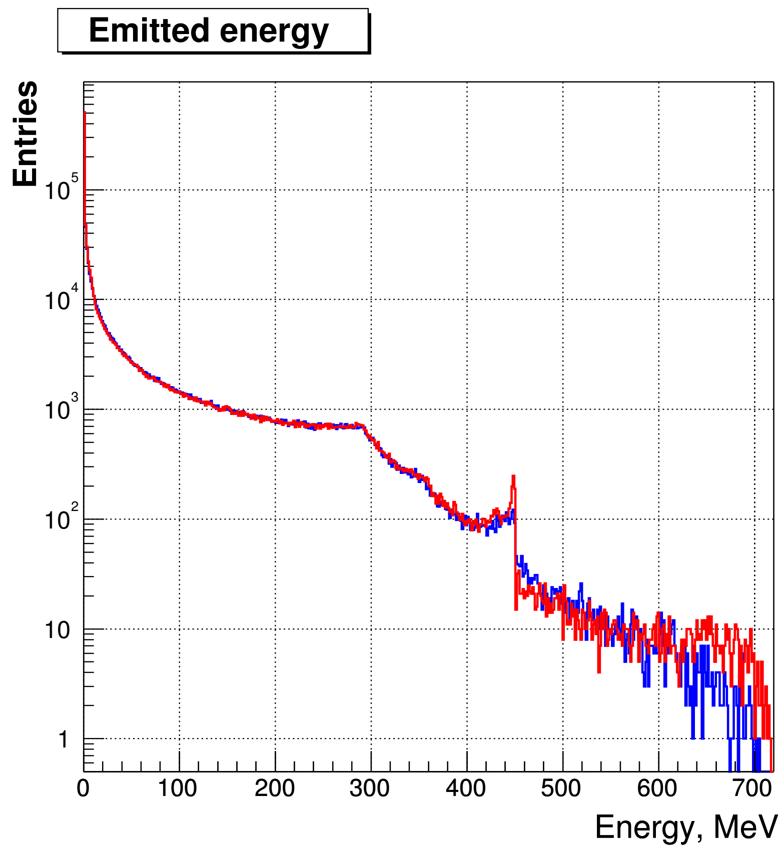
Angular distribution, $|\Delta\theta| < 1$ rad, $|\Delta\phi| < \pi$ rad

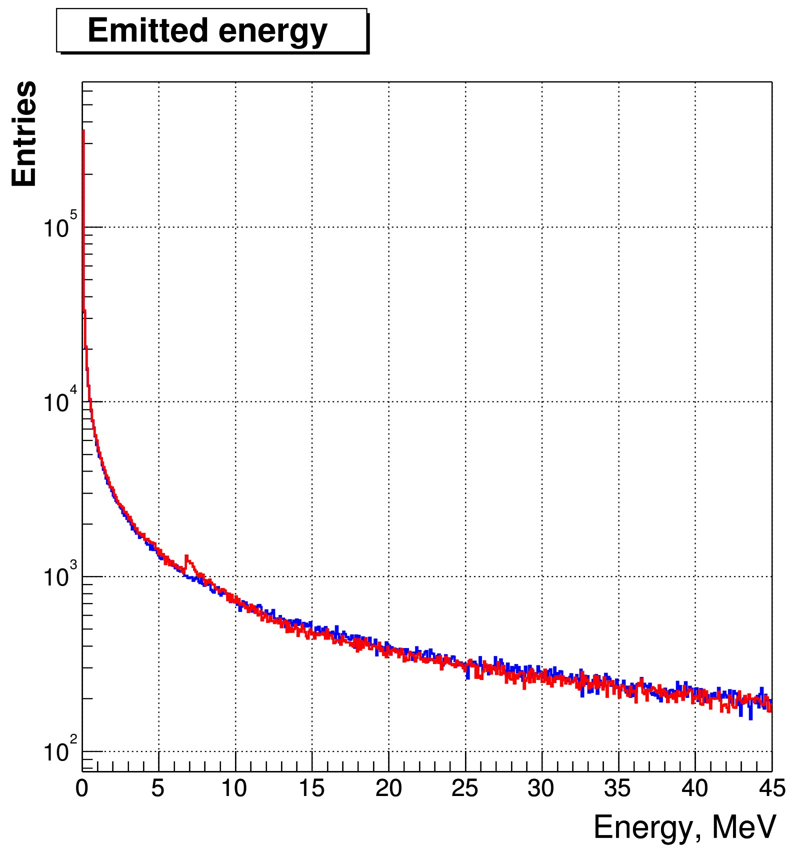




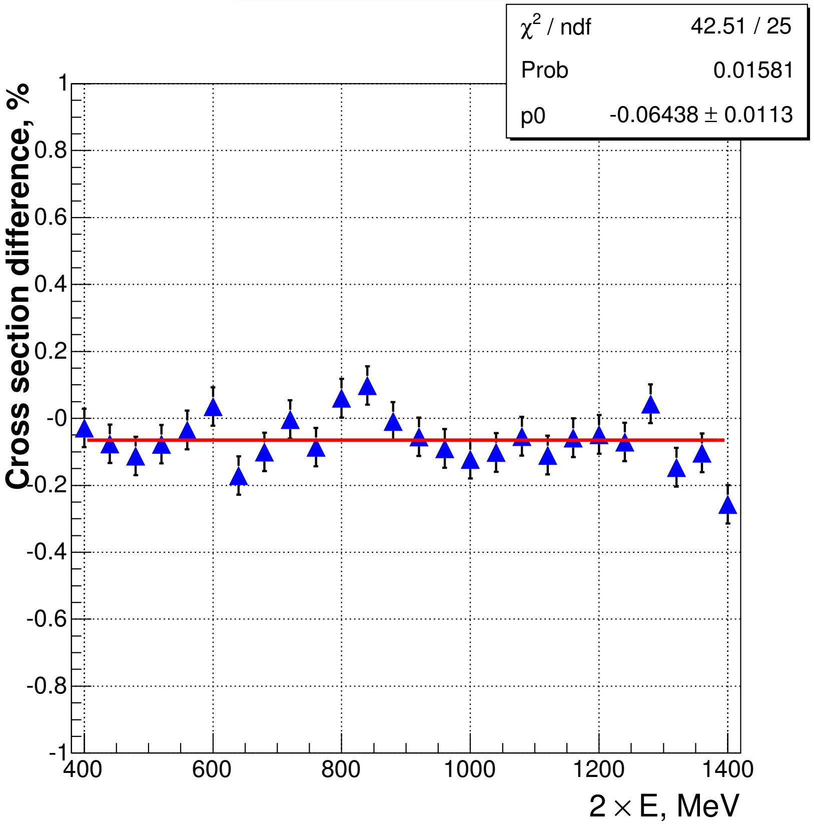
Comparison with BHWIDE

Emitted energy spectrum, $|\Delta\theta| < 1$ rad, $|\Delta\phi| < \pi$ rad

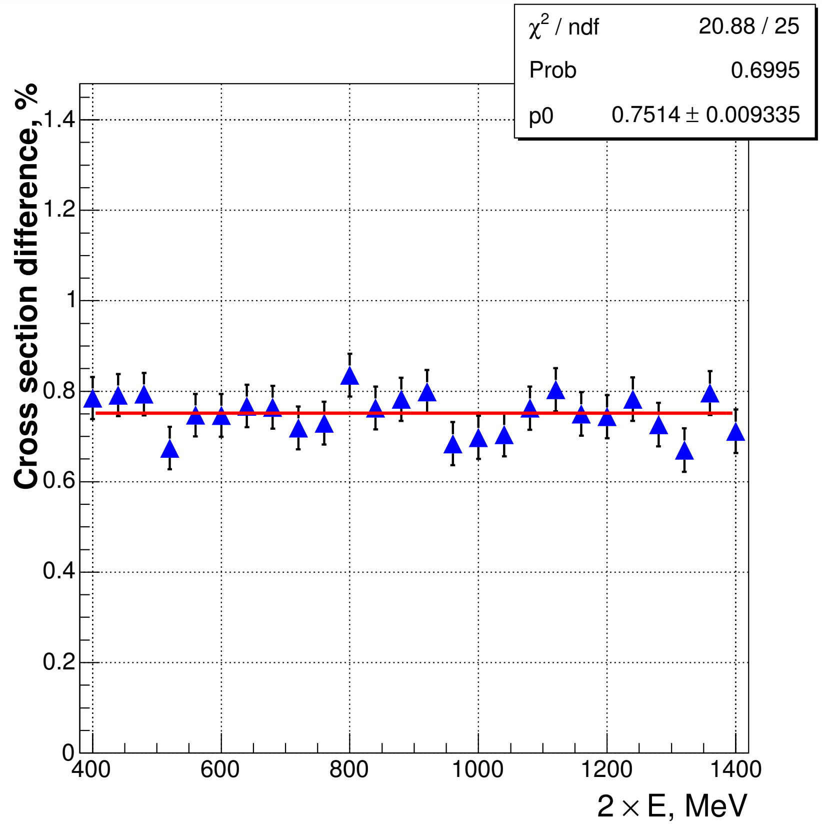




Comparison with BHWIDE & Babayaga

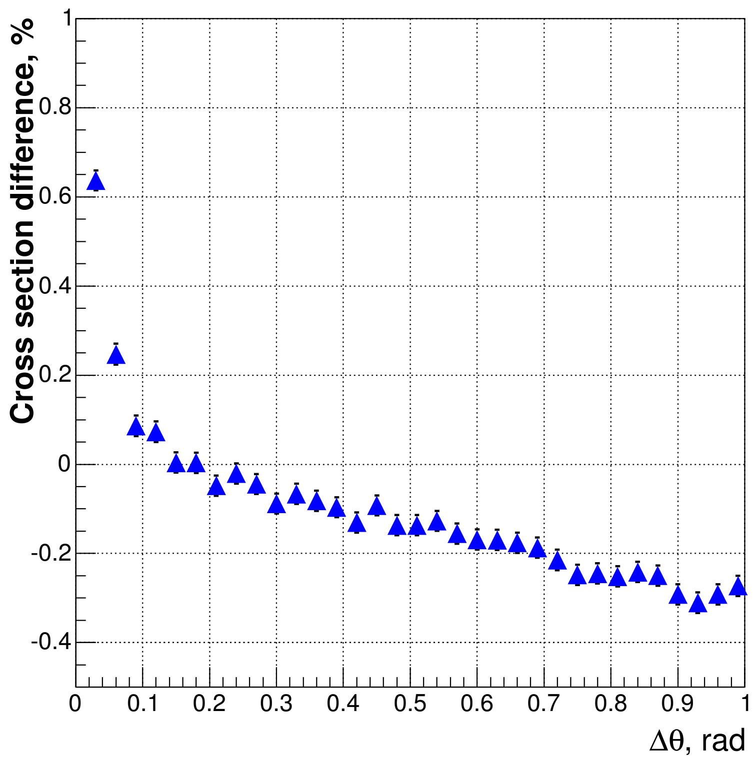


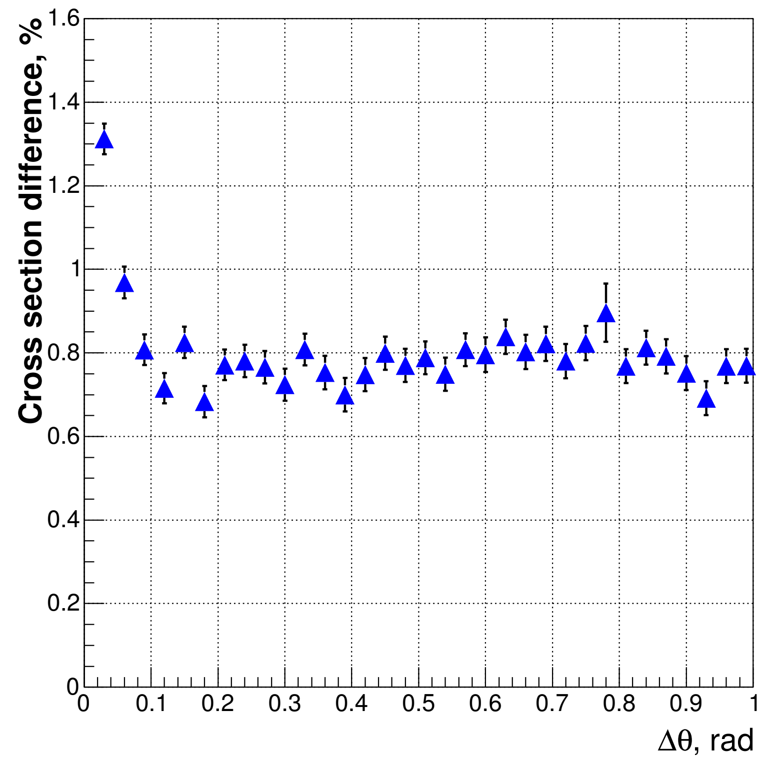
BHWIDE energy scan



Babayaga v.3.5 energy scan

Comparison with BHWIDE & Babayaga

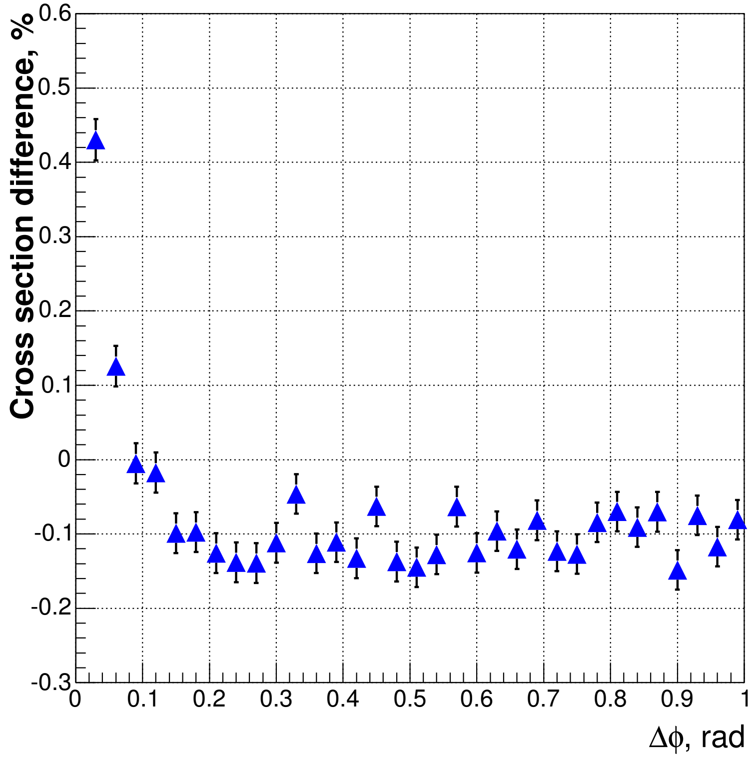




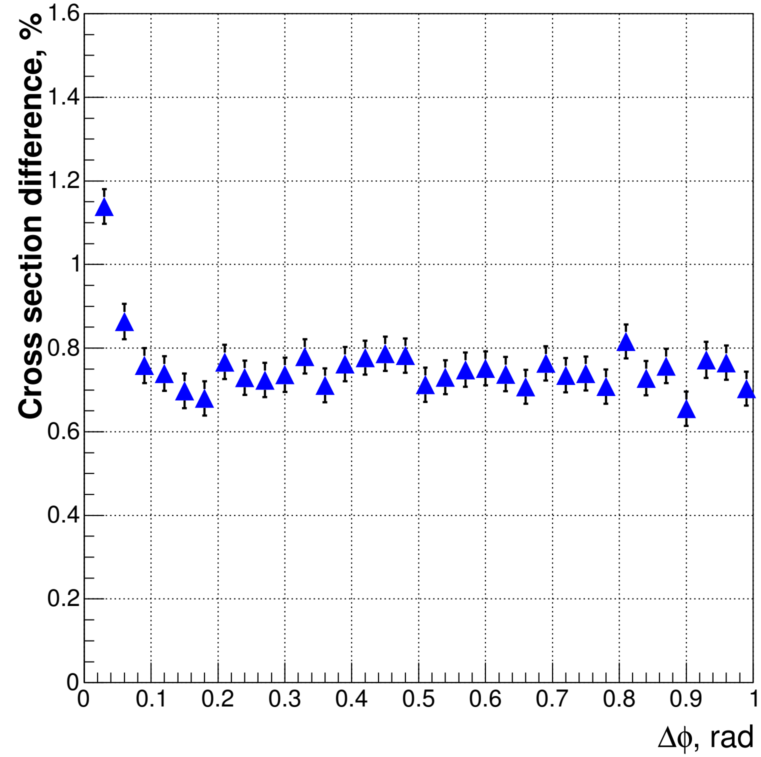
Cross section difference with BHWIDE depend on $\Delta\theta$

Cross section difference with Babayaga v.3.5 depend on $\Delta\theta$

Comparison with BHWIDE & Babayaga



Cross section difference with BHWIDE depend on $\Delta\phi$



Cross section difference with Babayaga v.3.5 depend on $\Delta\phi$

Muon pair production

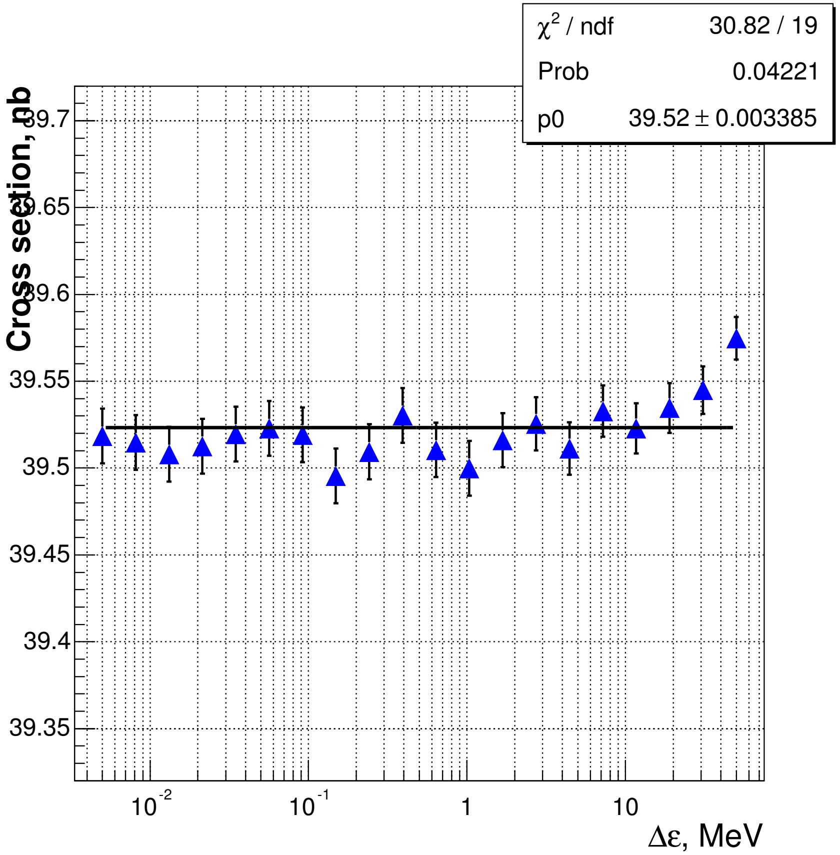
A.Arbuzov, E.Kuraev *et al.*, JHEP 97 10(1997) 001 Eur. Phys. J. C $\mathbf{46}$, 689 (2006)

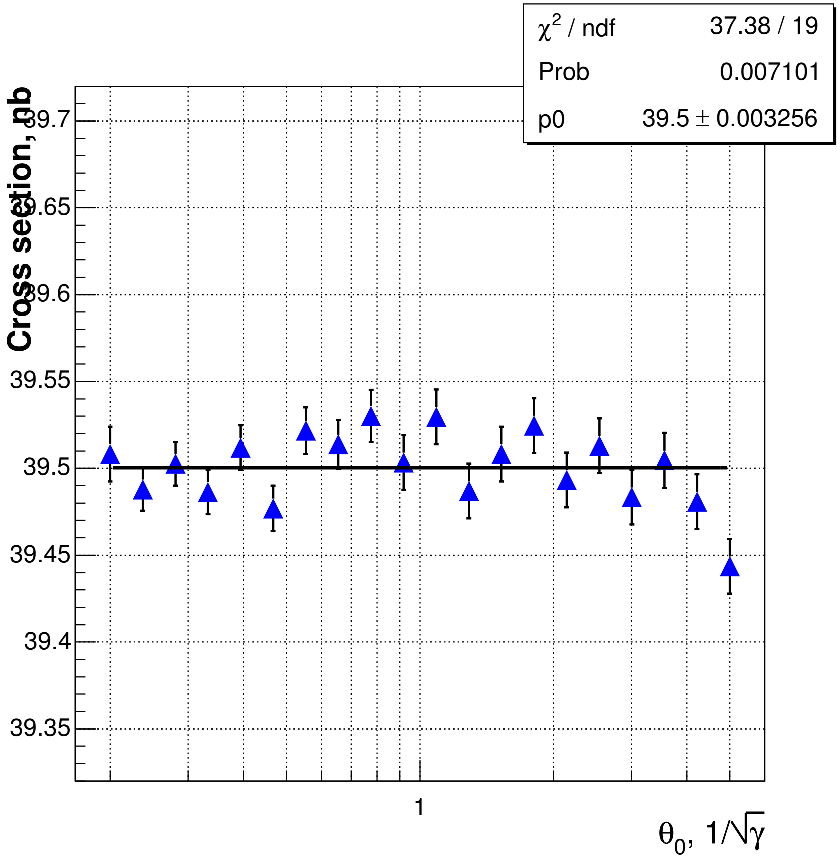
- The first order $\dfrac{\alpha}{\pi}$ is taken into account exactly.
- The contribution of higher orders was considered in the leading logarithmic approximation.

Estimated accuracy $\leq$ 0.2%

Dependence on auxiliary parameters

$$e^+e^- \to \mu^+\mu^-\gamma$$

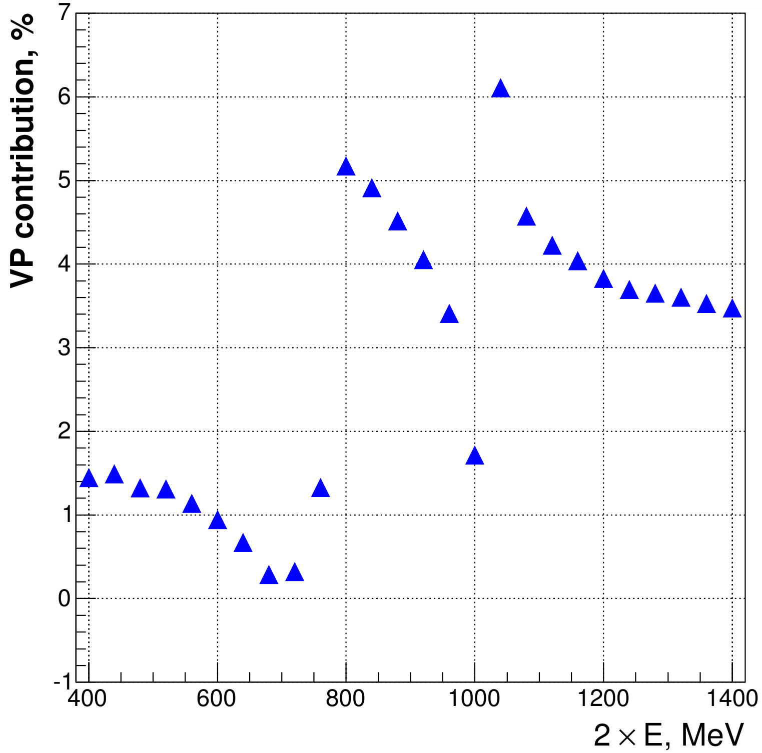




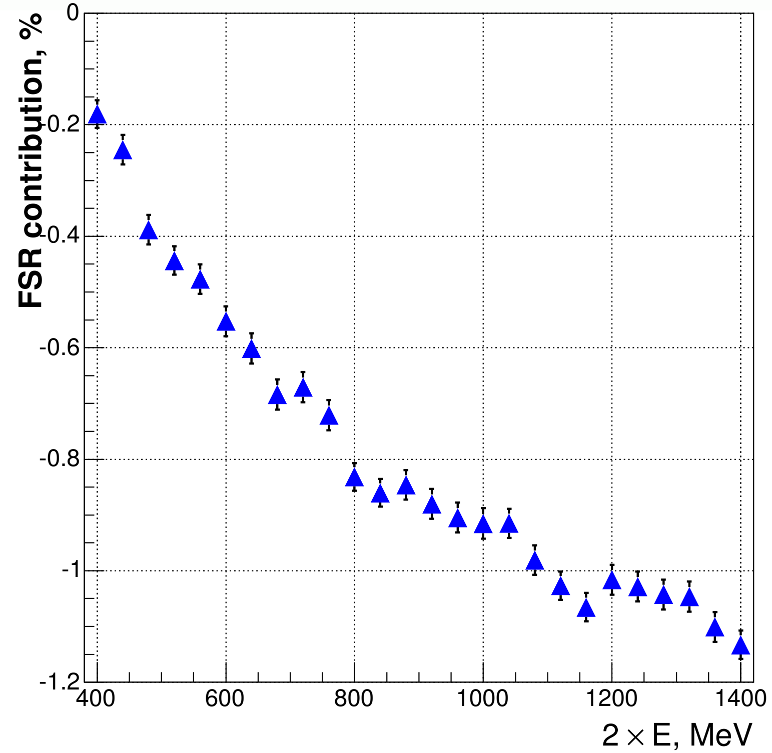
$\Delta\varepsilon$

θ_0 in $1/\sqrt{\gamma}$ units

Contribution to the $e^+e^- \to \mu^+\mu^-\gamma$ cross section

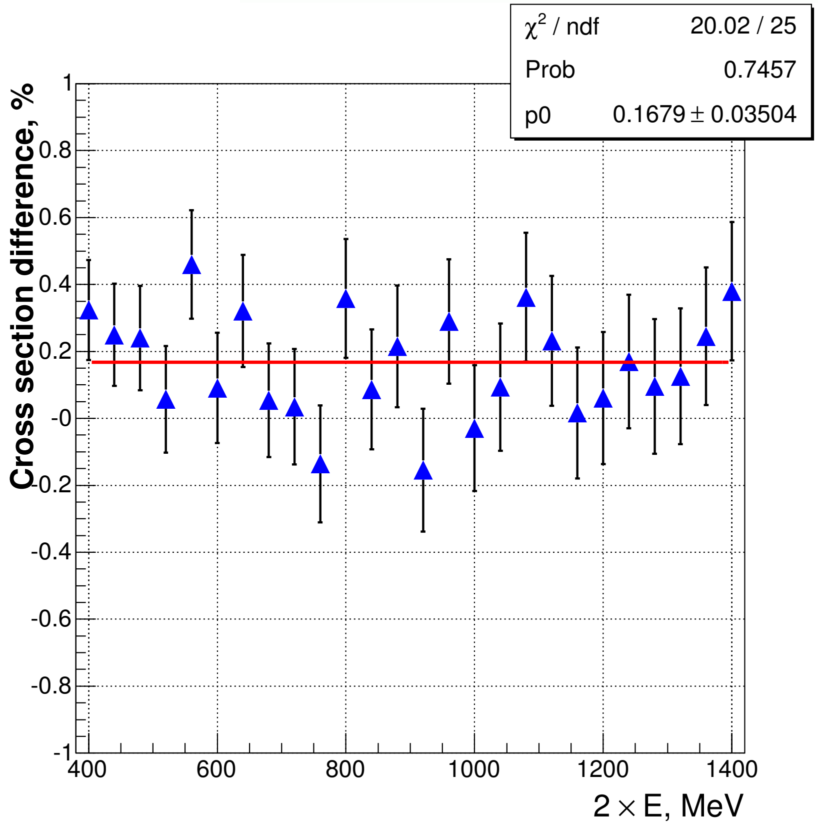


Vacuum polarization contribution

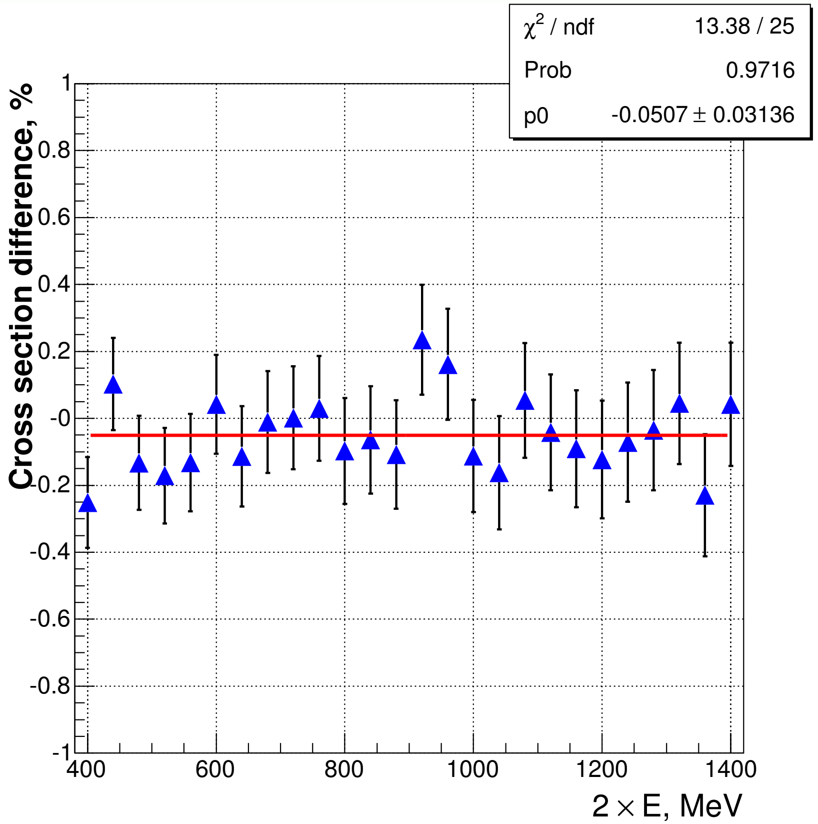


Cross section change by FSR vs energy

Comparison with KKMC



FSR is ON



FSR is OFF

Pion production

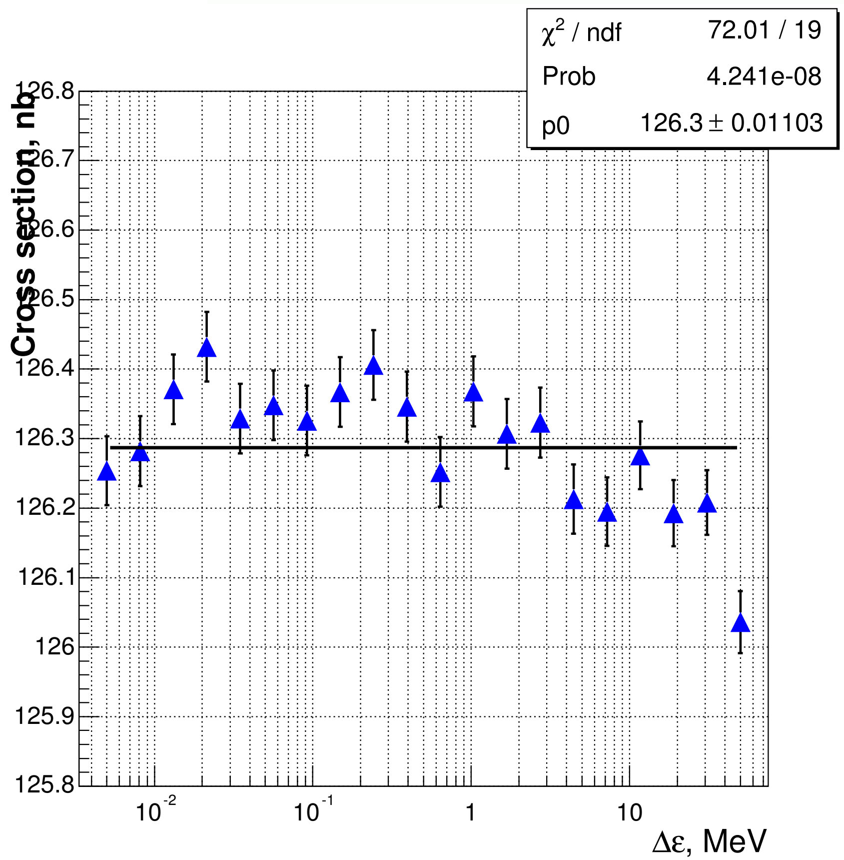
A.Arbuzov, E.Kuraev *et al.*, JHEP 97 10(1997) 006 Eur. Phys. J. C **46**, 689 (2006)

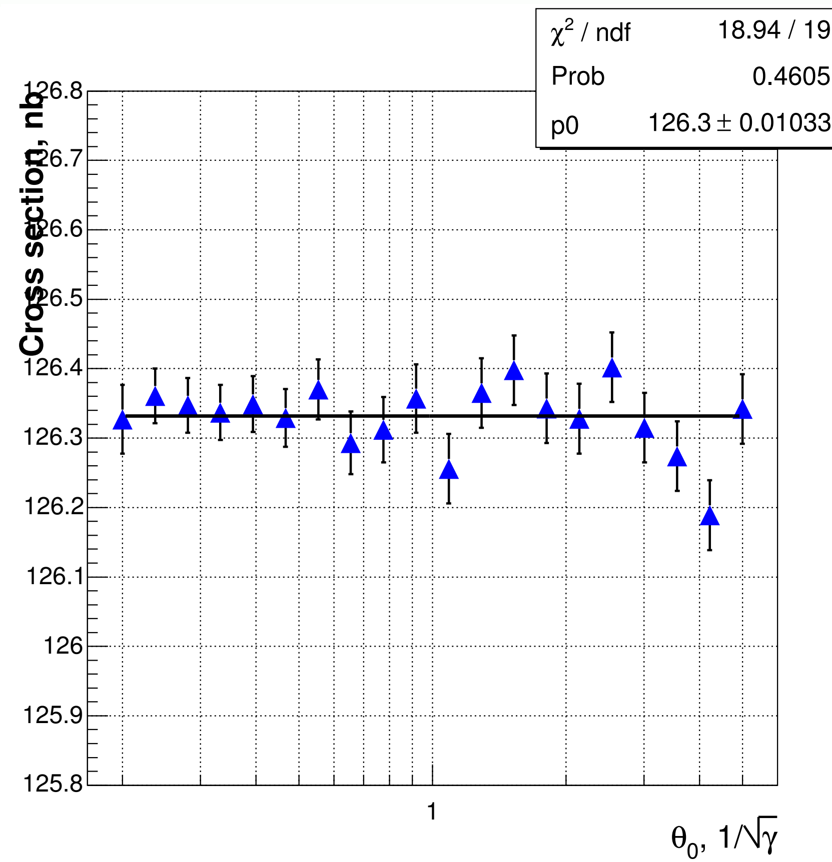
- The first order $\frac{\alpha}{\pi}$ is taken into account exactly.
- The contribution of higher orders was considered in the leading logarithmic approximation.
- Considering the pseudoscalar mesons as point like objects.
- Vacuum polarization corrections (by hadrons and leptons) are included in the pion form factor as usually.

Estimated accuracy $\leq$ 0.2%

Dependence on auxiliary parameters

$$e^+e^- \to \pi^+\pi^-\gamma$$



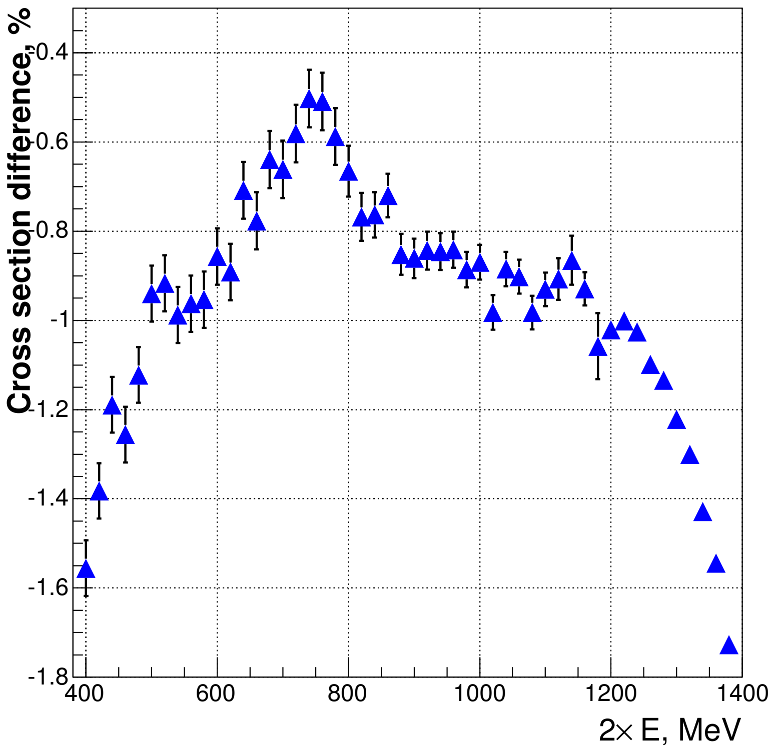


$\Delta\varepsilon$

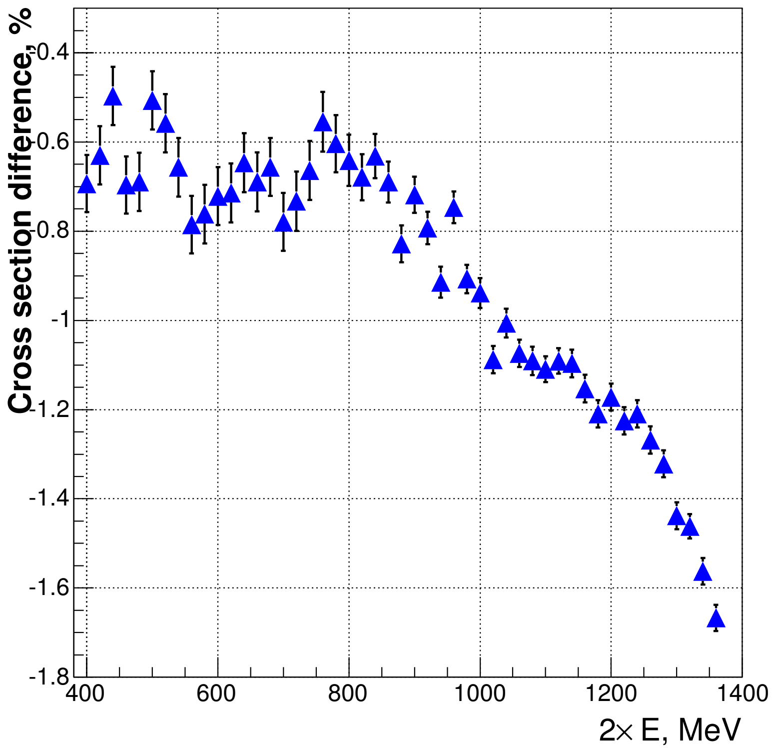
θ_0 in $1/\sqrt{\gamma}$ units

Comparison with Babayaga v3.5

Babayaga does not have FSR for pions, so FSR is switching in our program



FSR is ON



FSR is OFF

$\gamma\gamma$ *production*

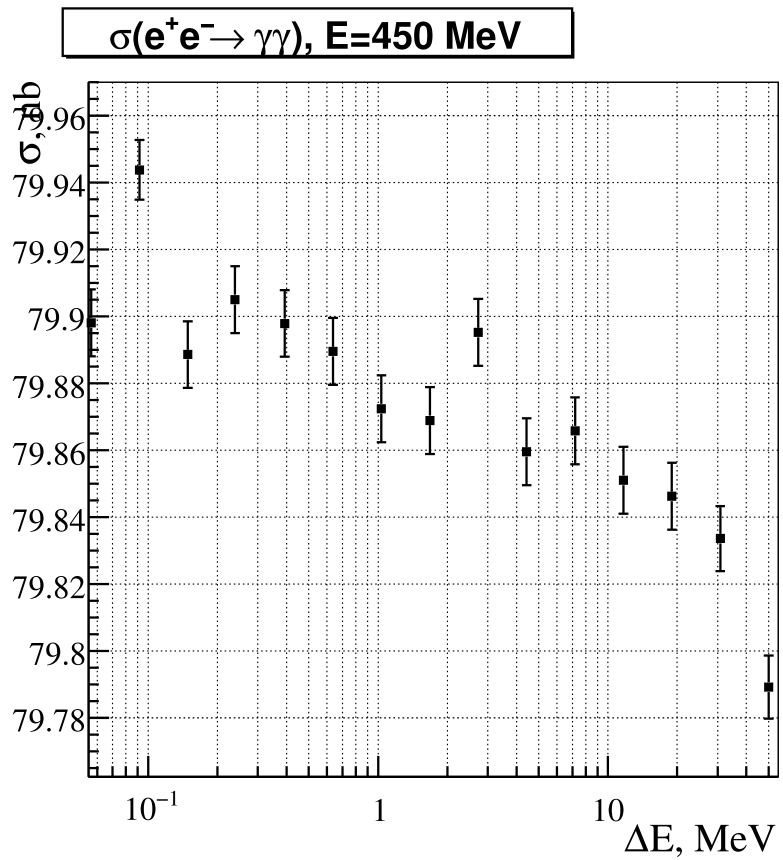
A.Arbuzov, E.Kuraev $et\ al.$, JHEP 97 10(1997) 001

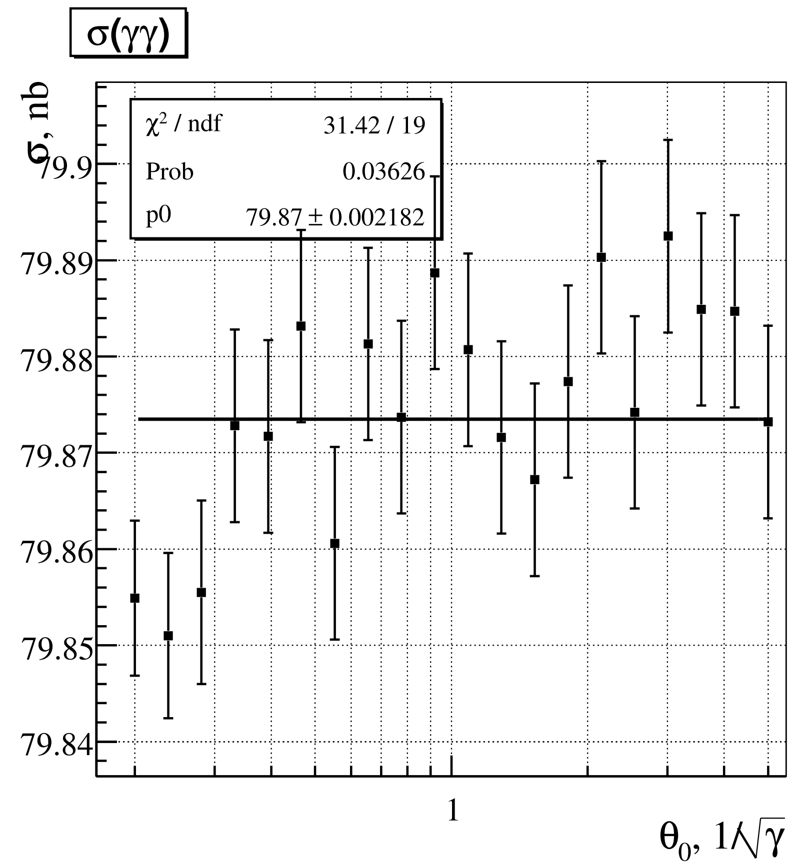
- The first order $\dfrac{\alpha}{\pi}$ is taken into account exactly.
- The contribution of higher orders was considered in the leading logarithmic approximation.
- No vacuum polarization $\rightarrow$ cross check with Bhabha luminosity determination

Estimated accuracy $\leq$ 0.2%

Dependence on auxiliary parameters

$e^+e^- \to \gamma\gamma\gamma$





$\Delta\varepsilon$

θ_0 in $1/\sqrt{\gamma}$ units

Photon vacuum polarization

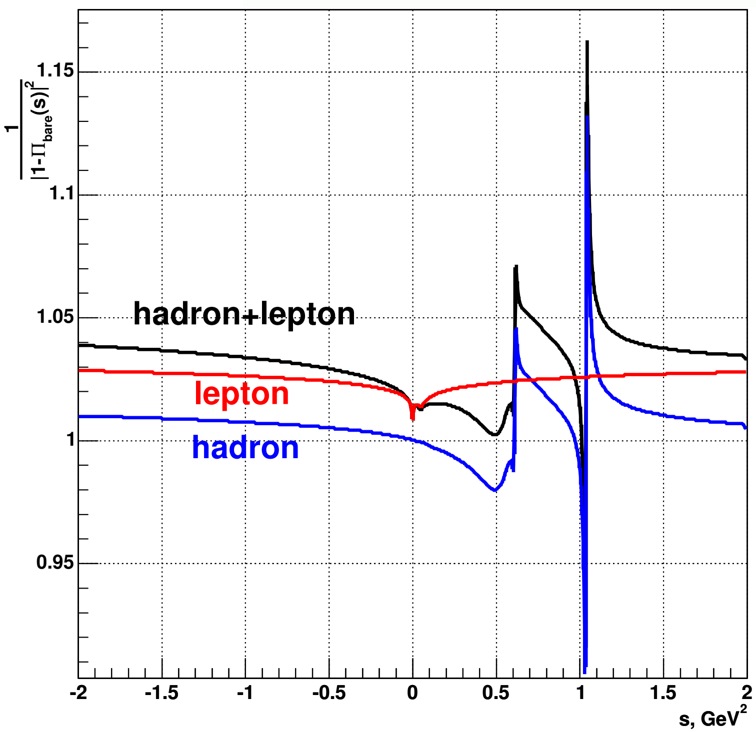
$$\Pi(s) = \Pi_l(s) + \Pi_h(s)$$

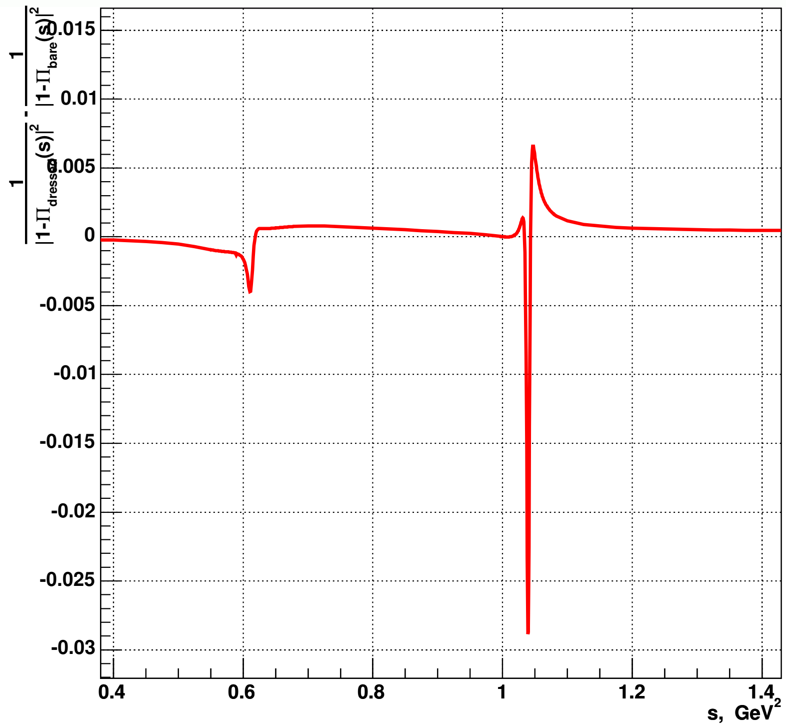
$$\Pi_l(s) = \frac{\alpha}{\pi}\Pi_1(s) + \left(\frac{\alpha}{\pi}\right)^2 \Pi_2(s) + \ldots$$

$$\Pi_h(s) = \frac{s}{4\pi^2\alpha}\left[\mathrm{PV}\int_{4m_\pi^2}^{\infty} \frac{\sigma^{e^+e^-\to\mathrm{hadrons}}(s')}{s'-s}\mathrm{d}s' - \mathrm{i}\pi\sigma^{e^+e^-\to\mathrm{hadrons}}(s)\right]$$

- Analytical expression of $\Pi_l(s)$ is well known.
- We used the most precise $e^+e^- \to \mathrm{hadrons}$ data for $\Pi_h(s)$ calculation.

Photon vacuum polarization





Contribution to VP from leptons and hadrons

Difference in VP when "dressed" and "bare" hadron cross-sections are used

Conclusion

- Codes for calculation of $e^+e^- \to e^+e^-\gamma$, $\mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$ cross-sections with precision $\leq$ 0.2% have been written and tested
- No dependencies on auxiliary parameters within claim precision in wide range
- Good agreement with BHWIDE for $e^+e^- \to e^+e^-\gamma$ process and with KKMC for $e^+e^- \to \mu^+\mu^-\gamma$ process have been shown
- No program for the $e^+e^- \to \pi^+\pi^-\gamma$ process with the same or better precision
- Vacuum polarization calculation is based on the most precise e^+e^- data
- Code can be downloaded at