Inclusive and Exclusive Cross Sections at Low Energy e^+e^- Colliders

Johann H. Kühn

I. Perturbative Predictions for R(s)

II. α_s from R(s)

III. m_c and m_b from R(s)

IV. Baryon Form Factors

V. Inclusive J/ψ Production
and Glue Balls

VI. Summary

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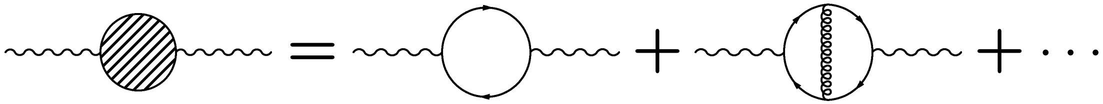
Computational Theoretical Particle Physics
SFB TR9

I. Perturbative Predictions for R(s)

remember: $R(s) = 12\pi\,\mathrm{Im}\Pi(q^2 = s + i\epsilon)$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\Pi(q^2) \equiv i \int dx\, e^{i\,q\,x} \langle T\, j_\mu(x) j_\nu(0)\rangle$$



$\Pi(q^2)$ sums all intermediate states

status of theory:

- massless limit

$$R = 3\sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi} + \#\left(\frac{\alpha_s}{\pi}\right)^2 + \#\left(\frac{\alpha_s}{\pi}\right)^3 + \#\left(\frac{\alpha_s}{\pi}\right)^4 + \cdots \right)$$

parton model	QED Källen+ Sabry 1955	Chetyrkin+... Dine+... Celmaster 1979	Gorishny, Kataev, Larin; Surguladze, Samuel 1991	Baikov, Chetyrkin, JK 2008

- full quark-mass dependence up to $\mathcal{O}(\alpha_s^2)$

$$R_q = 3Q_q^2 \left[\frac{v(3-v^2)}{2} + \frac{\alpha_s}{\pi} f_1\left(\frac{m_q^2}{s}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 f_2\left(\frac{m_q^2}{s}\right) + \cdots \right]$$

Källen, Sabry 1955 (for f_1)

Chetyrkin, JK, Steinhauser 1993 (for f_2)

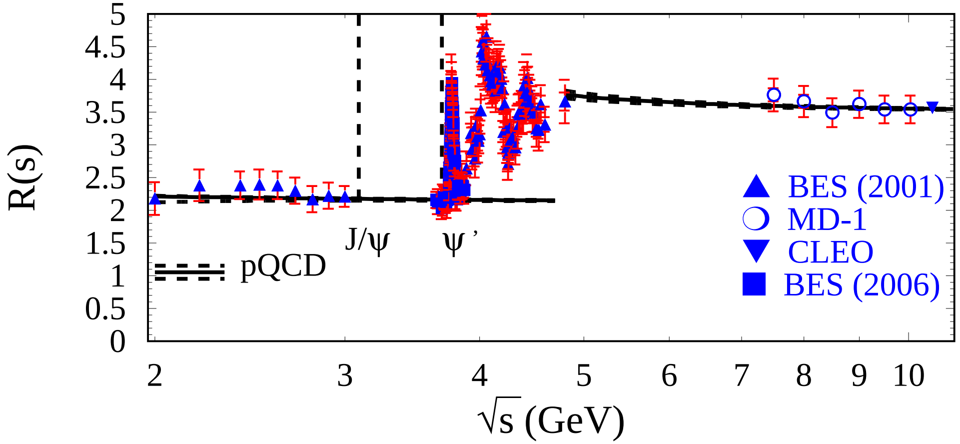
- expansion in m_q^2/s:

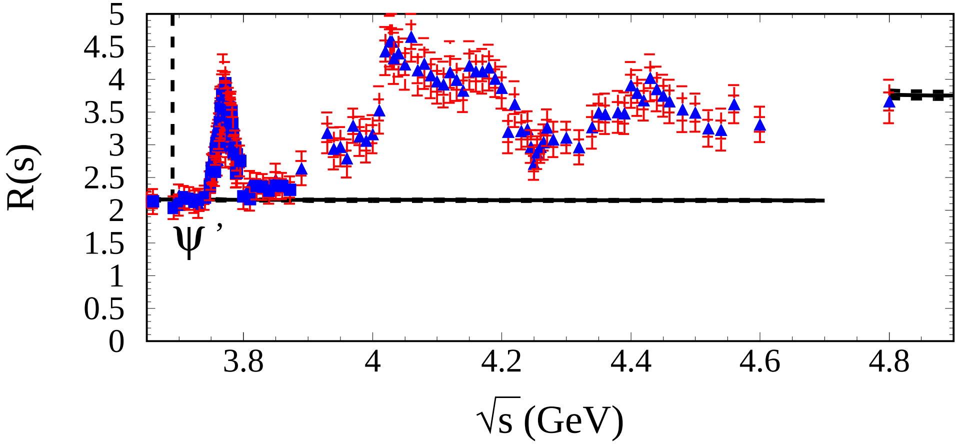
$\left(\frac{\alpha_s}{\pi}\right)^3$	$\frac{m_q^2}{s}$	Chetyrkin,JK	1990
$\left(\frac{\alpha_s}{\pi}\right)^3$	$\left(\frac{m_q^2}{s}\right)^2$	Chetyrkin,JK,Harlander	2000
$\left(\frac{\alpha_s}{\pi}\right)^4$	$\frac{m_q^2}{s}$	Baikov,Chetyrkin,JK	2003

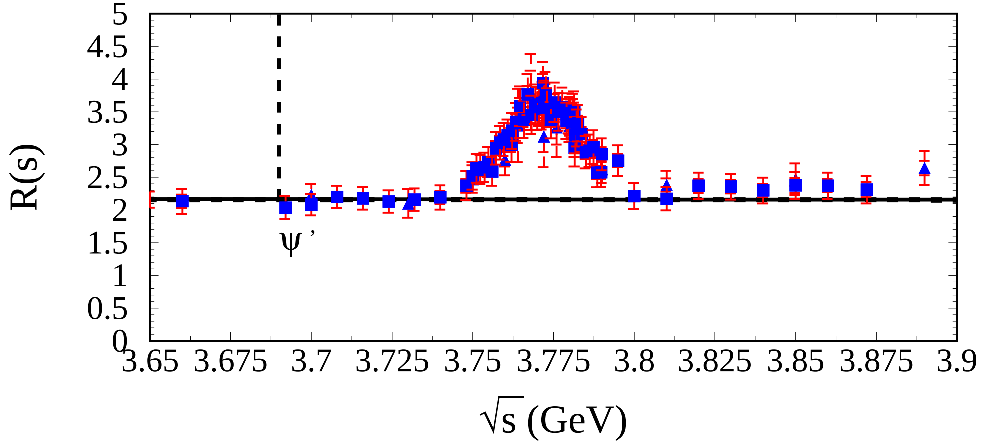
$\Rightarrow$ sufficient precision away from resonance region

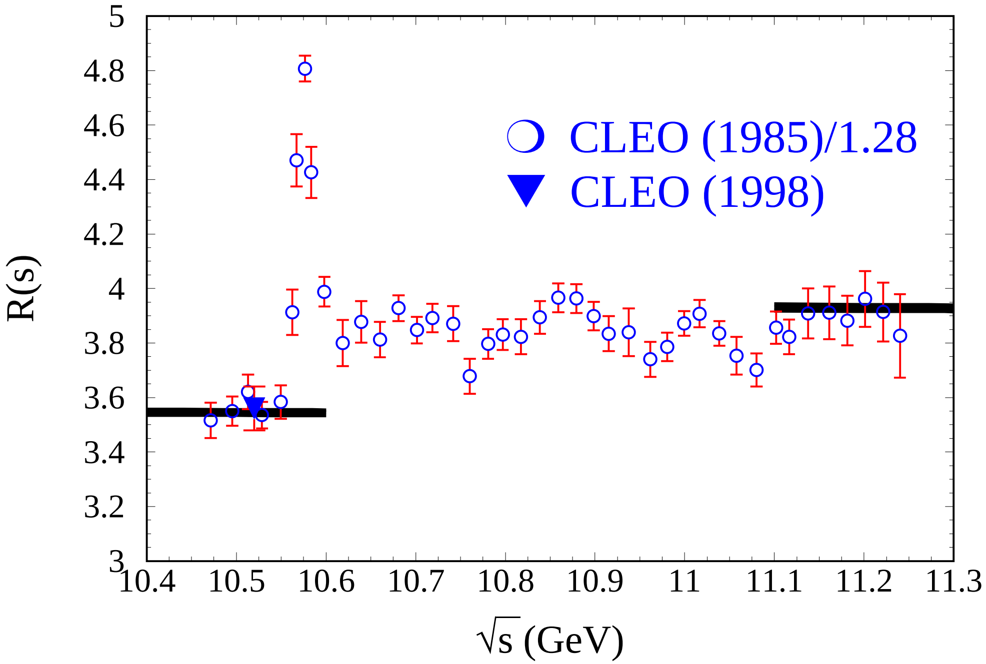
all formulas encoded in program rhad
by Harlander and Steinhauser (hep-ph/0212294)

II. α_s from R(s), Theory vs Experiment

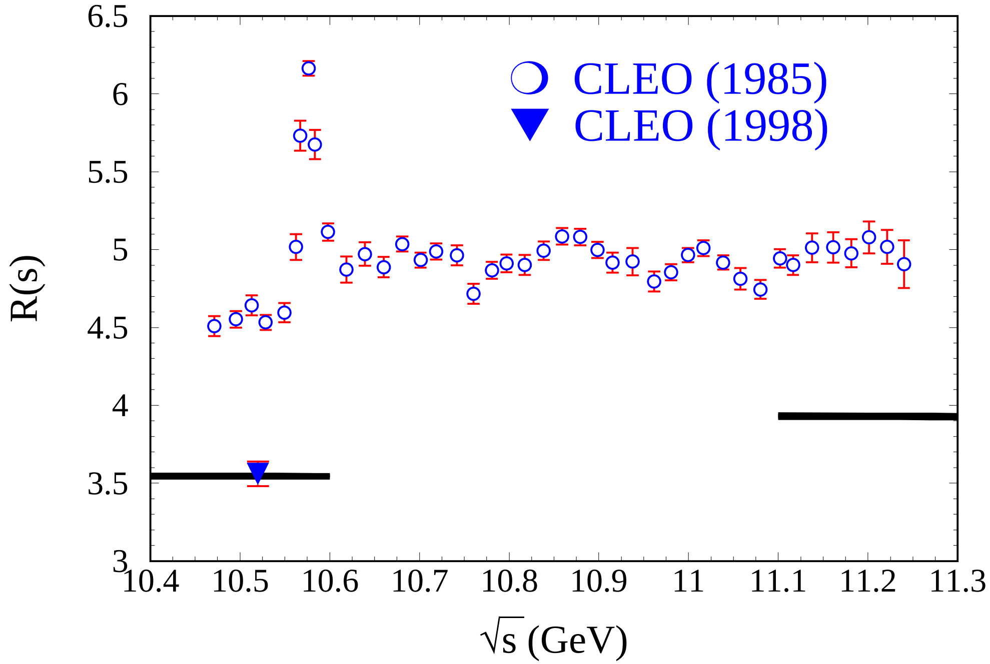




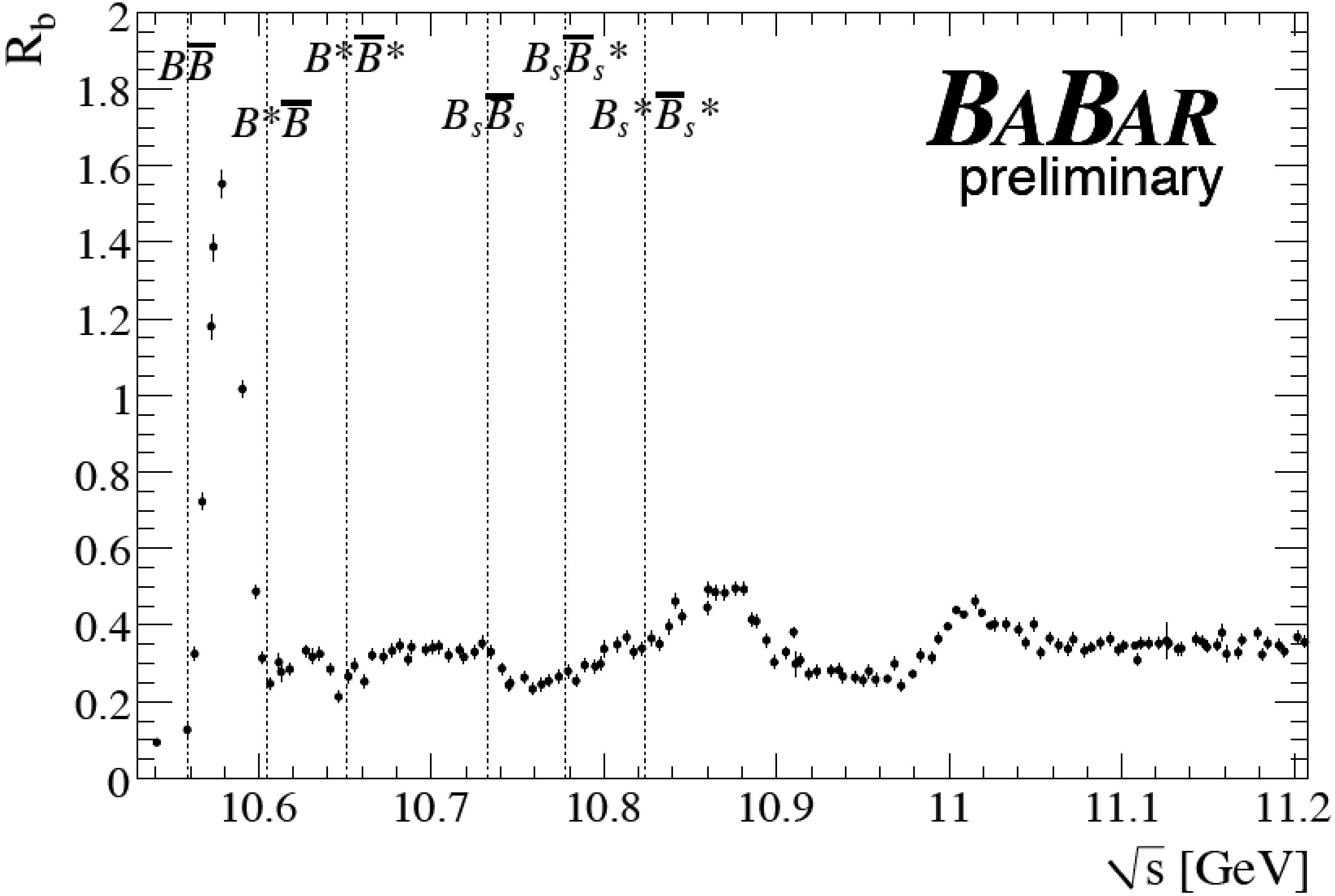




CLEO (1985)/1.28
CLEO (1998)
R(s)
√s (GeV)



CLEO (1985)
CLEO (1998)
R(s)
√s (GeV)



Theory: $R_b(11.2\,\mathrm{GeV}) = 0.265\,!$

Kinematic phase space threshold suppression $\sim v$

largely compensated by Coulomb enhancement:

Sommerfeld “rescattering” $\sim 4\pi/v$

- analysis from 2001 (Steinhauser, JK)

$$\alpha_s^{(3)}(3\,\text{GeV}) = 0.369\,{}^{+0.047}_{-0.046}\,{}^{+0.123}_{-0.130}$$

from BES(2000) below 3.73 GeV

$$\alpha_s^{(4)}(4.8\,\text{GeV}) = 0.183\,{}^{+0.059}_{-0.064}\,{}^{+0.053}_{-0.057}$$

from BES(2000) at 4.8 GeV

$$\alpha_s^{(4)}(8.9\,\text{GeV}) = 0.193\,{}^{+0.017}_{-0.017}\,{}^{+0.127}_{-0.107}$$

from MD1(1996) below 7 − 10 GeV

$$\alpha_s^{(4)}(10.52\,\text{GeV}) = 0.186\,{}^{+0.008}_{-0.008}\,{}^{+0.061}_{-0.057}$$

from CLEO(1998) 10.52 GeV

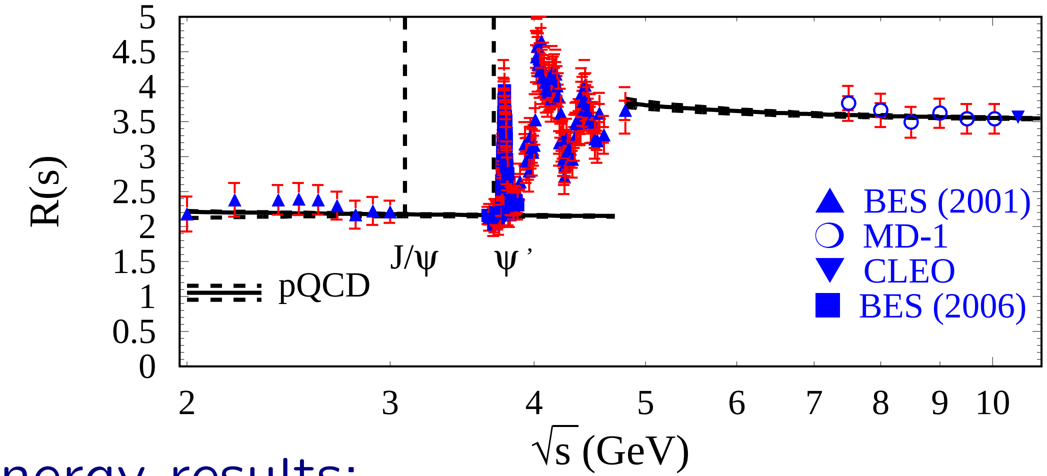
$\Rightarrow$ combine: $\alpha_s^{(4)}(5\,\text{GeV}) = 0.235\,{}^{+0.047}_{-0.047}$

$$\Rightarrow \qquad \alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$$

- analysis from 2007 (JK, Steinhauser, Teubner)

$$\alpha_s^{(4)}(9\,\text{GeV}) = 0.160 \pm 0.024 \pm 0.024$$

(based on CLEO 2007
6.964 – 10.538 GeV)



combine all low energy results:

$$\Rightarrow \alpha_s^{(4)}(9\,\text{GeV}) = 0.182^{+0.022}_{-0.025} \Rightarrow \alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$$

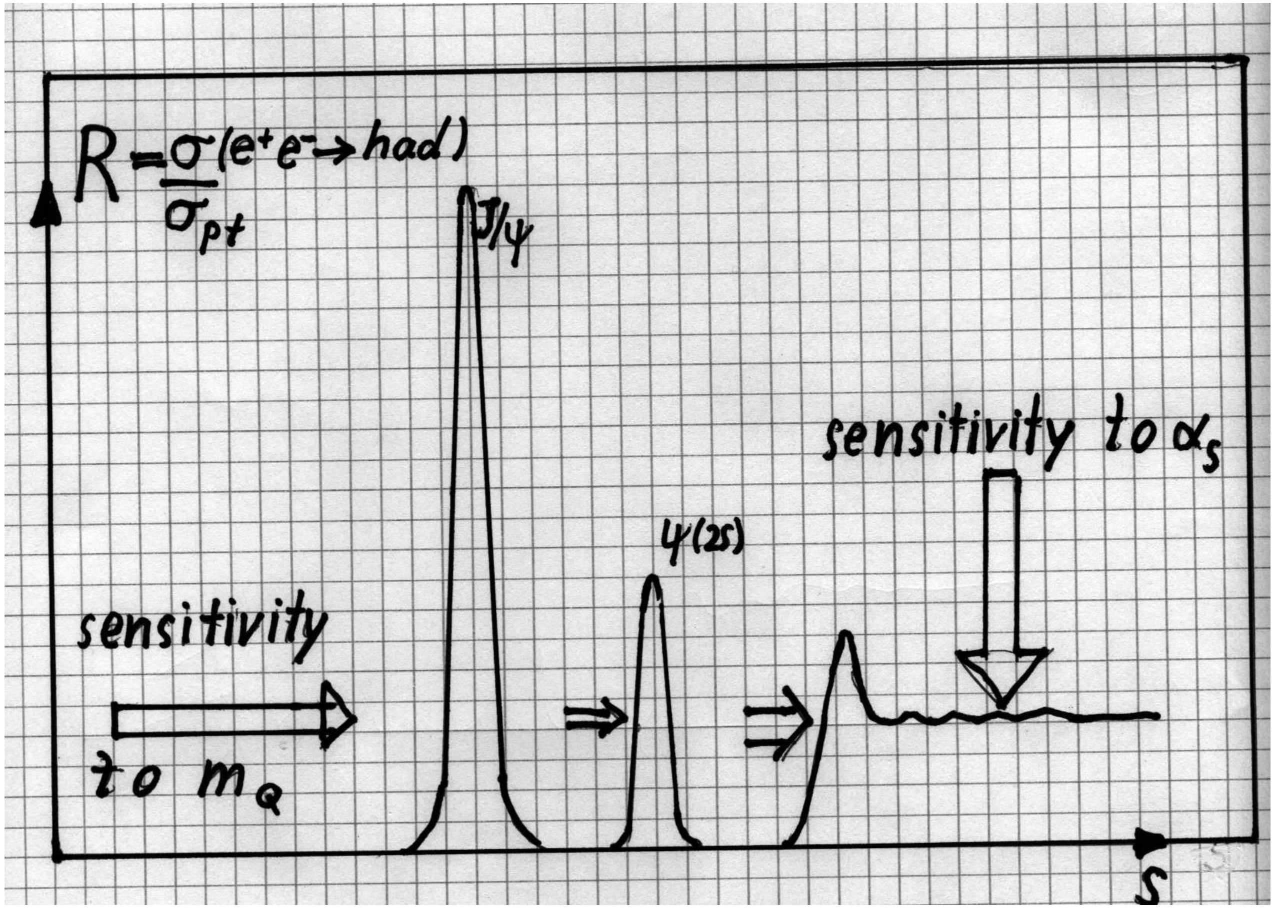
to be compared with

$$\alpha_s^{(5)}(M_Z) = 0.119 \pm 0.0026 \quad \text{from Z-decays}$$

aim: $\delta R/R < 0.5\%$!

III. m_c and m_b from R(s)

Main Idea (SVZ)



m_Q from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) = 12\pi\,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\, \Pi(q^2) \;\equiv\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\,\frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi}\bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + ...$$

Using calculated moments

$$\mathcal{M}_n^{\mathrm{th}} \equiv \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2} \right)^n \Pi_c(q^2) \Big|_{q^2=0} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_{\mathrm{c}}^2} \right)^n \bar{C}_n$$

and experiment

$$\mathcal{M}_n^{\mathrm{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\mathrm{exp}} = \mathcal{M}_n^{\mathrm{th}}$$

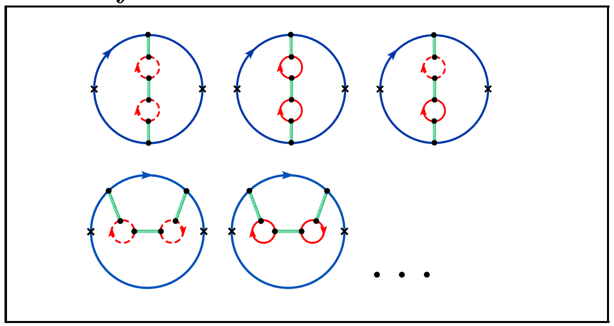
➪ m_{c}

Analysis in N^{3}LO

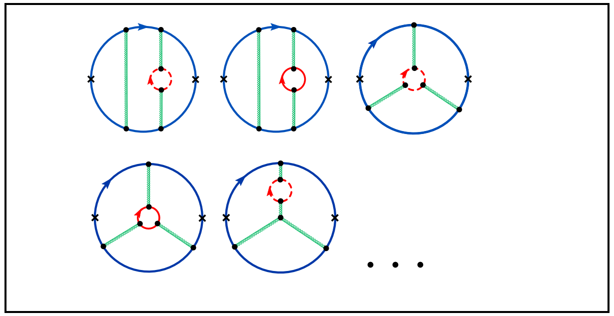
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical evaluation of master integrals

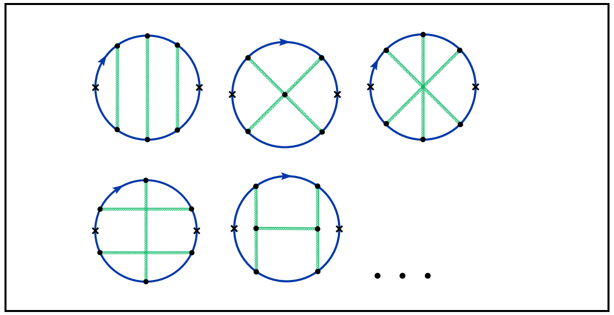
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



: heavy quarks, : light quarks,

n_f: number of active quarks

$\Longrightarrow$ About 700 Feynman-diagrams recall: $\Pi_Q(q^2) \;=\; Q_Q^2 \, \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \, z^n$

$\bar{C}_n$ depend on the charm quark mass through $l_{m\text{c}} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\begin{aligned} \bar{C}_n \;=\; & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m\text{c}}\right) \\ & + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m\text{c}} + \bar{C}_n^{(22)} l_{m\text{c}}^2\right) \\ & + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m\text{c}} + \bar{C}_n^{(32)} l_{m\text{c}}^2 + \bar{C}_n^{(33)} l_{ms}^3\right) \end{aligned}$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	− 0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	−3.4937	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

estimate $-6 < C_n^{(30)} < 6$, $\quad n = 3, 4$

Experimental input: $\int \frac{ds}{s^{n+1}} R_c(s)$

Contributions from

- narrow resonances: $R = \frac{9\,\Pi\, M_R\, \Gamma_e}{\alpha^2(s)}\,\delta(s - M_R^2)$
- threshold region ($2\,m_D$ – 4.8 GeV)
- perturbative continuum ($E \geq$ 4.8 GeV)

n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

↑ BES, SLAC — ↑ BES — ↑ BES! pQCD

Results (m_c)

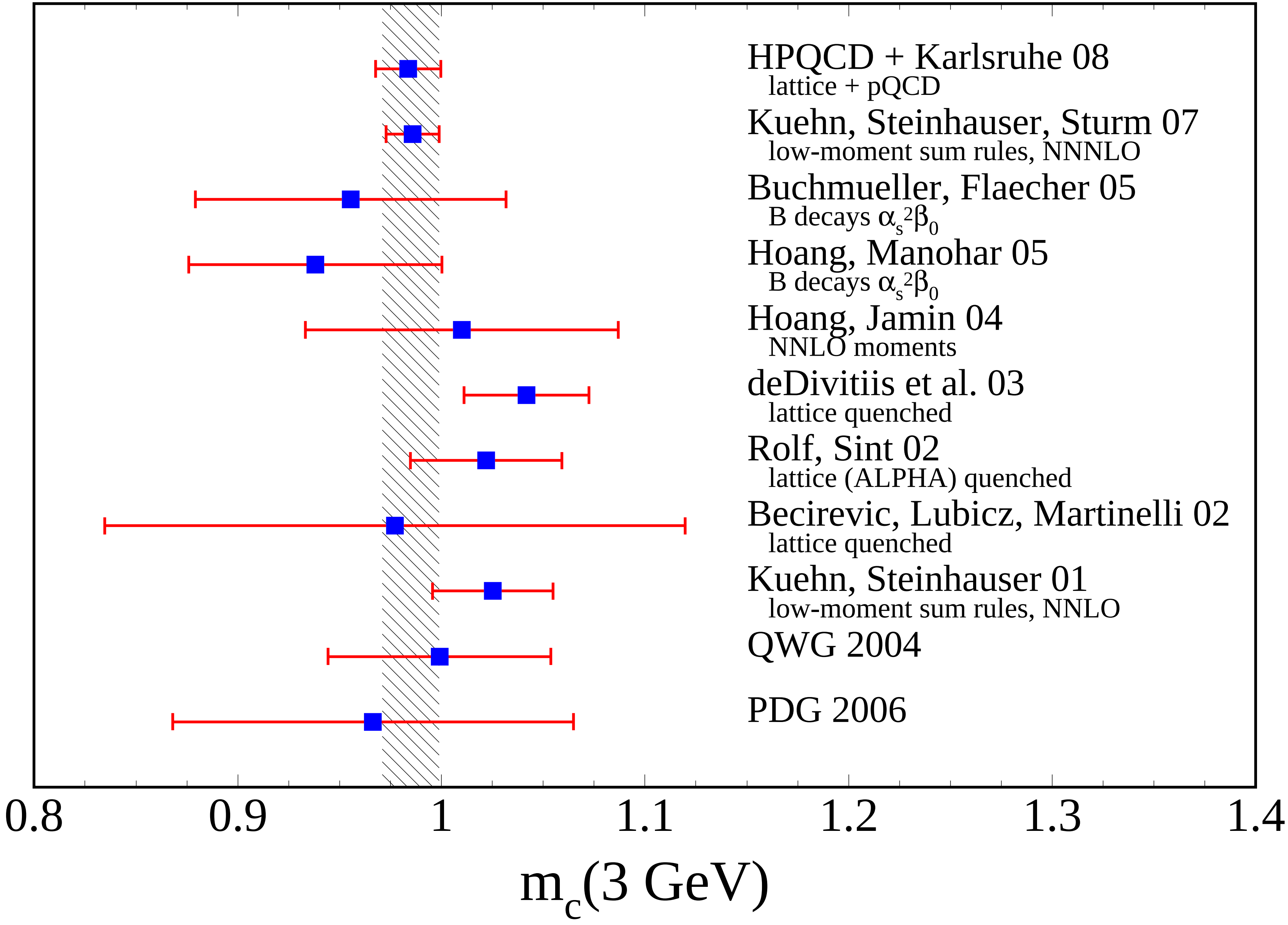
n	$m_c(3\ \text{GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.976	0.006	0.014	0.005	0.000	0.016	—	1.277
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1286 \pm 13\,\text{MeV}$

$n = 2$:

- $m_c(3\,\text{GeV}) = \begin{cases} 979 \pm 25\,\text{MeV} & \text{old, } C_2^{(3)} \text{estimated} \\ 976 \pm 16\,\text{MeV} & \text{new, } C_2^{(3)} \text{calculated} \end{cases}$
- $m_c(m_c) = 1277 \pm 16\,\text{MeV}$


HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2006
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV)

Results (m_b)

n	$m_\mathrm{b}(10\ \mathrm{GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_\mathrm{b}(m_\mathrm{b})$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.607	0.014	0.012	0.003	0.019	—	4.162
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_\mathrm{b}(10\mathrm{GeV}) = \begin{cases} 3609 \pm 25\,\mathrm{MeV}\ \text{old,}\ C_2^{(3)}\text{estimated} \\ 3607 \pm 19\,\mathrm{MeV}\ \text{new,}\ C_2^{(3)}\text{calculated} \end{cases}$
- $m_\mathrm{b}(m_\mathrm{b}) = 4162 \pm 19\,\mathrm{MeV}$
- $m_\mathrm{b}(m_\mathrm{t}) = 2701 \pm 18 \pm 14\,\mathrm{MeV}$
- $m_\mathrm{t}/m_\mathrm{b} = 59.8 \pm 1.3$

IV. Baryon Form Factors

matrix element of electromagnetic current

$$\langle N\overline{N}|J_\mu(0)|0\rangle \equiv -ie\cdot\bar{u}(q_2)\left(F_1^N(Q^2)\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N}\,[\gamma_\mu, \not{Q}\,]\right)v(q_1)$$

$F_1^N \widehat{=}$Dirac FF; $F_2^N \widehat{=}$Pauli FF

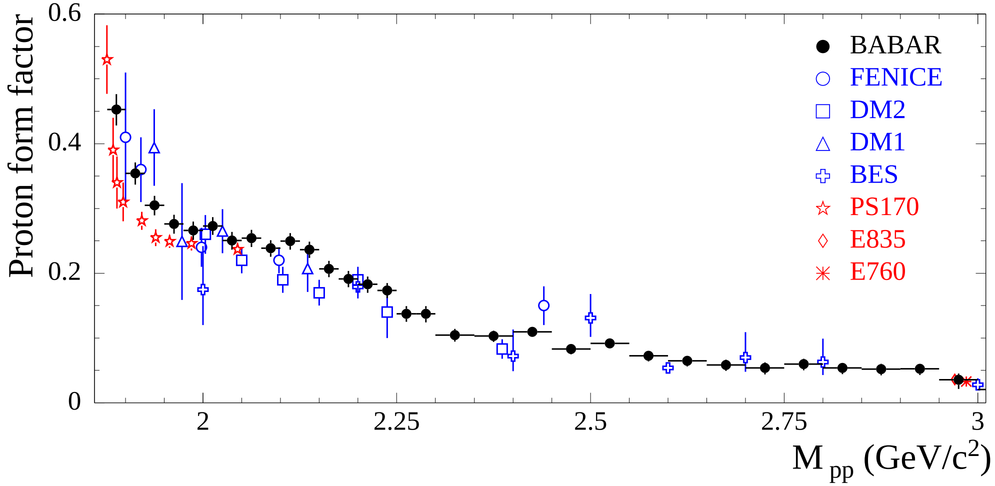
$G_M^N \equiv F_1^N + F_2^N \widehat{=}$magnetic FF; $G_E^N \equiv F_1^N + \frac{Q^2}{4m_N^2}F_2^N \widehat{=}$electric FF

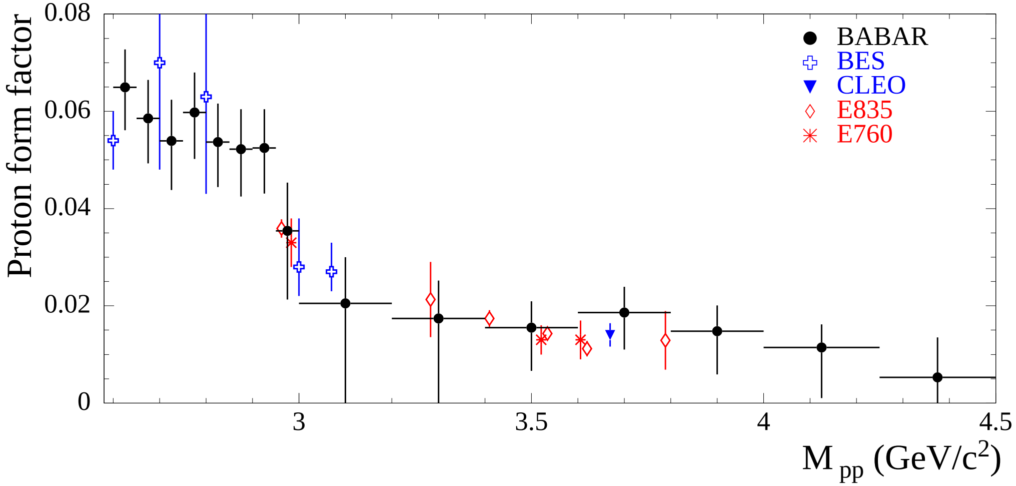
G_M and G_E can be separated via angular distributions

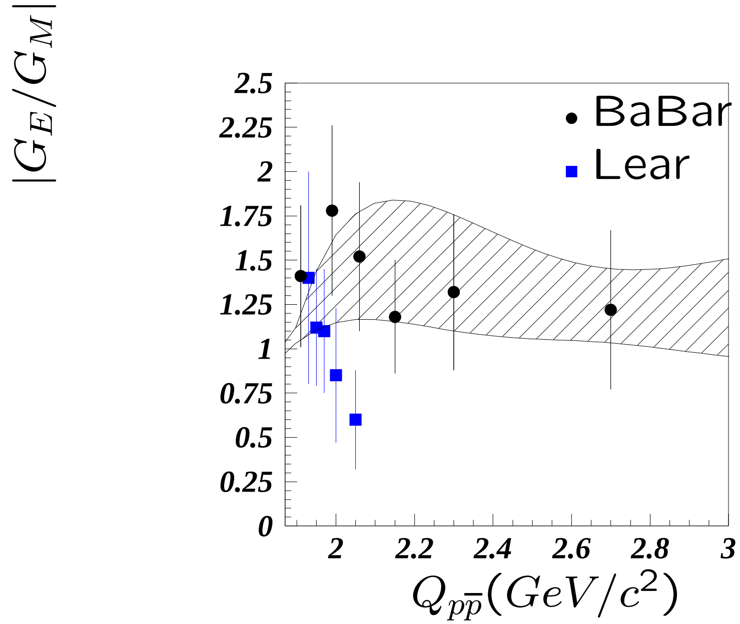
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta_N}{4Q^2}\left(|G_M^N|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E^N|^2\sin^2\theta\right) \ .$$

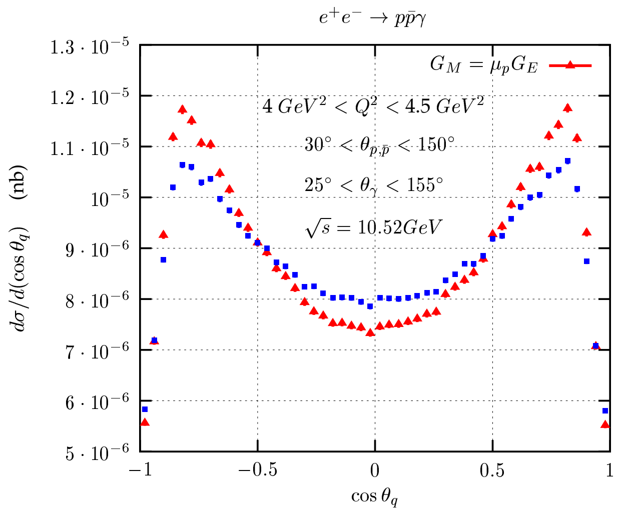
(conflicting results for $|G_e/G_M|$ at JLAB for spacelike q^2:

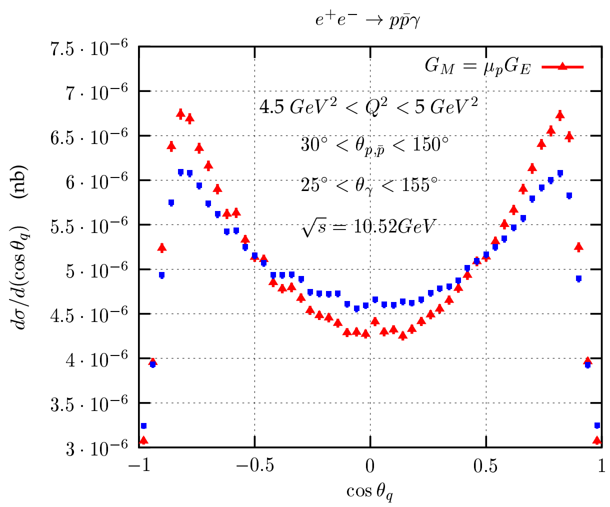
angular distribution (Rosenbluth separation) vs. polarization) results for timelike q^2 (should lead to important constraints also for negative q^2) using radiative return (PHOKHARA, BaBar)

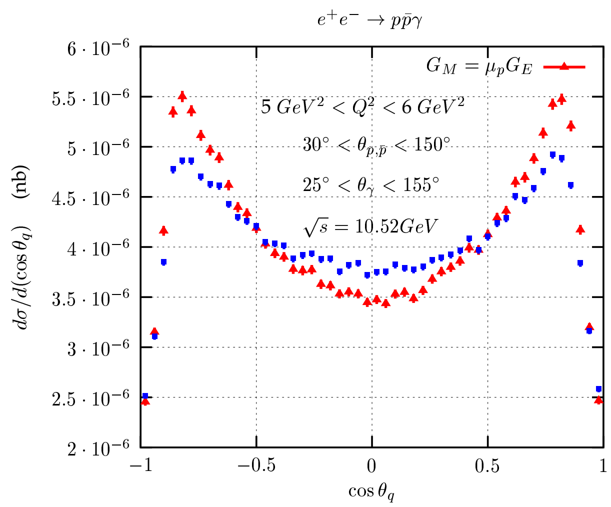












Predicition based on radiative return (PHOKHARA; Czyz, JK, Nowak, Rodrigo)

Interesting option for BESS or high luminosity B-factory spacelike q^2: scattering experiments

$\Rightarrow$ polarized electron beam and scattered proton

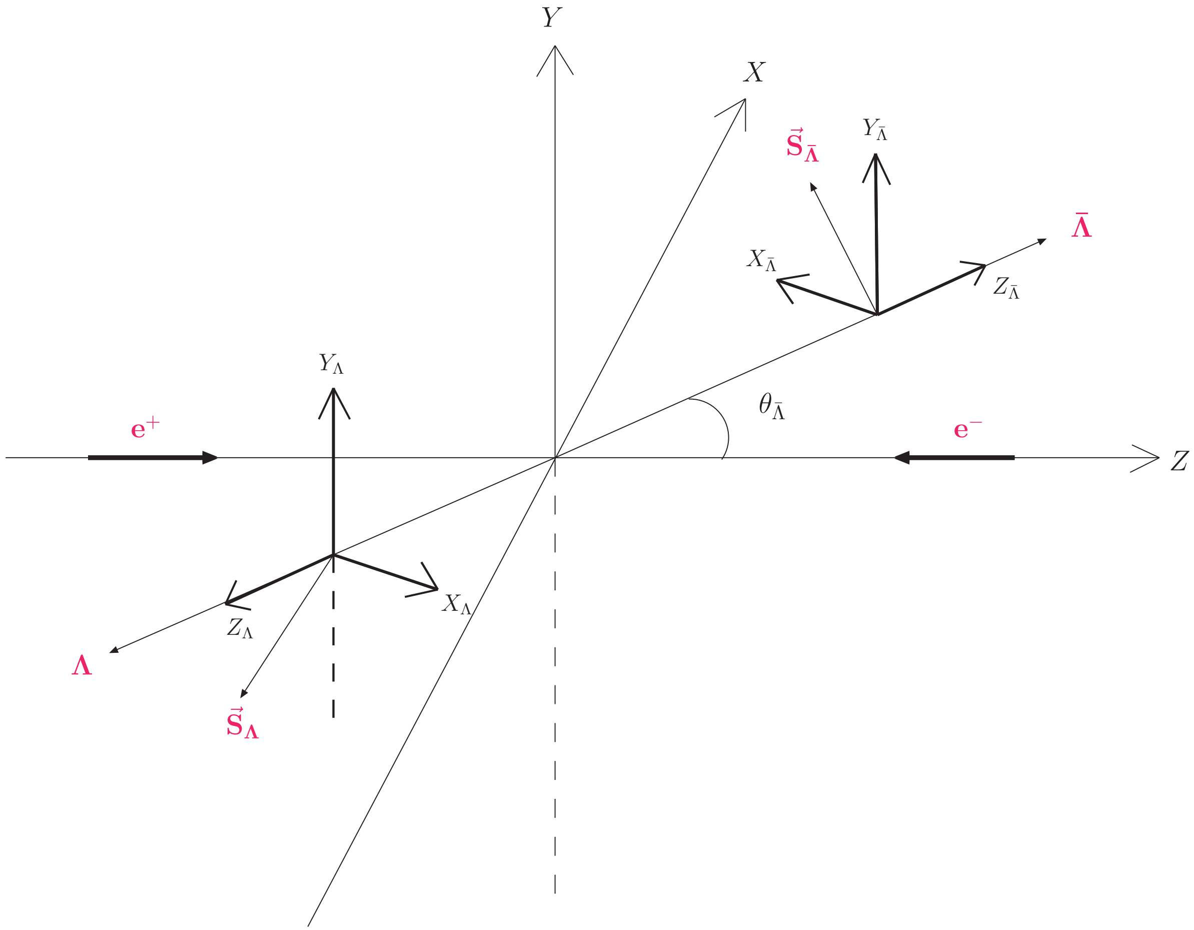
$$\vec{e}p \to e\vec{p}$$

timelike q^2: $e^+e^- \to N\overline{N}$

no polarized beams at present! polarization of N and $\overline{N}$?

$N \Rightarrow \Lambda \to p\pi$; weak decay: analyser!

$$\begin{aligned} d\sigma \sim L^0_{\mu\nu}H^{\mu\nu} = 4\pi^2\alpha^2 \Bigg\{ & |G_M|^2 \left(1+\cos^2\theta_{\bar\Lambda}\right) + \frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar\Lambda} \\ & + \frac{Im(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar\Lambda}) \left(S^y_\Lambda + S^y_{\bar\Lambda}\right) - \frac{Re(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar\Lambda}) \left(S^z_\Lambda S^x_{\bar\Lambda} + S^z_{\bar\Lambda} S^x_\Lambda\right) \\ & + \left(\frac{1}{\tau}|G_E|^2 + |G_M|^2\right) \sin^2\theta_{\bar\Lambda} \quad S^x_{\bar\Lambda} S^x_\Lambda + \left(\frac{1}{\tau}|G_E|^2 - |G_M|^2\right) \sin^2\theta_{\bar\Lambda} \quad S^y_{\bar\Lambda} S^y_\Lambda \\ & - \left(\frac{1}{\tau}|G_E|^2 \sin^2\theta_{\bar\Lambda} - |G_M|^2 \left(1+\cos^2\theta_{\bar\Lambda}\right)\right) S^z_{\bar\Lambda} S^z_\Lambda \Bigg\} \end{aligned}$$



correlation between S_Λ and $\overline{S_\Lambda}$ is sensitive to G_E, , G_M and relative phase

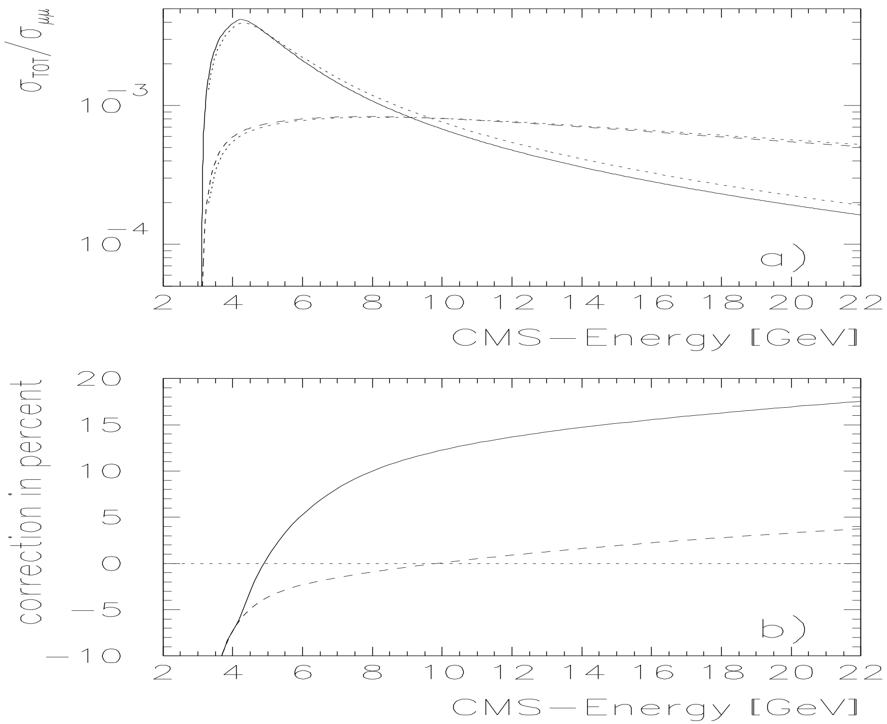
distributions: $\vec{S}_\Lambda \Rightarrow -0.642\,\vec{n}_{\pi^-}$; $\vec{S}_{\overline{\Lambda}} \Rightarrow +0.642\,\vec{n}_{\pi^+}$

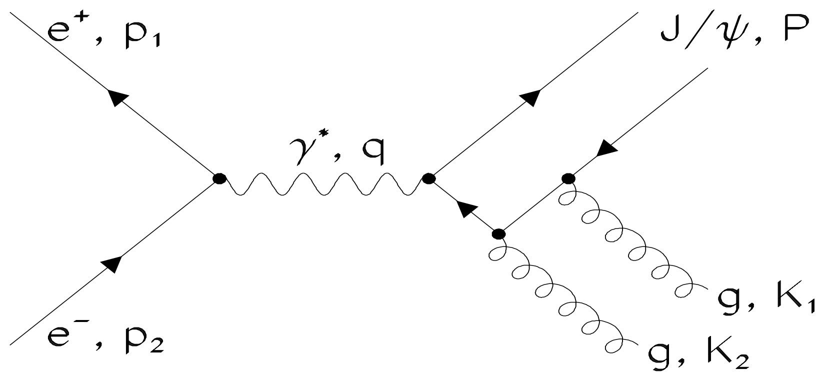
implemented in PHOKHARA (Czyz, Grzelinska, JK)

V. Exclusive Channels in $e^+e^- \to J/\psi + X$

High energies (inclusive J/ψ)

$e^+e^- \to J/\psi + gg$



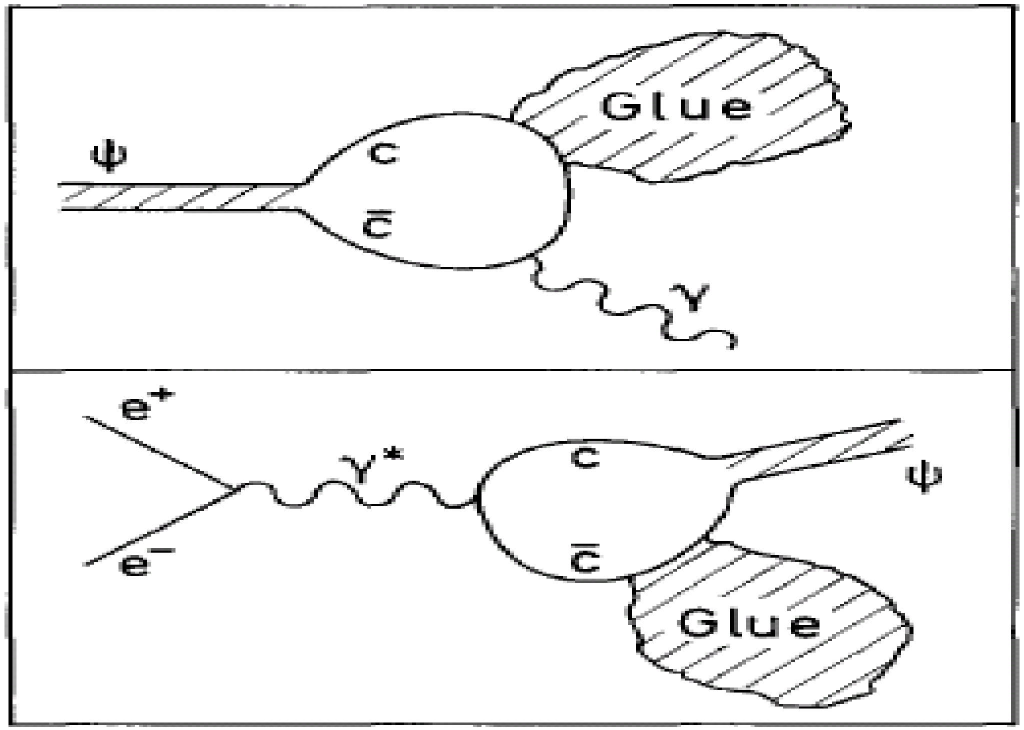


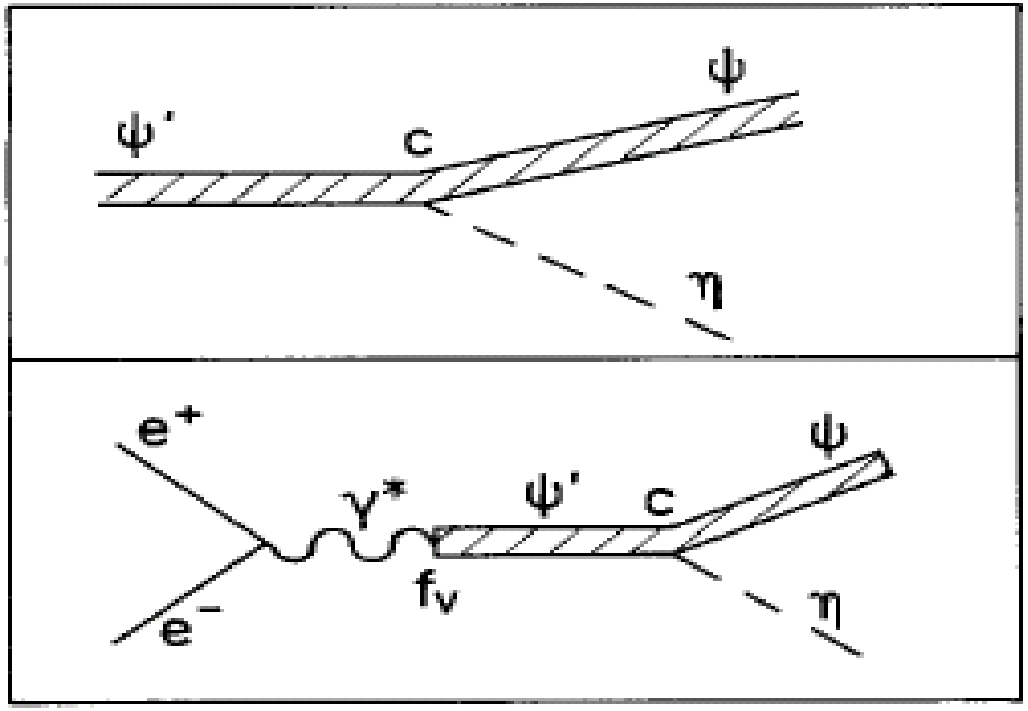
angular distributions, properties of gluon jets (Driesen, JK, Mirkes, 1993)

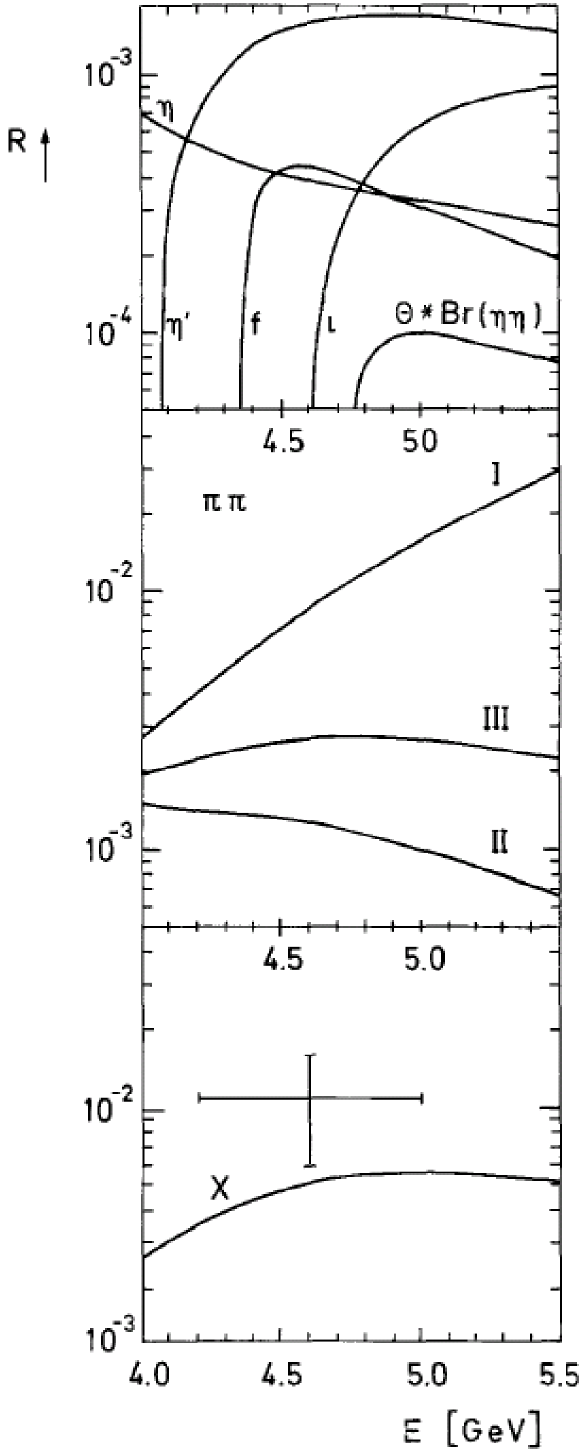
Low energies: nonperturbative physics! exclusive channels

$J/\psi \to \gamma$ Resonance $\Longleftrightarrow$ $\gamma^* \to J/\psi$+Resonance

Models







applicable to η, π^0, η', $\eta(1405/1475)$, glueballs, f_0, f_2;

dynamical assumptions required for $J/\psi\,\pi^+\pi^-$!

- clean signature: $e^+e^- \to J/\psi(\to \mu^+\mu^-)$+had
  and $m(\text{had}) \lesssim 2-3\,$GeV: analysis of glueballs
- radiative return: similar measurement at B-factory

(from JK, 1982)

VI. SUMMARY

- cross section measurements between 2 and 5 GeV

  $\Rightarrow$ α_s, m_c with high precision

- nucleon form factors G_E, G_M

  $\Rightarrow$ surprises for $q^2 < 0$; $q^2 > 0$?

- $e^+e^- \to J/\psi + X$

  $\Rightarrow$ interesting hadron dynamics

  $X =$ glue ball?