Systematic treatment of $\mathcal{O}\left(\alpha^2 L\right)$ QED corrections to Bhabha scattering

Andrej Arbuzov

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia

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Outline

- NLO QED version of the factorization theorem
- Ansatz for the master formula describing exclusive observables in $\mathcal{O}\left(\alpha^2 L^1\right)$
- Application to Bhabha scattering:
- 2-loop Soft + Virtual
- 2 Hard photons
- 1 Hard photon $\otimes$ 1-loop Soft + Virtual
- Pairs
- Outlook

QED Factorization Theorem

The QCD factorization theorem can be adopted for the QED case *e.g.* for Bhabha:

$$\mathrm{d}\sigma = \sum_{a,b,c,d=e^\pm,\gamma} \int_{\bar{z}_1}^{1} \mathrm{d}z_1 \int_{\bar{z}_2}^{1} \mathrm{d}z_2 \mathcal{D}^{\mathrm{str}}_{ae^-}(z_1) \mathcal{D}^{\mathrm{str}}_{be^+}(z_2) \Big[\mathrm{d}\sigma^{(0)}_{ab\to cd}(z_1, z_2) + \mathrm{d}\bar{\sigma}^{(1)}_{ab\to cd}(z_1, z_2)$$

$$+ \mathcal{O}\left(\alpha^2 L^0\right) \Big] \int_{\bar{y}_1}^{1} \frac{\mathrm{d}y_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{\mathrm{d}y_2}{Y_2} \mathcal{D}^{\mathrm{frg}}_{e^-c}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\mathrm{frg}}_{e^+d}\left(\frac{y_2}{Y_2}\right),$$

where $\sigma^{(0)}_{a+b\to c+d}$ is the Born-level partonic cross section, $\bar{\sigma}^{(1)}$ is the $\overline{\mathrm{MS}}$ subtracted $\mathcal{O}(\alpha)$ contribution to it,

$$\begin{aligned} \mathcal{D}^{\mathrm{str,frg}}_{ee}(z) &= \delta(1-z) + \frac{\alpha}{2\pi} d^{(1)}(z, \mu_0, m_e) + \frac{\alpha}{2\pi} L P^{(0)}(z) \\ &+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2} L^2 P^{(0)} \otimes P^{(0)}(z) + L P^{(0)} \otimes d^{(1)}(z, \mu_0, m_e) \right. \\ &+ \left. L P^{(1,\gamma)\mathrm{str,frg}}_{ee}(z) \right) + \mathcal{O}\left(\alpha^2 L^0, \alpha^3\right) \end{aligned}$$

QED Master Formula Ansatz

Using slicing in the photon energy, we cast the corrected cross section in the form

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \mathrm{d}\sigma^{(1)}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(1)}_{\mathrm{H}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} + \mathrm{d}\sigma^{(3)LO} + \ldots$$

For many observables we need to know the complete kinematics including hard photon angles, which are integrated over in the above QCD-like formula. Let us decompose the $\mathcal{O}\left(\alpha^2 L^{2,1}\right)$ hard γ radiation contribution

$$\mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} = \mathrm{d}\sigma^{(2)}_{\mathrm{HH(coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{HH(s-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(n-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(coll)}}$$

where slicing in the photon emission angle is applied:

- **"coll" means collinear photon(s) with $\vartheta_\gamma < \theta_0 \ll 1$,**
- **"n-coll" means non-collinear photon with $\vartheta_\gamma > \theta_0$,**
- **"HH(s-coll)" means semi-collinear kinematics, *i.e.* one collinear photon and one non-collinear**

Particular NLO contributions: Soft+Virt (1)

The combined effect of virtual and soft photonic corrections within the $\mathcal{O}\left(\alpha^2 L^1\right)$ can be obtained by convolution of the structure functions with the kernel cross section according to the general factorization theorem. Here one requires only one non-trivial convolution

$$\frac{\alpha}{2\pi} L \int\limits_{1-\Delta}^{1} \mathrm{d}z \int\limits_{0}^{1} \frac{\mathrm{d}x}{x} P^{(0)}\left(\frac{z}{x}\right) \mathrm{d}\bar{\sigma}^{(1)}(x)$$

This integral can be found for any relevant process as demonstrated in [A.A., E. Scherbakova, ZhETF Pis'ma 2006] **for the large-angle Bhabha case by getting $\mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}}$ in agreement with the complete $\mathcal{O}\left(\alpha^2\right)$ calculation.**

Particular NLO contributions: Soft+Virt (2)

$L = \ln \frac{M^2}{m_e^2}$ $\qquad$ $E_{\gamma_i} < \Delta E_{beam}$

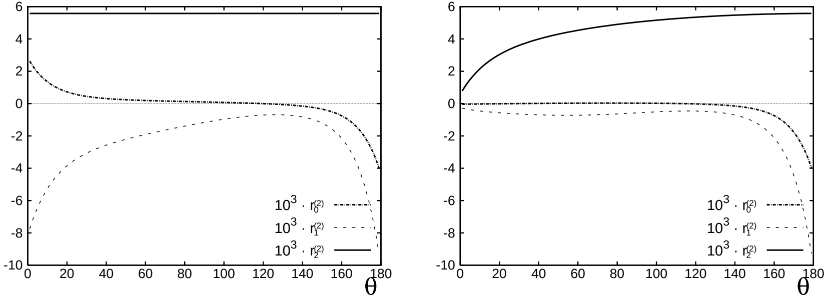

Figure: Soft and virtual second order photonic radiative corrections *versus* the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$ on the left side and $M = \sqrt{-t}$ on the right side.

Particular NLO contributions: H×H, H×(S+V)

Phase space of two hard photon emission (HH) is splitted into three regions:

1. **non-collinear:** $\theta_{1,2} > \vartheta_0$
   suited for Monte Carlo simulations
2. **semi-collinear:** $\theta_1 > \vartheta_0$ and $\theta_2 < \vartheta_0$
   in $\mathcal{O}\left(\alpha^2 L\right)$ has factorized form $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \otimes R_{\mathrm{H}}^{\mathrm{ISR,FSR}}(z)$
3. **collinear:** $\theta_{1,2} < \vartheta_0$ is described by the HH radiation factor convoluted with the Born

Emission of one hard photon in $\mathcal{O}\left(\alpha^2 L\right)$ can be sliced into two domains:

1. **non-collinear:** $\theta_\gamma > \vartheta_0$ as a product of two factors
   $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \times \delta_{\mathrm{Soft+Virt}}^{\mathrm{LO}}$
2. **collinear:** $\theta_\gamma < \vartheta_0$ is described by the collinear NLO H radiation factor (see below)

QED Collinear Radiation Factors in NLO (1)

A.A., E. Scherbakova, Phys. Lett. B 660 (2008) 37

$$\mathrm{d}\sigma[a(p_1)+b(p_2)\rightarrow c(q_1)+d(q_2)+\gamma(k\sim(1-z)p_1)]$$
$$=\mathrm{d}\hat{\sigma}[a(zp_1)+b(p_2)\rightarrow c(q_1)+d(q_2)]\otimes R_{\mathrm{H}}^{\mathrm{ISR}}(z)$$

Emission of collinear photons in FSR and ISR with conditions

$$\vartheta_\gamma<\vartheta_0,\qquad \frac{m}{E}\ll\vartheta_0\ll 1,\qquad l_0=\ln\frac{\vartheta_0^2}{4},\qquad \frac{E_\gamma}{E}>\Delta\ll 1$$

In $\mathcal{O}(\alpha)$ the result is well known:

$$R_{\mathrm{H}}^{\mathrm{ISR}}(z)=\frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{E^2}{m^2}-1+l_0\right)+1-z+\mathcal{O}\left(\frac{m^2}{E^2}\right)+\mathcal{O}\left(\vartheta_0^2\right)\right]$$

QED Collinear Radiation Factors in NLO (2)

Emission of two collinear photons (HH) in the same direction is described by a one-fold integral of results from [A.A. et al., Nucl. Phys. B 483 (1997) 83]:

$$R_{\mathrm{HH}}^{\mathrm{ISR}}(z) = \left(\frac{\alpha}{2\pi}\right)^2 L\biggl\{(L+2l_0)\biggl(\frac{1+z^2}{1-z}(2\ln(1-z)-2\ln\Delta-\ln z) + \frac{1+z}{2}\ln z - 1 + z\biggr) + \frac{1+z^2}{1-z}\Bigl(\ln^2 z + 2\ln z - 4\ln(1-z) + 4\ln\Delta\Bigr) + (1-z)\Bigl(2\ln(1-z) - 2\ln\Delta - \ln z + 3\Bigr) + \frac{1+z}{2}\ln^2 z\biggr\}$$

FSR factor is restored with help of the Gribov-Lipatov relation generalized for the collinear emission case:

$$R_{\mathrm{HH}}^{\mathrm{FSR}}(z) = -zR_{\mathrm{HH}}^{\mathrm{ISR}}\left(\frac{1}{z}\right)\Bigg|_{\ln\Delta\to\ln\Delta-\ln z;\ l_0\to l_0+2\ln z}$$

QED Collinear Radiation Factors in NLO (3)

Emission of one collinear hard photon accompanied by one-loop soft and virtual correction (H(S+V)) is received using the NLO QED splitting functions

$$R^{\mathrm{ISR}}_{\mathrm{H(S+V)}}(z) \otimes \mathrm{d}\hat{\sigma}(z) = \delta^{(1)}_{\mathrm{(S+V)}} R^{\mathrm{ISR}}_{\mathrm{H}}(z) \otimes \mathrm{d}\sigma^{(0)}(z)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 L \left[2\frac{1+z^2}{1-z}\left(\mathrm{Li}_2\,(1-z) - \ln(1-z)\ln z\right)\right.$$
$$\left. -(1+z)\ln^2 z + (1-z)\ln z + z \right] \otimes \mathrm{d}\sigma^{(0)}(z),$$

$$\delta^{(1)}_{\mathrm{(S+V)}} = \frac{\mathrm{d}\sigma^{(1)}_{\mathrm{Soft}} + \mathrm{d}\sigma^{(1)}_{\mathrm{Virt}}}{\mathrm{d}\sigma^{(0)}}\,,$$

where $\sigma^{(0)}(z)$ is the boosted Born cross section, and $\delta^{(1)}_{\mathrm{(S+V)}}$ is the relative $\mathcal{O}\left(\alpha\right)$ Soft + Virtual radiative correction with $E_\gamma^{\mathrm{Soft}} < \Delta E$. The corresponding FSR factor is received again using the Gribov-Lipatov relation.

Pair Corrections (I)

Leptonic and hadronic pair corrections are important for a number of precision observables. Exclusive treatment here is of ultimate importance. Monte Carlo has to be used for real or for hard pairs, then soft and virtual ones can be treated analytically (semi-analytically for the hadronic case).

Singlet and non-singlet NLO pair contributions in $\mathcal{O}\left(\alpha^2 L\right)$ to inclusive observables can be described within the QCD-like factorization approach.

But if we have a MC for hard pairs, we can extract analytically the soft+virtual part, so that

$$\mathrm{d}\sigma^{(2)}_{\mathrm{pair}} = \mathrm{d}\sigma^{(2)MC}_{\mathrm{H\ pair}} \ + \ \mathrm{d}\sigma^{(0)} \times \delta^{(2)}_{\mathrm{S+V\ \ pair}}$$

Pair Corrections (II)

For e^+e^- pairs the Soft+Virt part is provided by the Δ-part of non-singlet pair contribution to electron structure function

$$\delta^{(2)}_{\mathrm{S+V\ \ pair}} = 4\left(\frac{\alpha}{2\pi}\right)^2 \left\{ \frac{1}{3} L^2 \left(2\ln\Delta + \frac{3}{2} \right) + L\left(-\frac{20}{9}\ln\Delta - \frac{4\pi^2}{18} - \frac{1}{6} \right) \right\}$$

Note cancellation of $\mathcal{O}\left(\alpha^2 L^3\right)$ terms in the sum of virtual and soft pair contributions

Heavier pairs: separate virtual and real emission (for energies of a few GeV) because even soft muons and pions can be detected (*I guess*)

Code structure

According to dimension of integration:

- **1d** Born + Soft + Virt in $\mathcal{O}\left(\alpha^0, \alpha^1 L, \alpha^1, \alpha^2 L, \alpha^3 L^3\right)$
- **2d** 1 collinear hard photon in $\mathcal{O}\left(\alpha^1 L, \alpha^1, \alpha^2 L^2, \alpha^2 L, \alpha^3 L^3\right)$: 4 cases
- **3d** 2 collinear hard photons in different directions $\mathcal{O}\left(\alpha^2 L^2, \alpha^2 L, \alpha^3 L^3\right)$: 6 cases
- **4d(a)** 3 collinear hard photons in different directions $\mathcal{O}\left(\alpha^3 L^3\right)$: 4 cases
- **4d(b)** 1 non-collinear hard photon $\mathcal{O}\left(\alpha^1, \alpha^2 L\right)$
- **5d** 2 hard photons: collinear and non-collinear $\mathcal{O}\left(\alpha^2 L\right)$ new for MCJPG

Residual dependence on Δ and ϑ_0 is at the level 0.02% mainly from $\mathcal{O}\left(\alpha^1\right)$.

Approximations

- In $\mathcal{O}\left(\alpha^1\right)$ radiation factor terms of the order $\mathcal{O}\left(m^2/E^2\right)$ and $\mathcal{O}\left(\theta_0^2\right)$ can be restored if required
- Factorization scale dependence is considerable for LO, and rather small for NLO QED
- The collinear cone can be transformed into any other form
- $\mathcal{O}\left(\alpha^2 L^0\right)$ terms in particular kinematical domains are in general process-dependent and can be added
- Negatively weighted events within this approach are possible but not numerous

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- $\mathcal{O}\left(\alpha^2 L^0\right)$ contributions can be put into the same structure
- MCGPJ can be upgraded
- MC integrator for Bhabha scattering is ready for tuned comparisons