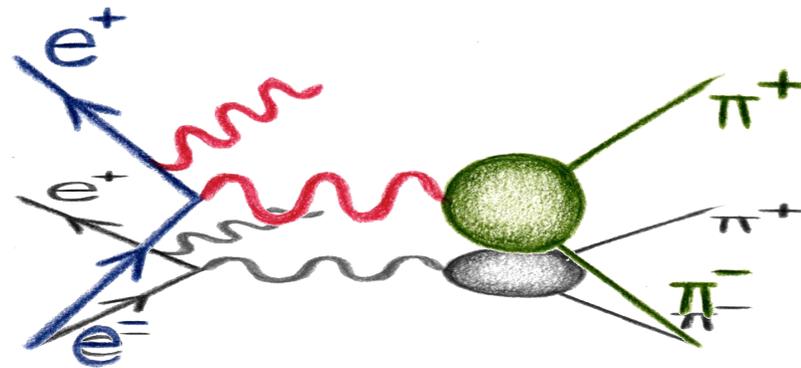


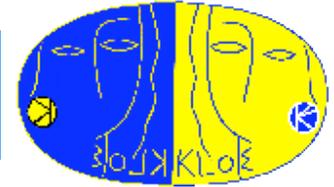
# Monte Carlo generators and radiative corrections used and needed in the KLOE ISR analyses



S. Müller  
(for the KLOE  $\pi\pi\gamma$ -group)

*Radio MONTE CARlow WG Meeting in Beijing, October 2008*

# Abs. Norm. vs ratio $\pi\pi\gamma/\mu\mu\gamma$



Measure the (ISR) - radiative cross section  $d\sigma_{\pi\pi\gamma(\gamma)}/dM_{\pi\pi}^2$ : using as normalization the integrated luminosity  $\int Ldt$  obtained from Bhabha events:

$$\frac{d\sigma_{\pi\pi\gamma(\gamma)}^{obs}}{dM_{\pi\pi}^2} = \frac{\Delta N_{Obs} - \Delta N_{Bkg}}{\Delta M_{\pi\pi}^2} \cdot \frac{1}{\epsilon_{Sel}} \cdot \frac{1}{\int Ldt}$$

Then extract  $\sigma_{\pi\pi}$  from  $d\sigma_{\pi\pi\gamma(\gamma)}/dM^2$  via theoretical radiator function  $H(s, M_{\pi\pi}^2)$ :

$$\sigma_{\pi\pi}(M_{\pi\pi}^2) \approx s \frac{d\sigma_{\pi\pi\gamma(\gamma)}(M_{\pi\pi}^2)}{dM_{\pi\pi}^2} \cdot \frac{1}{H(s, M_{\pi\pi}^2)}$$

**OR**

Obtain  $\sigma_{\pi\pi}$  from the ratio of  $d\sigma_{\pi\pi\gamma(\gamma)}/dM_{\pi\pi}^2$  /  $d\sigma_{\mu\mu\gamma(\gamma)}/dM_{\mu\mu}^2$  :

$$\sigma_{\pi\pi}^{Born}(M_{\pi\pi}^2) \approx \frac{d\sigma_{\pi\pi\gamma}^{obs}/dM_{\pi\pi}^2}{d\sigma_{\mu\mu\gamma}^{obs}/dM_{\mu\mu}^2} \sigma_{\mu\mu}^{Born}(M_{\mu\mu}^2)$$

# Small angle event selection



**2 pion tracks at large angles**

$$50^\circ < \theta_\pi < 130^\circ$$

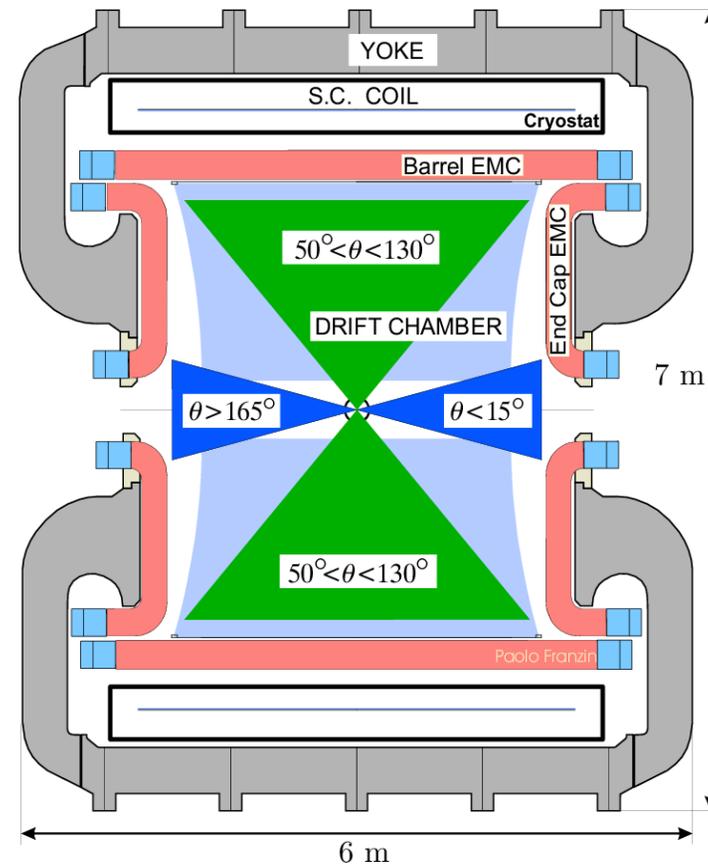
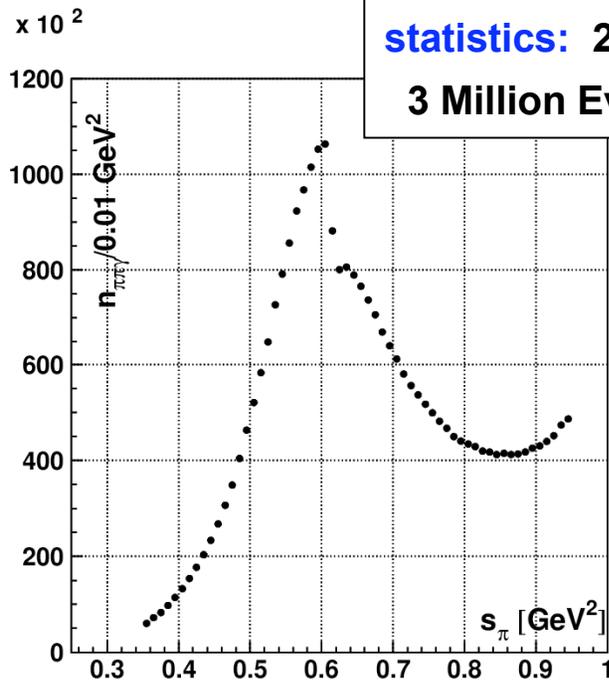
**Photons at small angles**

$$\theta_\gamma < 15^\circ \text{ or } \theta_\gamma > 165^\circ$$

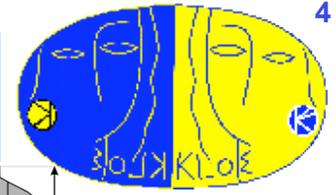
- ✓ high statistics for ISR events
- ✓ low relative FSR contribution
- ✓ suppression of  $\phi \rightarrow \pi^+\pi^-\pi^0$  background
- ⊖ Region below  $0.35 \text{ GeV}^2$  kinematically suppressed

→ photon momentum from kinematics:

$$\vec{p}_\gamma = \vec{p}_{\text{miss}} = -(\vec{p}_+ + \vec{p}_-)$$



# Large angle event selection

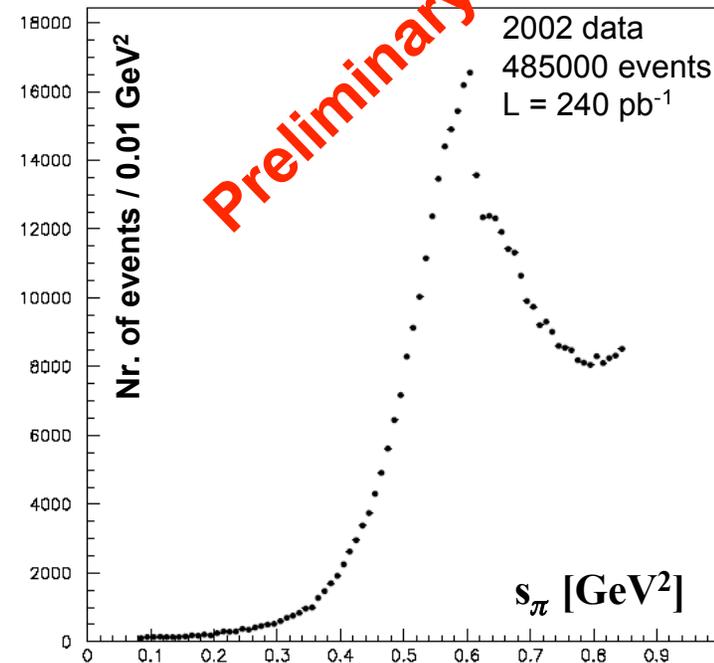
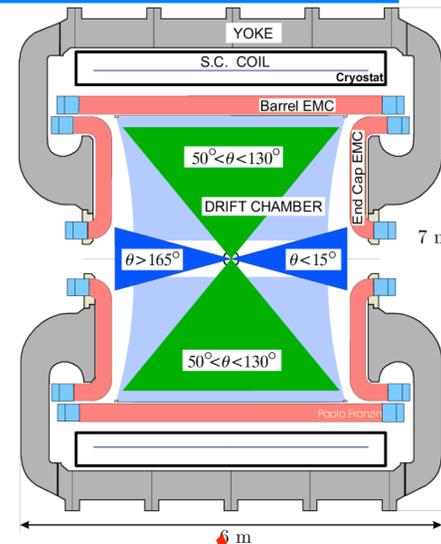


**2 pion tracks at large angles**

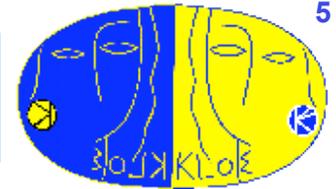
$$50^\circ < \theta_\pi < 130^\circ$$

**At least 1 photon with  $50^\circ < \theta_\gamma < 130^\circ$   
and  $E_\gamma > 50 \text{ MeV} \rightarrow$  photon detected**

- ✓ independent complementary analysis
- ✓ threshold region  $(2m_\pi)^2$  accessible
- ✓  $\gamma_{\text{ISR}}$  photon detected  
(4-momentum constraints)
- ✓ lower signal statistics
- ✓ larger contribution from FSR events
- ✓ large  $\phi \rightarrow \pi^+\pi^-\pi^0$  background contamination
- ✓ irreducible background from  $\phi$  decays ( $\phi \rightarrow f_0 \gamma \rightarrow \pi\pi \gamma$ )



# KLOE analyses summary

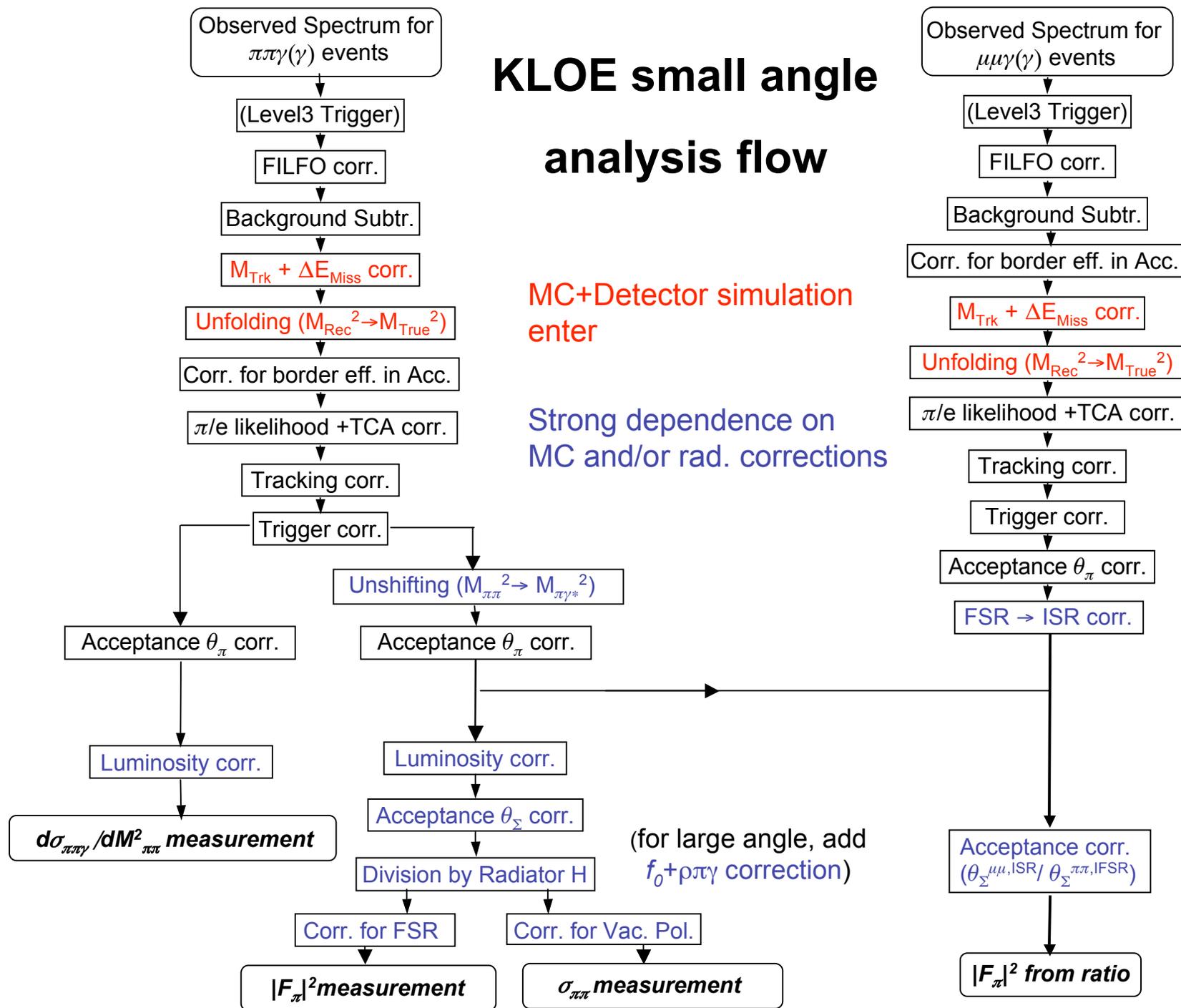


- **Small angle photon, absolute normalization with Bhabha events** ✓  
using  $140\text{pb}^{-1}$  of 2001 data      Phys. Lett. B 606 (2005) 12
- **Small angle photon, absolute normalization with Bhabha events** ✓  
using  $240\text{pb}^{-1}$  of 2002 data      arXiv: hep-ex/0809.3950    subm. to Phys. Lett. B
- Large angle photon, absolute normalization with Bhabha events  
using  $240\text{pb}^{-1}$  of 2002 data
- Large angle photon, absolute normalization with Bhabha events  
using  $200\text{pb}^{-1}$  of 2006 data (outside  $\phi$ -resonance)
- Small angle photon, normalization with  $\mu\mu\gamma$  events from data  
using  $240\text{pb}^{-1}$  of 2002 data
- *Large angle photon, normalization with  $\mu\mu\gamma$  events from data  
using  $240\text{pb}^{-1}$  of 2006 data (outside  $\phi$ -resonance)*
- *$|F_\pi|^2$  from  $\pi/\mu$  using coll. events for 4 points around  $M_\phi$*

In progress

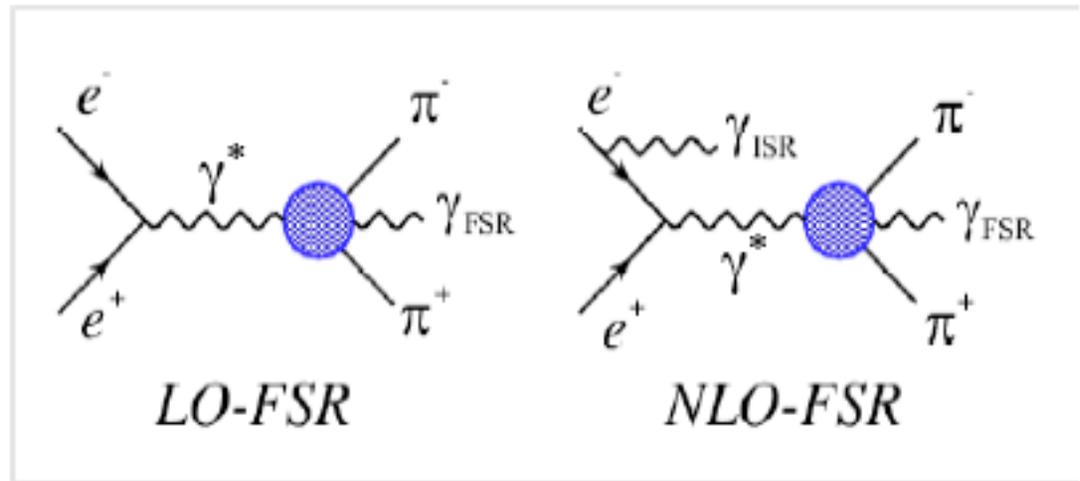
Future

# KLOE small angle analysis flow



# Final State radiation:

The cross section for  $e^+e^- \rightarrow \pi^+\pi^-$  has to be inclusive with respect to final state radiation events in order to evaluate  $a_\mu$ . We consider two kinds of FSR contributions:



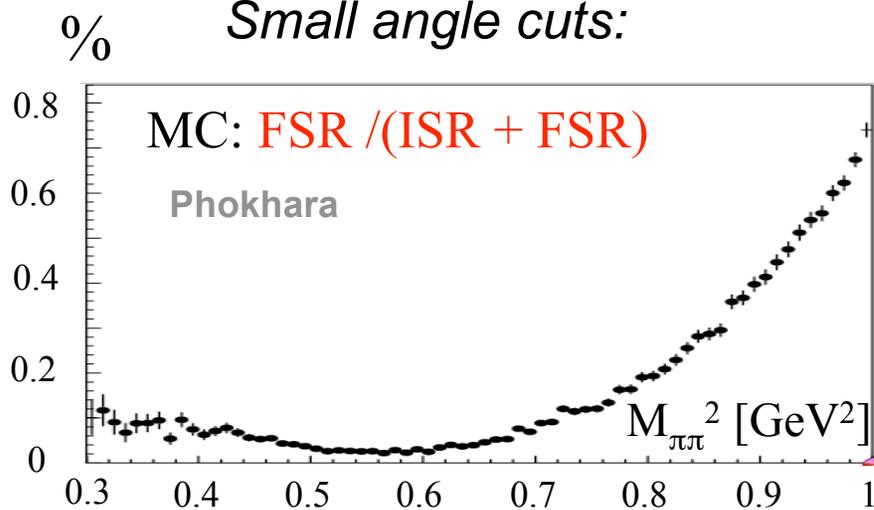
**LO-FSR:** No initial state radiation,  $e^+$  and  $e^-$  collide at the energy  $M_\phi = 1.02 \text{ GeV}$

This is clearly the dominant part, and it enters our spectrum since

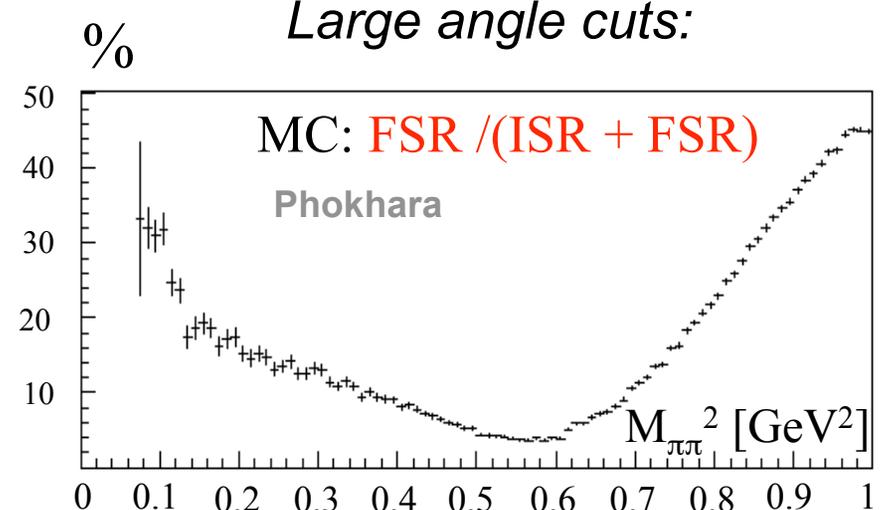
$$M_{\pi\pi}^2 < M_{\gamma^*}^2$$

**NLO-FSR:** Simultaneous presence of one photon from initial state radiation and one photon from final state radiation

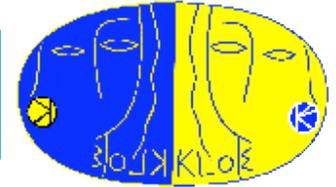
*Small angle cuts:*



*Large angle cuts:*

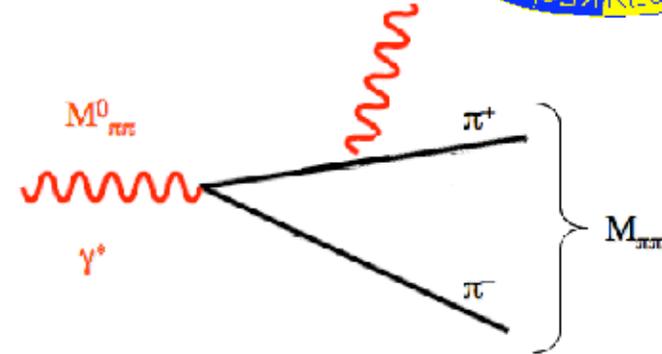


# „Unshifting“: $M^2_{\pi\pi} \rightarrow M^2_{\gamma^*}$



Photon emissions from the pions changes the measured value of  $M^2_{\pi\pi}$  from the invariant mass squared of the virtual photon produced in the  $e^+e^-$  collision,  $M^2_{\gamma^*}$

$$M^2_{\pi\pi} \leq M^2_{\gamma^*}$$



Use special version of PHOKHARA which allows to determine whether photon comes from initial or final state → build matrix which relates  $M^2_{\pi\pi}$  to  $M^2_{\gamma^*}$ .

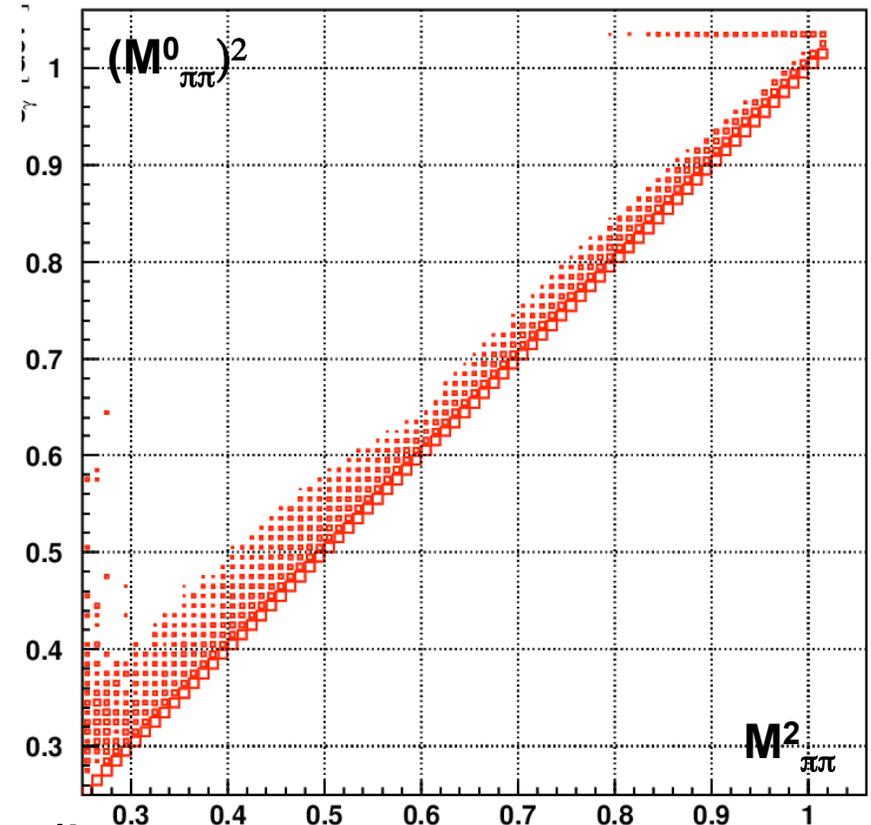
ISR only:

$$M^2_{\gamma^*} = M^2_{\pi\pi}$$

FSR photon present:

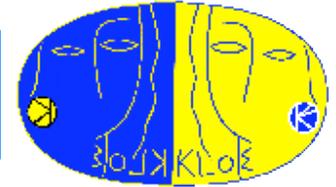
$$M^2_{\gamma^*} = M^2_{\pi\pi\gamma(\text{FSR})}$$

$e^+e^- \rightarrow \pi^+\pi^-\gamma_{\text{FSR}}$  events (“*no FSR*”) are “*unshifted*” to  $M^2_{\gamma^*} = 1.04 \text{ GeV}^2$

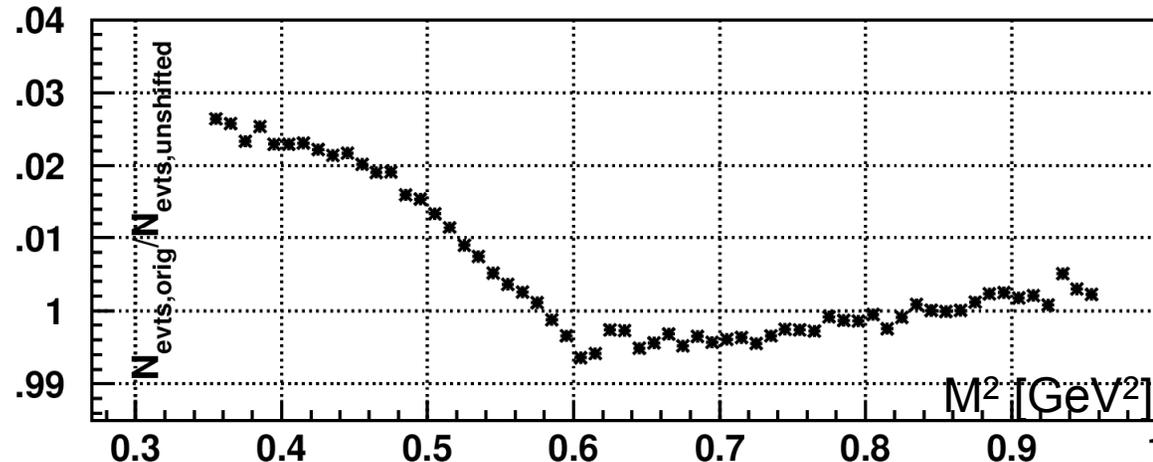


Would be nice to have also for  $\mu\mu\gamma$  channel!

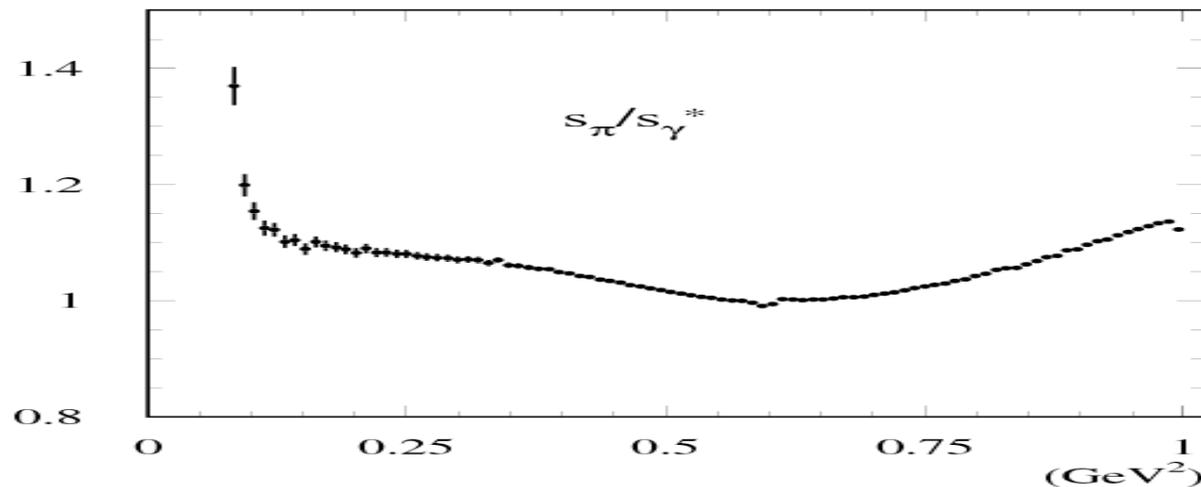
# „Unshifting“: $M^2_{\pi\pi} \rightarrow M^2_{\gamma^*}$



Relative increase of events with 1  $\gamma_{\text{ISR}}$  and 1  $\gamma_{\text{FSR}}$  over pure ISR events at low values of  $M^2_{\pi\pi}$  increases the effect in this region for small angle analysis.



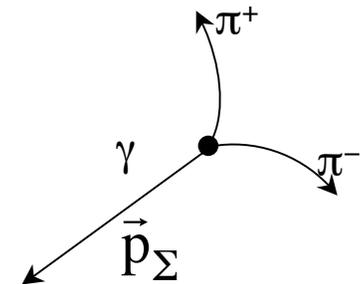
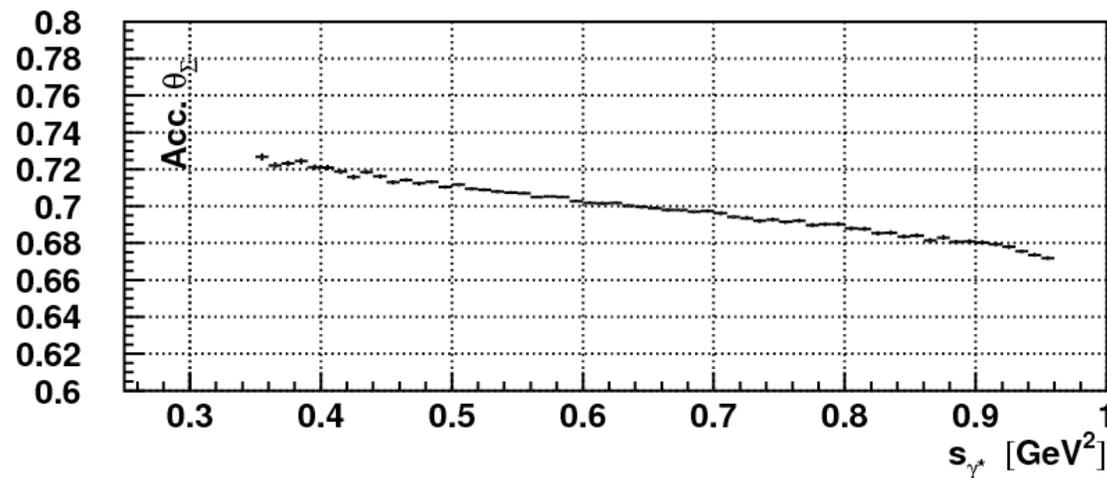
Effect is higher for large angle analysis due to the greater presence of FSR, especially at very high and very low  $M^2_{\pi\pi}$



# $\theta_\Sigma$ correction (Acceptance)

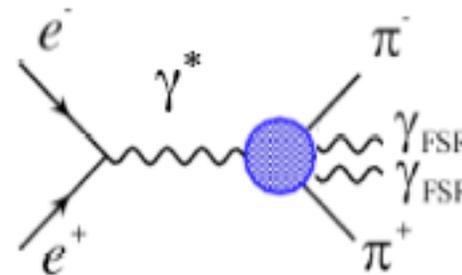


$\theta_\Sigma$  is the angle of the photon (system) obtained from the momenta of the two charged tracks in the small angle analysis:  $\vec{p}_\Sigma = \vec{p}_{\text{miss}} = -(\vec{p}_+ + \vec{p}_-)$

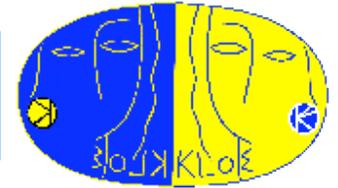


This acceptance depends on FSR (as does the acceptance for  $\theta_\gamma$  in the large angle analysis)  $\Rightarrow$  strong dependence on the implementation of FSR in PHOKHARA5/6/6.1...

Effect of second hard photon from FSR???



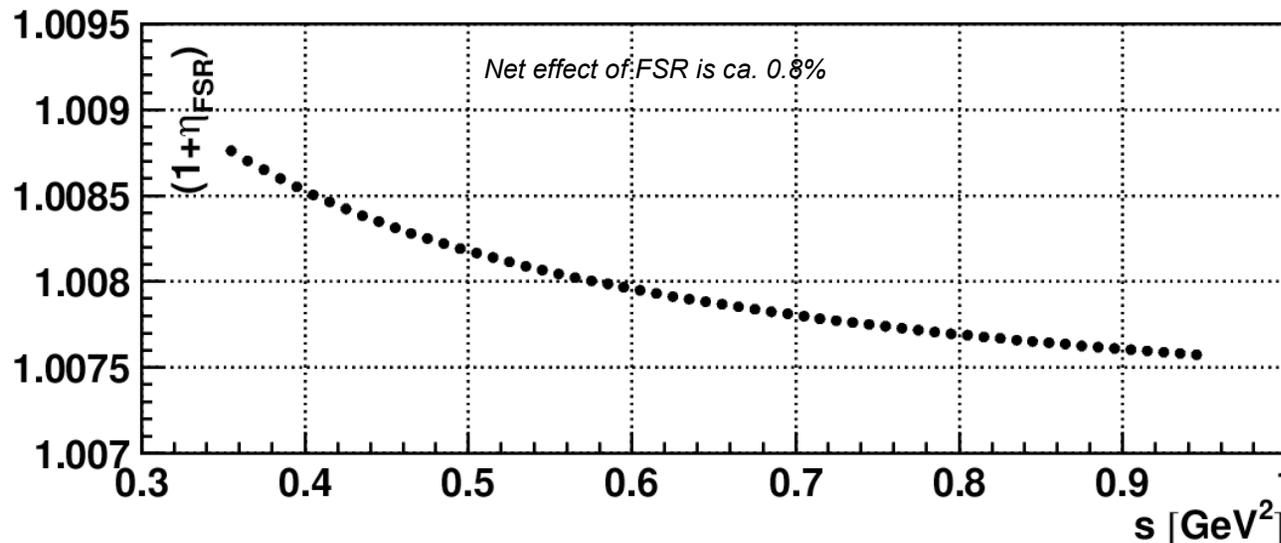
# Final State Radiation (FSR)



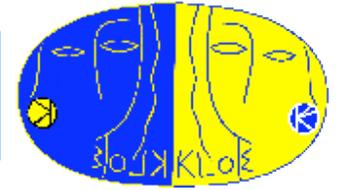
$\sigma_{\pi\pi}$  needs to be inclusive with respect to final state radiation when used in the dispersive integral. Therefore the analysis has been designed to provide a final spectrum which is inclusive in  $\text{FSR}@(\text{M}^2_{\gamma^*})$ .



Concerning the  $|F_\pi|^2$ , we undress the spectrum from FSR by dividing for  $(1+\eta_{\text{FSR}})$ , which is calculated assuming radiation from pointlike pions (sQED)



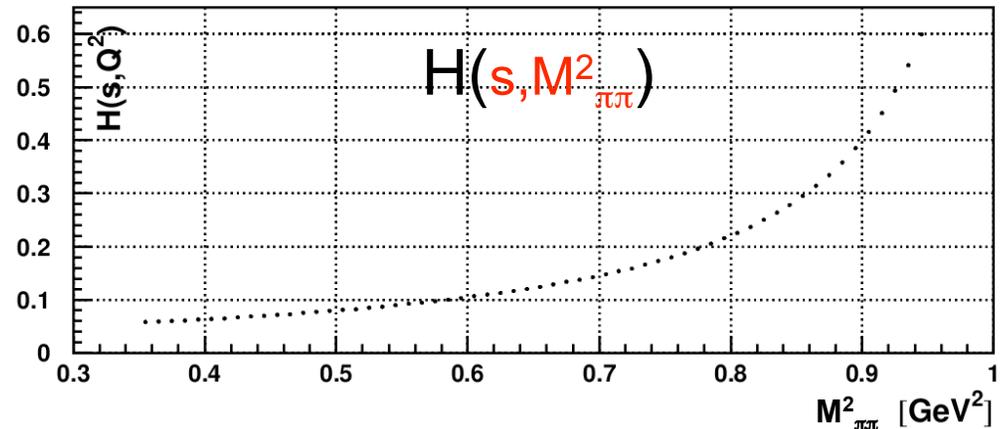
# Radiator function



- ISR-Process calculated at NLO-level  
*PHOKHARA* generator (Czyż, Kühn *et.al*)  
**Theoretical Precision: 0.5%**

**Biggest theor. uncertainty in SA analysis**

$$s \cdot \frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \sigma_{\pi\pi}(s) \times H(s, M_{\pi\pi}^2)$$



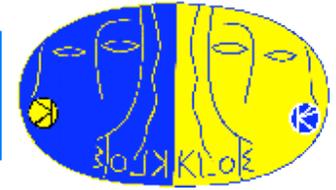
$s$  is the collider energy.

We obtain the radiator function technically by setting  $|F_{\pi}|^2=1$  in the PHOKHARA Monte Carlo generator, and generate ISR events inclusive in  $\theta_{\pi}$  and  $\theta_{\Sigma}$ :

$$H(s, M_{\pi\pi}^2) = s \times \frac{3M_{\pi\pi}^2}{\pi\alpha^2\beta_{\pi}^3} \times \frac{d\sigma_{\pi\pi\gamma}(M_{\pi\pi}^2)}{dM_{\pi\pi}^2} \Bigg|_{|F_{\pi}(M_{\pi\pi}^2)|^2=1}^{\text{MC}}$$

How well does the factorization of H work in the presence of FSR?

# Luminosity:



KLOE measures  $L$  with Bhabha scattering at large angles

F. Ambrosino et al. (KLOE Coll.)  
**Eur.Phys.J.C47:589-596,2006**

$55^\circ < \theta < 125^\circ$   
 acollinearity  $< 9^\circ$   
 $p \geq 400$  MeV

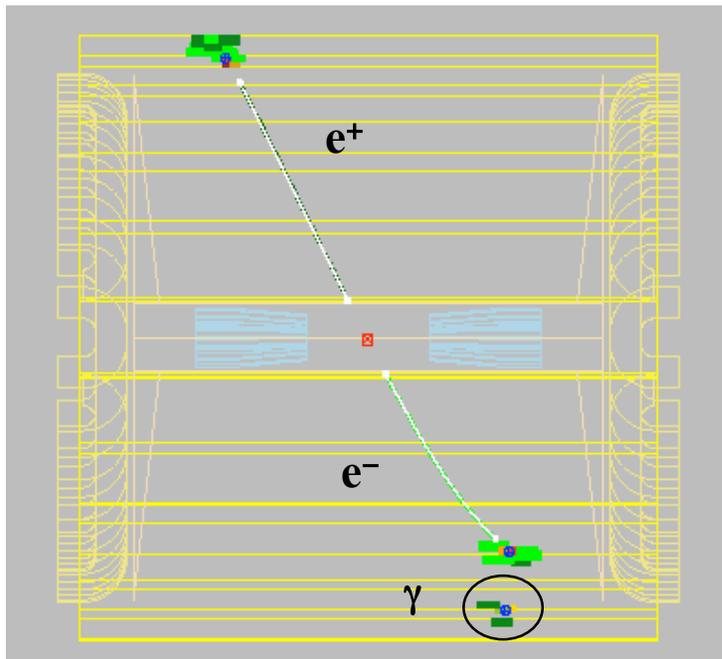
$$\int \mathcal{L} dt = \frac{N_{obs} - N_{bkg}}{\sigma_{eff}}$$

generator used for  $\sigma_{eff}$

**BABAYAGA (Pavia group):**

*C. M.C. Calame et al., NPB584 (2000) 459*

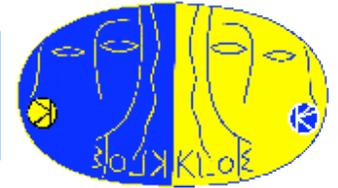
**New:** *C. M.C. Calame et al., NPB758 (2006) 22*



new version (**BABAYAGA@NLO**) gives  
 0.7% decrease in cross section,  
 and better accuracy: 0.1%

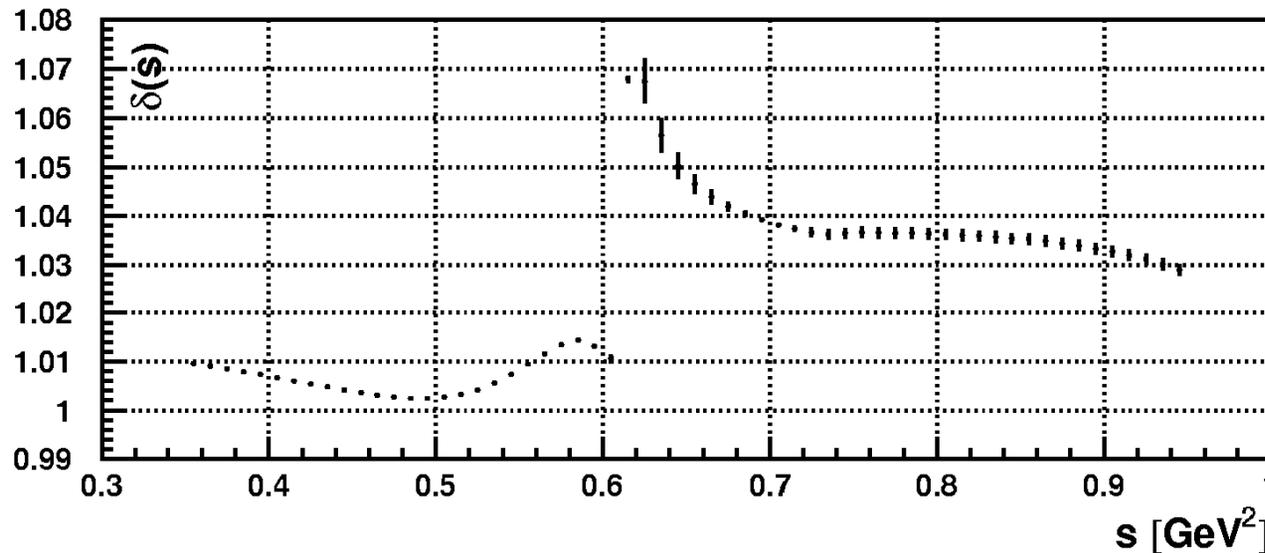
Systematics on Luminosity	
Theory	0.1 %
Experiment	0.3 %
TOTAL 0.1 % th $\oplus$ 0.3% exp = 0.3%	

# Vacuum Polarisation



For use in the dispersive integral for  $\Delta^{\pi\pi}a_\mu$ , one needs to subtract effects from vacuum polarization (VP) to obtain a *bare* cross section  $\sigma_{\pi\pi}^0$ :

$$\sigma_{\pi\pi}^0(s) = \sigma_{\pi\pi}^{\text{dressed}}(s) \left( \frac{\alpha(0)}{\alpha(s)} \right)^2 = \sigma_{\pi\pi}(s) / \delta(s)$$



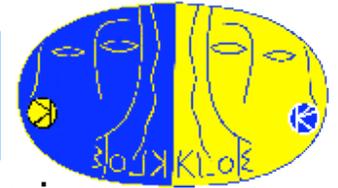
Points obtained from F. Jegerlehner's webpage  
(the only points which are publically available!)

Correction is applied only to the cross section  $\sigma_{\pi\pi}^0$  (not on  $\sigma_{\pi\pi\gamma}$  and  $|F_\pi|^2$ ).

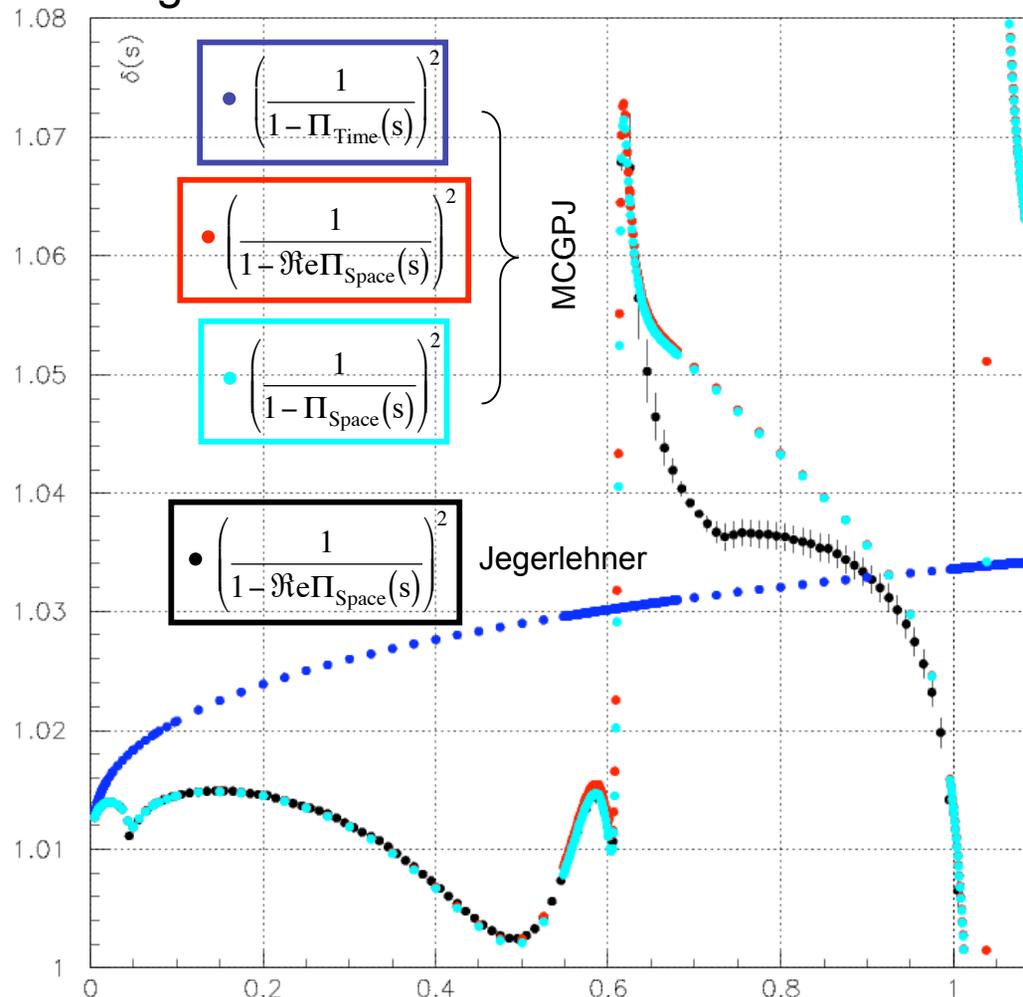
**NOT APPLIED** when comparing with  $|F_\pi|^2$  from CMD2/SND!

Error on VP points introduces an relative error on the value of  $\Delta^{\pi\pi}a_\mu$  of 0.1%.

# Vacuum Polarisation

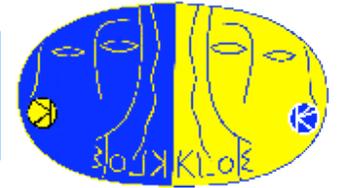


Confronting the VP points from F. Jegerlehner's webpage with the one in MCGPJ shows some disagreement between 0.6 and 1 GeV<sup>2</sup>:



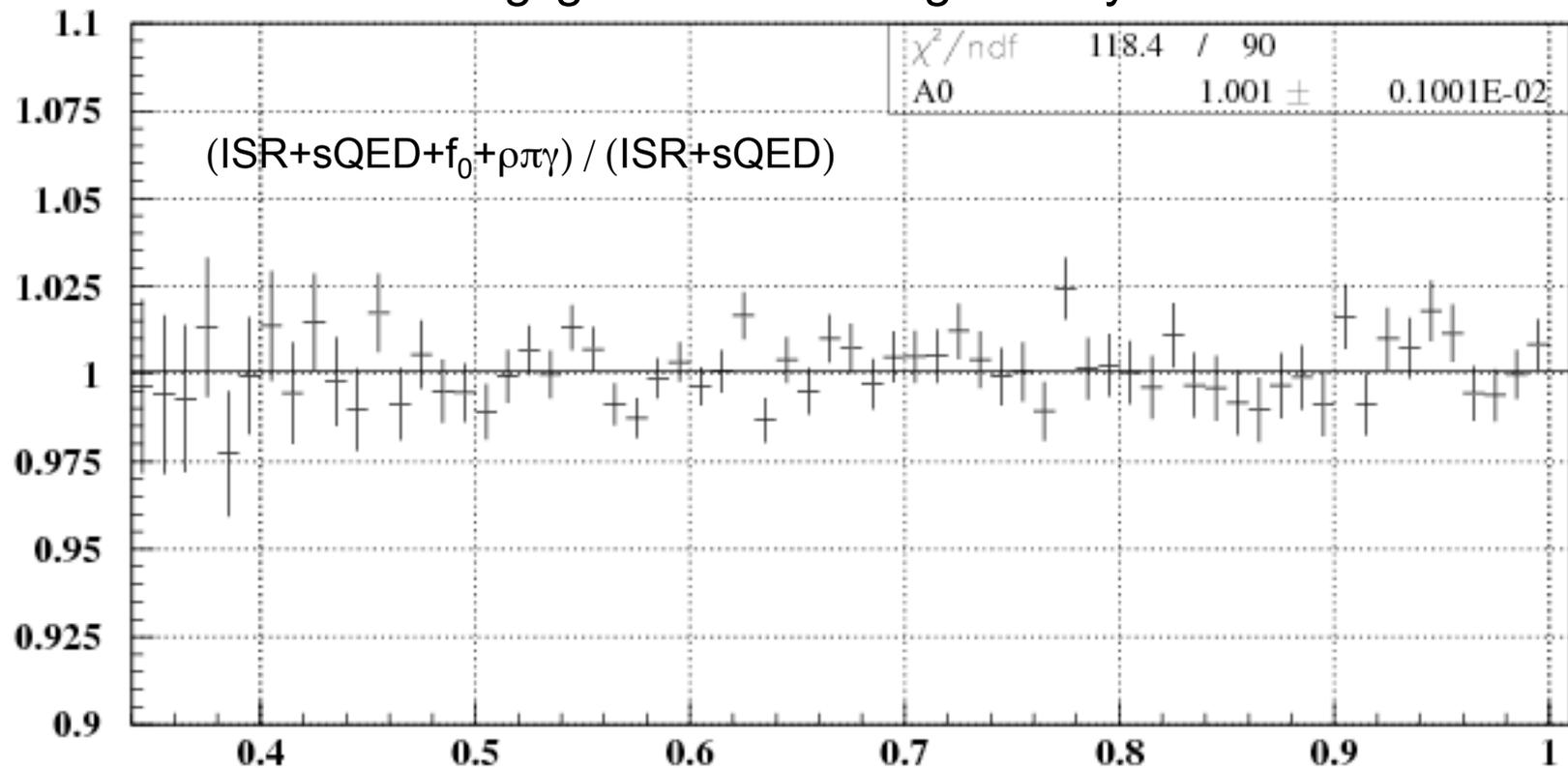
Fred is currently updating his function, as it does not yet include the recent results from BaBar in that region.

# $f_0 + \rho\pi$ correction



$\phi \rightarrow (f_0 + \sigma)\gamma \rightarrow \pi\pi\gamma$ : Checked with PHOKHARA 6.1 generator, latest Achasov model  
 $\phi \rightarrow \pi\rho \rightarrow \pi(\pi\gamma)$ : with parameters from KLOE  $f_0 \rightarrow \pi^0\pi^0$  analysis

Contribution negligible in small angle analysis:



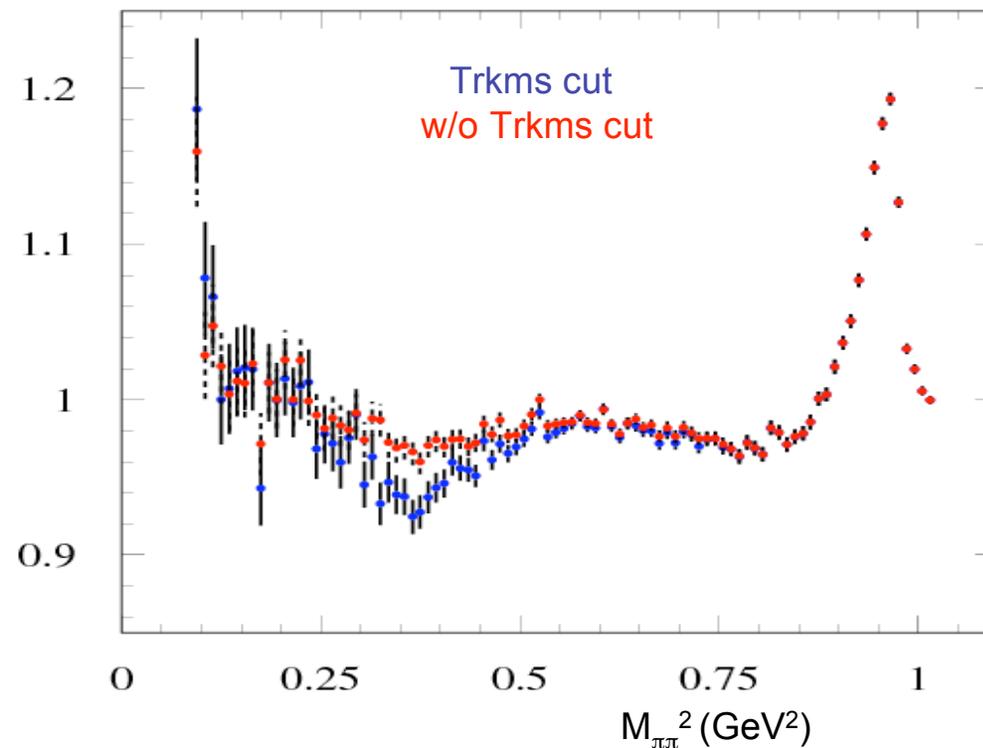
# $f_0 + \rho\pi$ correction



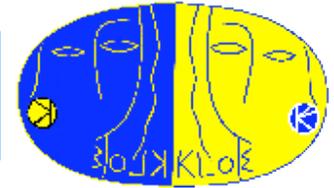
$\phi \rightarrow (f_0 + \sigma)\gamma \rightarrow \pi\pi\gamma$ : Checked with PHOKHARA 6.1 generator, latest Achasov model  
 $\phi \rightarrow \pi\rho \rightarrow \pi(\pi\gamma)$ : with parameters from KLOE  $f_0 \rightarrow \pi^0\pi^0$  analysis

Contribution relevant in large angle analysis:

$(\text{ISR} + \text{sQED} + f_0 + \rho\pi\gamma) / (\text{ISR} + \text{sQED})$



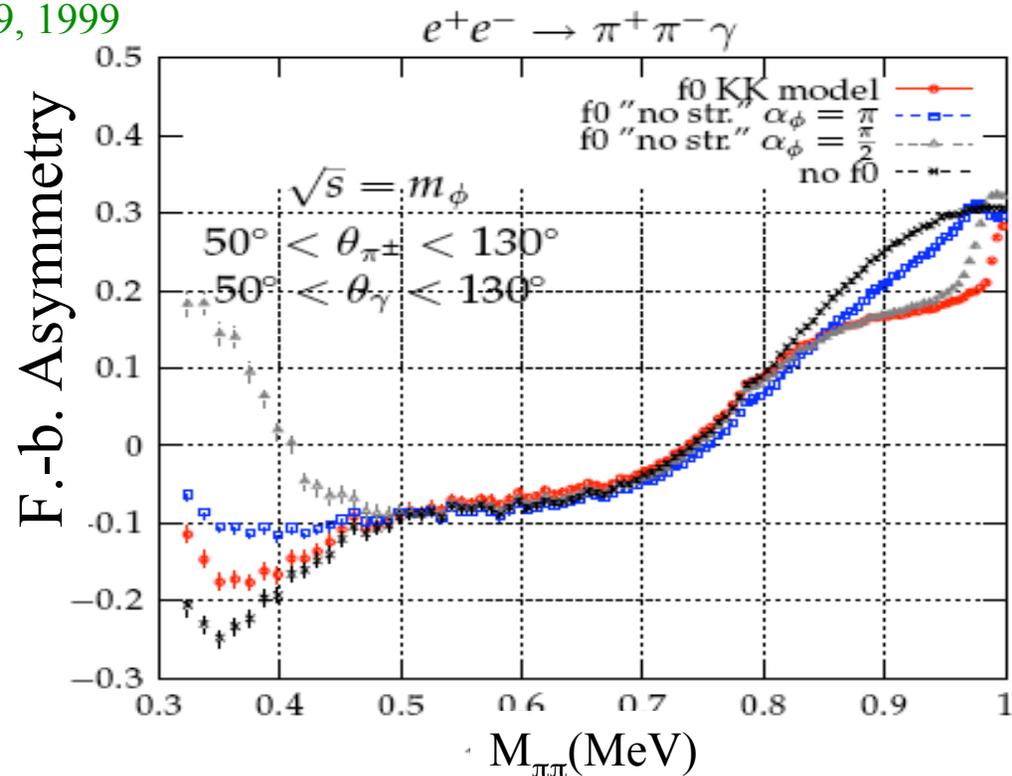
# $f_0 + \rho\pi$ correction



As the interference between ISR and (FSR+ $f_0$ ) leads to a non-vanishing forward-backward asymmetry, one can use this asymmetry to further optimize/scrutinize the model parameters

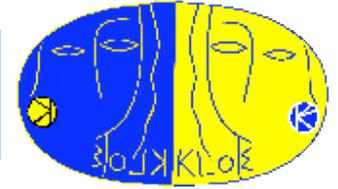
Binner, Kühn, Melnikov, Phys. Lett. B 459, 1999  
 Czyz, Grzelinska, Kühn, hep-ph/0412239

$$A = \frac{N(\theta^+ > 90^\circ) - N(\theta^+ < 90^\circ)}{N(\theta^+ > 90^\circ) + N(\theta^+ < 90^\circ)}$$



For this a (semi)analytical parametrization of the asymmetry as a function of the model parameters would be very welcome!!

# Conclusions



After the large improvement in the precision of the luminosity reference cross section with BABAYAGA@nlo, the biggest theoretical uncertainties for the small angle analyses come from the radiator function and the knowledge of FSR.

- Radiator function contributes 0.5% uncertainty

*Box graphs?*

*Factorization?*

- FSR enters in many places in the analysis (Unshifting,  $\theta_\Sigma, \dots$ )

*Second hard photon?*

*FSR for muons (PHOKHARA Omega)?*

The same holds for the large angle analysis, where in addition one needs to control the contribution from scalar mesons.

*(Semi)analytic description of asymmetry?*

Both large angle analyses using the absolute normalization with 2002 data („on-peak“) and 2006 data („off-peak“) are in a very advanced state.

*Manpower?*