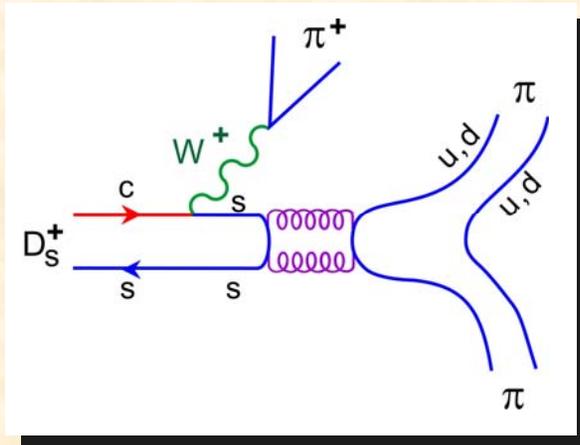
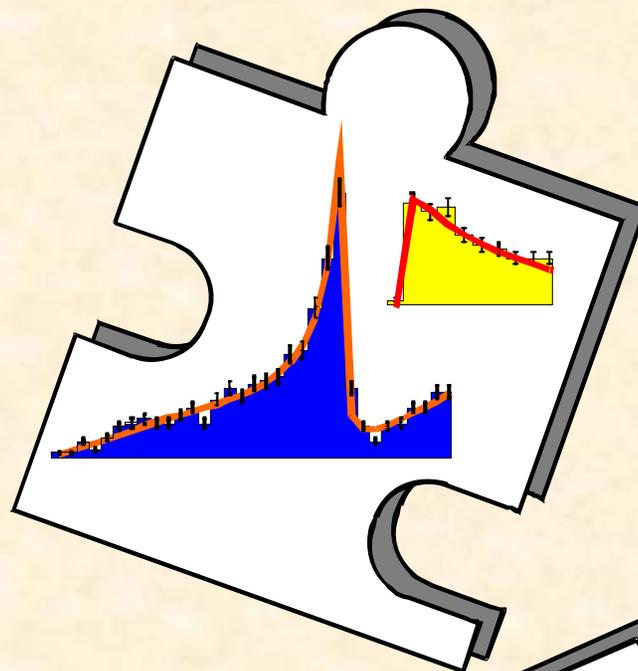
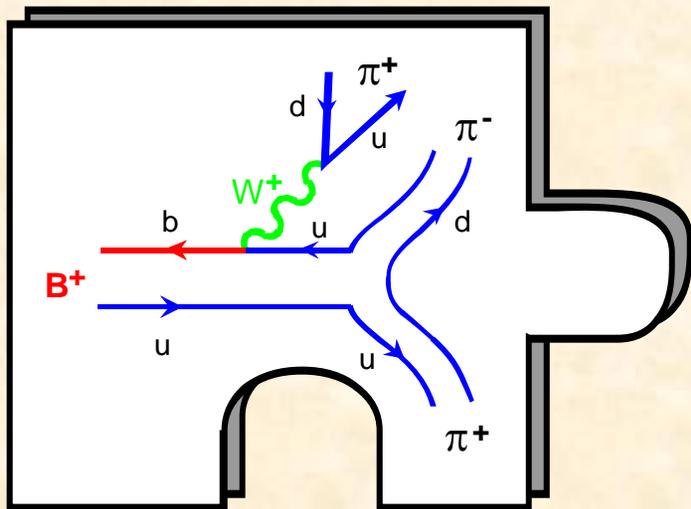


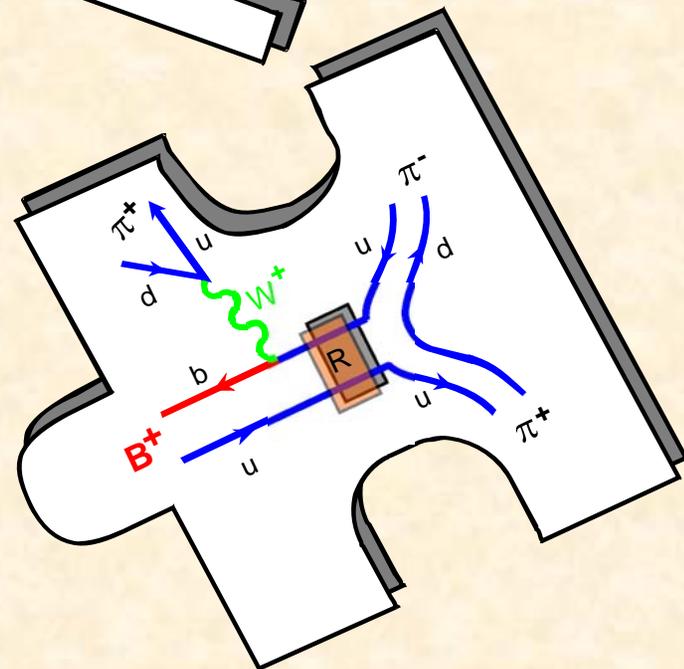
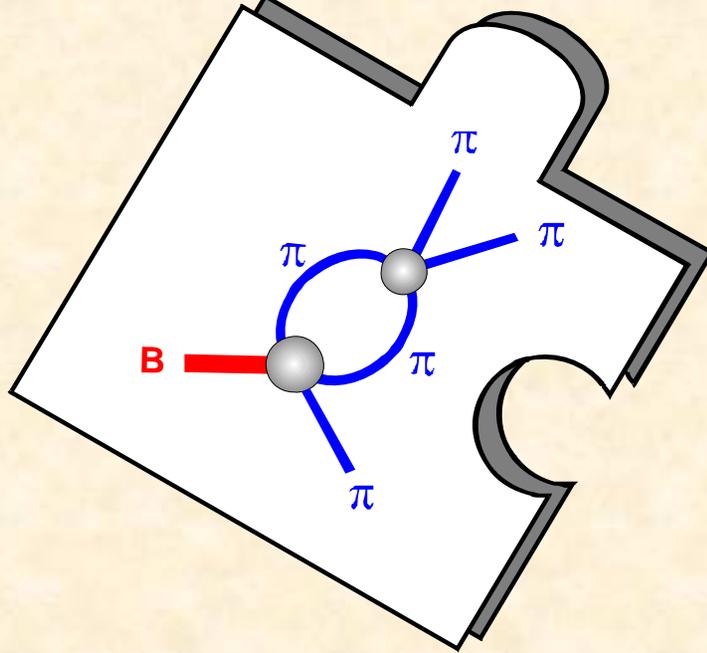
**What heavy flavour decays can teach us about light quark strong physics**



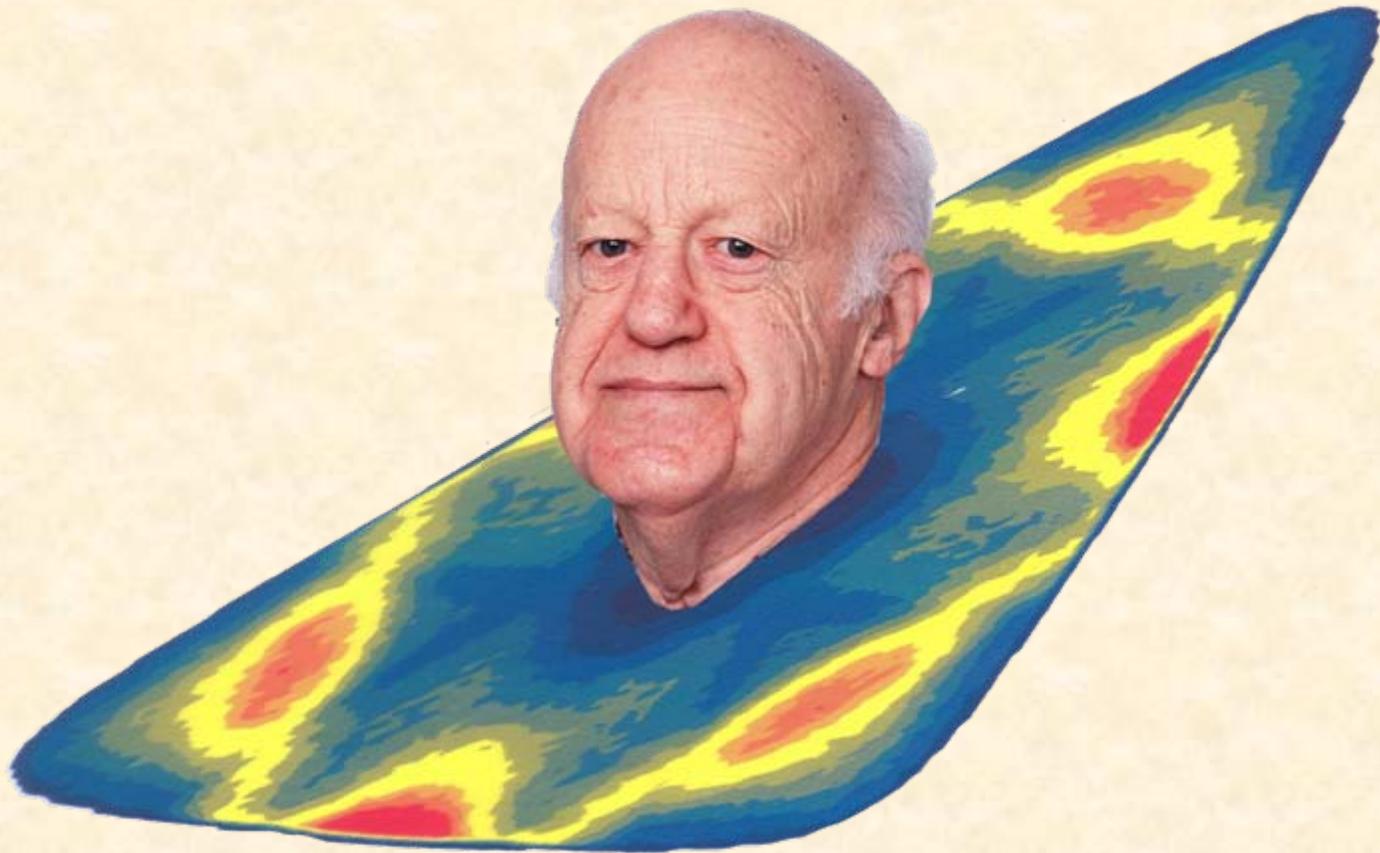
# Physics Puzzle

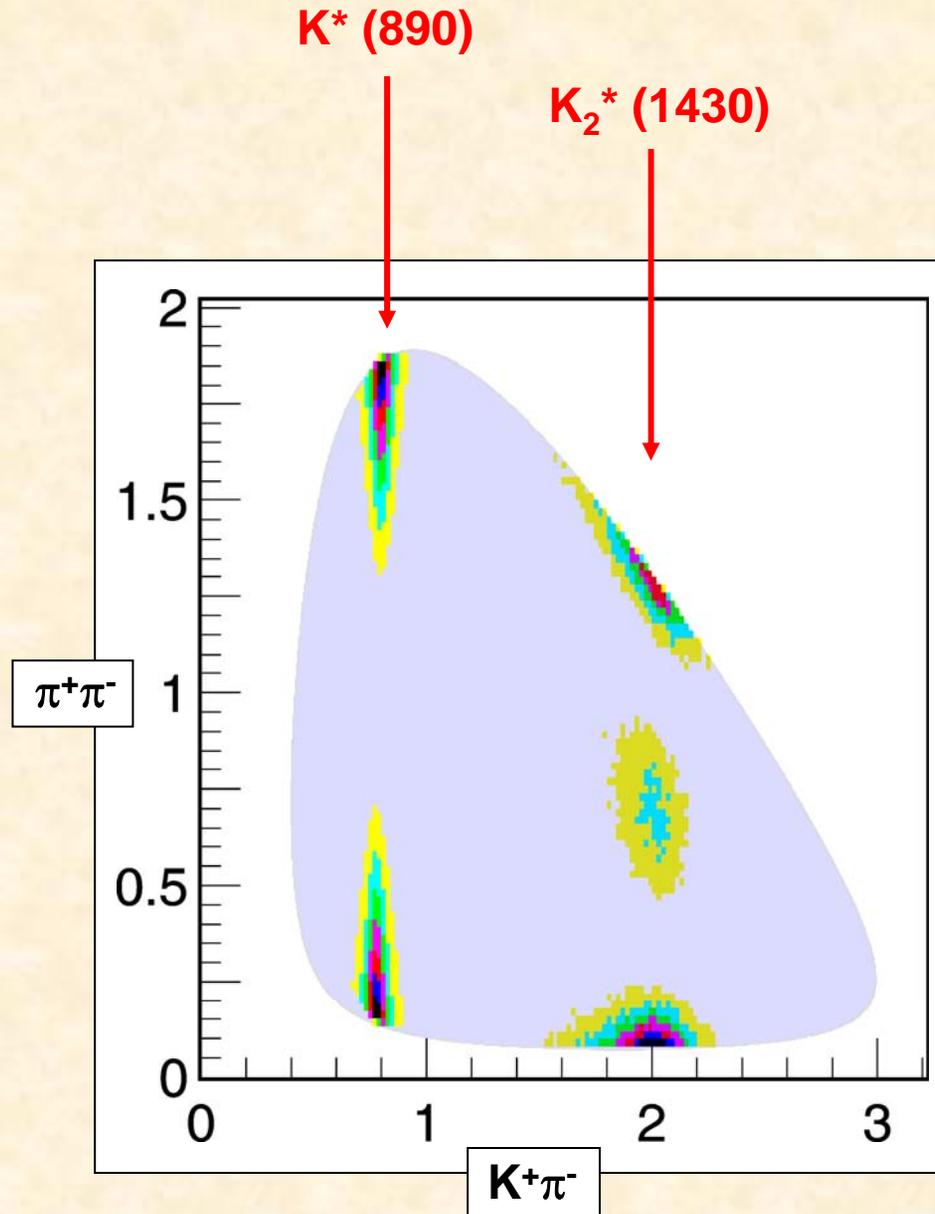


~~CP~~

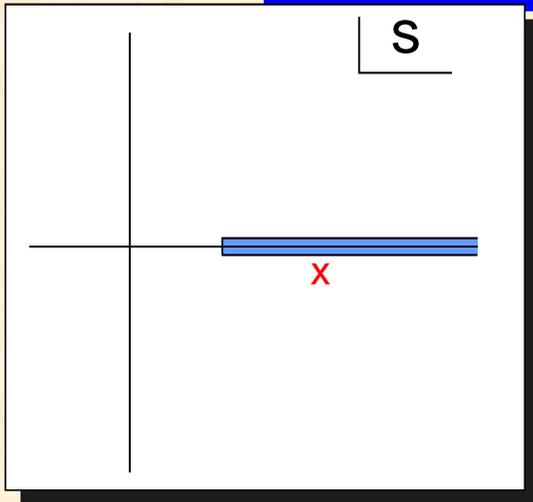
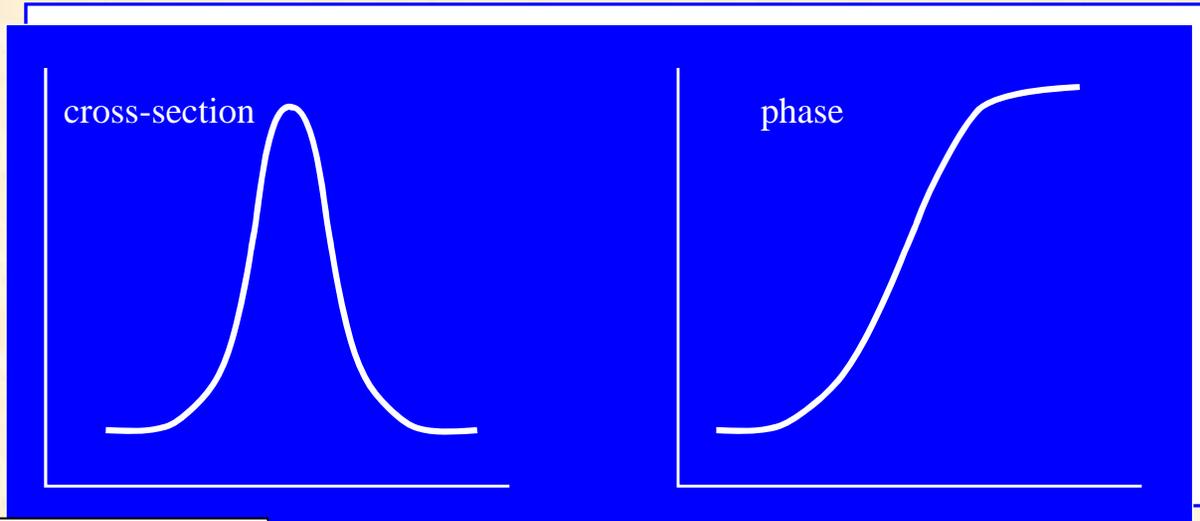


# Dalitz Analysis



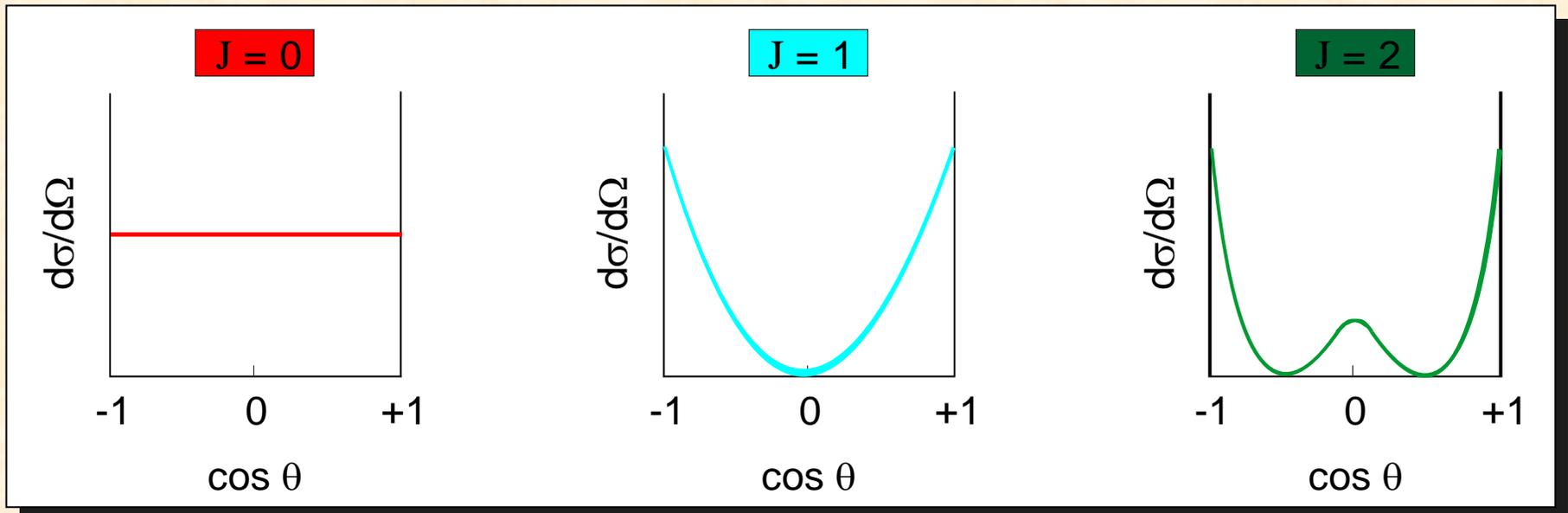
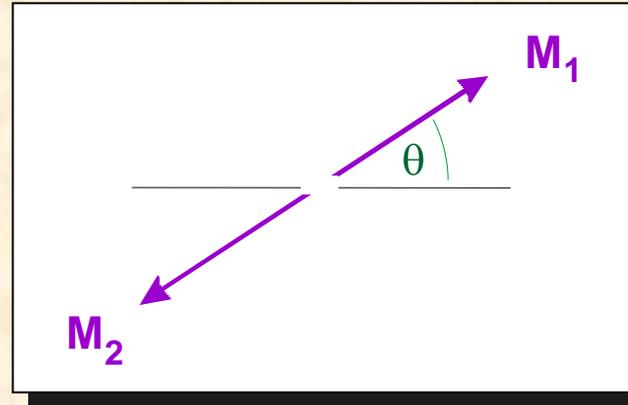


# Hadron states

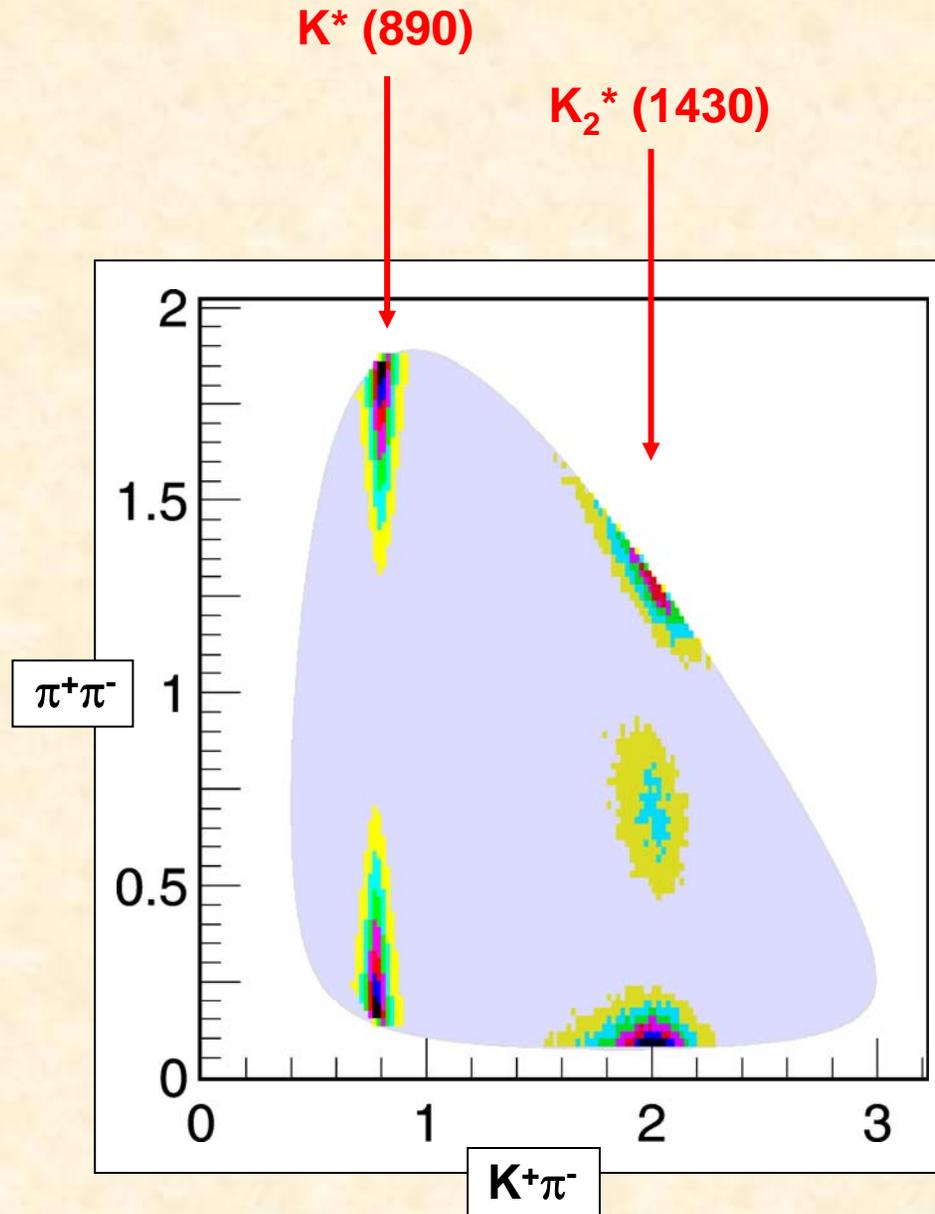


$$\frac{1}{M^2 - s - iM\Gamma}$$

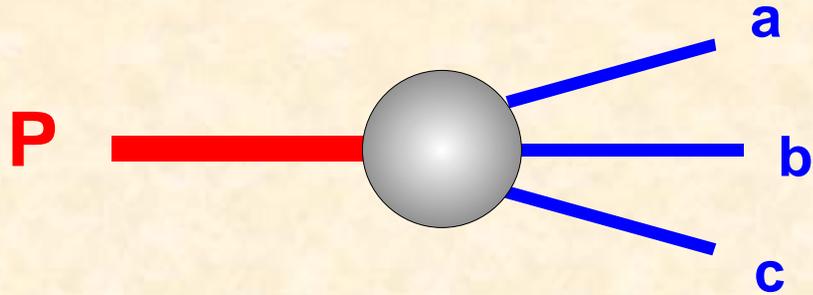
# SPIN ANALYSIS



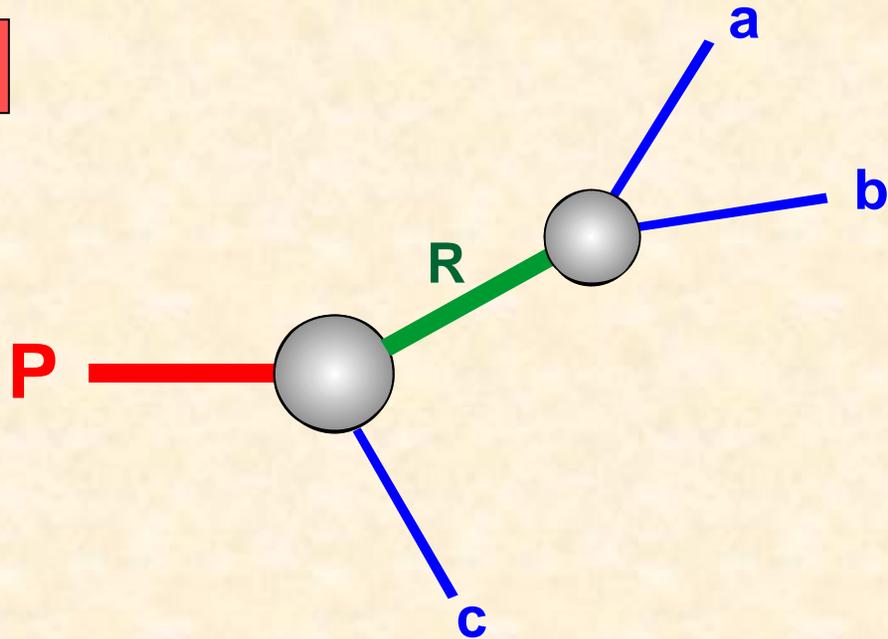
Spectroscopy: interplay of **poles & zeros**



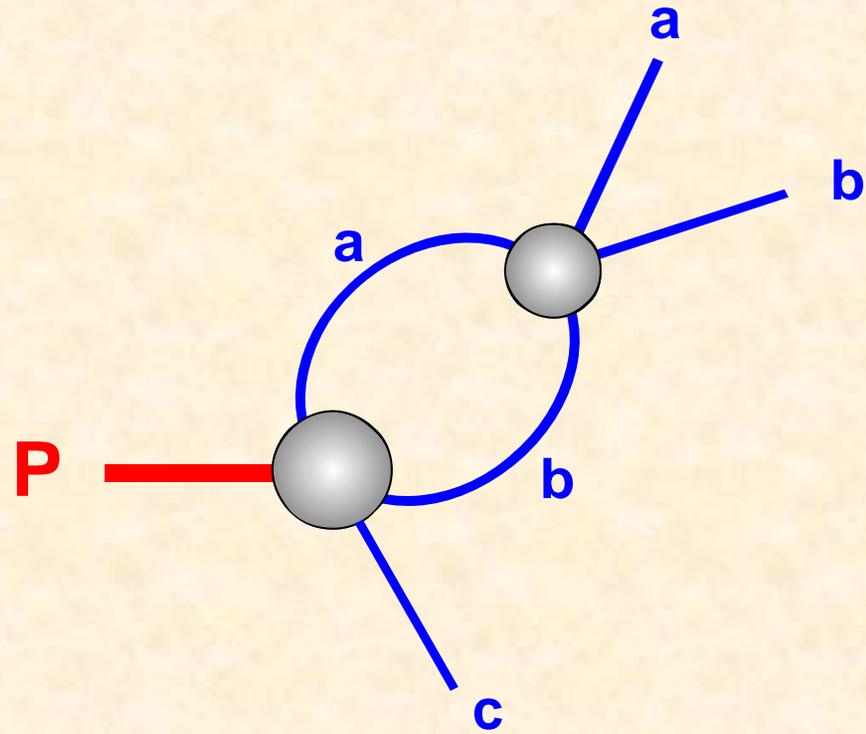
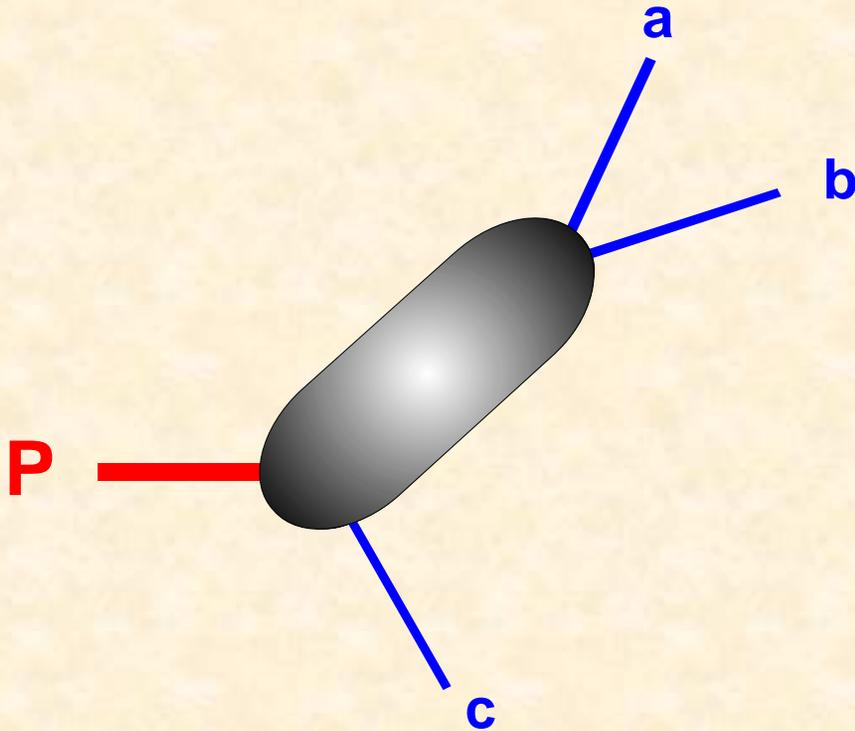
**P** → **a b c**



isobar picture

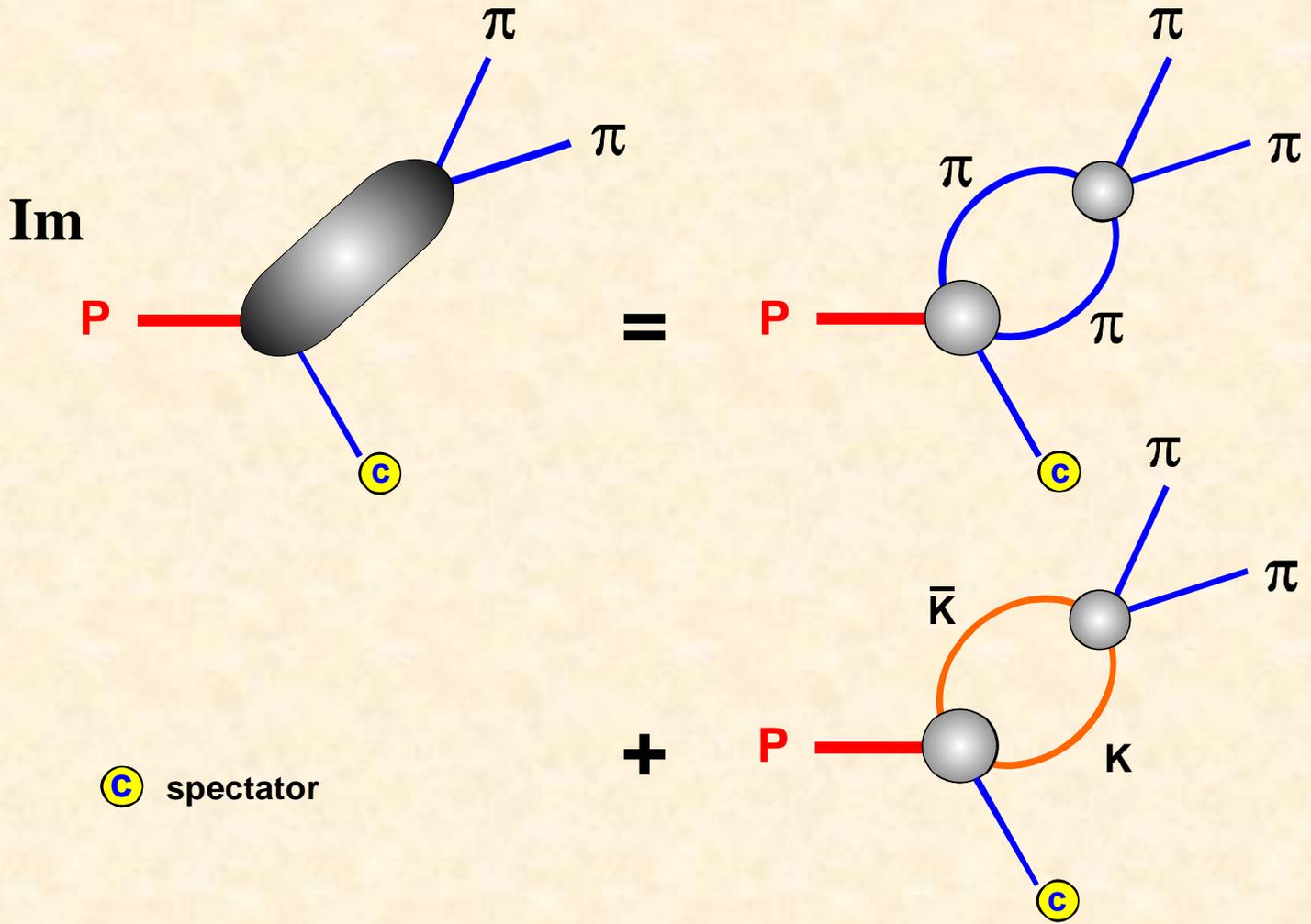


**P** → **a b c**

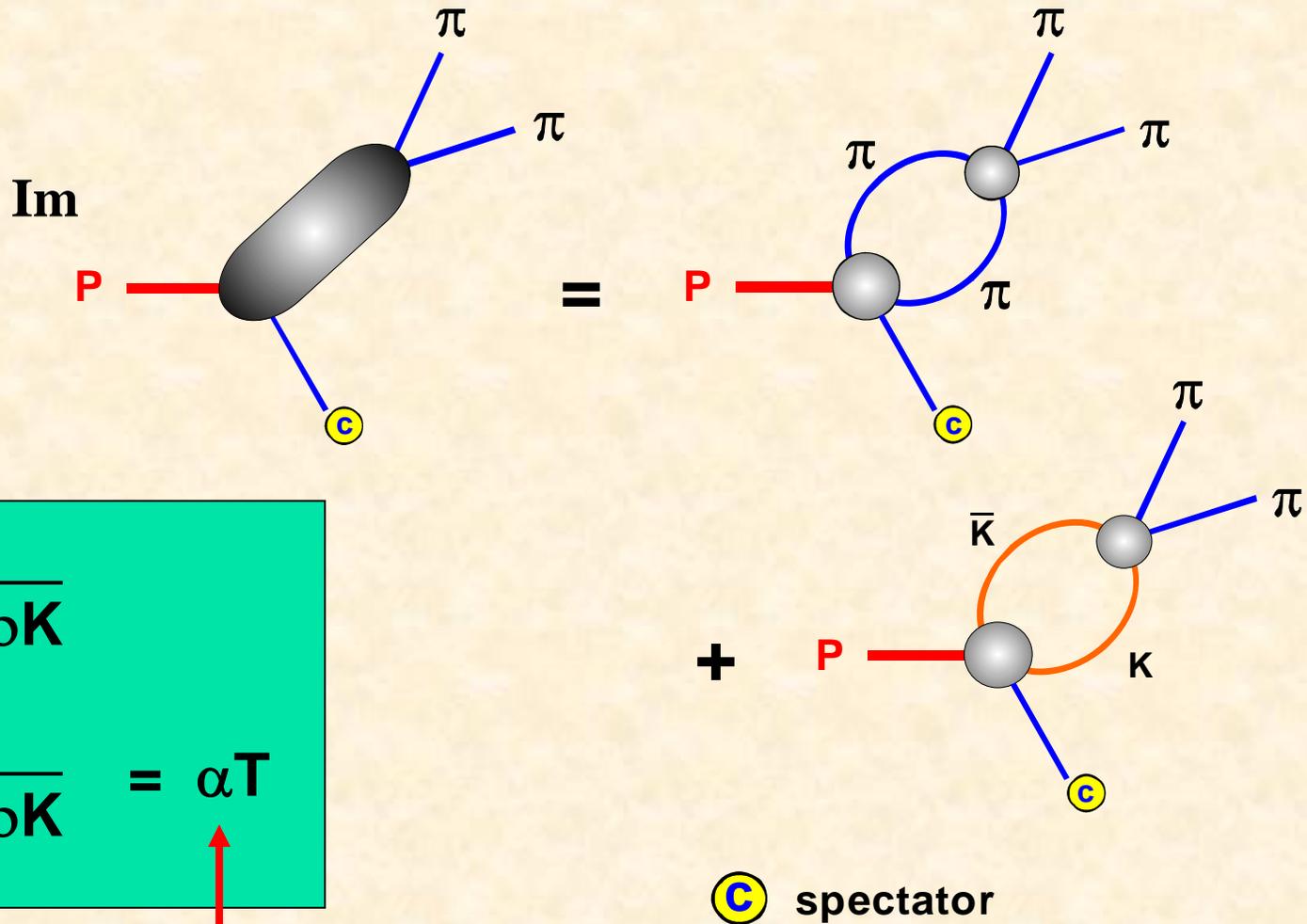


**unitarity connects to hadronic scattering**

# Unitarity for $P \rightarrow \pi\pi (c)$



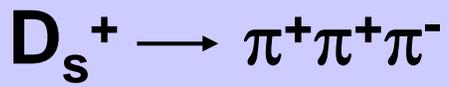
# UNITARITY : decays in spectator picture



$$T = \frac{K}{1 - i\rho K}$$

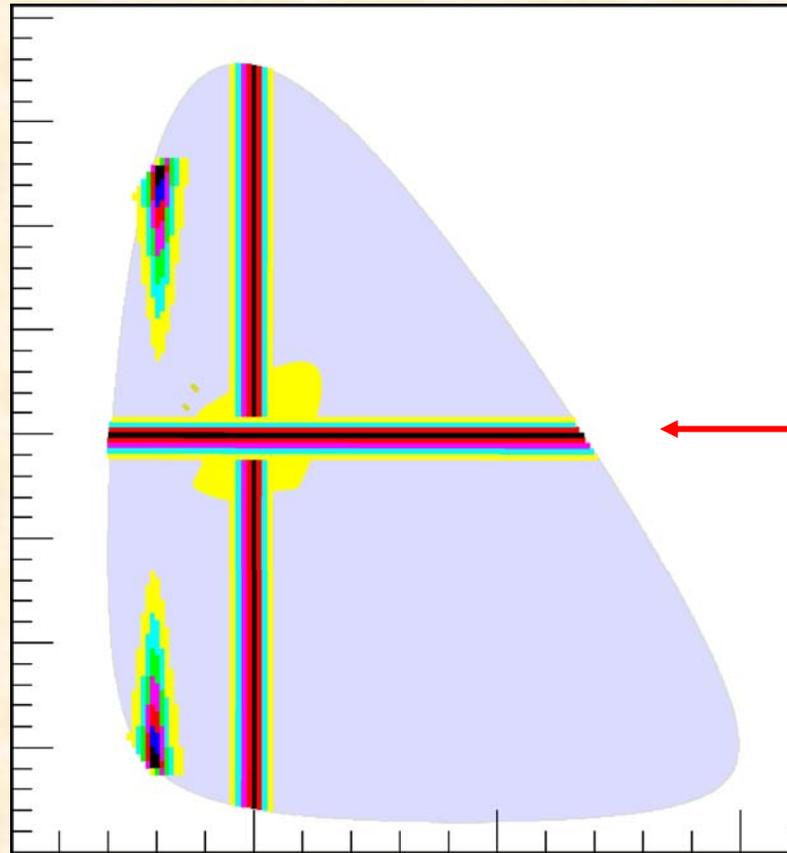
$$F = \frac{P}{1 - i\rho K} = \alpha T$$

↑  
coupling function



E791

$\pi^+\pi^-$

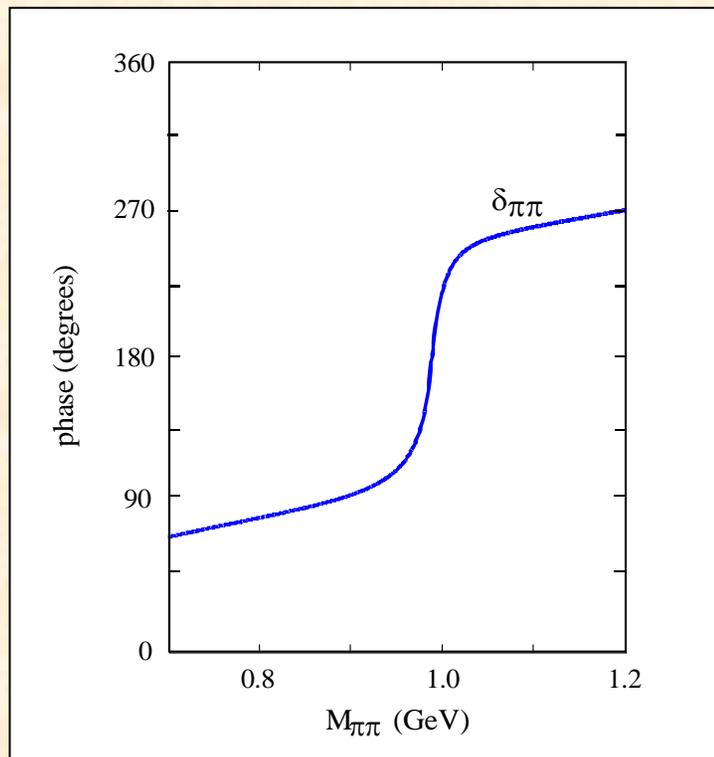
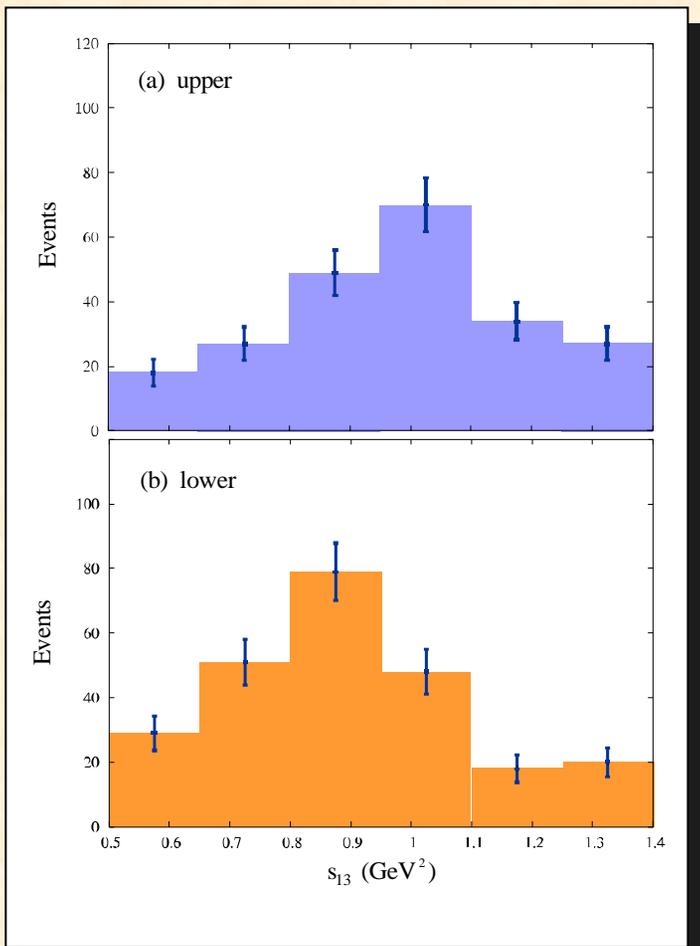
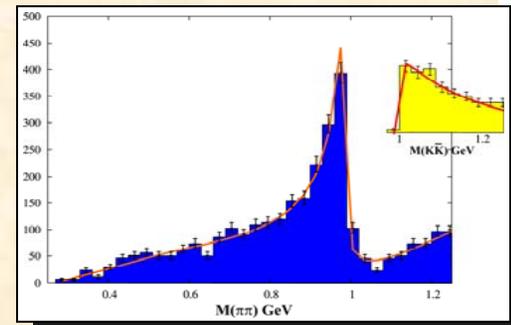
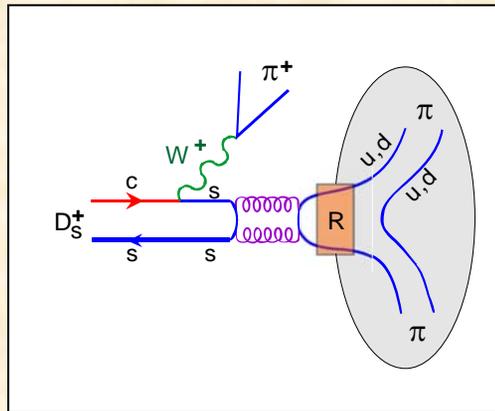
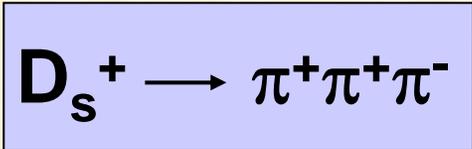


$f_0(980)$

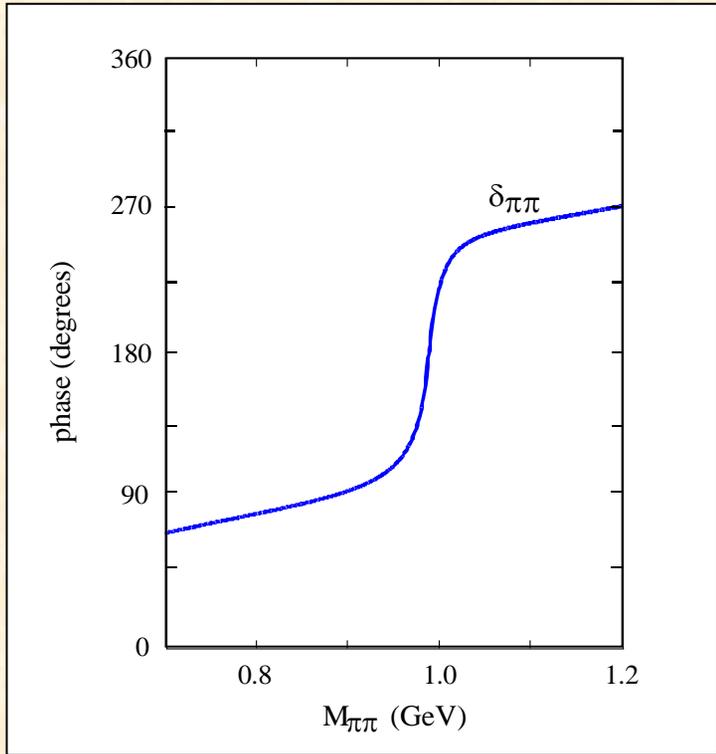
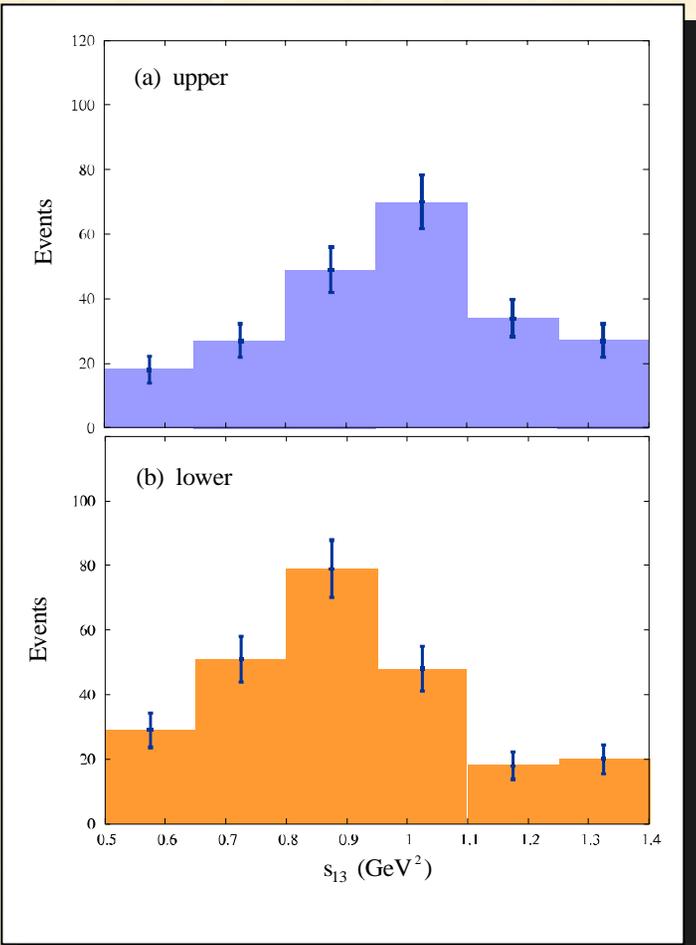
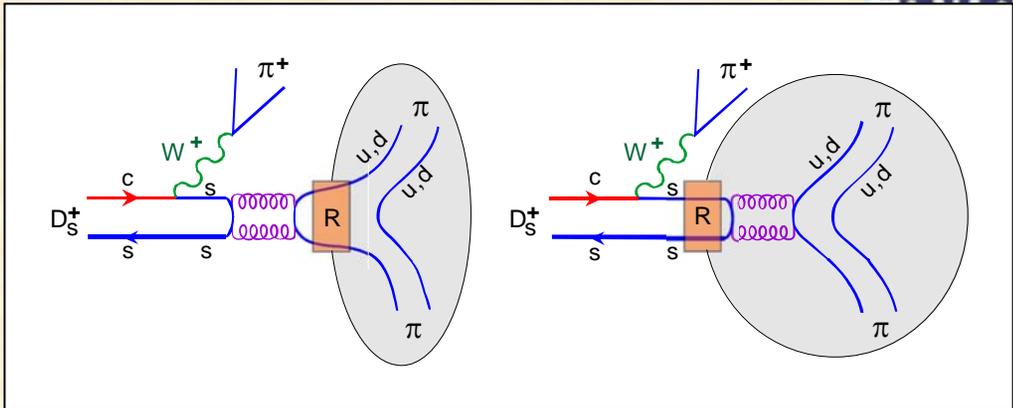
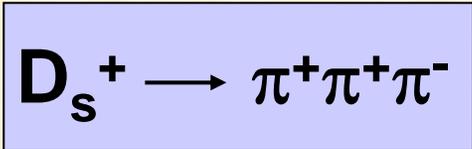
$f_0(980)$

$\pi^+\pi^-$

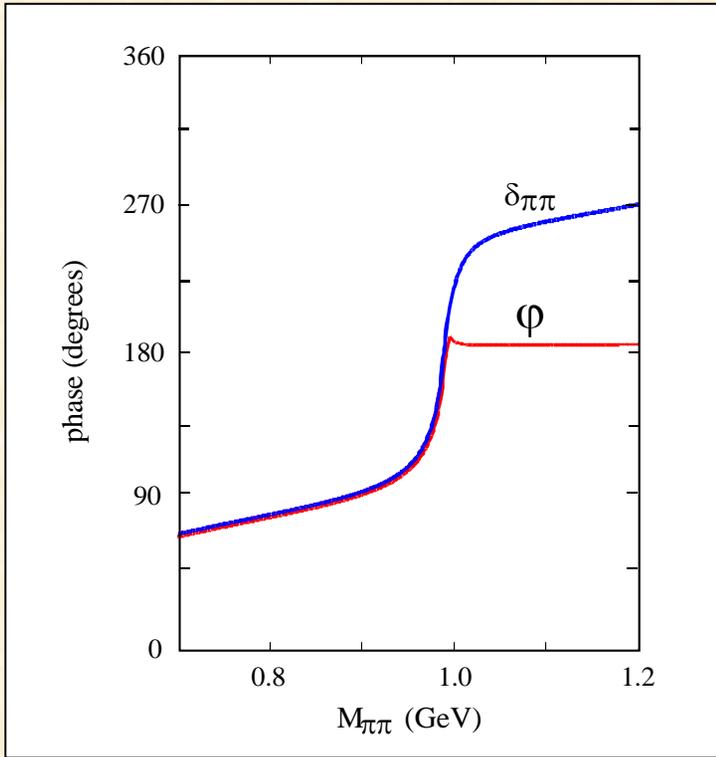
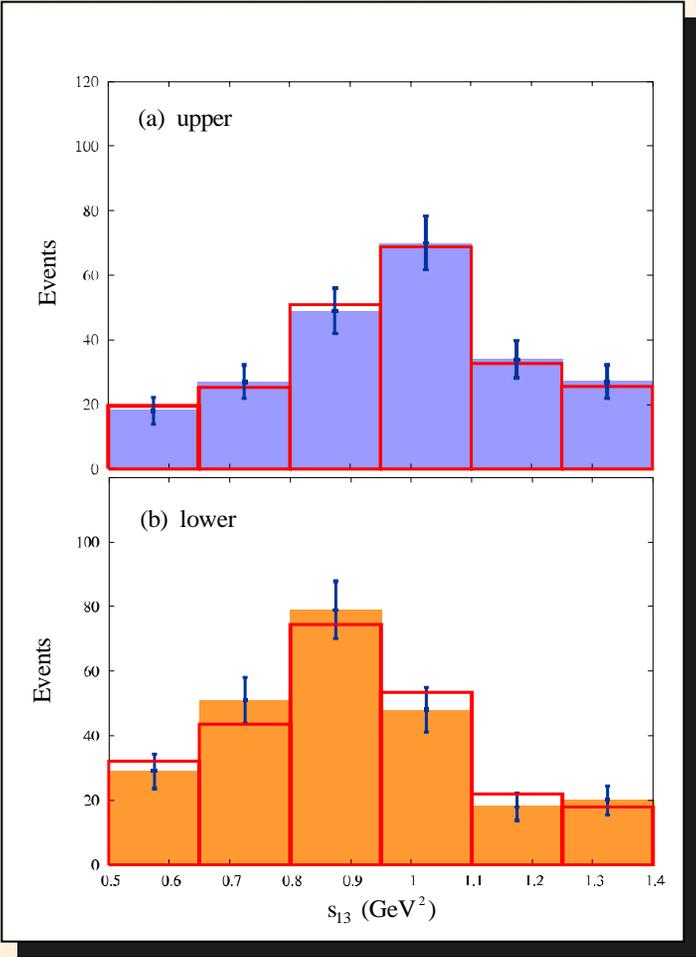
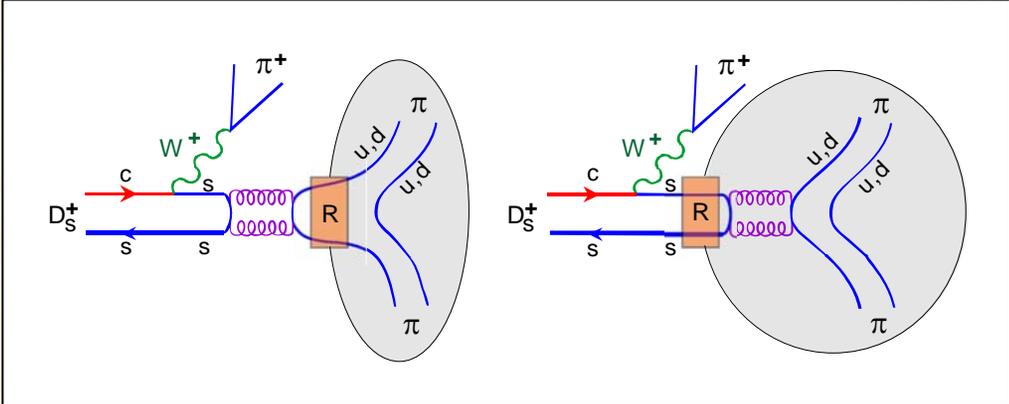
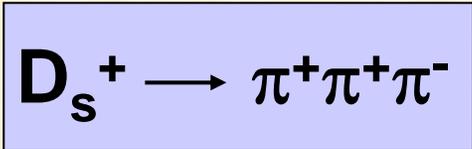
Bediaga & Miranda

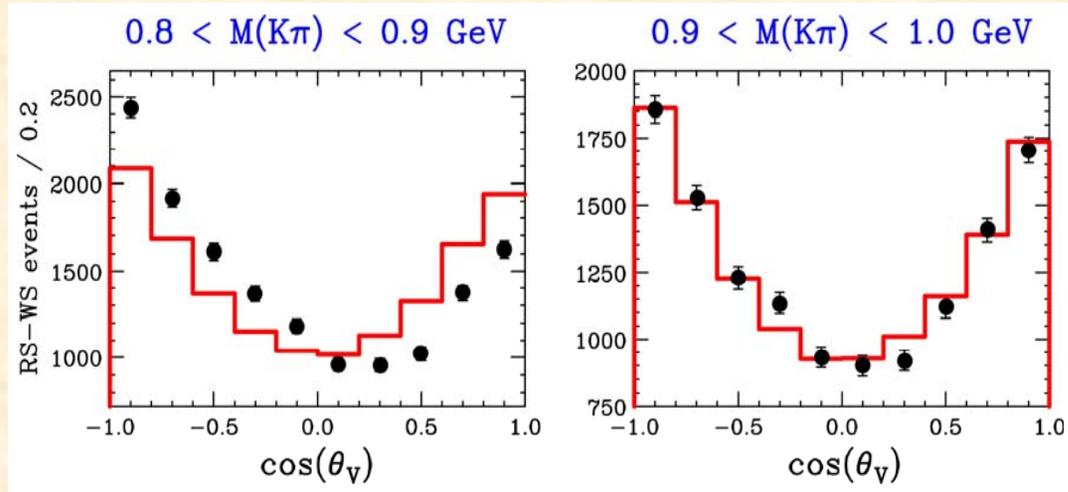


**Bediaga & Miranda**



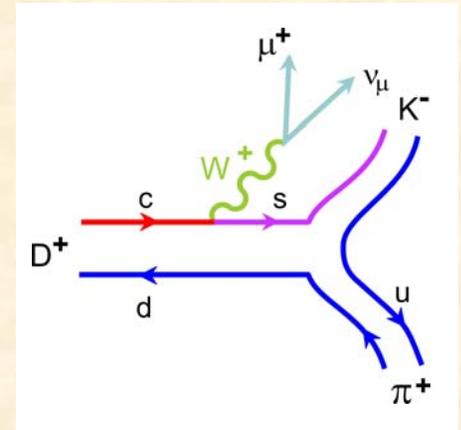
**Bediaga & Miranda**



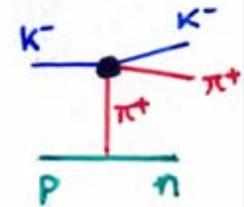
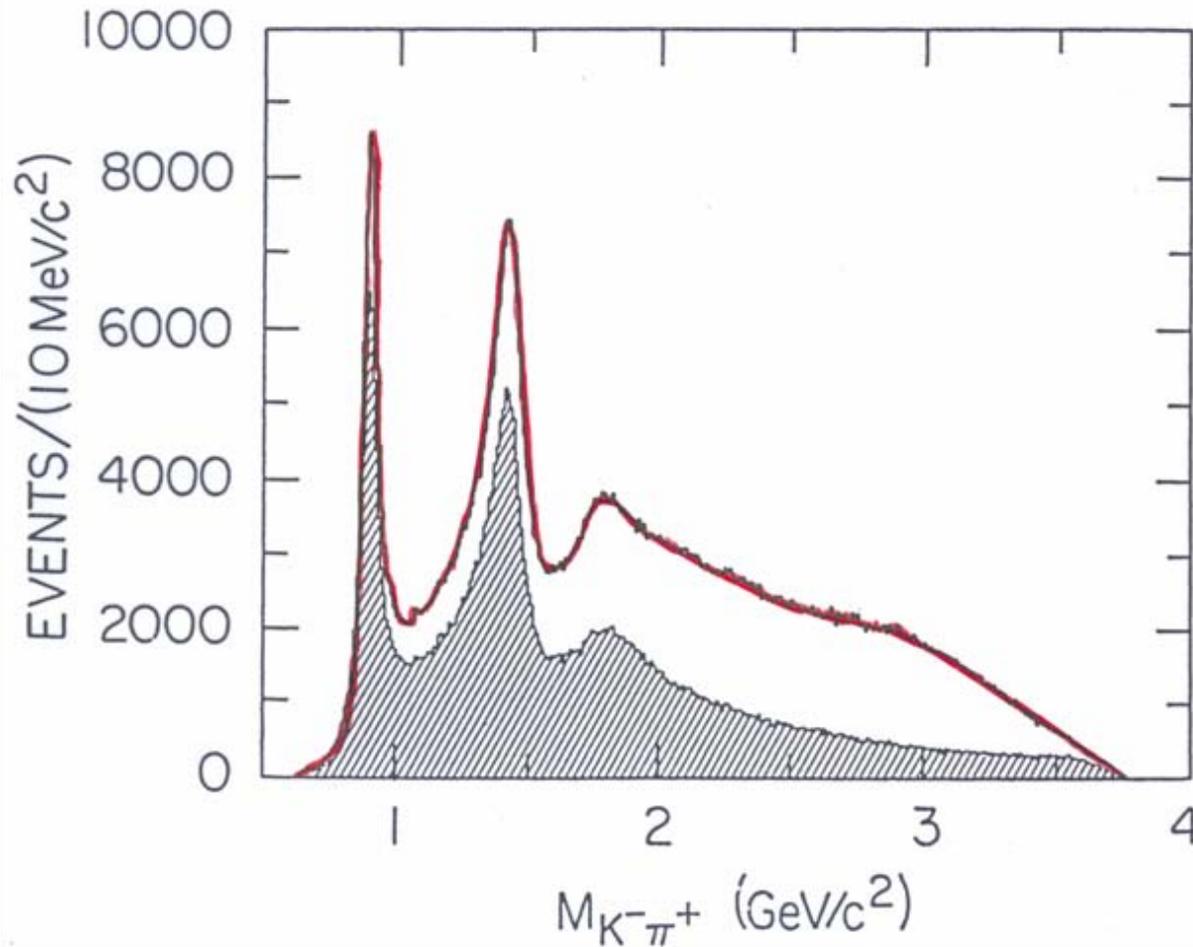


$$\mathcal{F}(D \rightarrow (K\pi)\mu\nu; s) = \mathcal{F}_{sl}^{1/2}(s) + \mathcal{F}_{sl}^{3/2}(s)$$

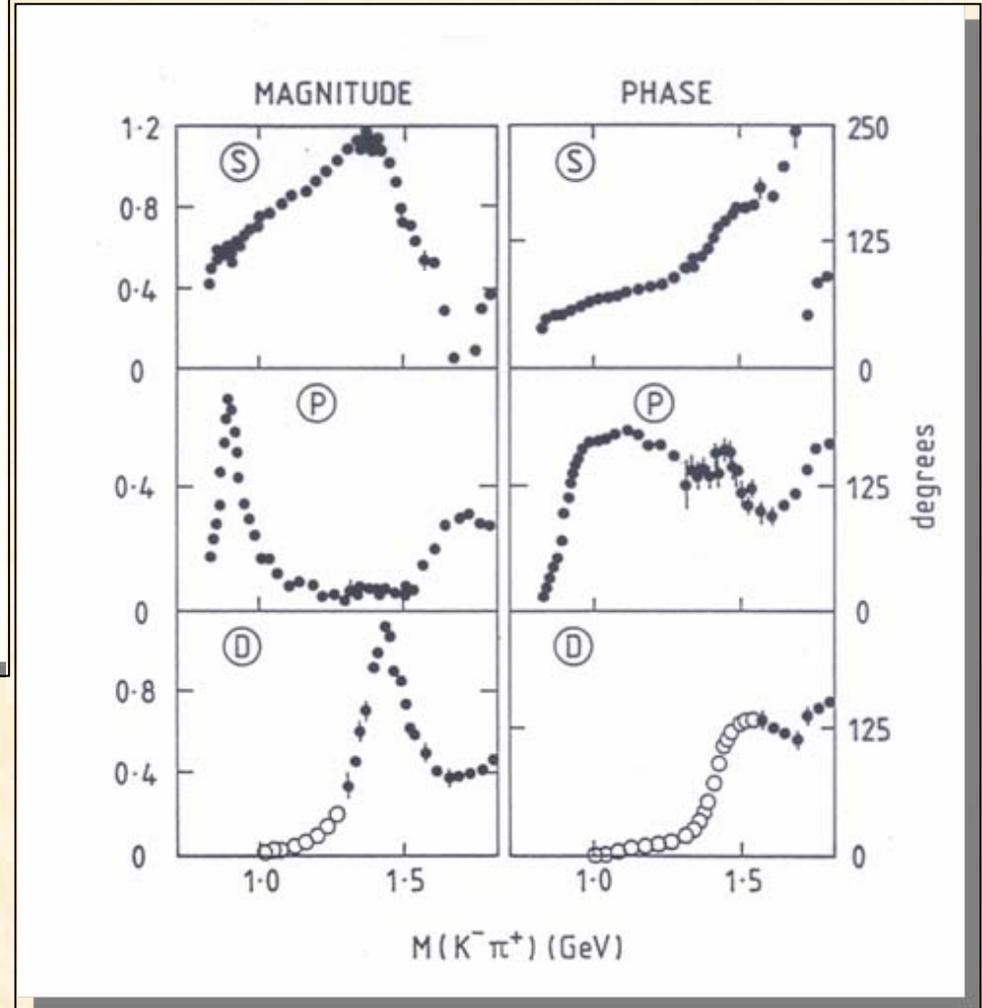
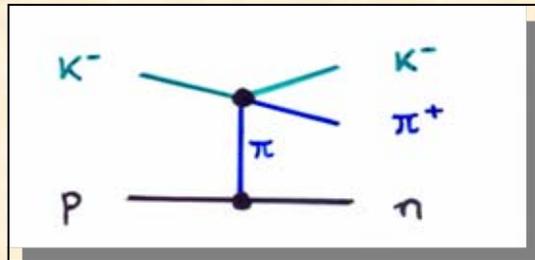
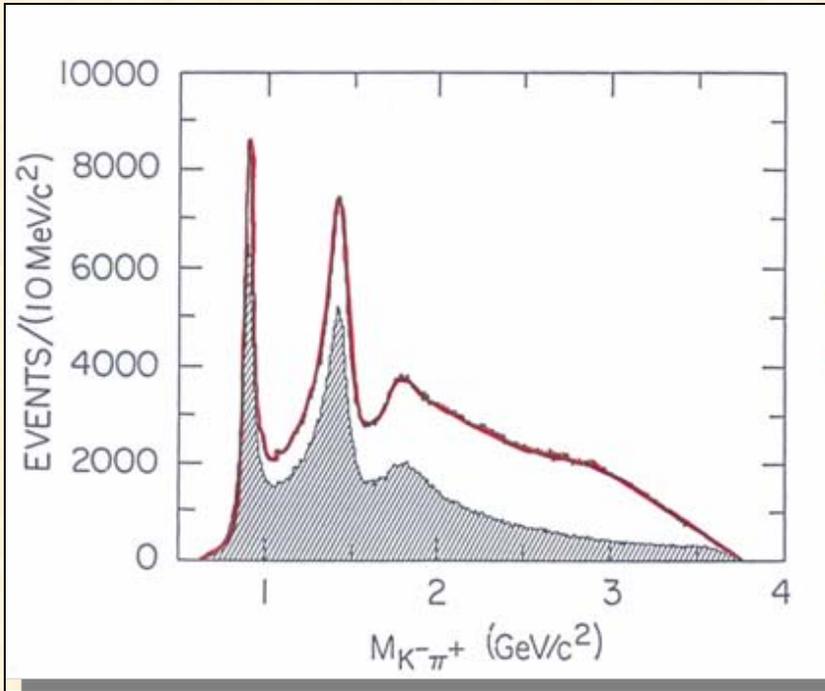
$$\mathcal{F}_{sl}^I(s) = |\mathcal{F}_{sl}^I(s)| \exp[i\delta^I(s)]$$



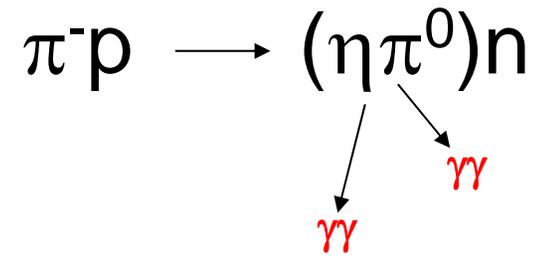
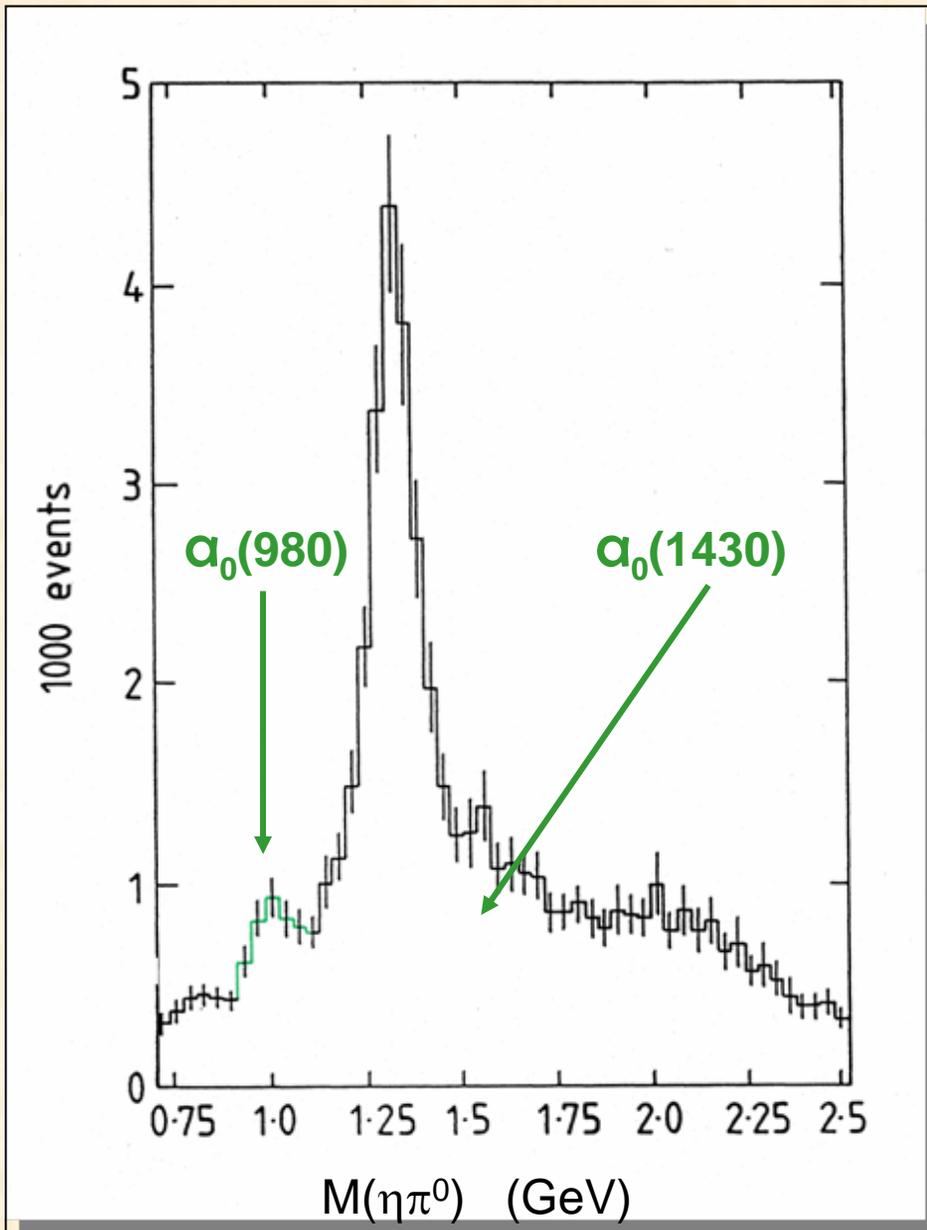
LASS:  $K^-p \rightarrow K^- \pi^+ n$



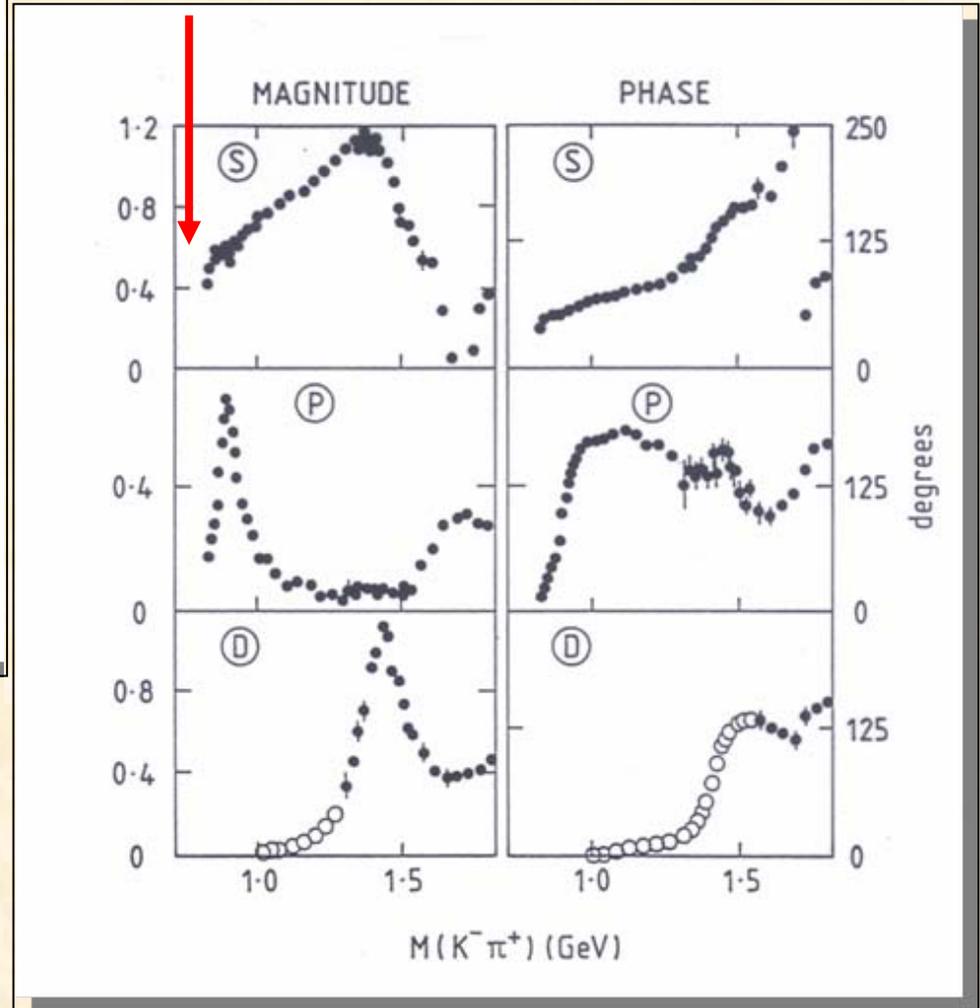
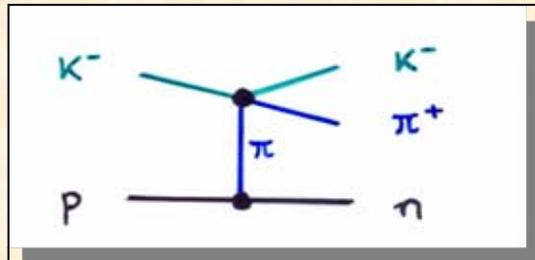
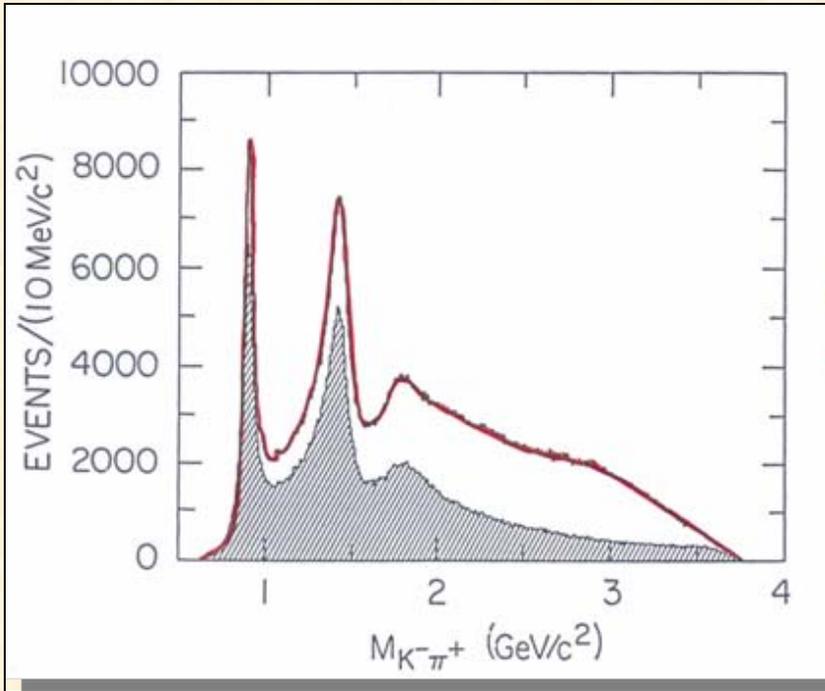
# LASS: $K^-p \rightarrow K^- \pi^+ n$



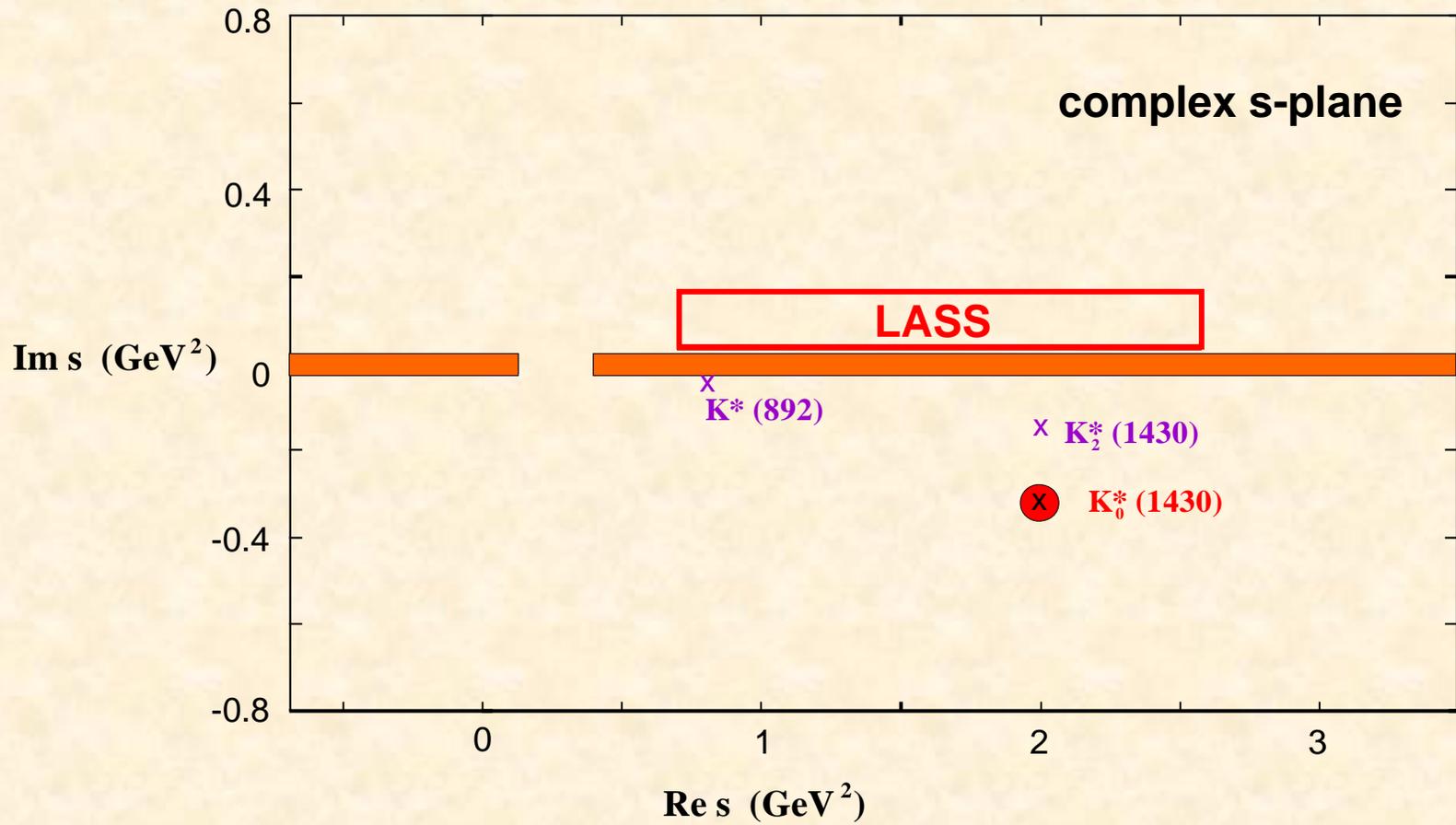
# GAMS



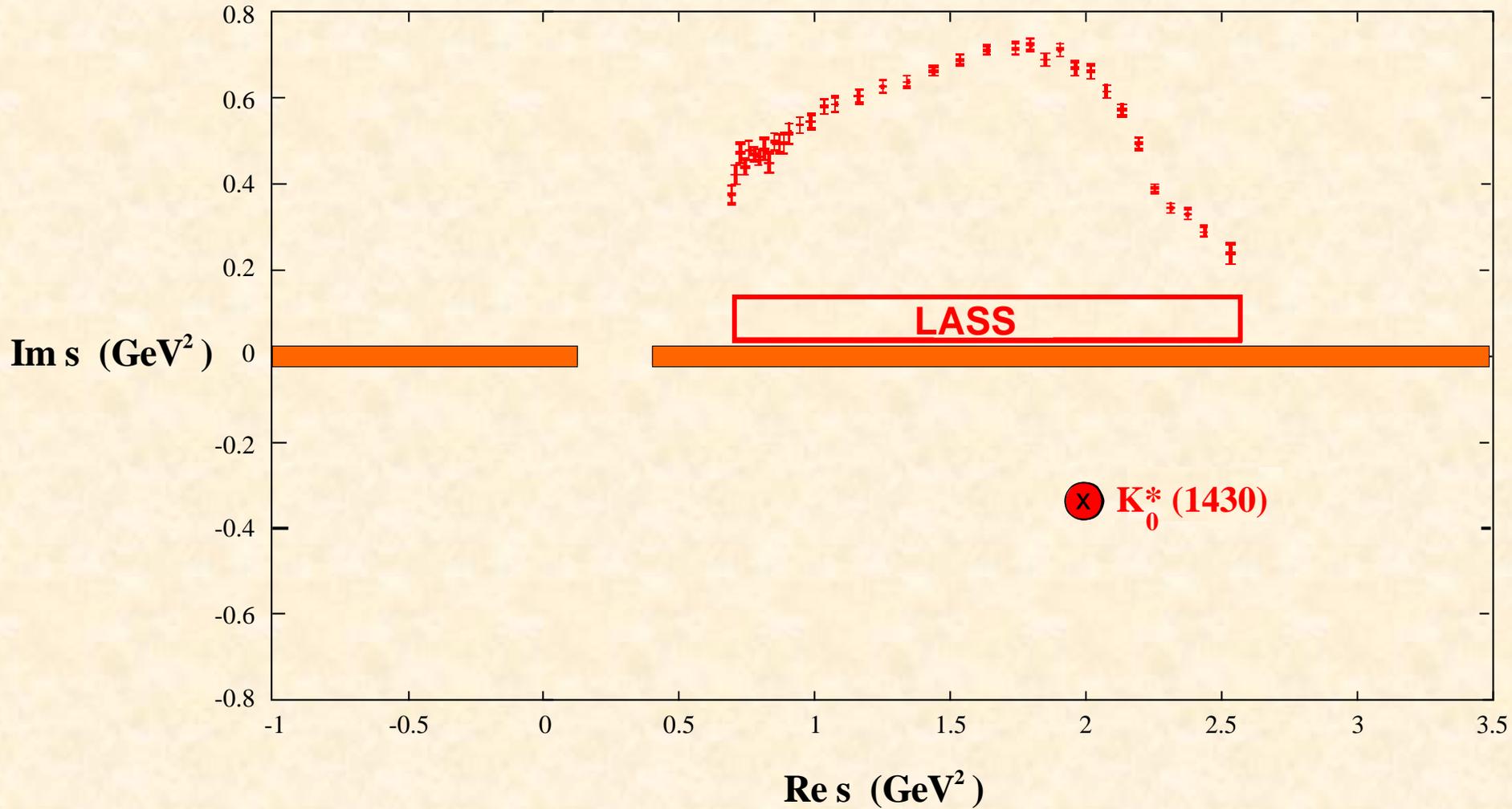
# LASS: $K^-p \rightarrow K^- \pi^+ n$



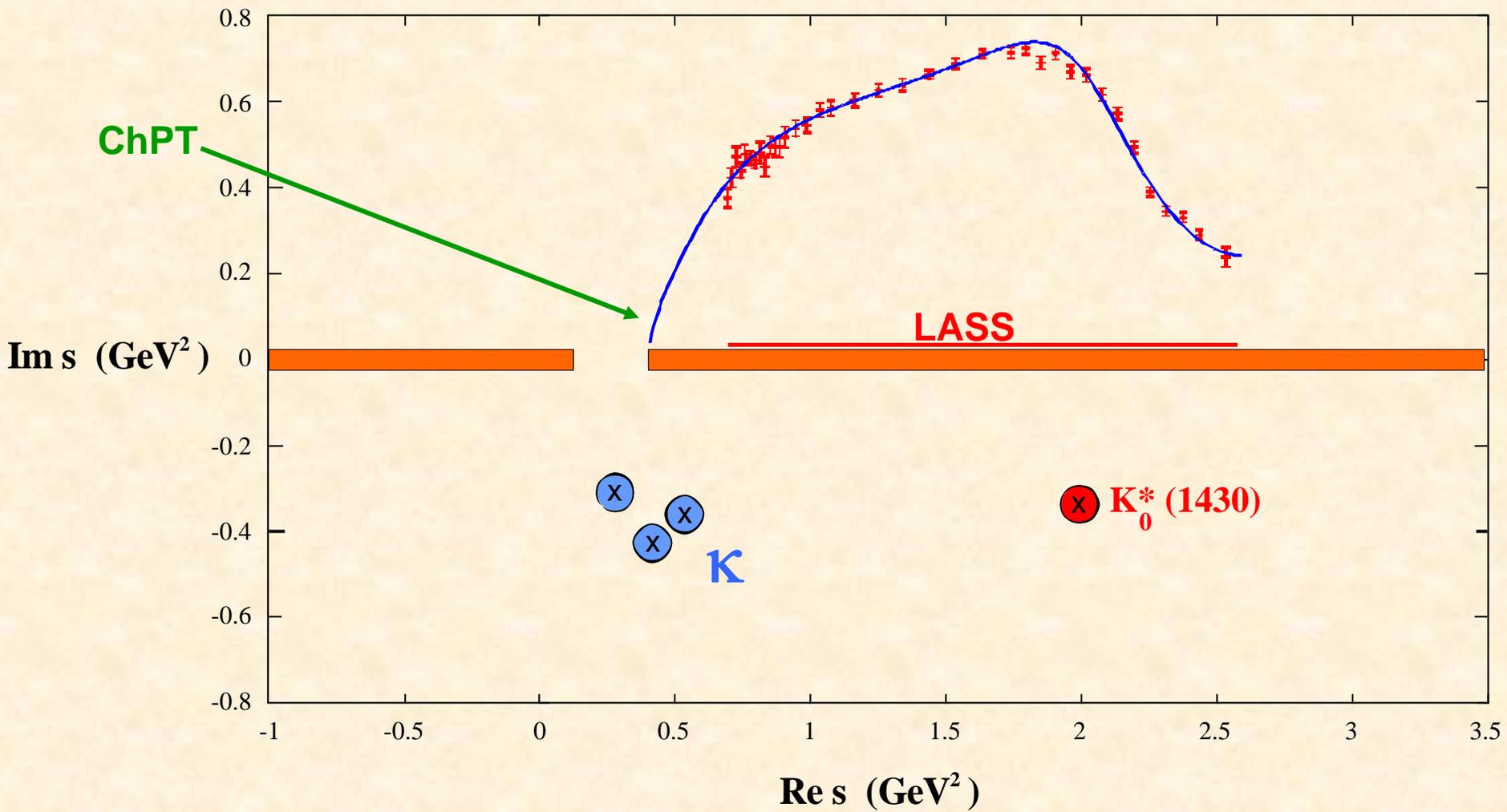
$\pi K : I = 1/2, J$

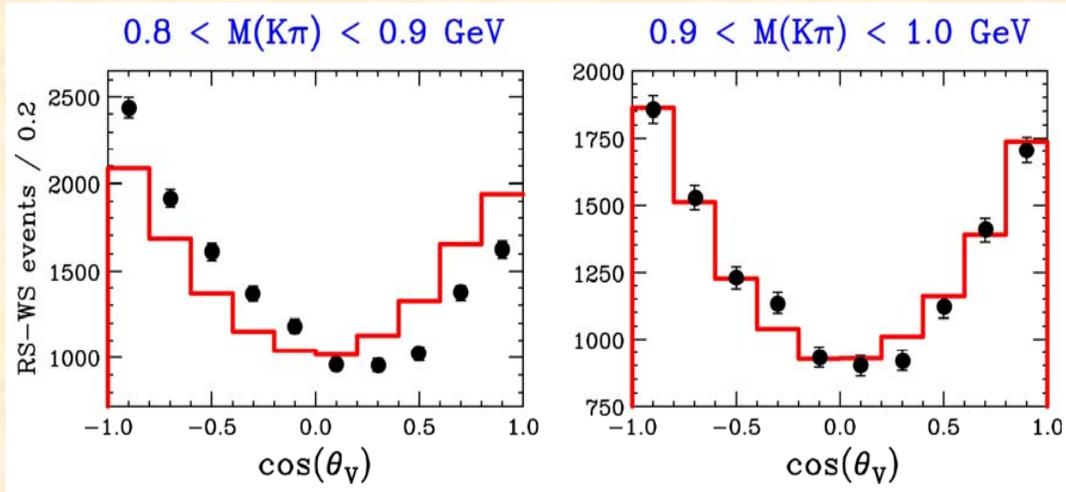


# $\pi K : I = 1/2, J = 0$



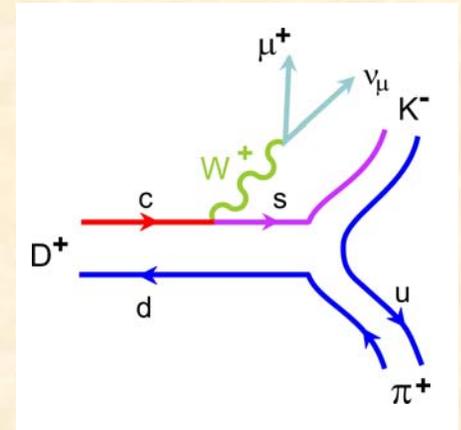
# $\pi K : I = 1/2, J = 0$

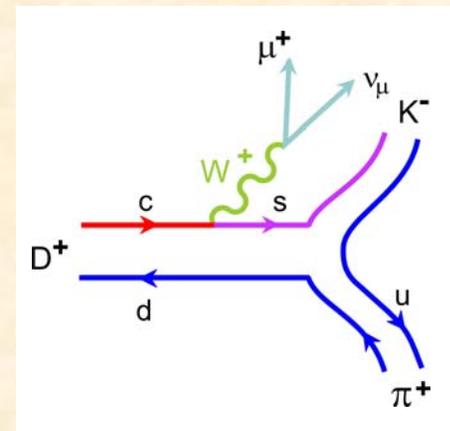
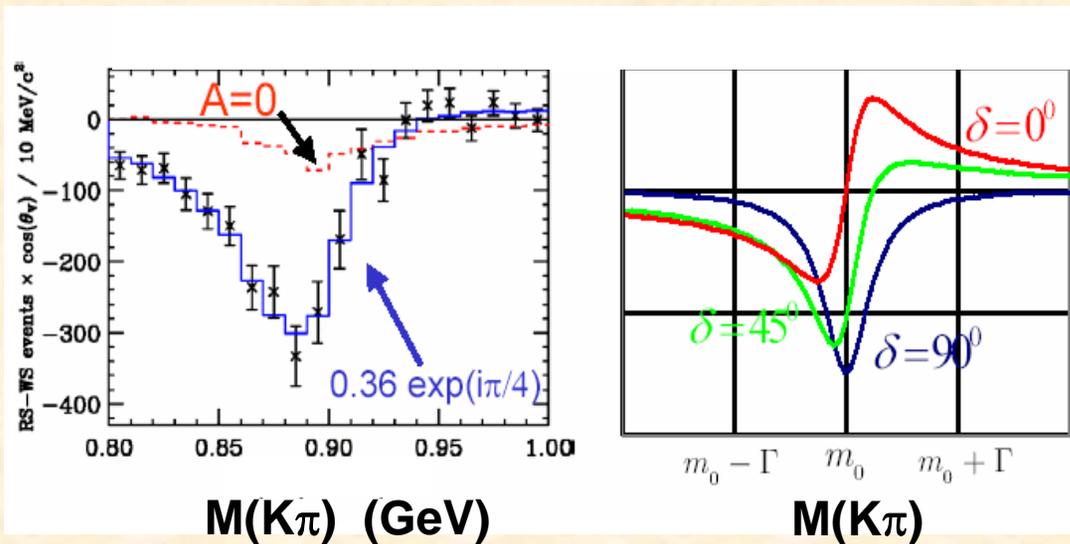
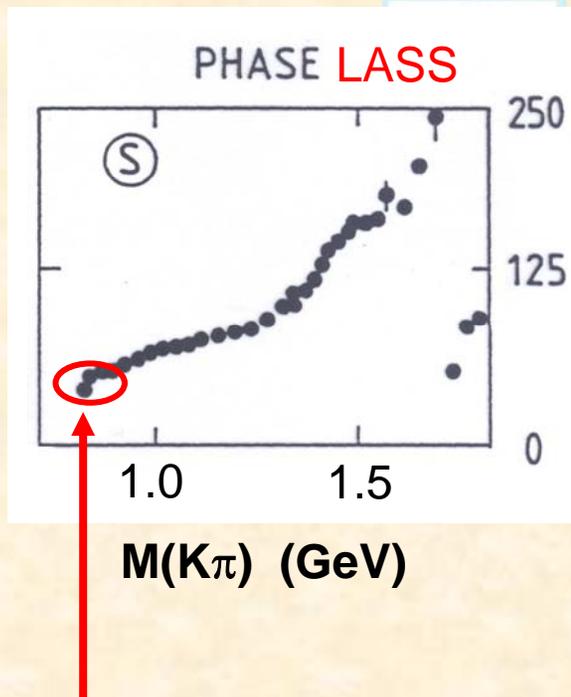
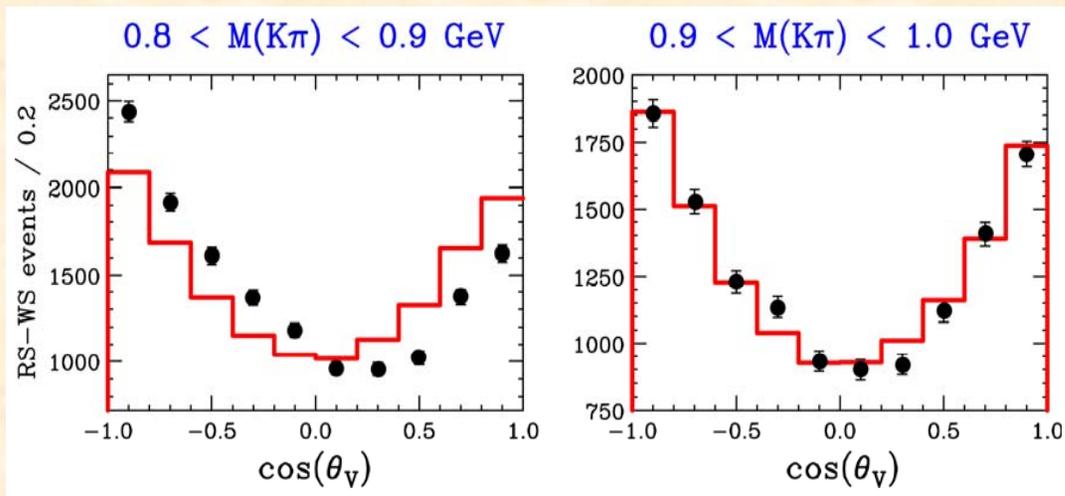


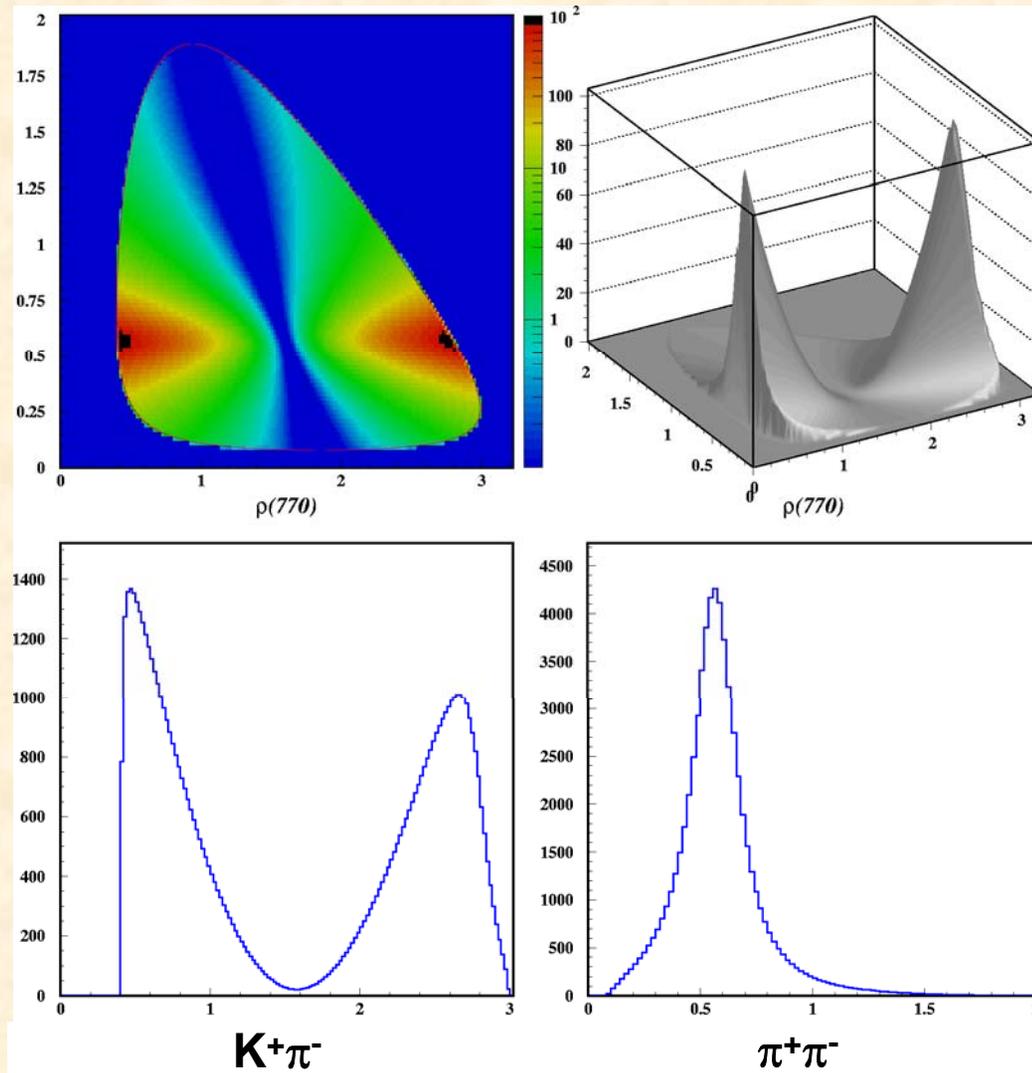


$$\mathcal{F}(D \rightarrow (K\pi)\mu\nu; s) = \mathcal{F}_{sl}^{1/2}(s) + \mathcal{F}_{sl}^{3/2}(s)$$

$$\mathcal{F}_{sl}^I(s) = |\mathcal{F}_{sl}^I(s)| \exp[i\delta^I(s)]$$

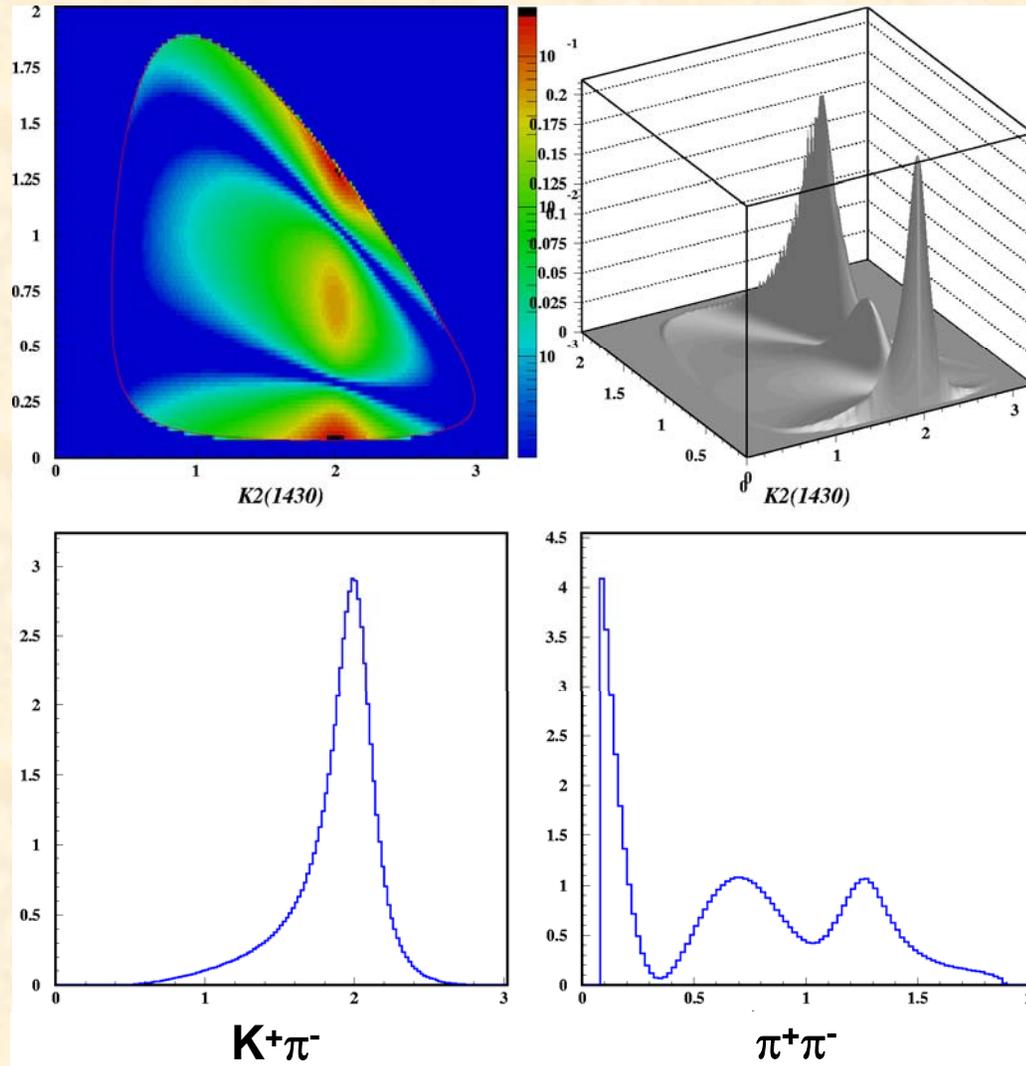





$$\rho(770)$$


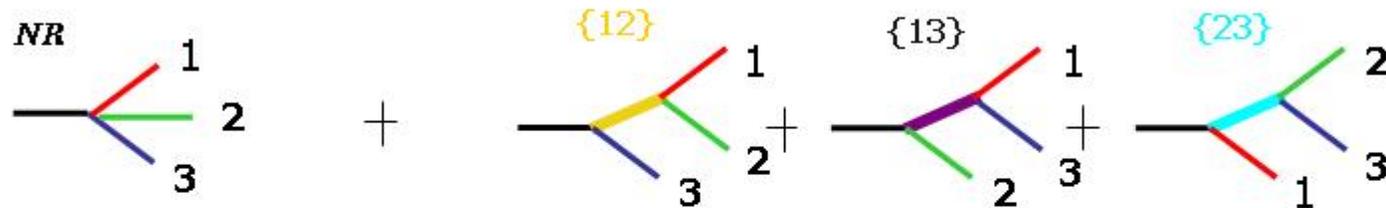


$K_2^*(1430)$



# “Traditional” Dalitz Plot Analyses

- The “isobar model” has been widely used, with Breit-Wigner resonant terms, over the past 15 years.



- Amplitude for channel  $\{ij\}$ :

$$A_{ij} = \underbrace{d_0 e^{i\delta_0}}_{\text{NR Constant}} + \sum_R d_R e^{i\delta_R} A(s_{ij}) \times F_0^D(q, r_D) F_J^R(p, r_R) M_J(p, q)$$

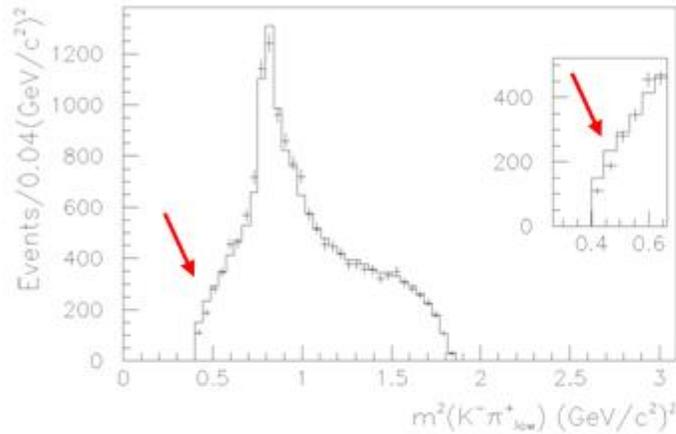
D form factor
R form factor
spin factor

- Each resonance “R” (mass  $M_R$ , width  $\Gamma_R$ ) assumed to have form

$$A_R(s_{ij}) = [m_R^2 - s_{ij} - im_R \Gamma(p, r_R)]^{-1}$$

$p, q$  are momenta in  $ij$  rest frame  
 $r_D, r_R$  meson radii

# E791 $D^+ \rightarrow K^- \pi^+ \pi^+$

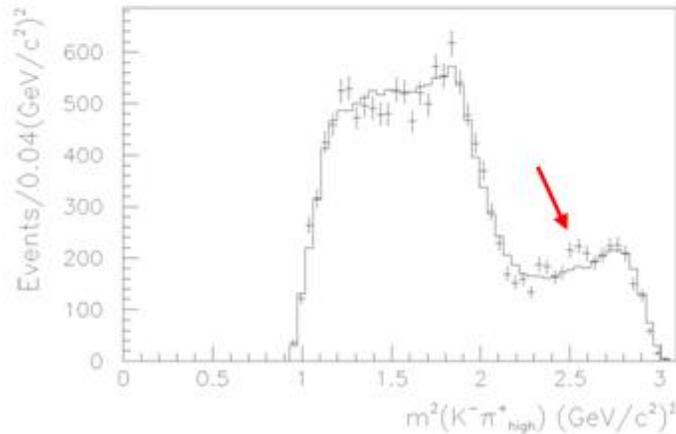


$D^+$

→

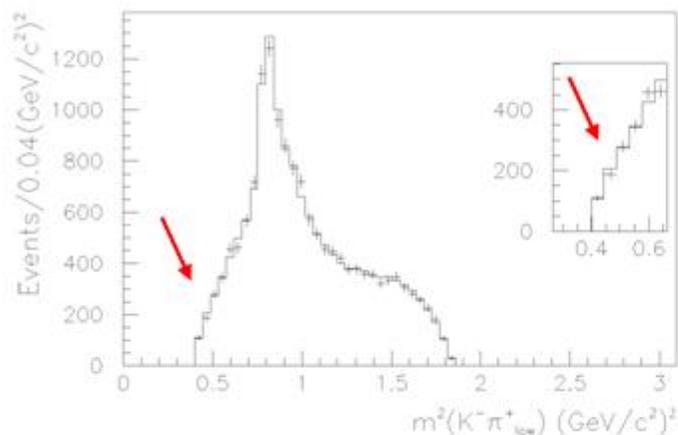
non resonant	$90.0 \pm 2.6\%$	$0^\circ$ (fixed)
$K^*(890)\pi^+$	$13.8 \pm 0.5\%$	$54 \pm 2^\circ$
$K_0^*(1430)\pi^+$	$30.6 \pm 1.6\%$	$109 \pm 2^\circ$
$K_2^*(1430)\pi^+$	$0.4 \pm 0.1\%$	$33 \pm 8^\circ$
$K_1^*(1680)\pi^+$	$3.2 \pm 0.3\%$	$66 \pm 3^\circ$
<hr/>		
$\sim 138\%$		

$\chi^2/\text{d.o.f.} = 2.7$



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# E791 $D^+ \rightarrow K^- \pi^+ \pi^+$

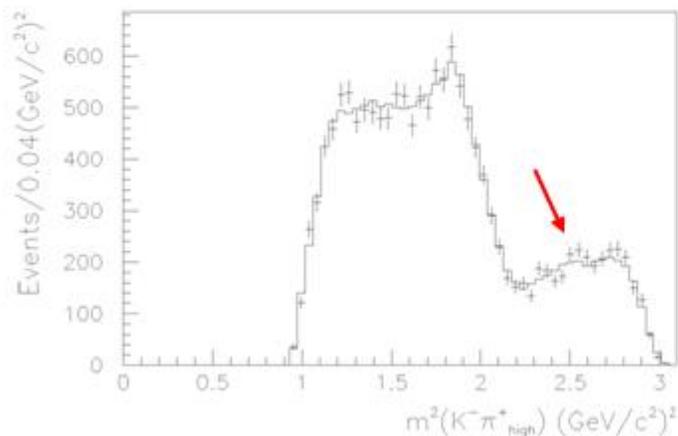


$D^+$

→

non resonant	$13.0 \pm 5.8 \pm 2.6\%$	$349 \pm 14 \pm 8^\circ$
" $\kappa$ " $\pi^+$	$47.8 \pm 12.1 \pm 3.7\%$	$187 \pm 8 \pm 17^\circ$
$K^*(890)\pi^+$	$12.3 \pm 1.0 \pm 0.9\%$	$0^\circ$ (fixed)
$K_0^*(1430)\pi^+$	$12.5 \pm 1.4 \pm 0.4\%$	$48 \pm 7 \pm 10^\circ$
$K_2^*(1430)\pi^+$	$0.5 \pm 0.1 \pm 0.2\%$	$306 \pm 8 \pm 6^\circ$
$K_1^*(1680)\pi^+$	$2.5 \pm 0.7 \pm 0.2\%$	$28 \pm 13 \pm 15^\circ$

$\sim 89\%$



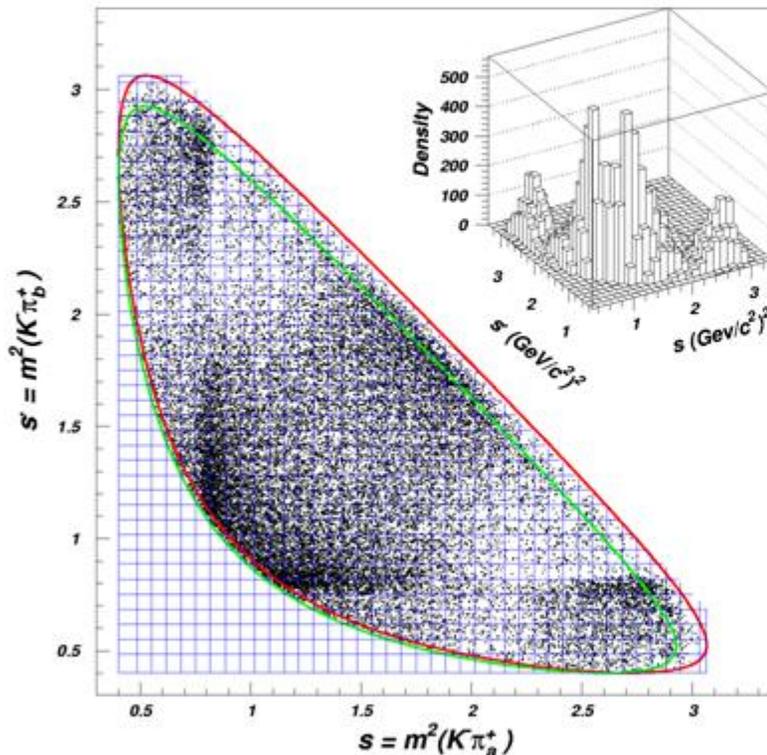
$\chi^2/\text{d.o.f.} = 0.73$   
(95%)

Probability

$M_\kappa = 797 \pm 19 \pm 42 \text{ MeV}/c^2$   
 $\Gamma_\kappa = 410 \pm 43 \pm 85 \text{ MeV}/c^2$

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# E791 $D^+ \rightarrow K^- \pi^+ \pi^+$ Dalitz Plot



- Most interesting feature:
  - $K^*(892)$  bands dominate
  - Asymmetry in  $K^*(892)$  bands→ Interference with large  $s$ -wave component
- Also:
  - Structure at  $\sim 1430$  MeV/c<sup>2</sup> mostly  $K_0^*(1430)$
  - Some  $K_2^*(1420)$ ? or  $K_1^*(1410)$ ??
  - Perhaps some  $K_1^*(1680)$ ?
- So
  - At least the  $K^*(892)$  can act as interferometer for  $s$ -wave
  - Perhaps other resonances can fill in some gaps too.

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# $s$ -wave from $D^+ \rightarrow K^-\pi^+\pi^+$ Dalitz Plot?

- Divide  $m^2(K^-\pi^+)$  into slices
- Find  $s$ -wave amplitude in each slice (two parameters)
  - Use remainder of Dalitz plot as an interferometer

$$\frac{d^2\Gamma}{ds_{12}ds_{13}} \propto |\mathcal{S} + (\mathcal{P} + \mathcal{D})|^2$$

- For  $s$ -wave:
  - Interpolate between  $(c_k, \gamma_k)$  points:

$$\mathcal{S} = \text{Interp}(c_k e^{i\gamma_k}) \times F_0^D(q, r_D) F_0^R(p, r_R)$$

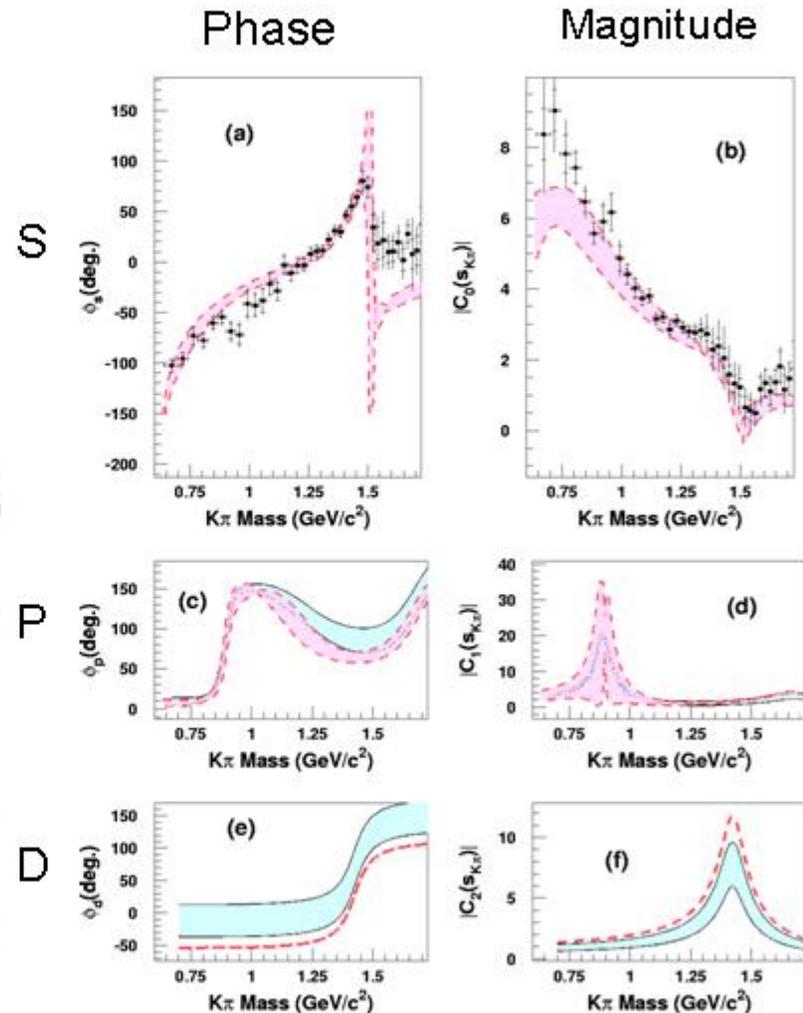
- Model  $\mathcal{P}$  and  $\mathcal{D}$

$\mathcal{S}$  ("partial wave")

# Fit E791 Data for $s$ -wave

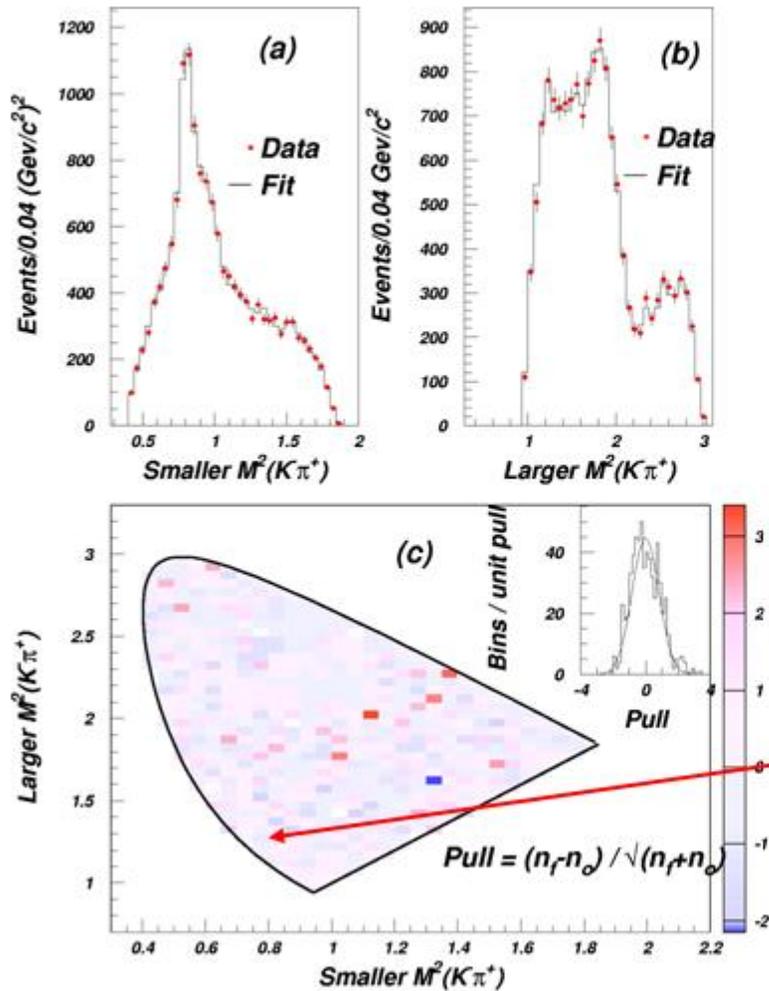
Float P and D parameters and find S:

- General appearance similar to isobar model fit:
  - Magnitudes at low mass differ
  - Phases above  $K_0^*$ (1430)
- Tests with many MC samples of this size (15K events), produced to simulate the isobar model, produce similar differences in  $\sim 15\%$  of the cases
- Major source of systematic uncertainty:
  - Contribution of reference waves in region between  $K^*(892)$  and  $K^*(1680)$ .



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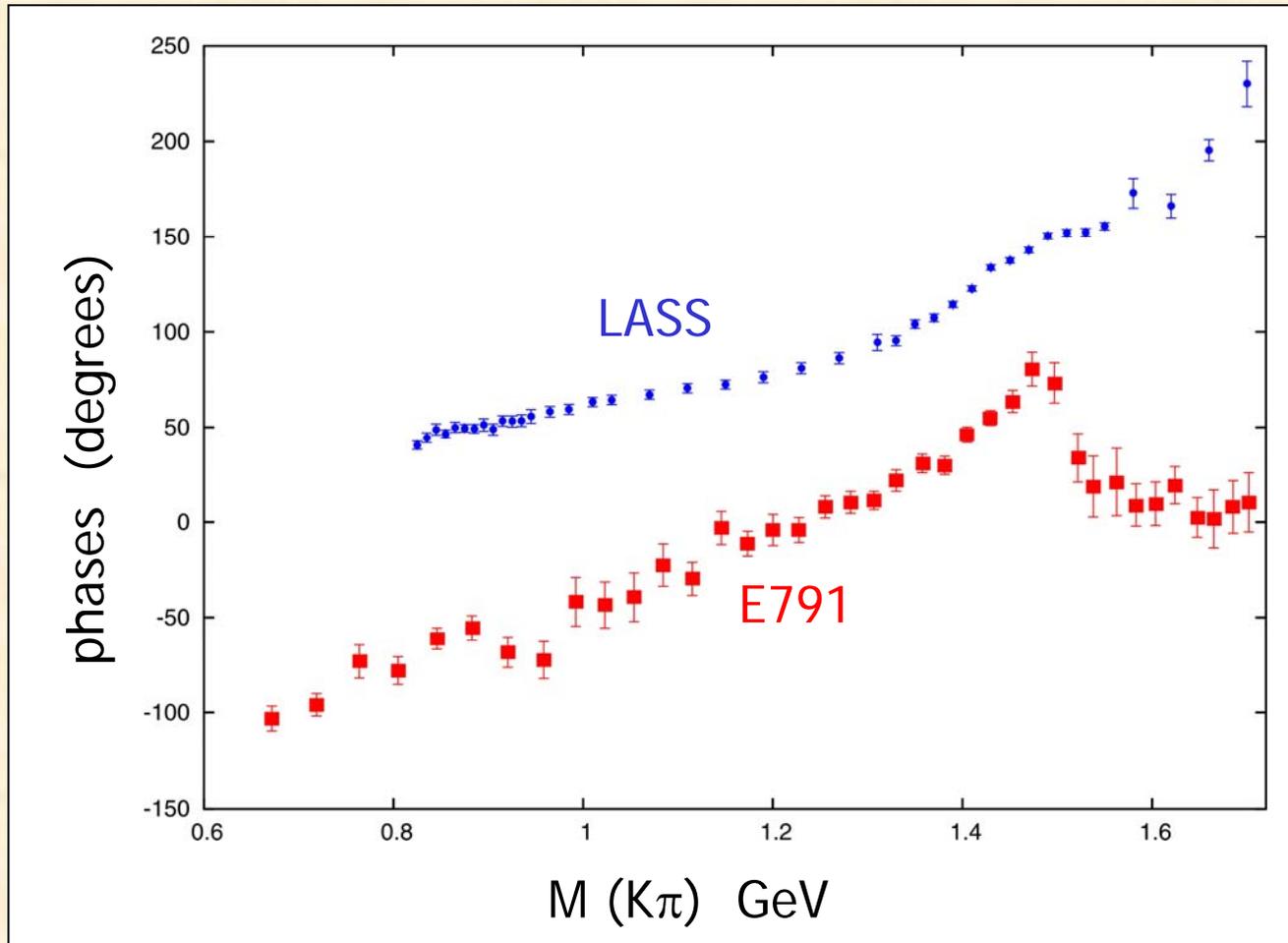
# Comparison with Data



$$\chi^2/\text{NDF} = 272/277 \text{ (48\%)}$$

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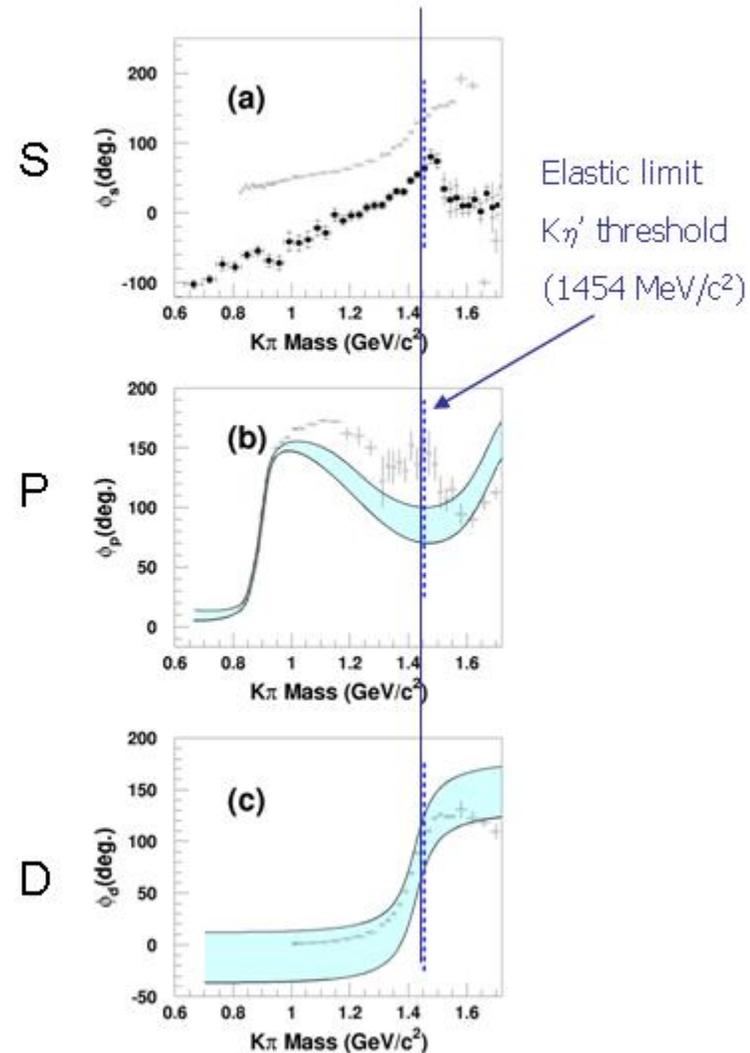
# E791 $\nu$ elastic scattering (LASS)

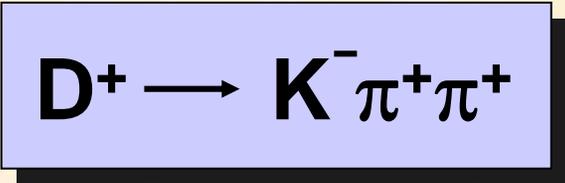


# Watson Theorem - a direct test

Phases for S, P and D waves are compared with those from LASS.

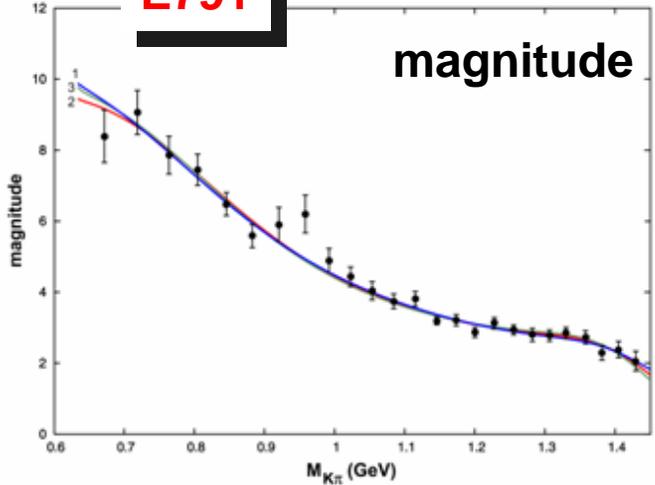
- $s$ -wave phase  $\phi_s$  for E791 is shifted by  $-75^\circ$  wrt LASS.
  - $\phi_s$  energy dependence differs below  $1100 \text{ MeV}/c^2$ .
- $\phi_p$  does not match well between  $K^*(892)$  and  $K^*(1680)$  resonances
- $\phi_d$  match is excellent up to elastic limit.



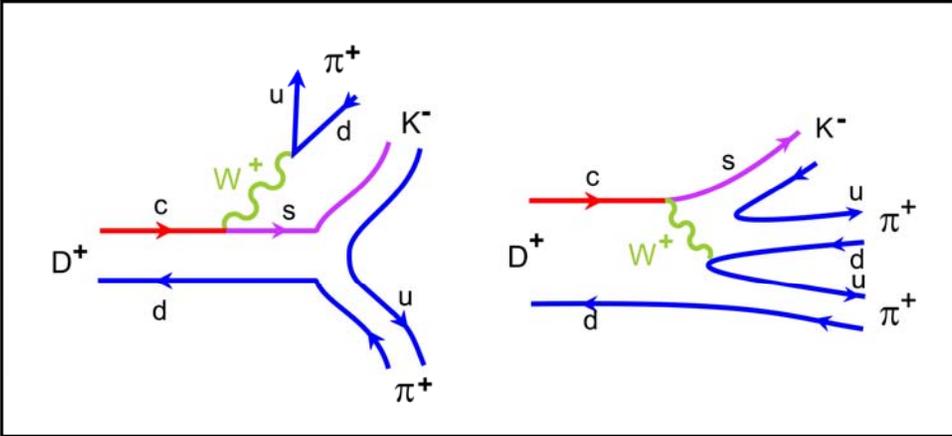
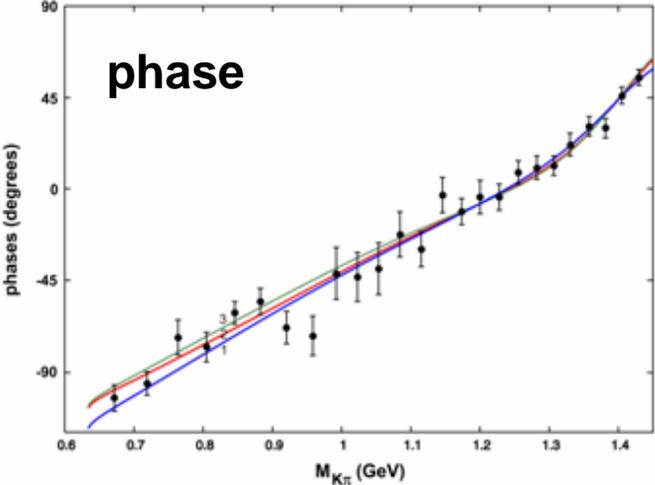


E791

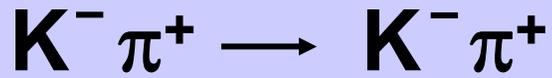
magnitude



phase



$K\pi$  sector



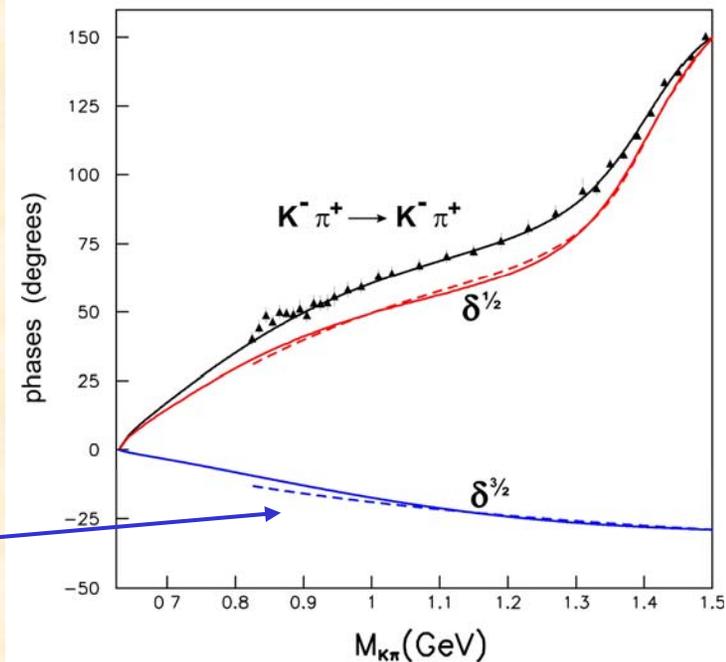
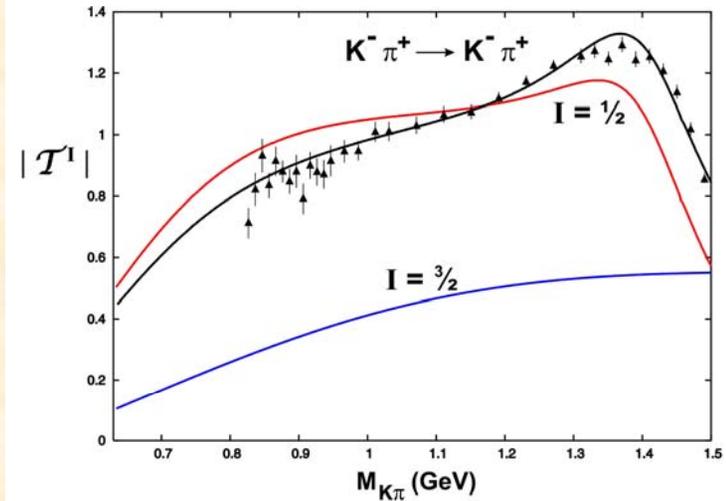
$$\mathcal{T}^I(s) = \frac{1}{\rho} \sin \delta^I \exp(i\delta^I)$$

$$\rho = 2k/\sqrt{s}$$

$$\mathcal{T}(K^- \pi^+ \rightarrow K^- \pi^+; s) =$$

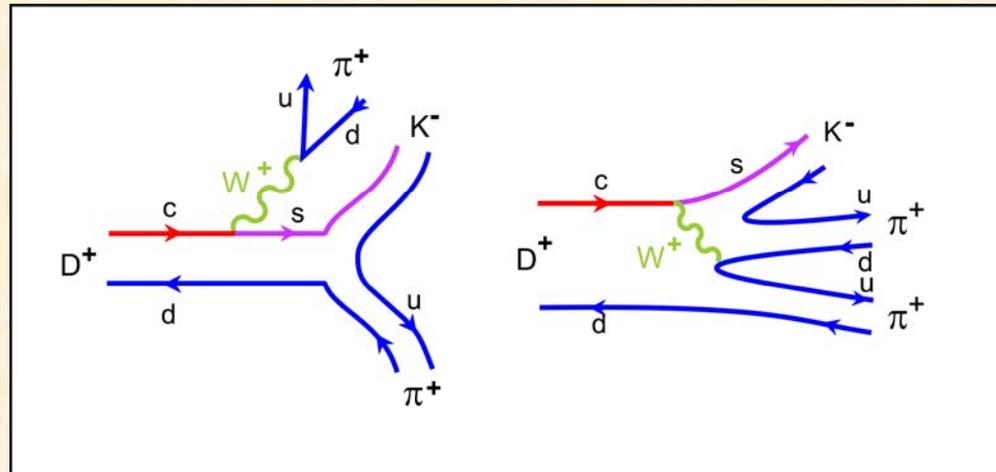
$$\frac{2}{3\rho} \left[ \sin \delta^{1/2} \exp(i\delta^{1/2}) + \frac{1}{2} \sin \delta^{3/2} \exp(i\delta^{3/2}) \right]$$

Descotes-Genon et al.



$$D^+ \longrightarrow K^- \pi^+ \pi^+$$

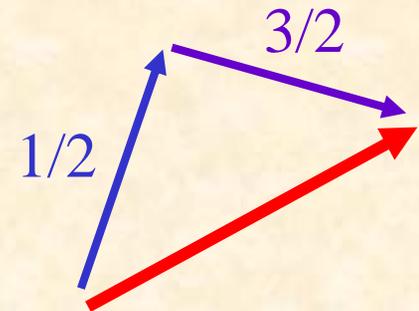
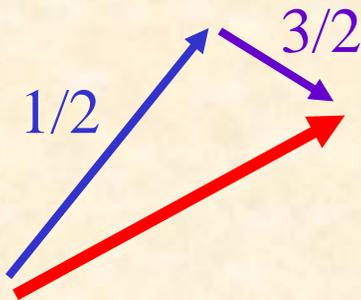
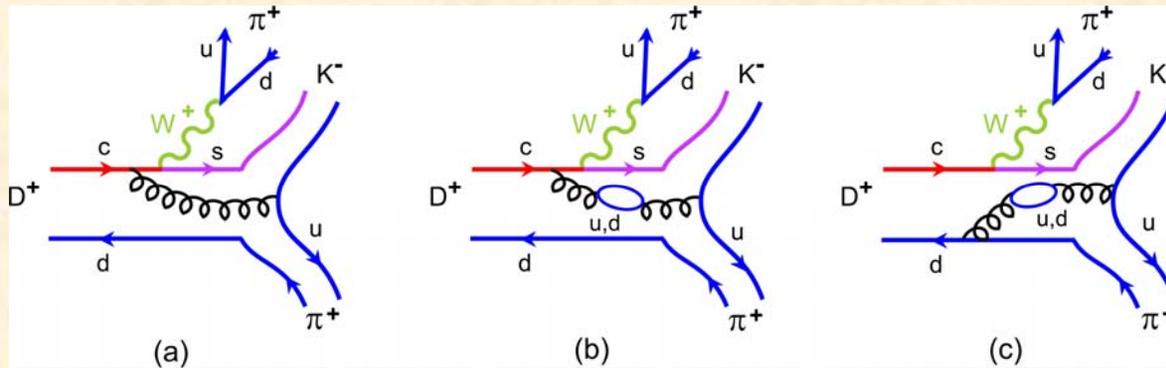
$$\mathcal{A} = \mathcal{F}^{1/2} + \mathcal{F}^{3/2}$$



$$\mathcal{F}^I(s)_{had} = | \mathcal{F}_{had}^I(s) | \exp [i\delta^I(s) + i\beta_I]$$



$$\mathcal{A} = \mathcal{F}^{1/2} + \mathcal{F}^{3/2}$$

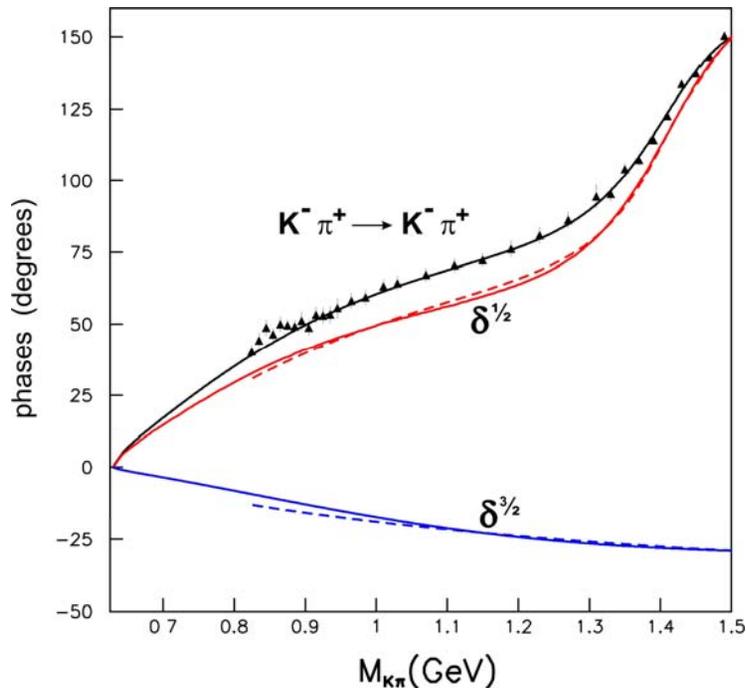


$$\mathcal{F}^I(s)_{had} = |\mathcal{F}_{had}^I(s)| \exp [i\delta^I(s) + i\beta_I]$$



$$\mathcal{F}^{1/2}(E) = \mathcal{A} \frac{\sin(\delta^{3/2}(E) + \beta_{3/2} - \phi(E))}{\sin(\delta^{3/2}(E) - \delta^{1/2}(E) - \beta_{1/2} + \beta_{3/2})} \exp[i(\delta^{1/2}(E) + \beta_{1/2})]$$

$$\mathcal{F}^{3/2}(E) = \mathcal{A} \frac{\sin(\delta^{1/2}(E) + \beta_{1/2} - \phi(E))}{\sin(\delta^{1/2}(E) - \delta^{3/2}(E) + \beta_{1/2} - \beta_{3/2})} \exp[i(\delta^{3/2}(E) + \beta_{3/2})]$$



$$\beta_{1/2} = \phi(E_r) - \delta^{1/2}(E_r) + m\pi, \quad \beta_{3/2} = \phi(E_r) - \delta^{3/2}(E_r) + n\pi$$



$$\mathcal{F}^{1/2}(E) = \mathcal{A} \frac{\sin(\delta^{3/2}(E) + \beta_{3/2} - \phi(E))}{\sin(\delta^{3/2}(E) - \delta^{1/2}(E) - \beta_{1/2} + \beta_{3/2})} \exp[i(\delta^{1/2}(E) + \beta_{1/2})]$$

$$\mathcal{F}^{3/2}(E) = \mathcal{A} \frac{\sin(\delta^{1/2}(E) + \beta_{1/2} - \phi(E))}{\sin(\delta^{1/2}(E) - \delta^{3/2}(E) + \beta_{1/2} - \beta_{3/2})} \exp[i(\delta^{3/2}(E) + \beta_{3/2})]$$

$$\beta_{1/2} = \phi(E_r) - \delta^{1/2}(E_r) + m\pi, \quad \beta_{3/2} = \phi(E_r) - \delta^{3/2}(E_r) + n\pi$$

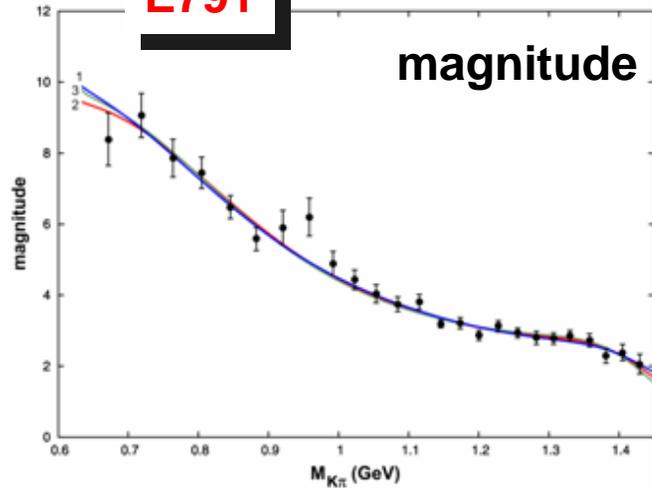
$$\mathcal{F}^{1/2}(E) = \mathcal{A} \frac{\sin(\phi(E) - \phi(E_r) - \delta^{3/2}(E) + \delta^{3/2}(E_r))}{\sin(\delta^{1/2}(E) - \delta^{3/2}(E) - \delta^{1/2}(E_r) + \delta^{3/2}(E_r))} \exp[i(\delta^{1/2}(E) - \delta^{1/2}(E_r) + \phi(E_r))]$$

$$\mathcal{F}^{3/2}(E) = \mathcal{A} \frac{\sin(\delta^{1/2}(E) - \delta^{1/2}(E_r) - \phi(E) + \phi(E_r))}{\sin(\delta^{1/2}(E) - \delta^{3/2}(E) - \delta^{1/2}(E_r) + \delta^{3/2}(E_r))} \exp[i(\delta^{3/2}(E) - \delta^{3/2}(E_r) + \phi(E_r))]$$

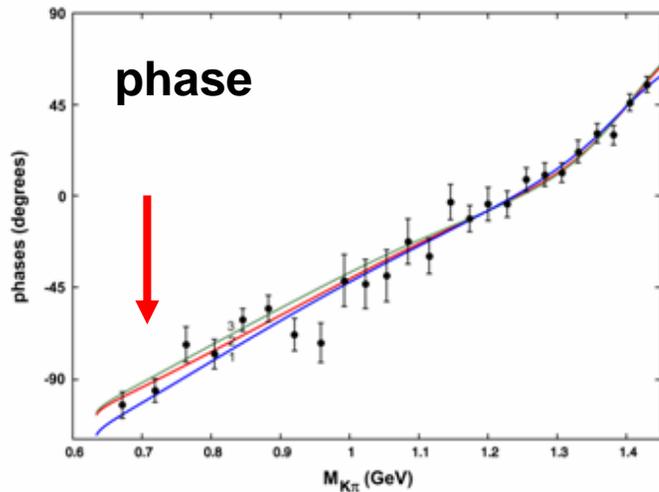


E791

magnitude

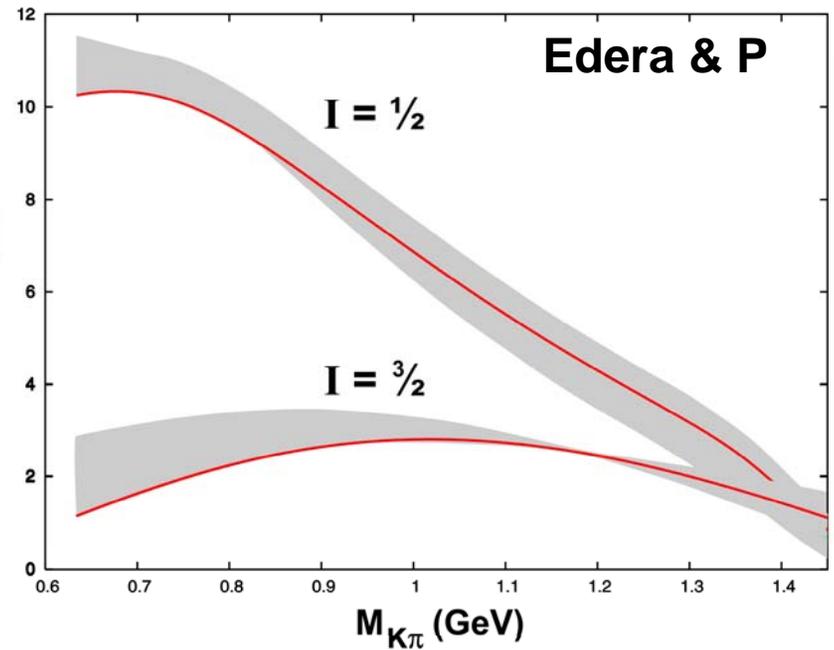


phase



Edera & P

$|\mathcal{F}^I|$



# Scalar mesons



$f_0(600)$

$f_0(980)$

$f_0(1710)$

$K_0^*(1430)$

$f_0(1370)$

$f_0(1500)$

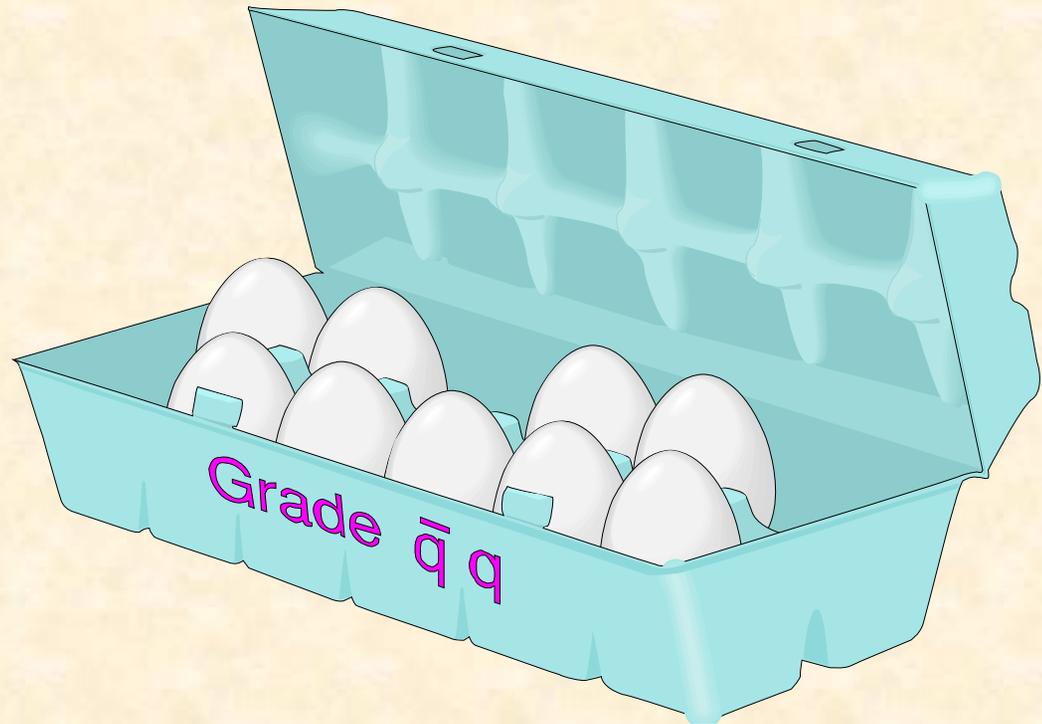
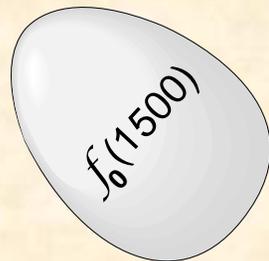
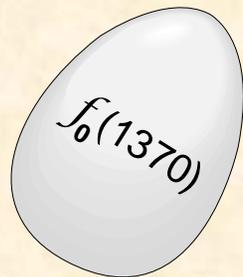
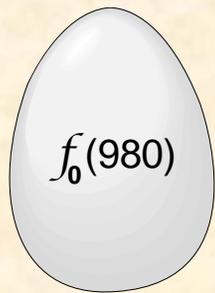
$K(800)$

$a_0(980)$

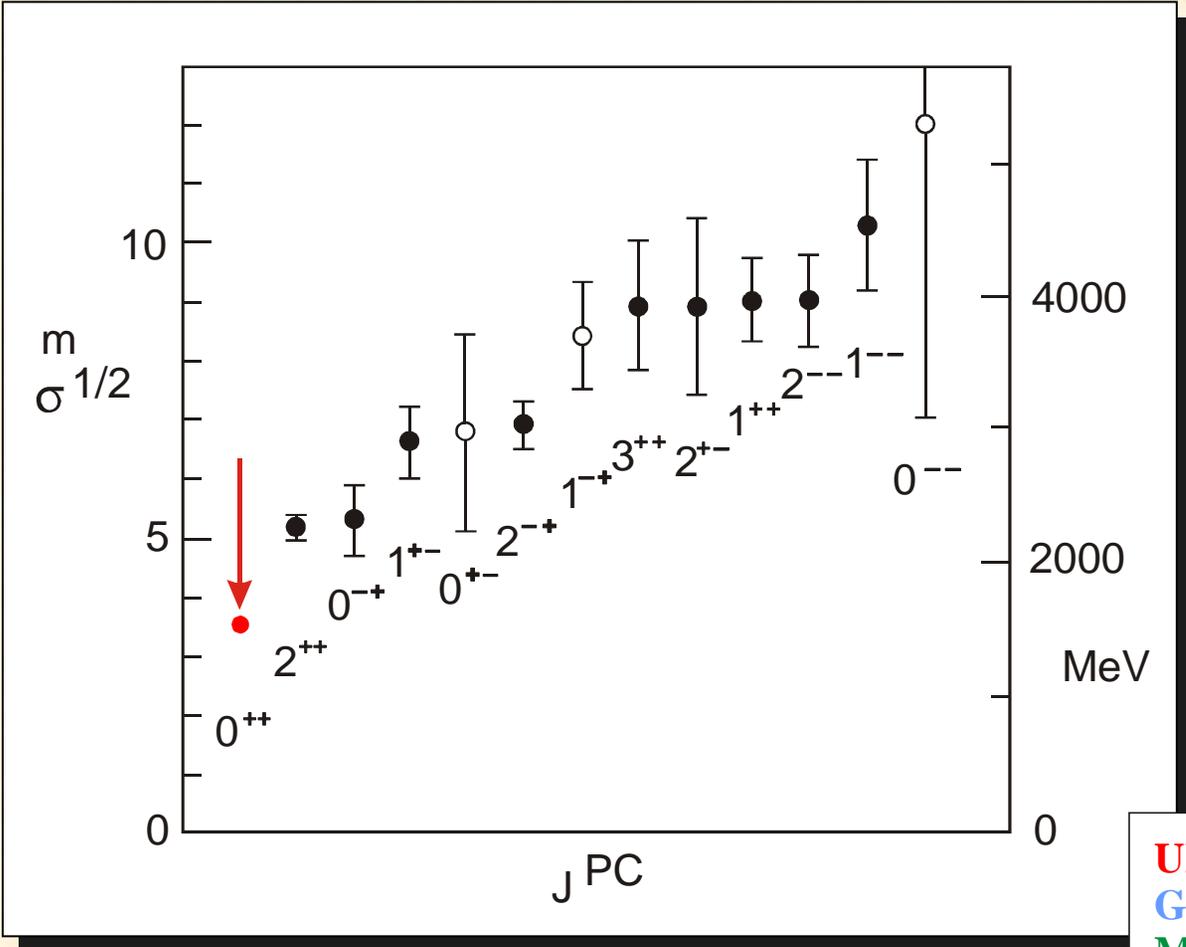
$a_0(1450)$

$K_0^*(1950)$

Which  $f_0$  is in which nonet?



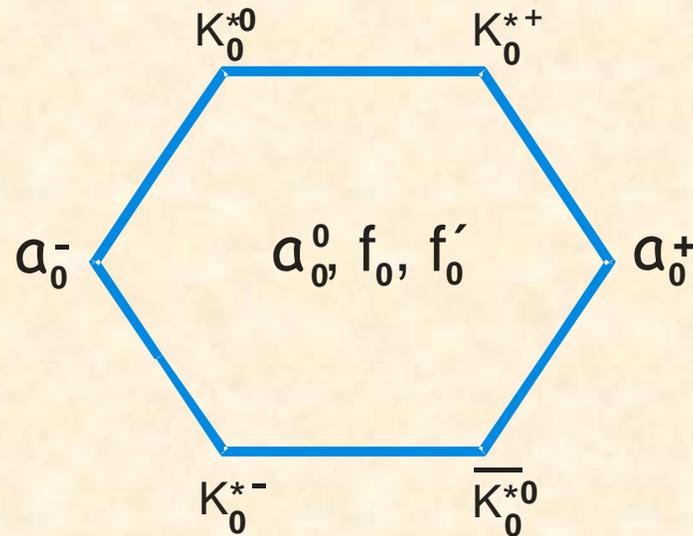
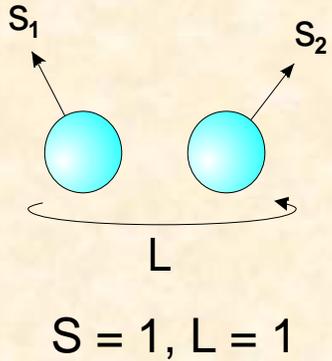
# glueball spectrum in a world without quarks



<b>UKQCD</b>	m = {	<b>1568 ± 89</b>
<b>GF11</b>		<b>1740 ± 71</b>
<b>MP</b>		<b>1630 ± 100</b>
<b>GF11 (reanal)</b>		<b>1648 ± 58</b>

first find the  
scalar meson multiplet

$$J^{PC} = 0^{++}$$

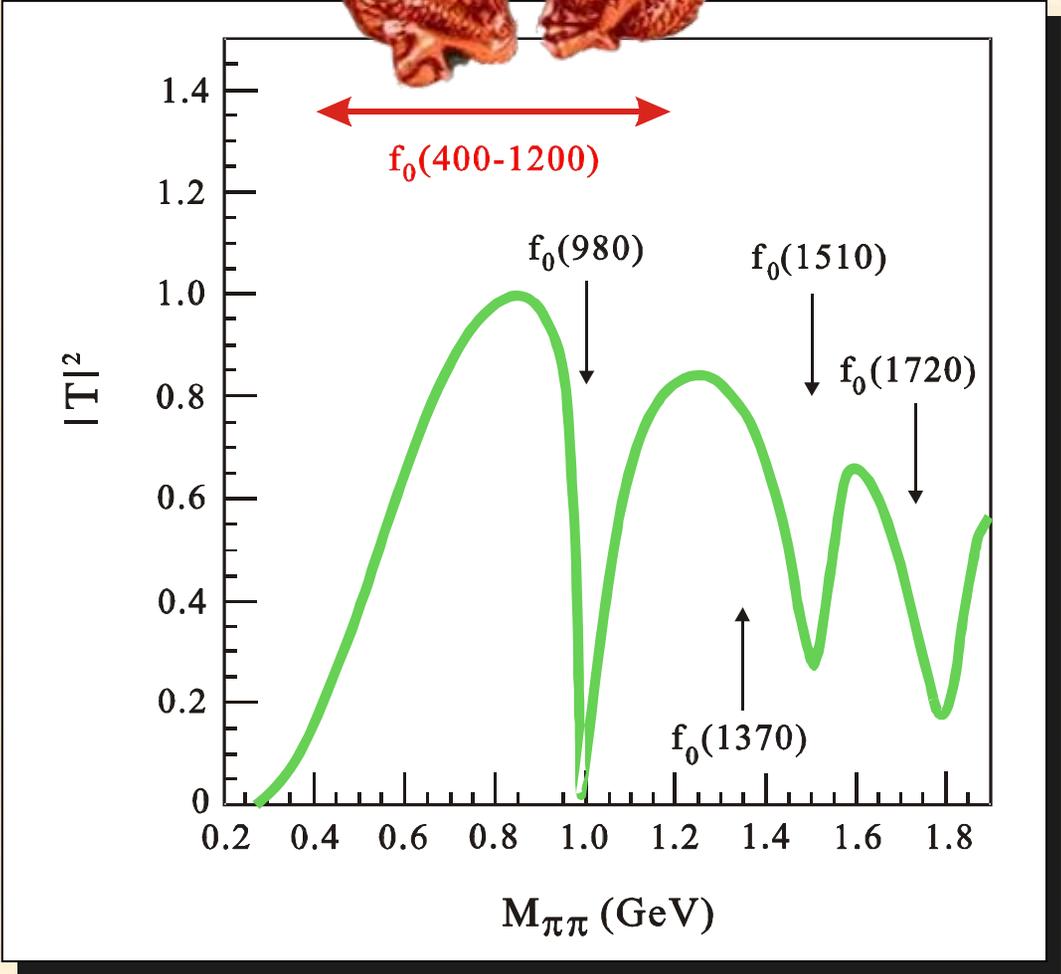


then find the extras

$$I = J = 0$$



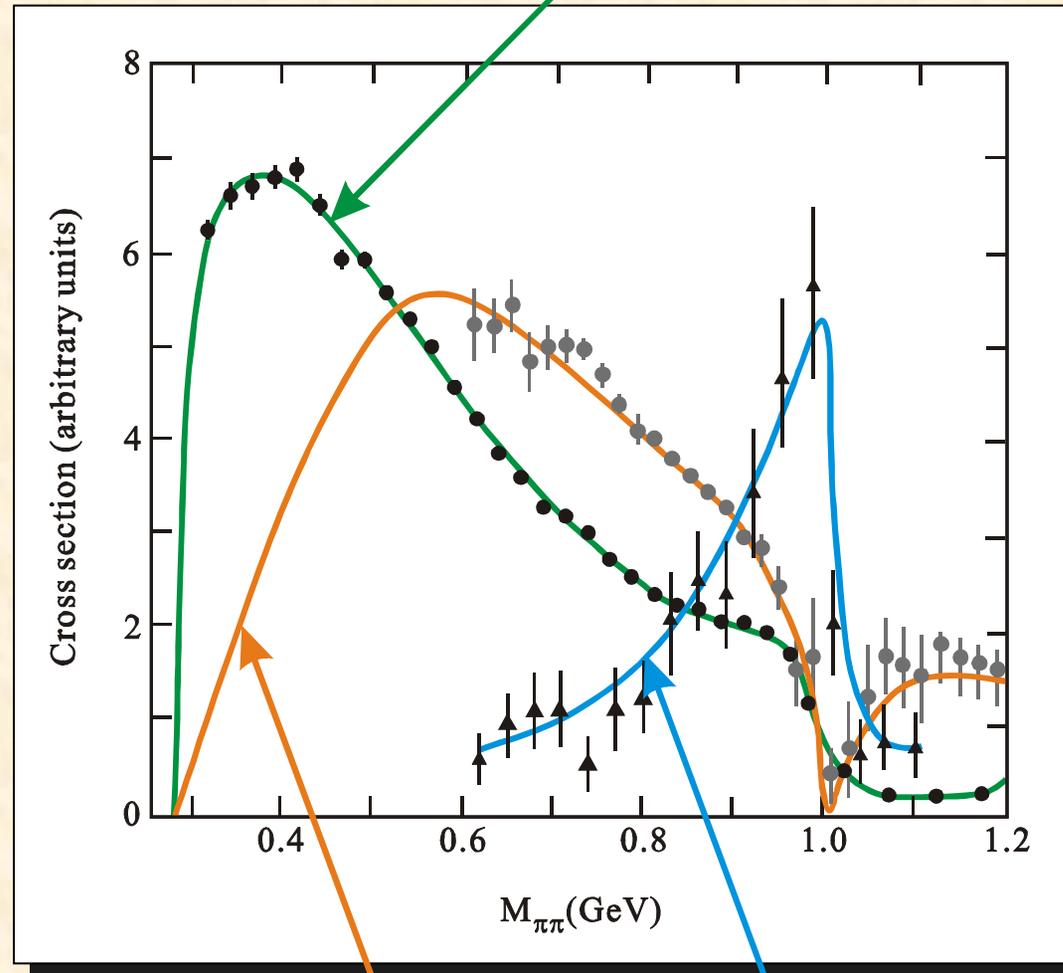
Minkowski  
& Ochs



# $\pi\pi$ sector in scattering & decays



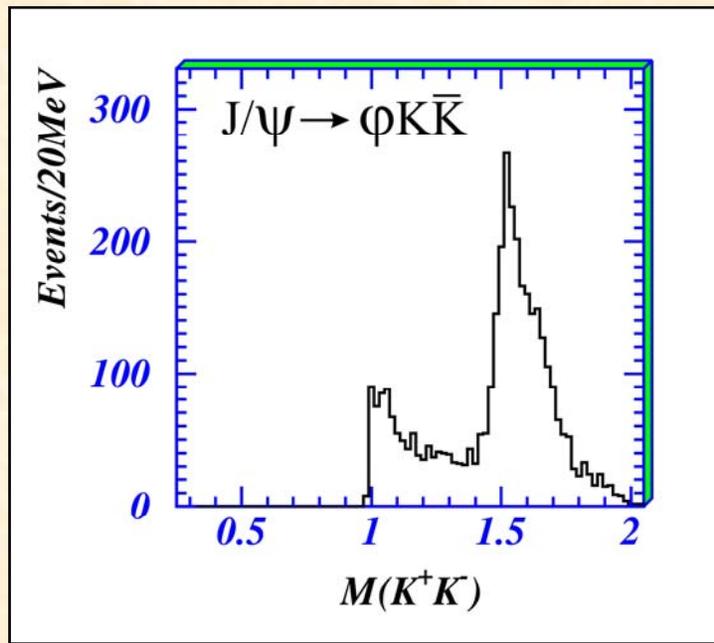
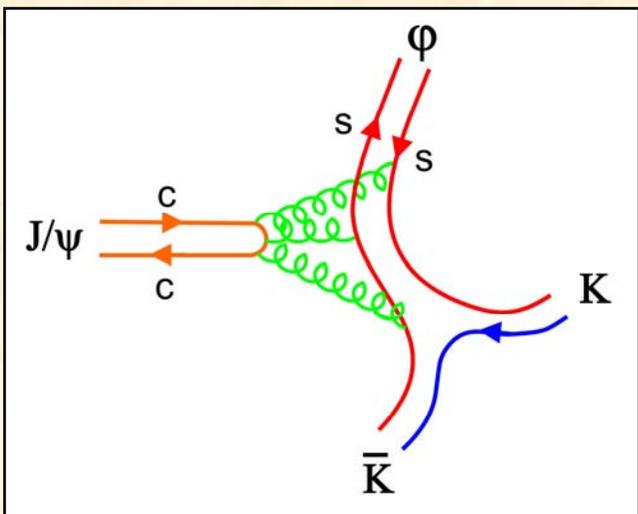
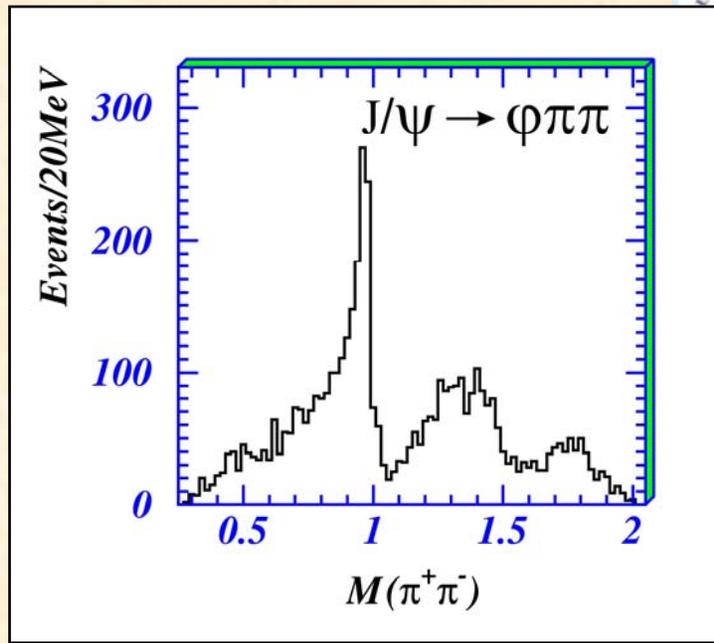
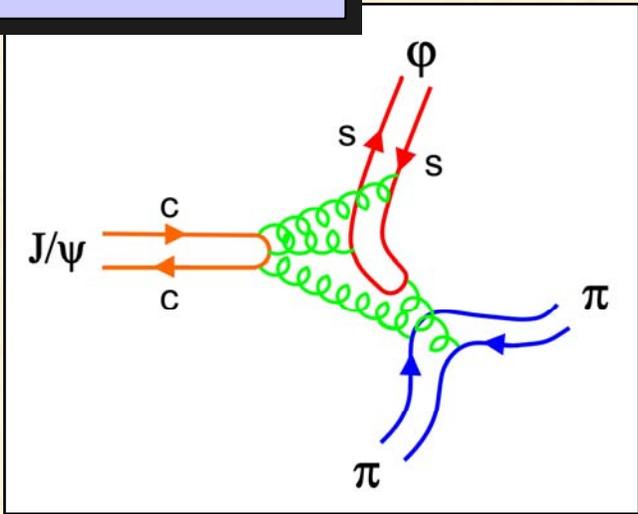
$pp \rightarrow pp(\pi\pi)$



$\pi\pi \rightarrow \pi\pi$

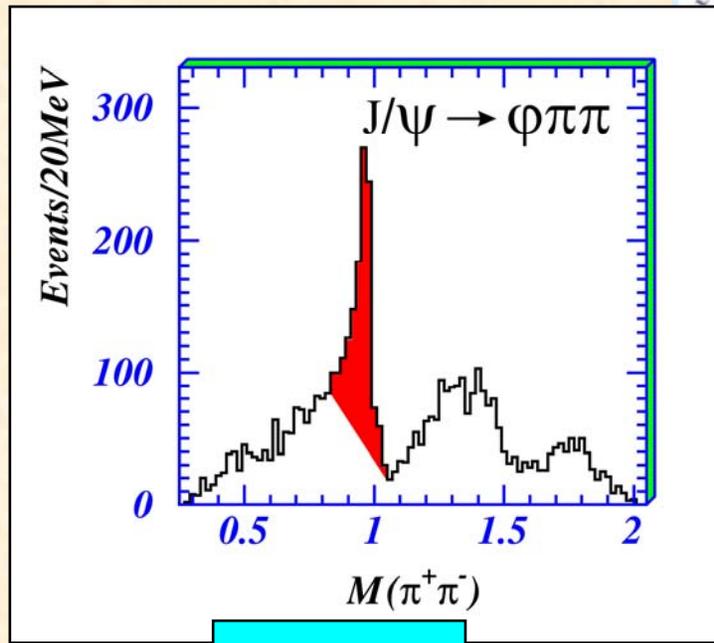
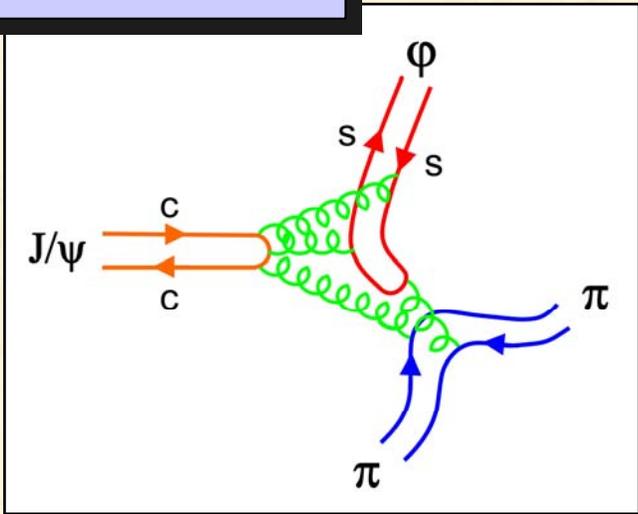
$\psi \rightarrow \phi(\pi\pi)$

# $J/\psi \rightarrow \phi M^+M^-$

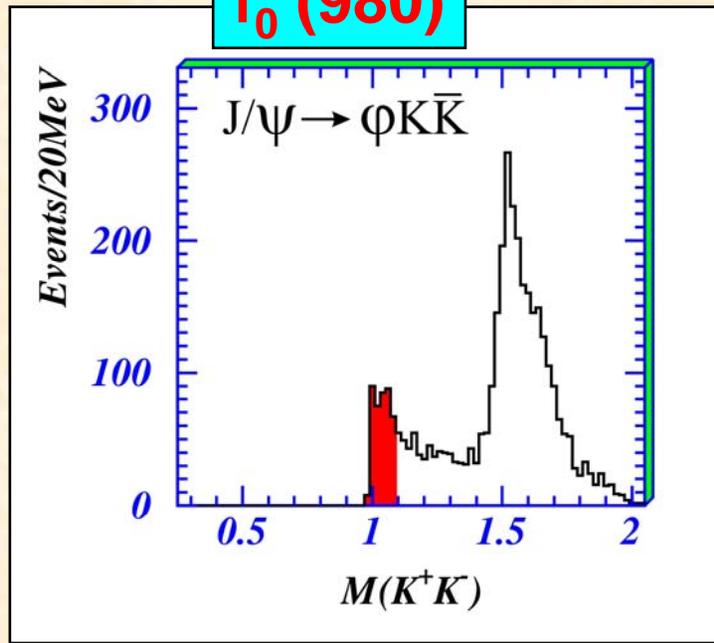
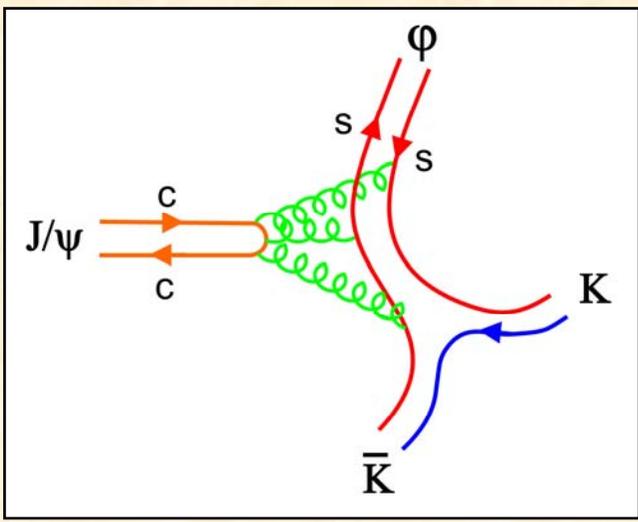


BES

# $J/\psi \rightarrow \phi M^+M^-$



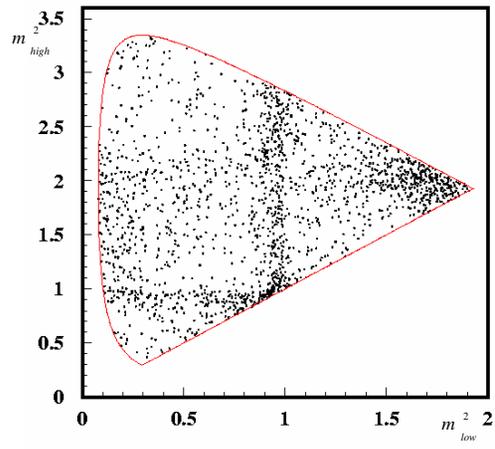
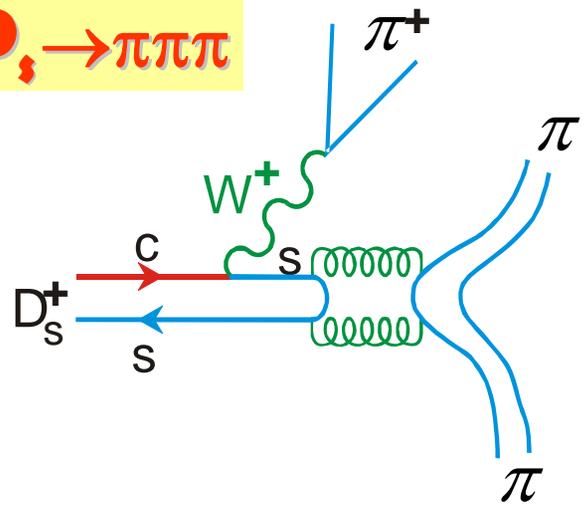
**$f_0(980)$**



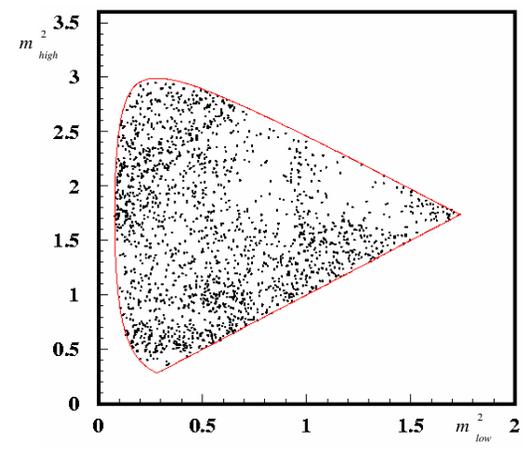
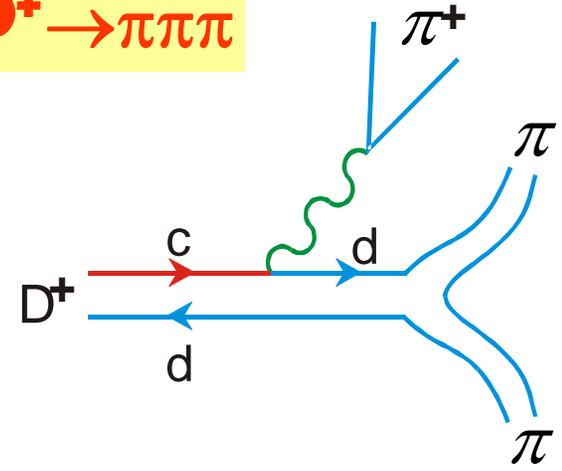
**BES**



**$D_s^+ \rightarrow \pi\pi\pi$**

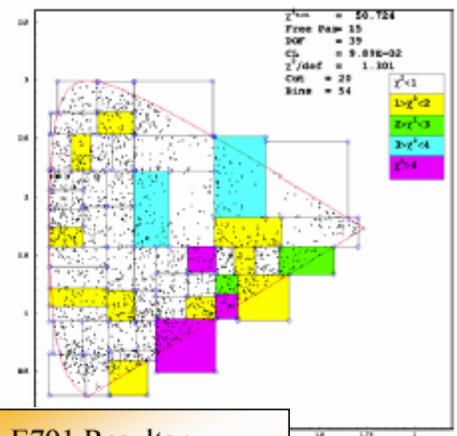
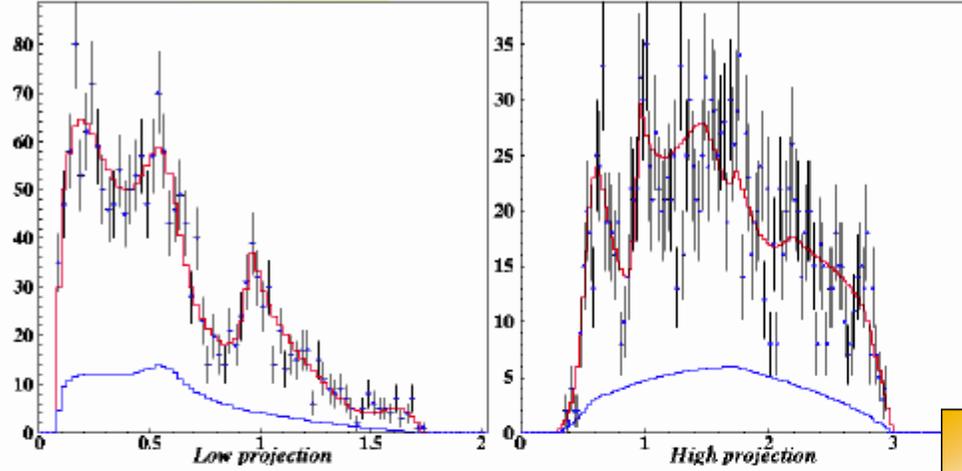


**$D^+ \rightarrow \pi\pi\pi$**



Preliminary

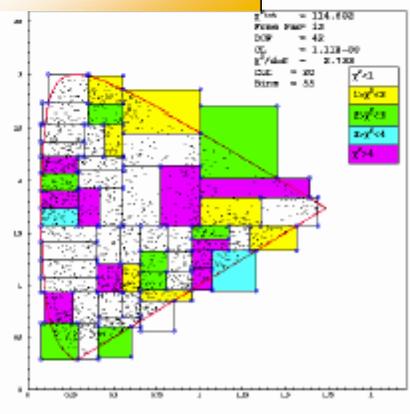
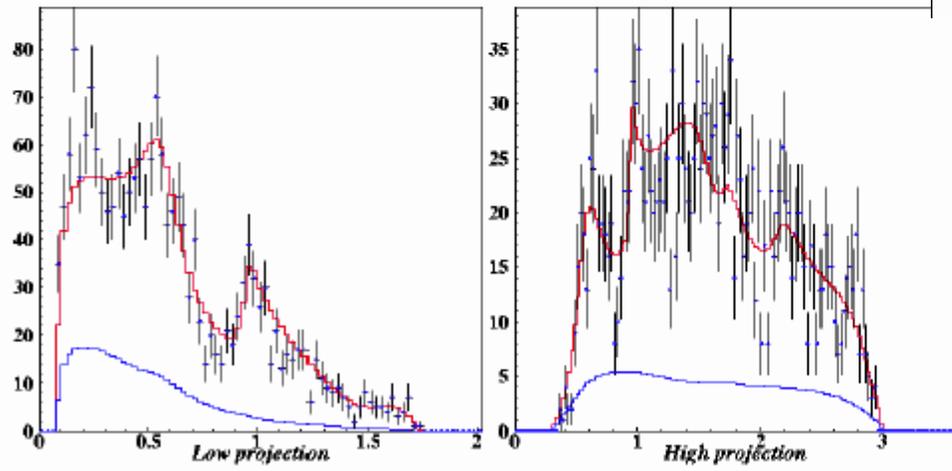
With  $f_0(400)$



E791 Results :  
 $m = 478^{+24}_{-23} \pm 17 \text{ MeV}$   
 $\Gamma = 324^{+42}_{-40} \pm 21 \text{ MeV}$

C.L. ~ 10%

Without  $f_0(400)$



~ 10<sup>-8</sup> %

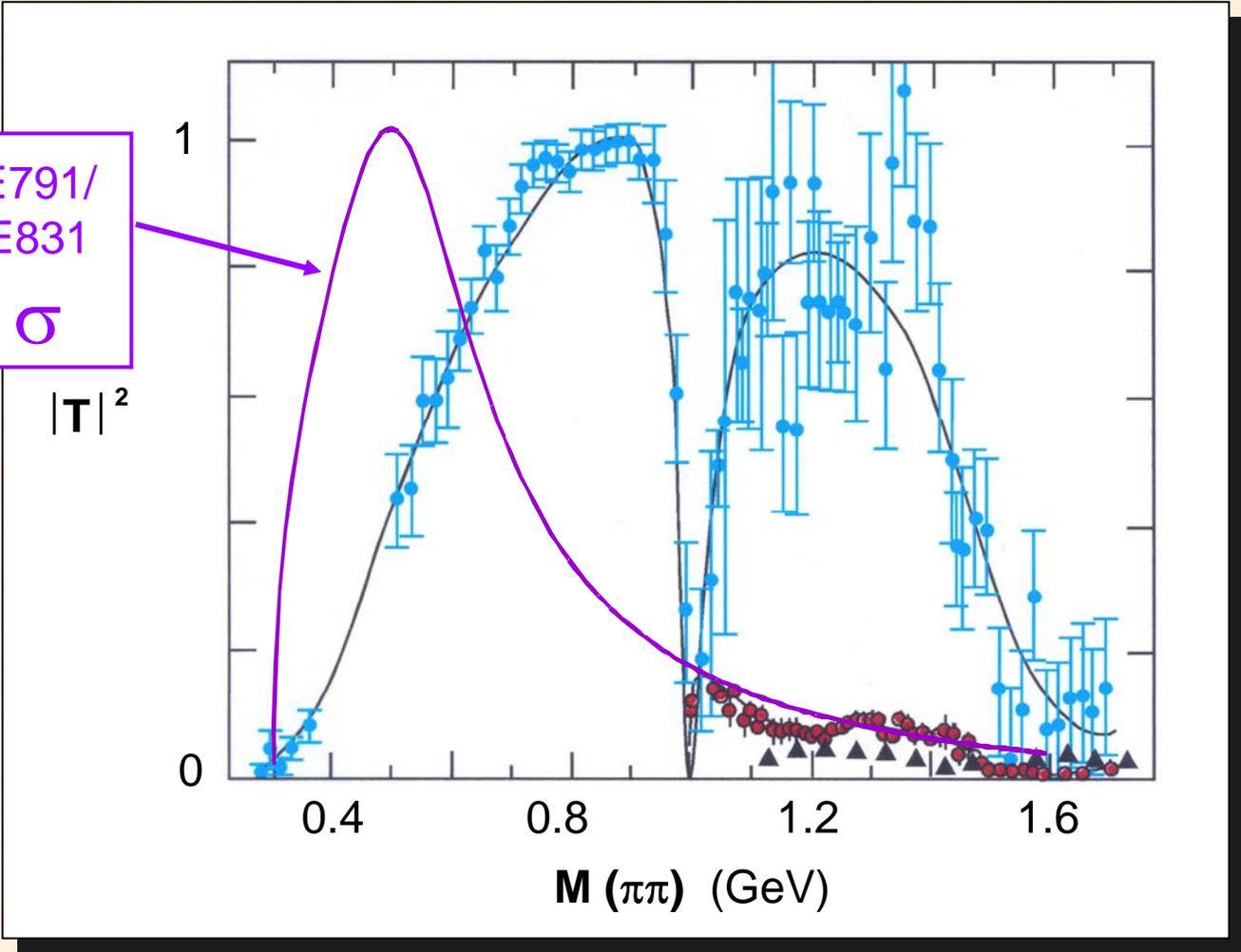


Sandra Malvezzi - Dalitz plot in the charm sector



# I = J = 0

E791/  
E831  
 $\sigma$



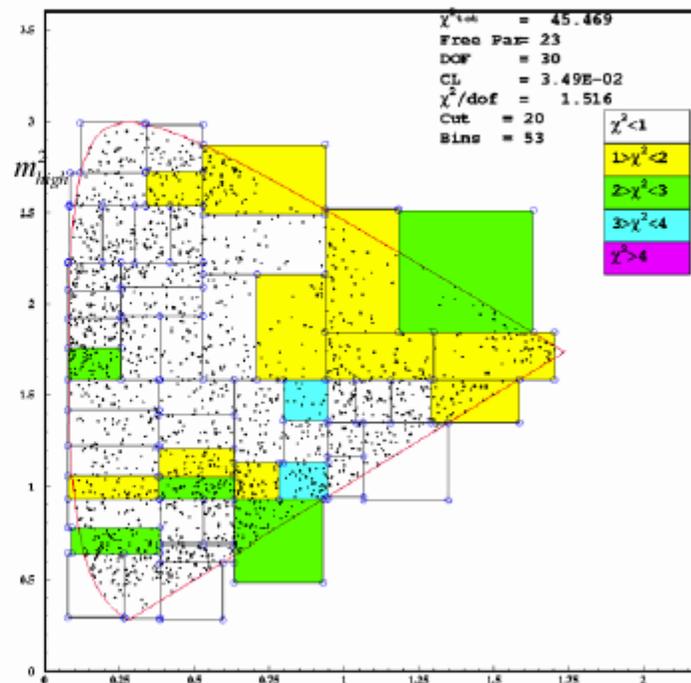
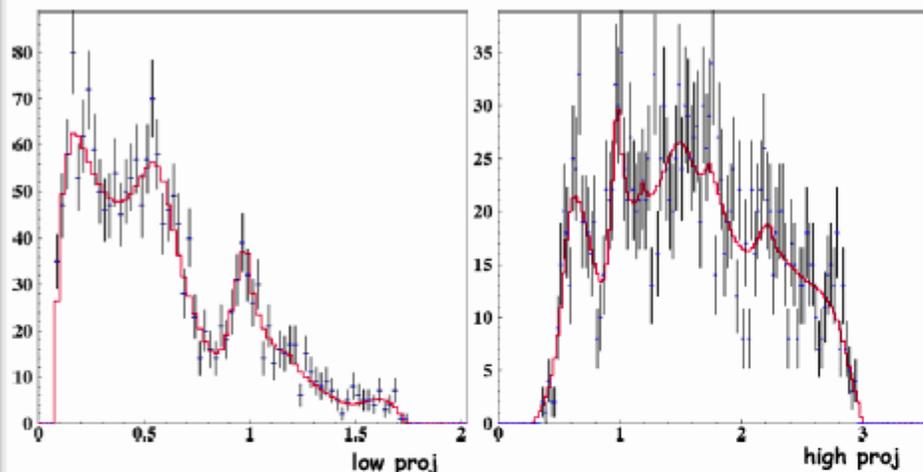
●  $\pi\pi \rightarrow \pi\pi$

●  $\pi\pi \rightarrow K\bar{K}$

▲  $\pi\pi \rightarrow \eta\eta$

$D^+ \rightarrow \pi\pi\pi$

Preliminary



fit fractions

phases

$\Gamma_{\text{swave}}$	$= 0.6647 \pm 0.0416$	$( 101.8 \pm 22.5)$
$\Gamma_{\rho(770)}$	$= 0.2116 \pm 0.0436$	$( 0.0 \pm 0.0)$
$\Gamma_{f_2(1275)}$	$= 0.1143 \pm 0.0142$	$(-113.0 \pm 9.0)$

$$\sum_r f_r \sim 99\%$$

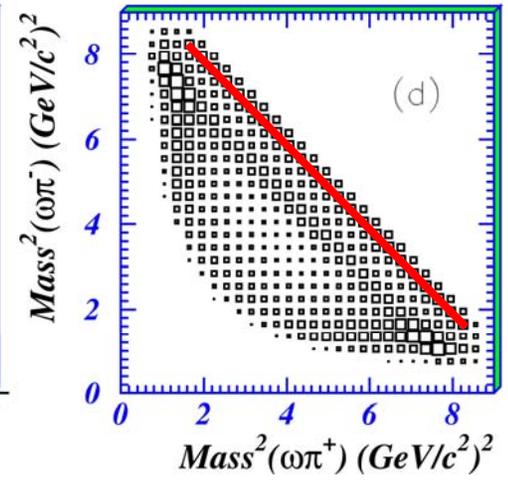
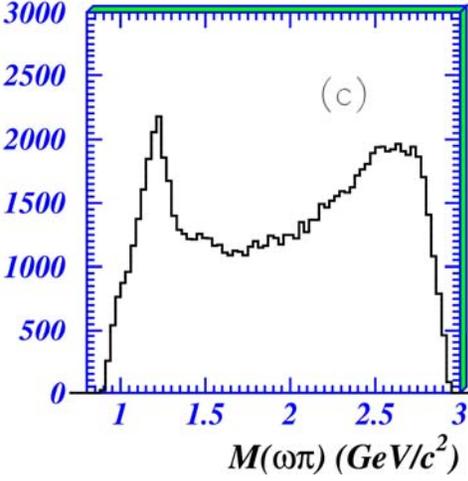
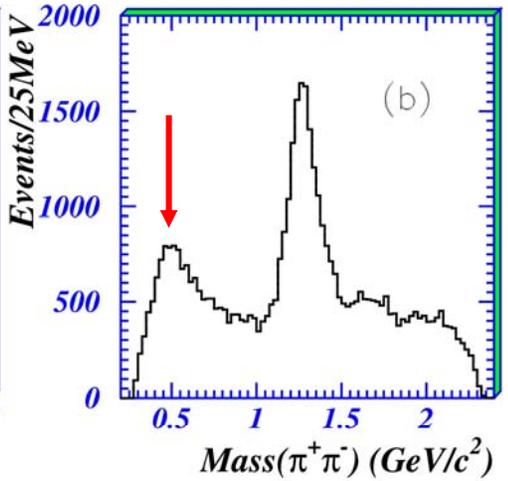
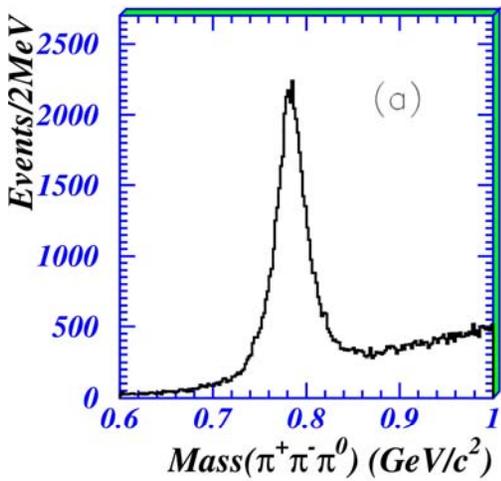
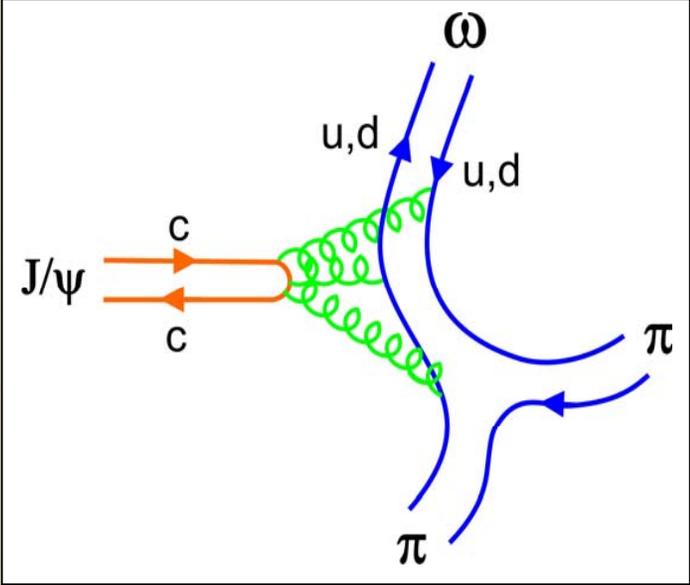


Sandra Malvezzi - Dalitz plot in the charm sector

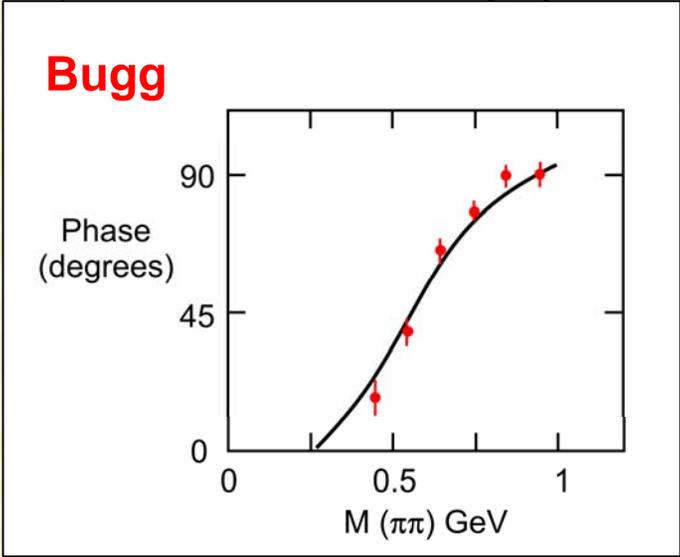
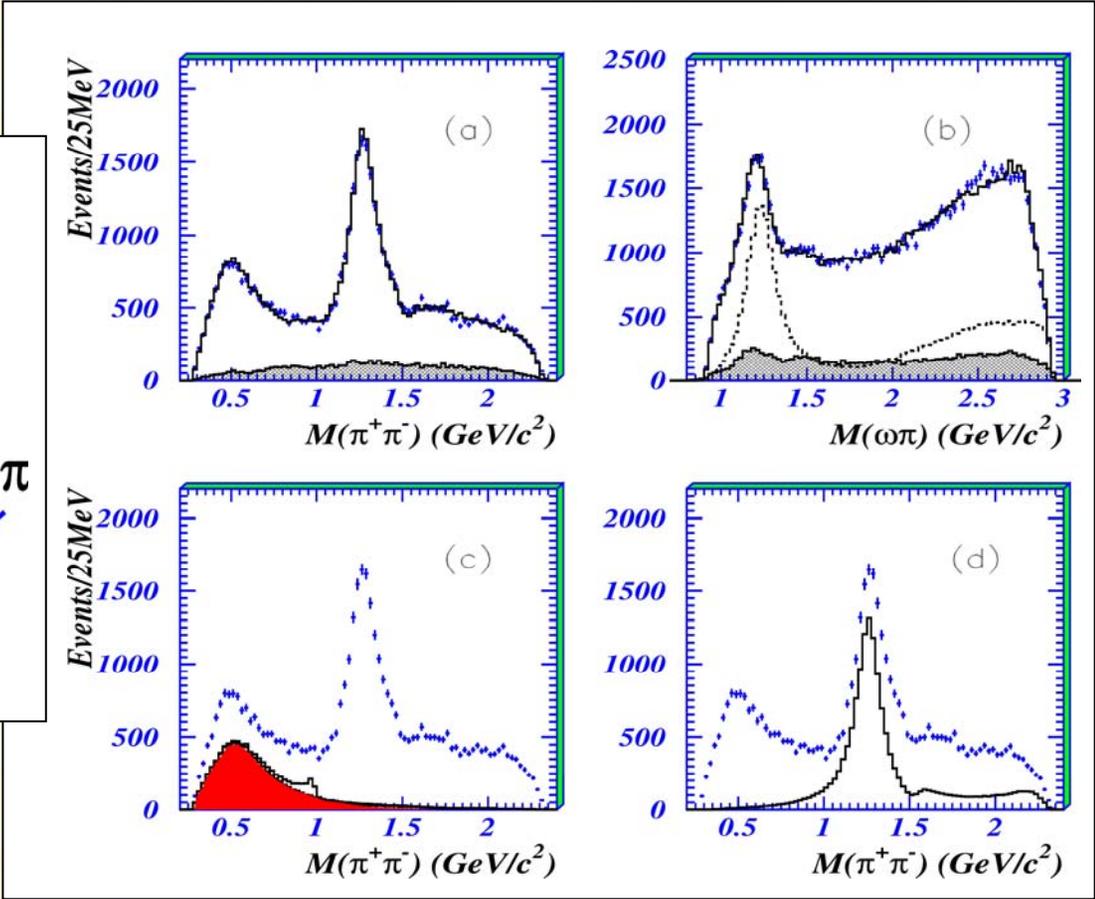
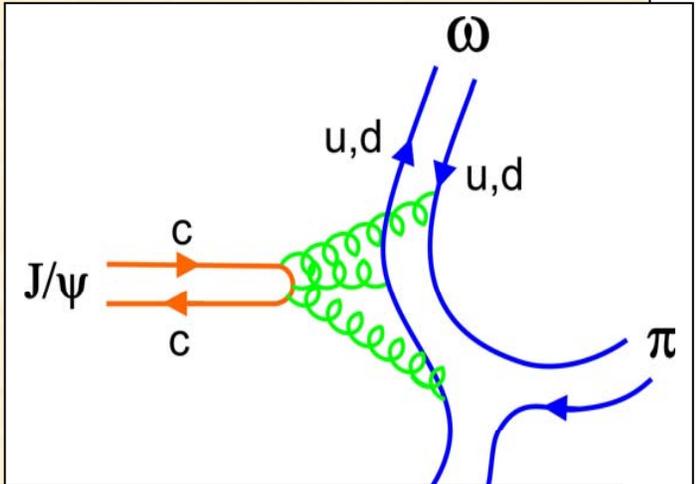
$m_{\text{low}}^2$

# $J/\psi \rightarrow \omega \pi^+ \pi^-$

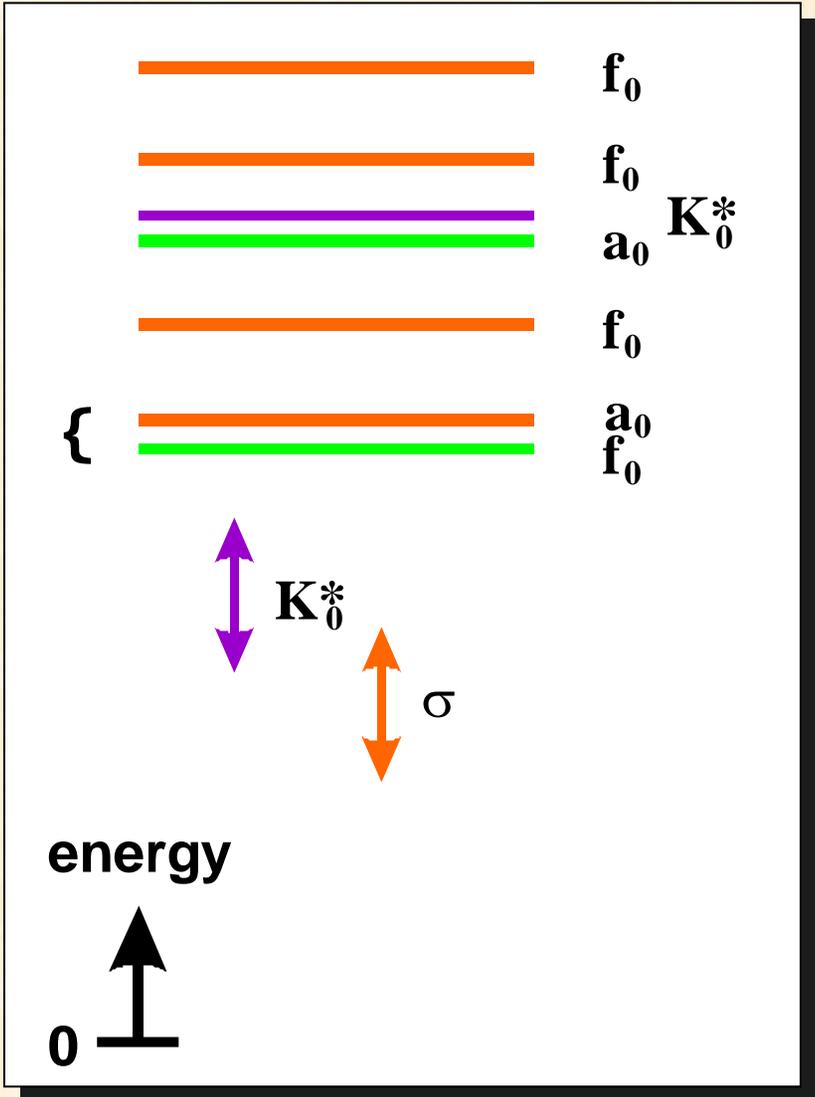
BES



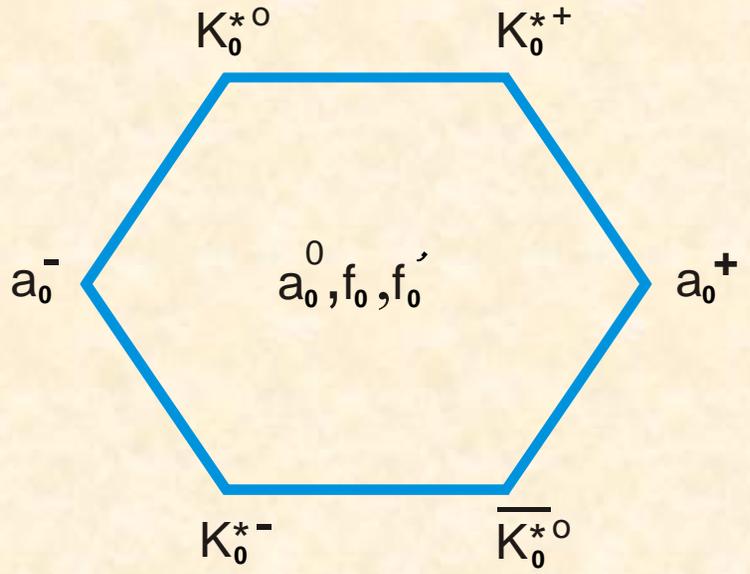
$J/\psi \rightarrow \omega \pi^+ \pi^-$



# Scalar multiplet



$$J^{PC} = 0^{++}$$



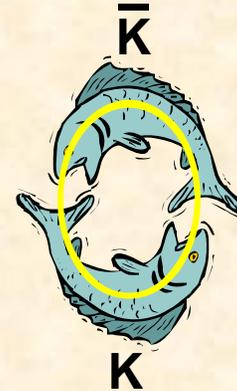
# quark model = hadron world?



$\phi$



+



$$\frac{1}{m_0^2 - s}$$



$$\frac{1}{M^2 - s - iM\Gamma}$$

# quark model = hadron world?

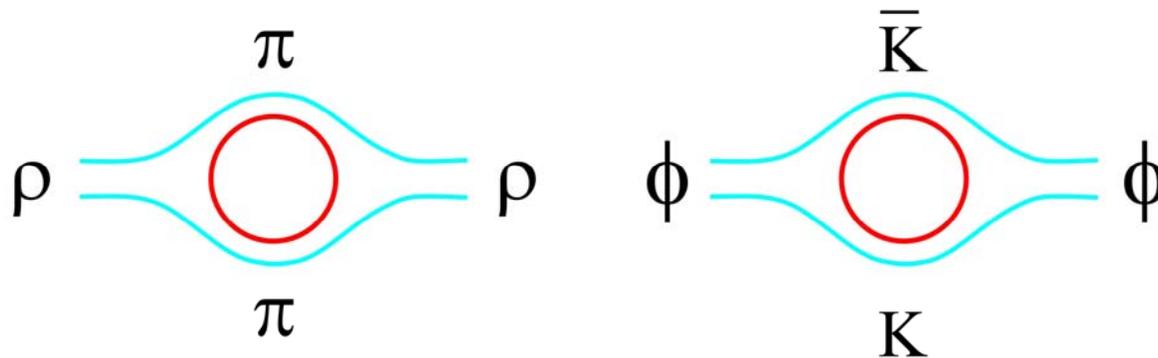
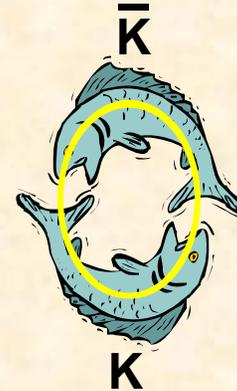


$\phi$



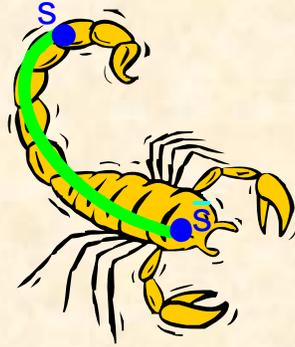
+

$\frac{1}{N_c}$

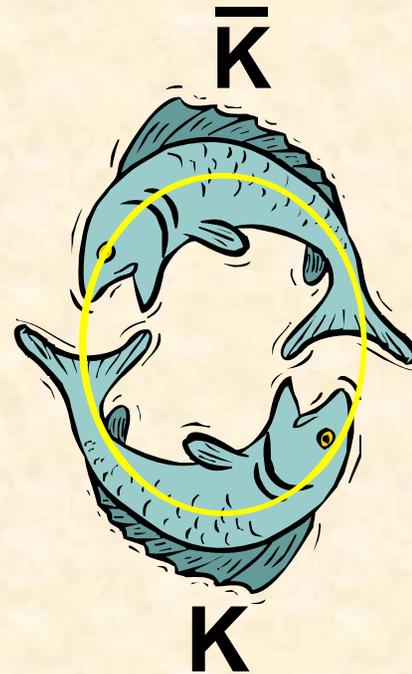


**unquenching unimportant**

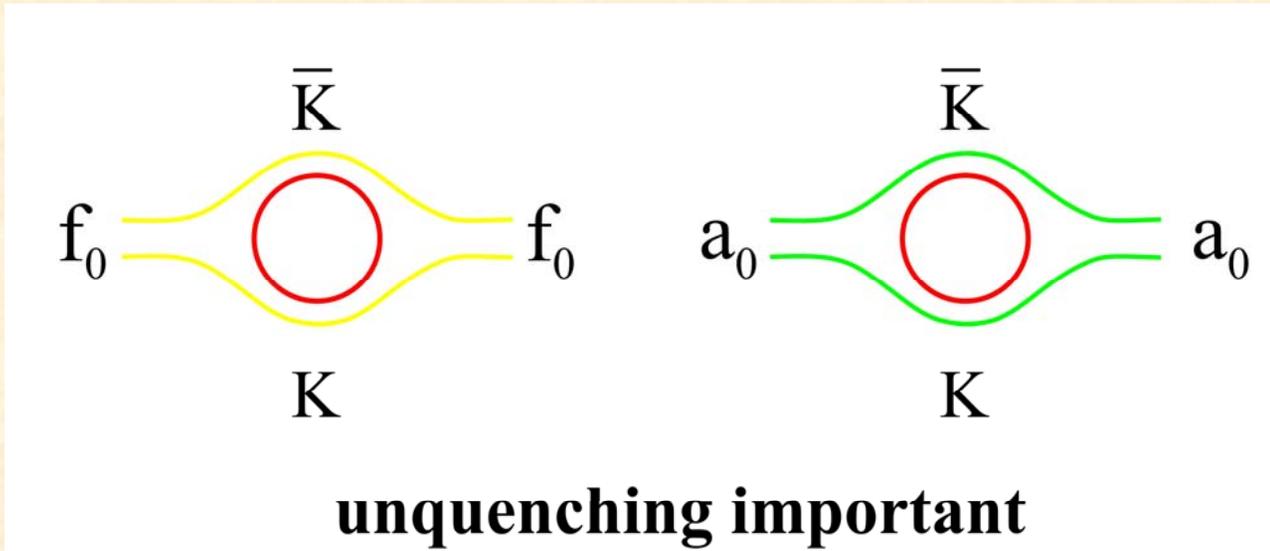
$f_0$



+



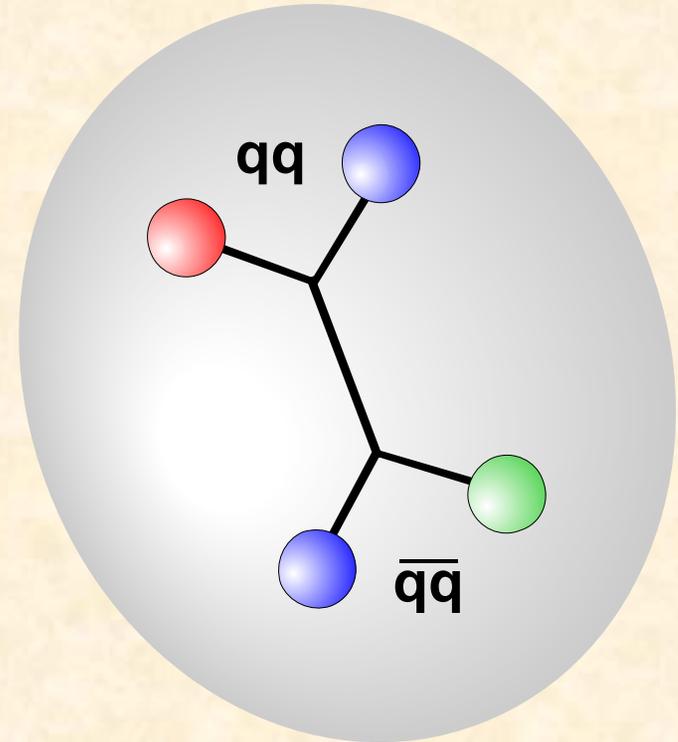
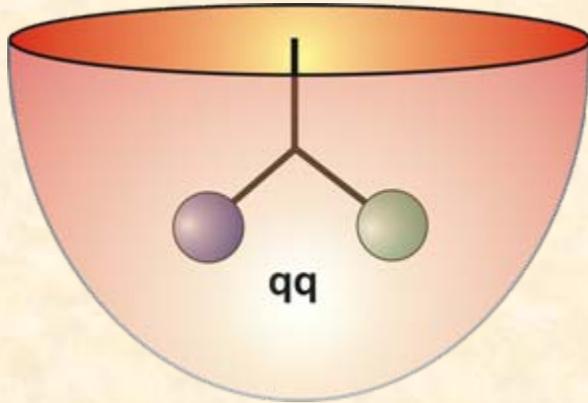
40%



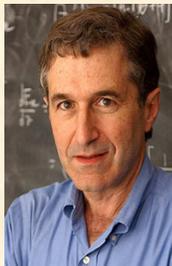
# diquarks: colour



## tetraquark



Jaffe & Wilczek



## Scalar diquarks

[ud]

[us]

[ds]

[cd]

[cu]

[cs]

# Scalar meson multiplets



**$q\bar{q}$**

**$q\bar{q}q\bar{q}$**

- $\bar{s}s$  —————  $f_0$
- $\bar{s}n$  —————  $K_0$
- $\bar{n}n$  —————  $a_0/f_0$

- $\bar{s}s\bar{n}n$  —————  $a_0/f_0$
- $\bar{s}n\bar{n}n$  —————  $K_0$   $\kappa$
- $\bar{n}n\bar{n}n$  —————  $f_0$   $\sigma$





# Scalar meson multiplets

$q\bar{q}$

$q\bar{q}q\bar{q}$

$\bar{s}s$  —————  $f_0$   
 $\bar{s}n$  —————  $K_0$   
 $\bar{n}n$  —————  $a_0/f_0$

$\bar{s}s\bar{n}n$  —————  $a_0/f_0$   
 $\bar{s}n\bar{n}n$  —————  $K_0$   $\kappa$   
 $\bar{n}n\bar{n}n$  —————  $f_0$   $\sigma$

Jaffe

Maiani, Piccinini, Polosa, Riquer

# Scalar meson multiplets

$q\bar{q}$

$q\bar{q}q\bar{q}$

$\bar{s}s$  —————  $f_0$   
 $\bar{s}n$  —————  $K_0$   
 $\bar{n}n$  —————  $a_0/f_0$

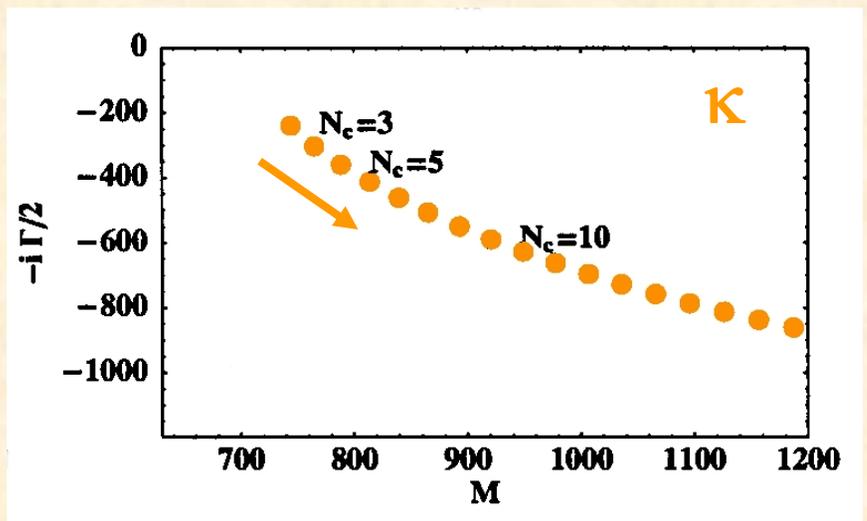
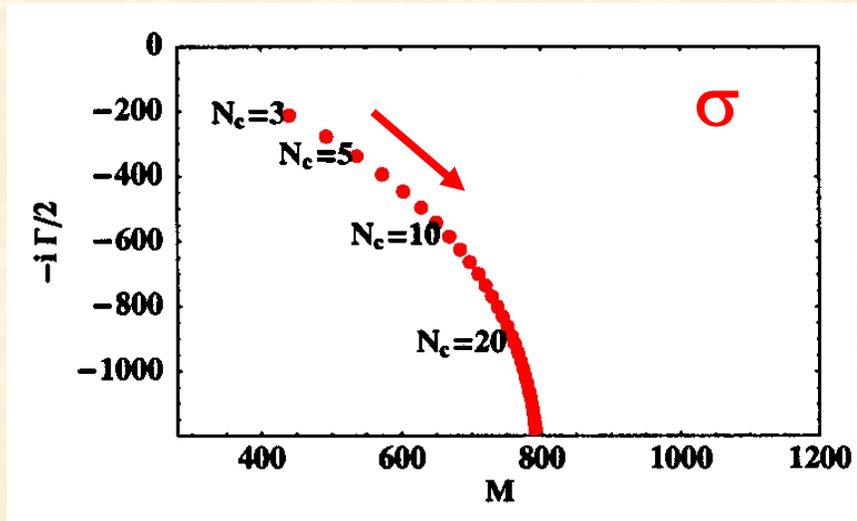
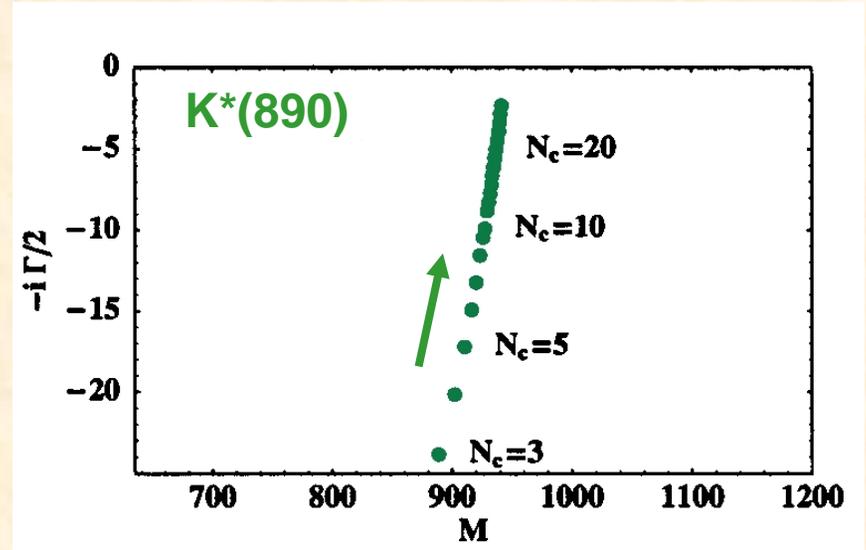
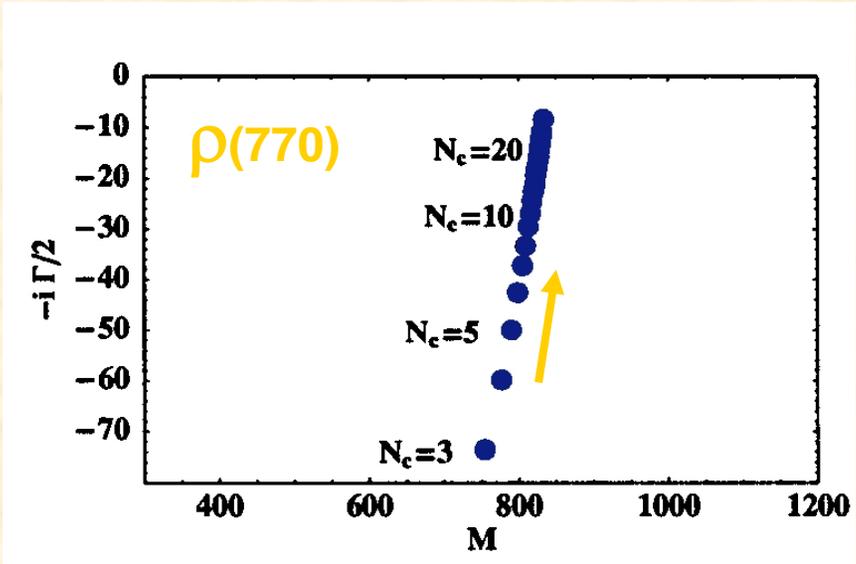
$\bar{s}s\bar{n}n$  —————  $a_0/f_0$   
 $\bar{s}n\bar{n}n$  —————  $K_0$   $\kappa$   
 $\bar{n}n\bar{n}n$  —————  $f_0$   $\sigma$

$N_c$  large  $\rightarrow$  stable

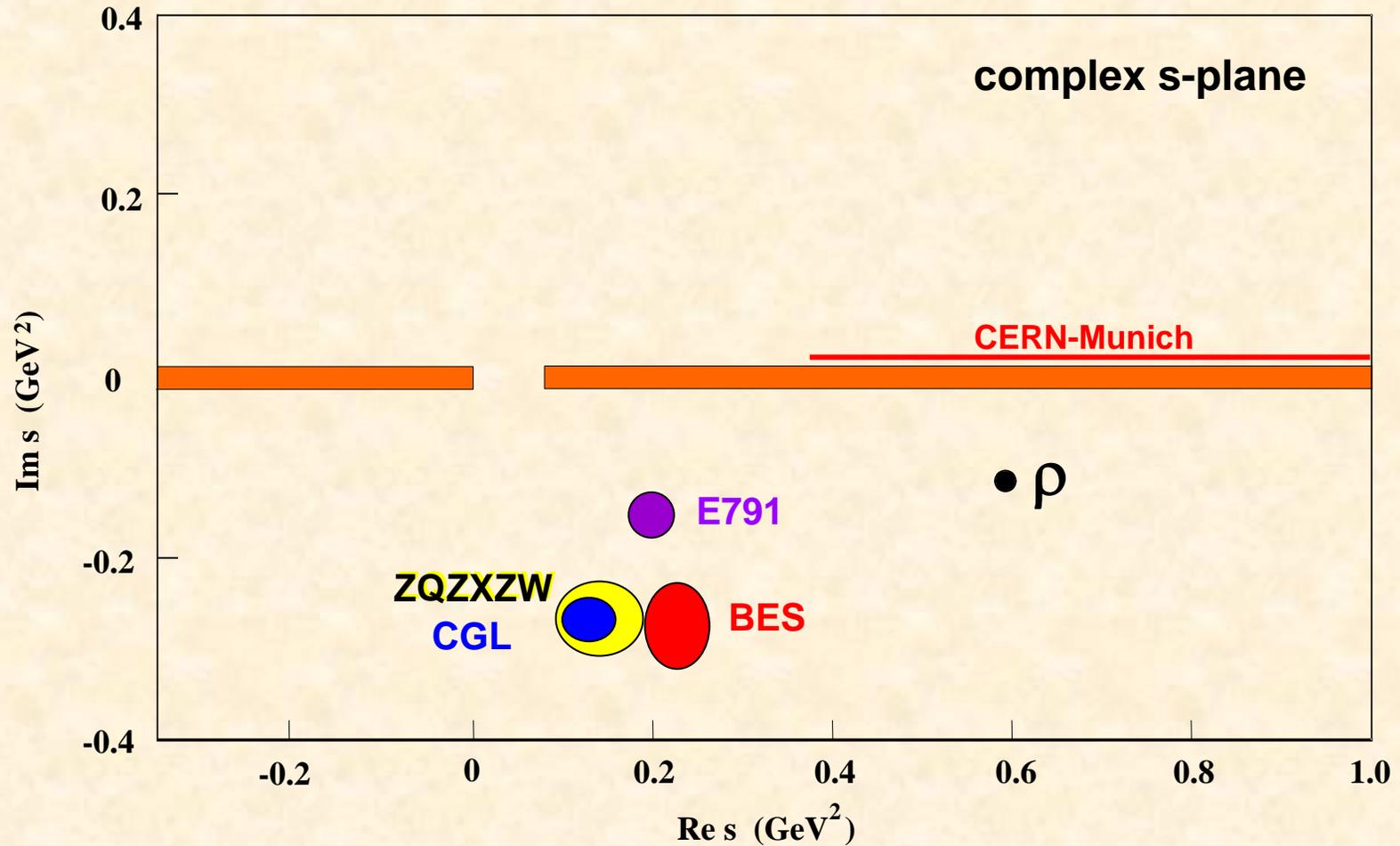
$N_c$  large  $\rightarrow$  meson continuum

$$N_c \rightarrow \infty$$

Pelaez  
Jaffe

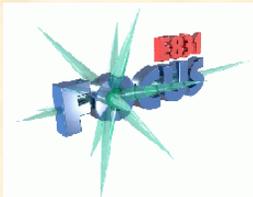


$\pi\pi : I = 0, J = 0$





**To learn about the Higgs sector of QCD**  
demands a global Dalitz analysis of  $J/\psi$ , B/D decays,  $\gamma\gamma$ , ...  
in Comprehensive Analyses



**LNF Spring Institute  
May-July 2005**