How soft a soft gluon should be

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Giulia Pancheri
INFN Frascati

In collaboration with R.M. Godbole, A. Grau and Y.N. Srivastava

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Soft Quanta Resummation

- In QED one never questions how soft a soft photon is
- the limit $k_{\gamma} \longrightarrow 0$ is well defined
- one routinely sums soft photons an d integrates their momenta down to zero

How about QCD?

Soft photons in QED

- The soft photon resummed distribution can be obtained both perturbatively and semi-classically Yennie, Frautschi and Suura, 1964
- To obtain it semi-classically is rather simple (E.P.T.,1968)

- The picture of a detector (experimentalist, according B. Touschek) as counting soft photons is unrealistic
- One rather observes an unbalance of energy and momentum between initial and final state

The Radiative Correction work with Bruno Touschek

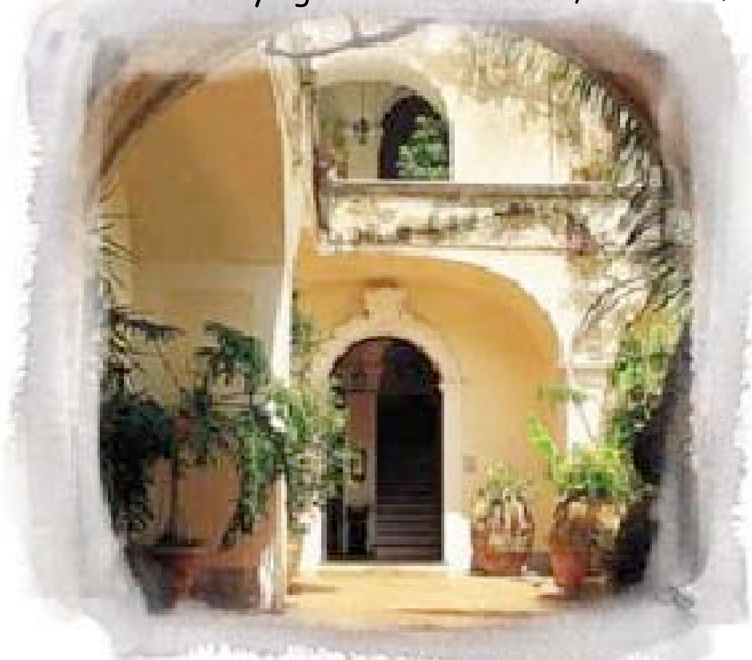
This is an example of what he called "earning our bread and butter". According to his philosophy, to earn this bread and butter we had to take care of the "administration of the Radiative Corrections". His main message at the time was that straightforward perturbation theory does not lend itself easily to dealing with the flood of soft photons which emerge from a high energy collision between charged particles. This, of course, was true at Adone's energies and still is, indeed much more so, at LEP and beyond.

.

To overcome this difficulty, he had developed a very elegant formalism, which he derived from the covariant formulation of the Bloch-Nordsieck theorem. Using this formalism one can sum all the soft massless quanta emitted by a semi-classical source. Present day soft gluon summation techniques use either this same formalism or a slightly different version of it, the coherent state formalism developed by Greco and Rossi during the same time, again under Touschek's inspiration. The Bloch-Nordsieck summation technique subsequently developed in two different directions. One corresponds to straightforward QED applications, like infrared radiative corrections to the cross-section for producing J/ψ resonance or for producing Z^o at LEP. The other corresponds to study the infrared structure of non-abelian theories and utilize the technique to sum soft gluons in QCD where the problem of higher order corrections is much more severe.

The Holiday to Positano: : September 1966

Touschek was staying at Palazzo Murat, with his family.



Bruno Touschek's humanity and warmth gave us young researchers one of the best times we all had with him, the holiday in Positano, in September 1966. On August 19 of that year, Touschek, Etim and myself, we had just finished the Radiative Correction paper and Touschek suggested that the whole group join him in early September in Positano, where he was going to stay for a month with his family. We all went driving down, in Paolo Di Vecchia's "cinquecento" .

The experience was memorable. In the morning, we would board a boat with a "marinaio" who took us to isolated coves, not otherwise reachable. We carried sandwiches and swam all day with Touschek and his family until the "marinaio" would come and take us back to the village in the evening. We would have dinner in some relatively simple place, where Touschek had made friends with proprietors and customers alike. I do not recollect, myself, the content of our conversations in Positano, just the obvious pleasure of living that came from Bruno and his desire to share it with us. We stayed only a few days but those few days have left us with one of Bruno's most serene and happy images.

Resummation (semiclassical)

$$d^4P(K) = \Sigma \Pi_k \{n_k\} \delta^4(K - \Sigma n_k k) d^4K$$

$$\uparrow$$

Poisson Distributions

$$d^4P(K) = d^4K \int e^{-iK\cdot x - \Sigma \bar{n}_k (1 - e^{-ik\cdot x})} d^4x$$

In the continuum limit $\Rightarrow$

$$d^4P(K) = d^4K \int e^{-iK\cdot x - h(x)} d^4x$$

$$h(x) = \int d^3\bar{n}_k (1 - e^{-ik\cdot x}) =$$

$$= \frac{\alpha}{(2\pi)^2} \int \frac{d^3k}{2k} \left|\frac{\Sigma p_{i\mu}}{p_i \cdot k}\right|^2 (1 - e^{-ik\cdot x})$$

About Radiative Corrections

He liked to say that the picture of an experimenter as of one counting soft photons is not entirely realistic, since he does not see single photons but rather an imbalance of energy and momentum between the incident and emergent particles. On the other hand existing perturbation theory works in a representation in which the number of photons is diagonal and the emission of any additional photons requires a further step in the perturbation procedure

QED Applications

•Energy distribution:

Calculation of $dP(K_0)$ is easy because □$_{QED}$ is outside integration

$$K_0^{\beta}(m_i, p_{i0})$$

•Transverse Momentum Distribution:

must be done either

numerically or

through approximations

relevant only for large QED

$$d^2P(K_\perp)$$

From Ugo Amaldi's memories

In Autumn '77, Bruno was at CERN for a sabbatical leave and we had many more occasions to discuss physics…
He told me very explicitly in this conversation that he now believed that the future of accelerators would have been based on antiproton-proton and proton-proton collisions. He was worried not only about the radiation in electron rings, but also about the fact that the very high energy in a lepton dresses itself more and more. ..In the 1960's he had introduced what he called the Bond factor, 0.07, which is just the factor in front of the Weizäcker-Williams formula, when you take into account the energy of Adone. There is a log of energy divided by the mass of the electron and it happened to be 0.07 and he was calling it Bond factor, and he was saying "Of course, Bond factor increases with the logarithm of energy and so leptons get dressed very much in high energy"

IN QCD

- Soft gluons are resummed with same technique
- The current has a color index
- For singlet final state or totally inclusive the final state has no color and summation over color indices simply introduces the Casimir coefficient
  - C_F=4/3 (emission from quarks)
  - C_A=3 from gluons

But the main difference is that α_s is not a constant

How important are soft gluons ?

- If the soft gluon spectrum is cut off at the lower end and one never reaches k_t=0, they are not so important in the overall energy dependence

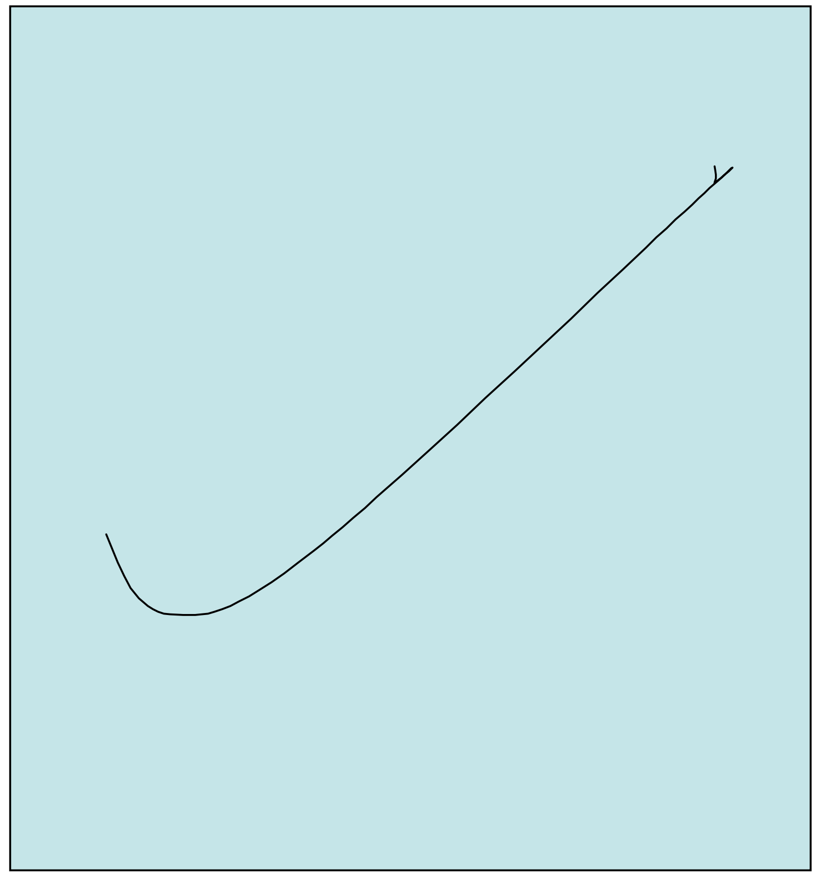
but

- If you let the integral down to k_t=0, you may encounter very strong effects depending how you model $\alpha_s(k_t)$

as k_t=0

Where can one see soft gluons in actions

- Intrinsic transverse momentum of partons
- Total cross-sections
- Decay constants
- Form factors
- All the processes where the partonic transverse degrees of freedom play a role



Partons transverse momentum

•In QCD, if the partons are collinear with the parent hadron, they will emerge from the collision still back-to-back

•This picture was implemented by initial state gluon radiation from quarks and gluons already in 1978 for the Drell-Yan process

Y.L. Dokshitzer, D. Diakonov and S.I. Troian, 1978, and soon after G. Parisi and R. Petronzio,1979.

The soft gluon transverse momentum distribution

$$\frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\mathbf{K}_\perp\cdot\mathbf{b}-h(b)}$$

with

$$h(b) = \int d^3\bar{n}_g(k)[1 - e^{-i\mathbf{k}_\perp\cdot\mathbf{b}}]$$

and with

$$d^3\bar{n}_g(k) = \frac{d^3k}{2k_0} \sum_{i,j=colors} |j^{\mu,i}(k)j_{\mu,j}(k)|$$

Two different limits

$$h(b) = \int d^3\bar{n}_g(k)[1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}]$$

- Small $k_\perp b$ → **Intrinsic transverse momentum**
- Large $k_\perp b$ → **Sudakhov FF**

For large kb one can neglect e^{ikb}

$$h(b) \approx S(b) = \int d^3\bar{n}_g(k) = \int \frac{d^3k}{2k_0} \sum_{colors} |j^{\mu,i}(k) j_{\mu,j}(k)|$$

Botts & Sterman, 1989

- Integrating from 1/b to a scale Q
- Using LO expression for $\alpha_s(k^2)$

$$h(b) \approx \frac{4C_F}{(11 - 2n_f/3)} ln\frac{Q^2}{\Lambda^2} ln\frac{ln(Q^2/\Lambda^2)}{ln(1/b^2\Lambda^2)}$$

Intrinsic transverse momentum

For small K_{perp} however, the approximate expression is not sufficient to reproduce the observed tranverse momentum distribution in various hadronic processes and an intrinsic transverse momentum is introduced artificially.

$$h(b) = b^2 p^2_{\perp int} + S(b)$$

How the soft gluon transverse momentum distribution generates the intrinsic transverse momentum

for $$k_\perp \cdot b \approx 0$$

$$h(b) \approx \int d^3\bar{n}_g(k)(k_\perp \cdot b)^2$$

$$\boxed{h(b) \approx b^2 < k_\perp^2 >}$$

To proceed further
One needs
to know α_s in the
Infrared region

Where the K=0 limit matters :Decay constants

- In hadronic exclusive decays there is no-transverse momentum imbalance and the relevant quantity to calculate is $\Pi(0)$

$$\Pi(0) \equiv \frac{d^2P(K_\perp)}{d_\perp^K}\Big|_{K_\perp=0} = \int d^2\vec{b}e^{-h(b)}$$

$$h(b) = \int d^3\bar{n}_k(1 - e^{-i\vec{b}\cdot\vec{k}_\perp})$$

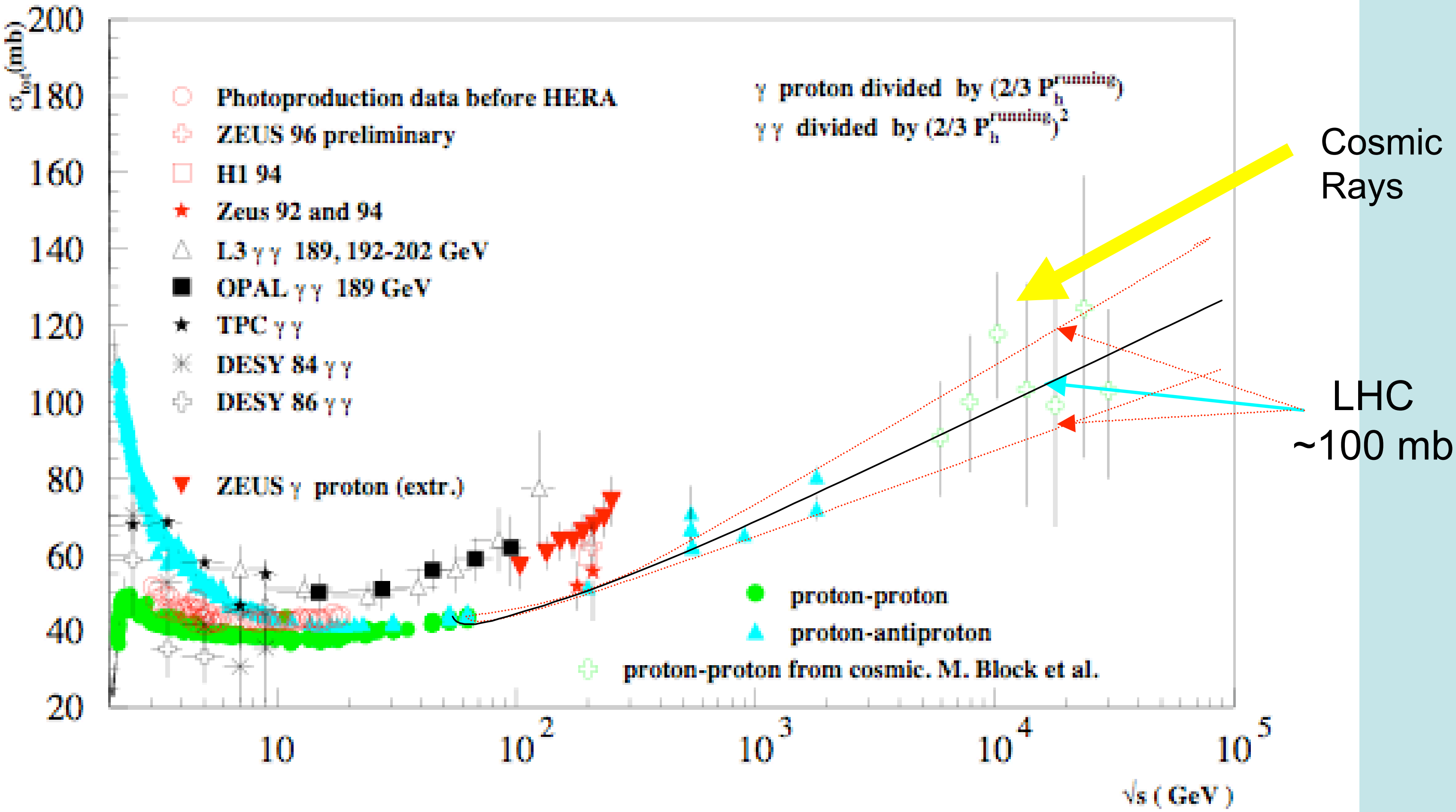
$k_\perp = 0$ is important here!

Total cross-sections and Bloch-Nordsieck Gluon summation

Or

How to build a realistic model implementing soft gluon K_t distribution in the energy dependence of total cross-section

Uncertainties in proton proton



OUTLINE

• Existing data on proton and photon total cross-sections are compared to a QCD model for inelastic collisions with

•hard parton parton scattering
•soft gluon effects a' la Bloch-Nordsieck for b-distribution of partons inside the hadrons

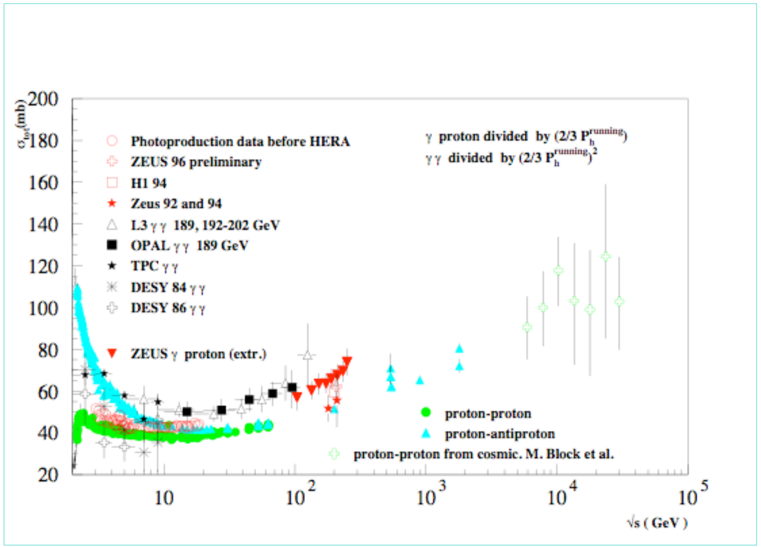
• One can see how
  • QCD minijets drive the rise of all total cross-sections
  • the energy dependent soft gluon emission softens the rise of minijets alone
  •the infrared behaviour of α_s influences the energy dependence of total cross-sections

Comparing the energy dependence of pp, pγ, γγ total cross-sections

To compare them scale with

- quark content factor :
  2/3 to go
  from proton to photon

- Vector Meson Dominance factor



$$P_{VMD} = \Sigma_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2}$$

P_{VMD} ~ 1/240   F. Halzen (1982)

Some differences in
- Normalization
- Initial decrease
- Slope of rise with energy

The traditional Regge-Pomeron picture does not seem to work from proton to photons

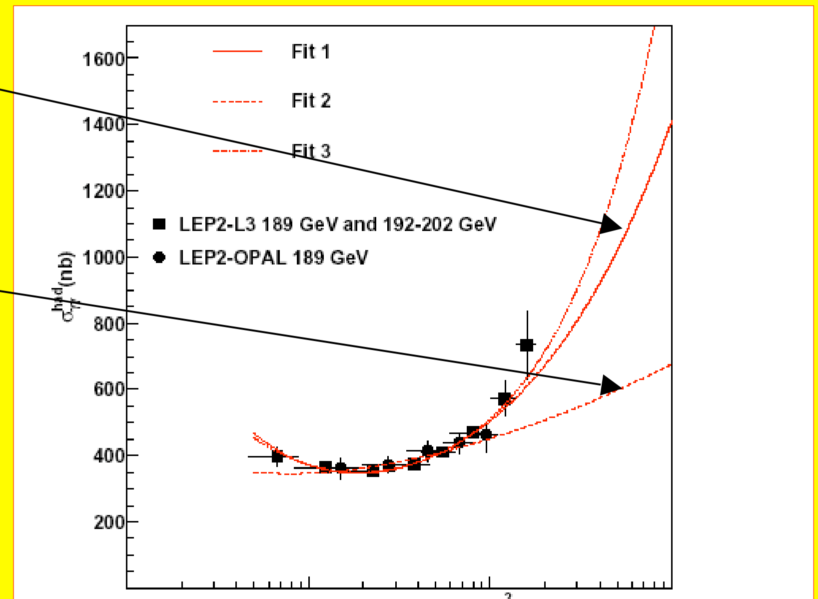
With A.de Roeck, A. Grau and RM Godbole **JHEP 0306:061,2003**

- Fit 1: C=0, □=0.250

  (for proton 0.093)
- Fit 2 : C=0 □=0.093
- Fit 3 : two rising powers, C not 0

  □'=0.418,

  □=0.093



$$\sigma_{total} = As^{\epsilon} + Bs^{-\eta} + Cs^{\epsilon'}$$

All present models for total cross-sections have parameters, either for the low (soft) energy part or the high energy or both

Soft : one fits the data in pp with power laws (Regge) and then extrapolates to gamma p

- Parameters for pp and pbar-p :
  - power exponents
  - normalization
- Parameters for gamma p : normalization (VMD+QPM)

High Energy : one can use power laws (Pomeron/s) and/or QCD jets or “QCD” inspired behaviour

- Power laws should not change from protons to photons
- In QCD cum eikonal, parameters like minimum jet transverse momentum should not change, while different parton densities and parton content may indicate that protons are different from photons

What QCD says about energy dependence in total cross-sections

- Perturbative QCD can be used when α_{strong}/π is small, practically for parton momenta around 1-2 GeV
- As the hadrons c.m. energy increases from 5 to 10^4 GeV in the c.m., the flux of perturbative partons of small x will increase=>the cross-section from such processes will increase

Perturbative QCD provides a natural mechanism for the increase of total cross-sections

The Eikonal Model can easily incororate QCD

- It ensure unitarity and analiticity in the calculation of σ_{tot}

$$f(\theta) = \int d^2\vec{b}e^{i\vec{b}\cdot\vec{q}}[1 - e^{i\chi(b)}]$$

BUT

$$\sigma^{tot}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-\chi_I(b,s)}cos(\chi_R)]$$

- It requires input of the spatial distribution of matter inside colliding hadrons

The simplest eikonal minijet model

The simplest formulation which
incorporates the assumption of QCD driven
rising cross-sections uses factorization of b-distribution from
energy dependent QCD driven terms

$$\sigma^{\rm tot}_{pp(\bar{p})} = 2\int d^2\vec{b}[1 - e^{-\chi_I(b,s)} cos(\chi_R)]$$

$$2\chi_I(b,s) \equiv n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}]$$

$$A_{ab}(b) \equiv A(b; k_a, k_b) = \frac{1}{(2\pi)^2}\int d^2\vec{q} e^{iq\cdot b}\mathcal{F}_a(q,k_a)\mathcal{F}_b(q,k_b)$$

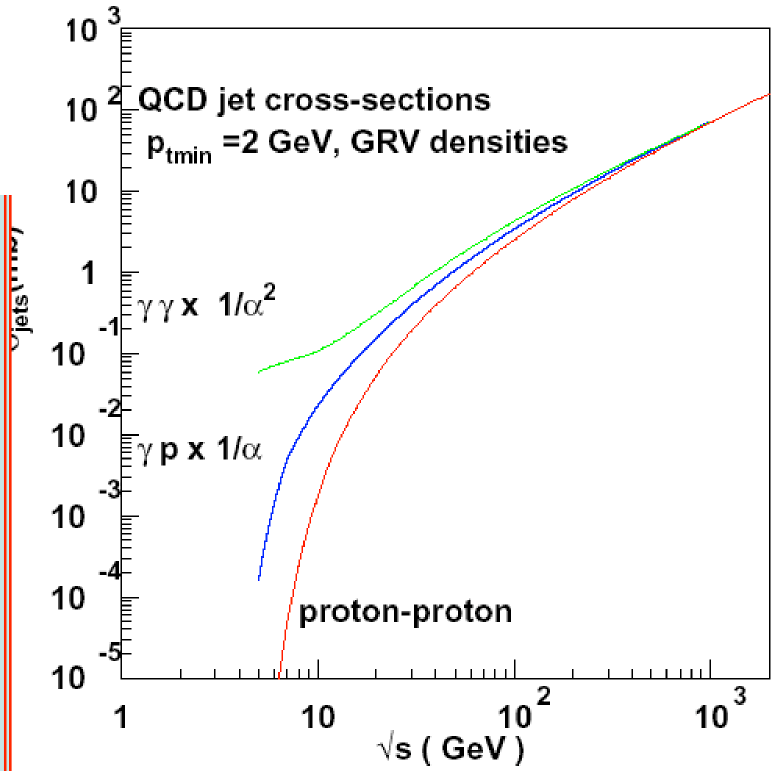
$\mathcal{F}_i(q,k_i)$ are the e.m. form factors

How QCD drives the rise of total cross-sections

$$\sigma_{jet}(s, p_{tmin}) = \int_{p_{tmin}} d^2\vec{p}_t \frac{d\sigma^{QCD}_{jet}(s, p_t)}{d^2\vec{p}_t}$$

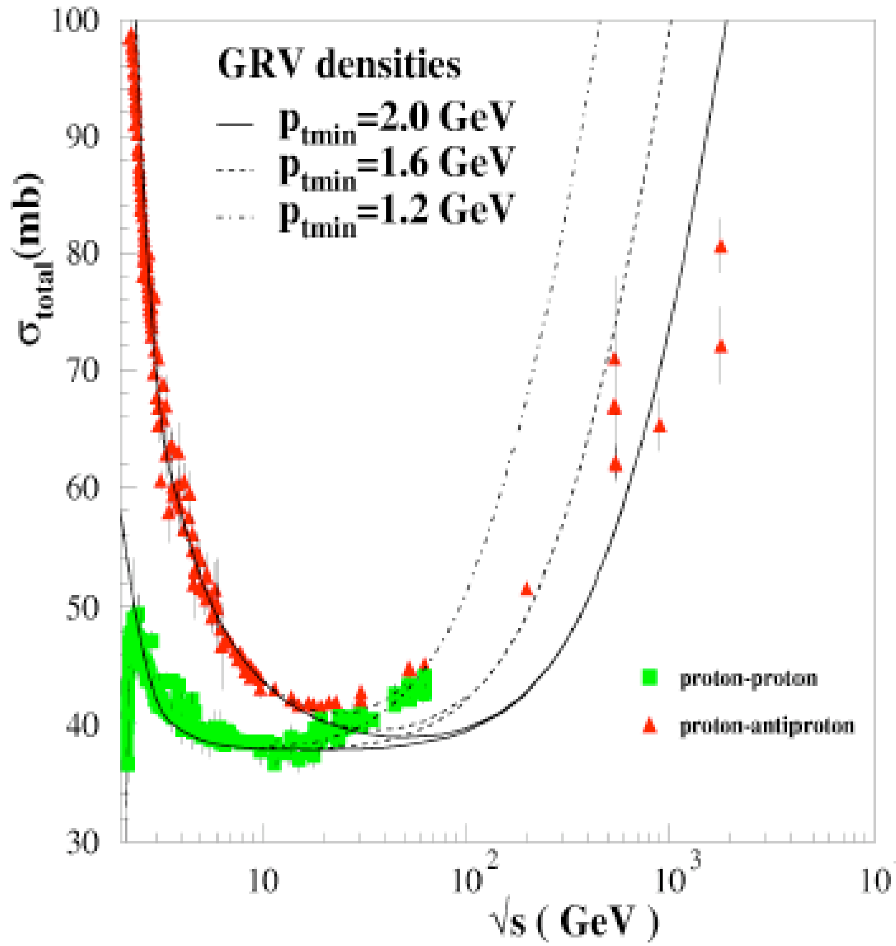
- As the parton flux increases with energy, integrated jet cross-sections increase rapidly with energy
- At low energy the quarks content for γ and protons is different
- With GRV, the gluon content is the same

N.B. σ_{jet} depends strongly on p_{tmin}



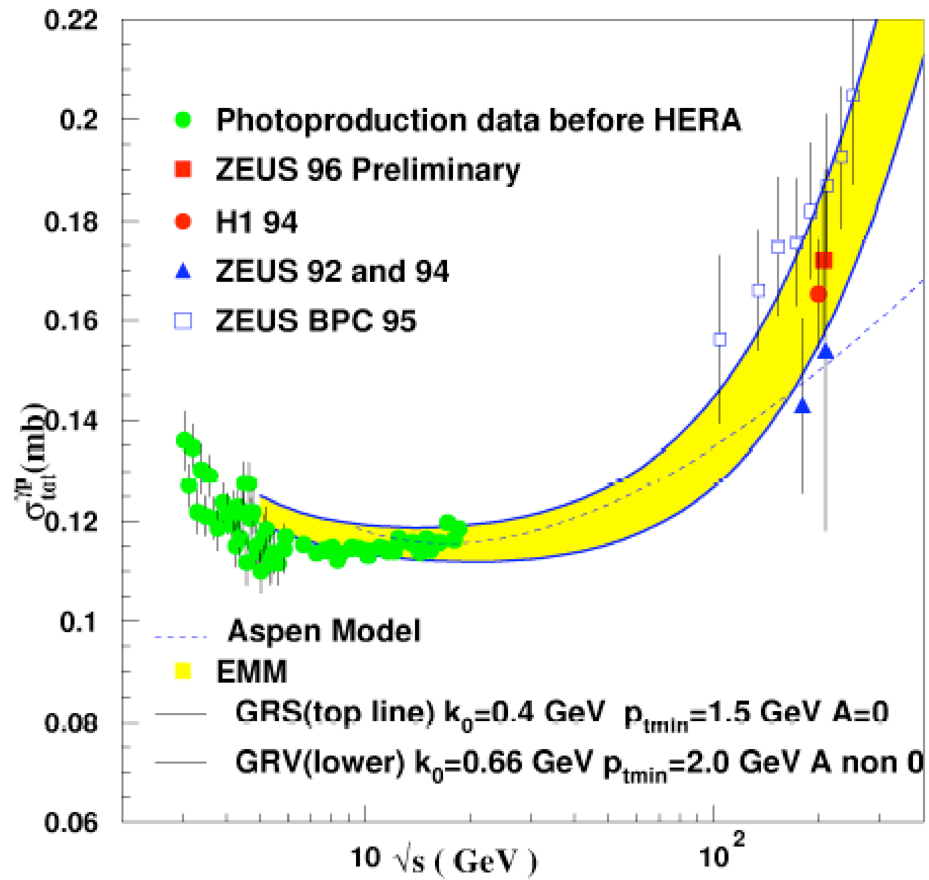
Eikonal + minijets + form factors does not work well

- It is possible to obtain the early rise with a p_{tmin} ~ 1 GeV
- It is possible to get the Tevatron points with p_{tmin} ~2 GeV
- There is no p_{tmin} who gives both the early dramatic rise and subsequent gentler increase



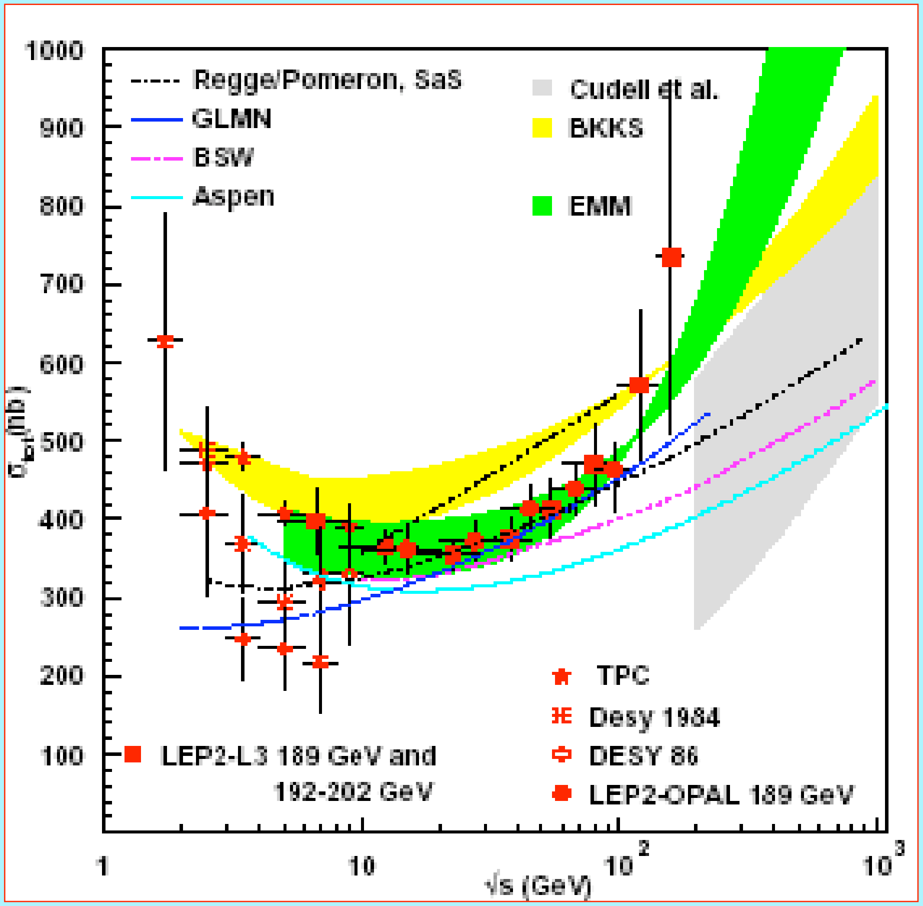
Uncertainties at HERA

- Data still have a large range of uncertainty
- Eikonal Minijet models (QCD) show fast rise
- Aspen Model (M.Block, E. Gregores, F. Halzen, G.P.,) is happy with a slower rise



Uncertainties in photon-photon

Already at √s=500 Gev predictions differ by a factor 5



Why such differences for photons ?

- Photon-photon cross-sections use input from proton data, both pp and □p
- Uncertainties from proton cross-sections and lack of parameter free guidance from theoretical models lead to large variations
- Choice of model :QCD or Regge-Pomeron exchanges or factorization a' la Gribov ?
- And anyway which QCD model?

A more realistic eikonal minijet model

- A physical approach to total x-sections based on

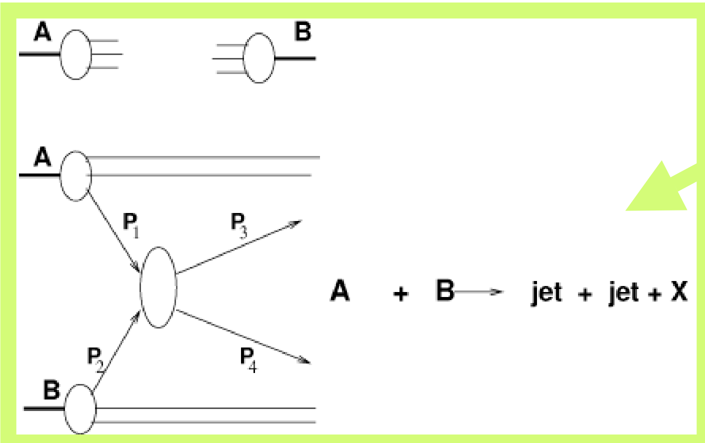
Minijets $p_t > p_{tmin}$ ← To drive the rise

+

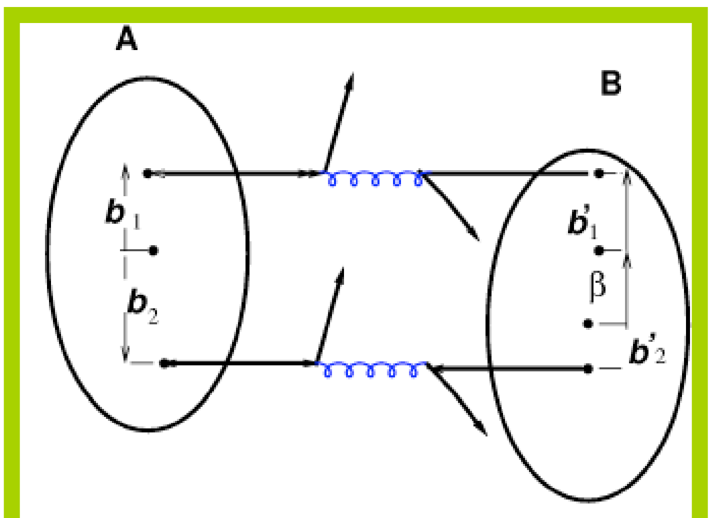
Soft gluon resummation down to $k_t = 0$ ← To tame the rise

QCD model for total cross-sections: Minijets, eikonal formalism and Bloch-Nordsieck resummation

QCD minijets drive rise of σ_{tot}

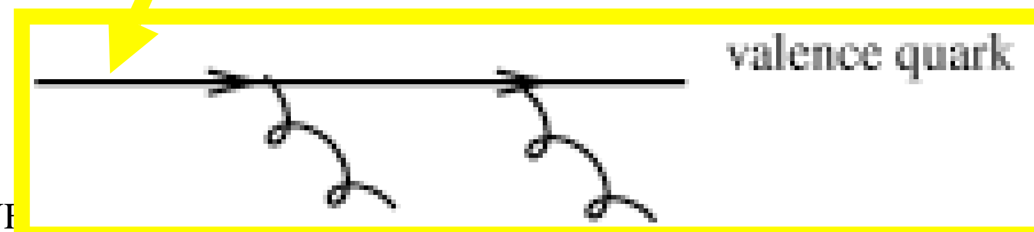


Overlap in b-space and Eikonal representation ensure unitarity



Transverse Overlap of the hadrons

Soft emission tames the rise with energy through increasing acollinearity



(At least) a two scale problem: p_T and k_T

Hard scale	Soft scale
p_t^{jet}	$k_t^{\text{soft gluons}}$

- For parton-parton scattering the scale is p_t^{jet} can be as low as 1-2 GeV

- For soft gluon emission from hard partons the scale is of order 20% of the hard scale
- it depends on x parton and p_t^{jet}

How QCD induces a decrease in the cross-sections as the energy increases

The number of collisions depends

- on the total parton-parton cross-section (minijets)

- on the parton acollinearity

Initial State soft gluon emission produces a

- parton acollinearity K_t

- $d^2P(K_t) = \int d^2 K_t \, \mathbf{e}^{\,i\, K_t \cdot b} \, e^{-h(b,s)}$

- $h(b,s) = \int d^3 n_g(k_t)[1 - \mathbf{e}^{\,-i\, k_t \cdot b}]$

- acollinearity is energy dependent

A(b) from soft gluon emission

- $A(b,s) \sim$ Fourier transform of $d^2P(K_t)$

$$\sim \quad e^{-h(b,s)}$$

$$A(b,s) = \frac{e^{-h(b,s)}}{\int d^2b\, e^{-h(b,s)}} \qquad \int A(b,s)\, d^2b = 1$$

Initial State Soft Gluon Radiation and Transverse Acollinearity

The Initial state transverse momentum distribution from soft gluon radiation

$$d^2P(K_t)$$

has been around for a long time

We wish to exploit it in order to change the violent rise due to minijets with p_{tmin}~1 GeV into a softer behaviour

The Energy Dependence of Soft Gluon Emission

Qualitatively

- As the energy increases, colliding partons on the average carry more energy
- soft gluons emitted from harder partons can carry away more momentum
- The overall acollinearity of initial partons increases
- The rise of number of collisions due to minijets is tamed by initial straggling of partons

Quantitatively?

- For each two parton process with x_1 and x_2 and jet p_t in final state, calculate maximum k_t allowed kinematically to soft gluon emission
- We approximate and take averages for realistic calculations

With A(b,s)=e$^{-h(b,s)}$ an extra energy dependence in the total cross-sections comes from h(b,s)

- $h(b,s) = \int_{k_{min}}^{k_{max}} d^3\bar{n}_{gluons}(k)\ [1 - e^{ik_t \cdot b}]$

- $k_{max} \Longrightarrow average\ over\ densities\ \Uparrow\ as\ \sqrt{s}\ \Uparrow$

- $k_{min} = 0$ in principle but one needs a model for

$$\alpha_s(k_t) \text{ as } k_t \to 0$$

ENERGY DEPENDENCE OF SOFT GLUON EMISSION

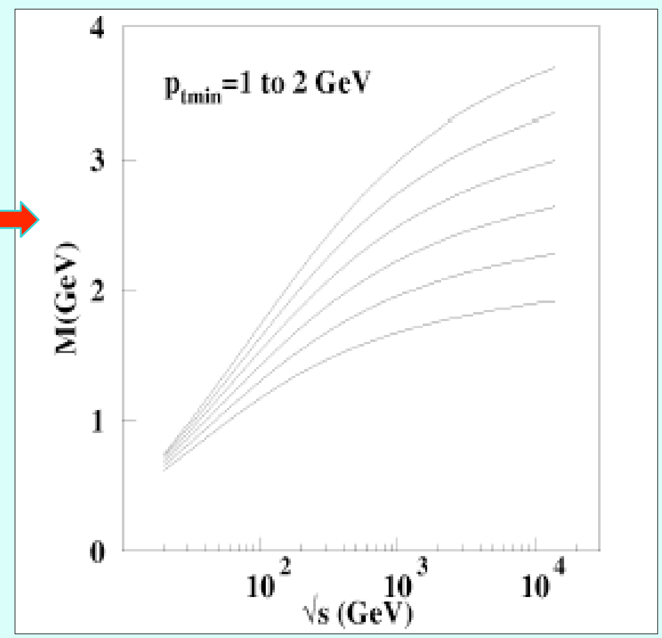
Maximum energy allowed to single gluon emission is obtained from

- exact kinematics
- average over densities

$$q_{max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{Q^2}{\hat{s}}\right)$$

$$M \equiv < q_{max}(s) > =$$

$$\frac{\sqrt{s}}{2}\frac{\sum_{i,j}\int \frac{dx_1}{x_1} f_{i/a}(x_1)\int \frac{dx_2}{x_2} f_{j/b}(x_2)\sqrt{x_1 x_2}\int dz(1-z)}{\sum_{i,j}\int \frac{dx_1}{x_1} f_{i/a}(x_1)\int \frac{dx_2}{x_2} f_{j/b}(x_2)\int(dz)}$$



CHOOSING α_s

α_s could be frozen , i.e. α_s (0)=constant

or

it could be singular but integrable

- Of course a singular α_s induces more acollinearity

Two models for α_s

A formulation inspired by the Richardson potential

$$\bar{\alpha}_s(k_\perp^2) = \frac{12\pi}{(33-2N_f)}\frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda_{QCD}})^{2p}]}$$

A frozen α_s as in Halzen (1980) or Altarelli, Greco, Martinelli(1984)

$$\alpha_s = \frac{12\pi}{(33-2n_f)\ \ln[a+k^2/\Lambda^2]}$$

A SINGULAR □s

$$\tilde{\alpha}_s(k_\perp^2) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda_{QCD}})^{2p}]}$$

- for $K_\perp \gg \Lambda_{QCD}$ $\quad \tilde{\alpha}_s \to \alpha_s^{AF}$
- for $K_\perp \ll \Lambda_{QCD}$ $\quad \tilde{\alpha}_s \to (k_\perp^2)^{-p}$

If p is smaller than 1 the integral can be done

Why a singular α_s?

With singular (but integrable) α_s

- $h(b,s) \sim b^2$ constant (actually b^{2p} , $p \sim 1$)

- $d^2P(K_t) \sim e^{-K_t^2}$ i.e. soft gluons induce an intrinsic transverse momentum

The frozen α_s has no such effect

Eikonal Minijet Model +Bloch-Nordsiek (BN) summation

For protons

Low energy parameters

- Normalization
- Low energy impact parameter distribution (b-distribution)

High Energy parameters

- Minimum jet transverse momentum
- Parton densities
- Infrared behaviour of a_s for *soft* gluon emission resummation in k_t (linked to partonic b-distribution)

The Low Energy Parametrization in Phys.Rev.D60:114020,1999

$$n(b,s) = n(b,s)_{soft} + n(b,s)_{hard}$$

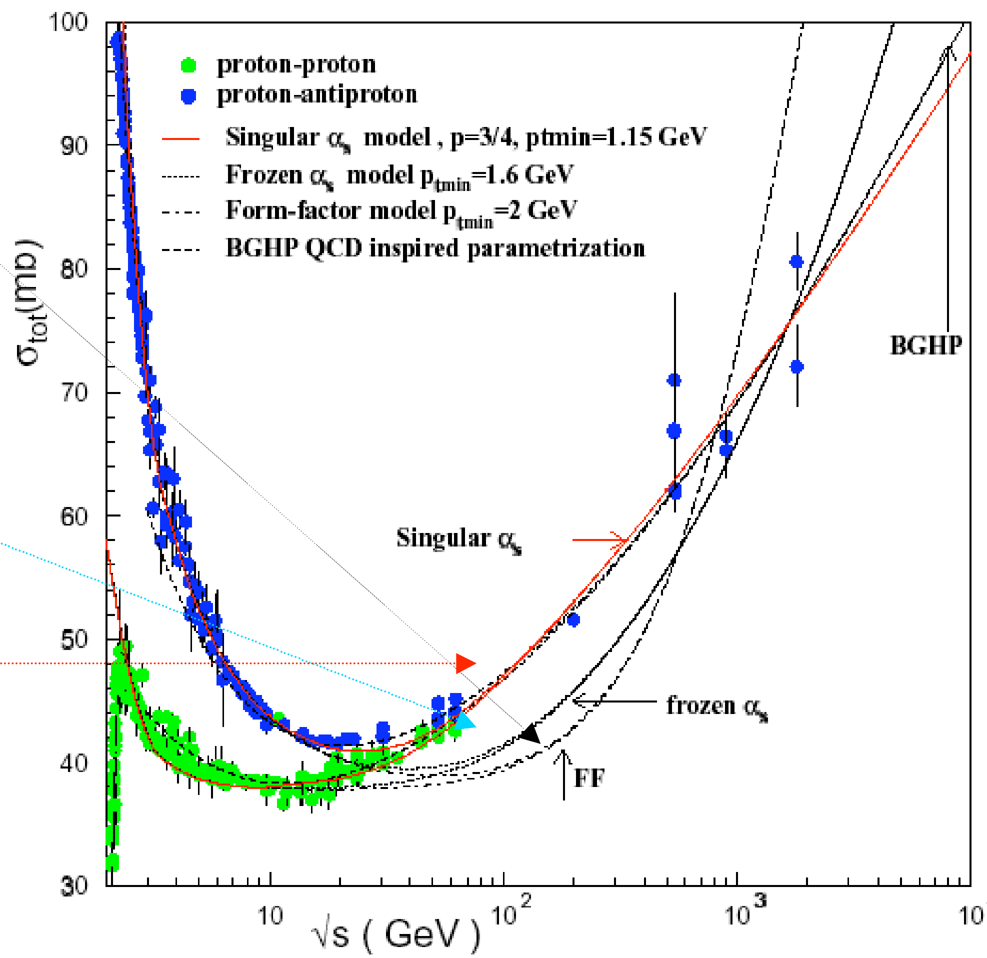
$p_t > p_{tmin}$

- At low energy minijets are negligible
- $n(b,s) \sim n(b,s)_{soft}$
- $n(b,s)_{soft} = A(b)\sigma_{soft}$
- A(b) from e.m. Form factors

Low energy σ_{soft} fitted with a constant and two decreasing powers of energy (a total of 5 parameters for pp and pbarp)

Eikonal with minijets + soft gluon emission resummed a' la BN

- The Form factor model for A(b) is the worst
- The frozen α_s model is slightly better but soft emission is not sufficient to reproduce both early and later rise
- For singular α_s soft emission does the job



Can the energy dependence induced by $A_{BN}(b,s)$ be exploited further?

- With a rising minijet cross-section, A_{BN} softens the rise

- With a constant cross-section, as we expect in the low energy region, A_{BN} can induce a decrease like in proton-proton or contribute to the decrease like in proton-antiproton

Soft gluons and the low energy parametrization hep-ph/0408355

$$n(b,s) = n(b,s)_{soft} + n(b,s)_{hard}$$

$p_t < p_{tmin}$ $\qquad$ $p_t > p_{tmin}$

We assume that soft gluon emission takes place in both regions but of course for soft gluons one always has

$k_t < q_{max} \sim 10\text{-}20\%\ p_t$

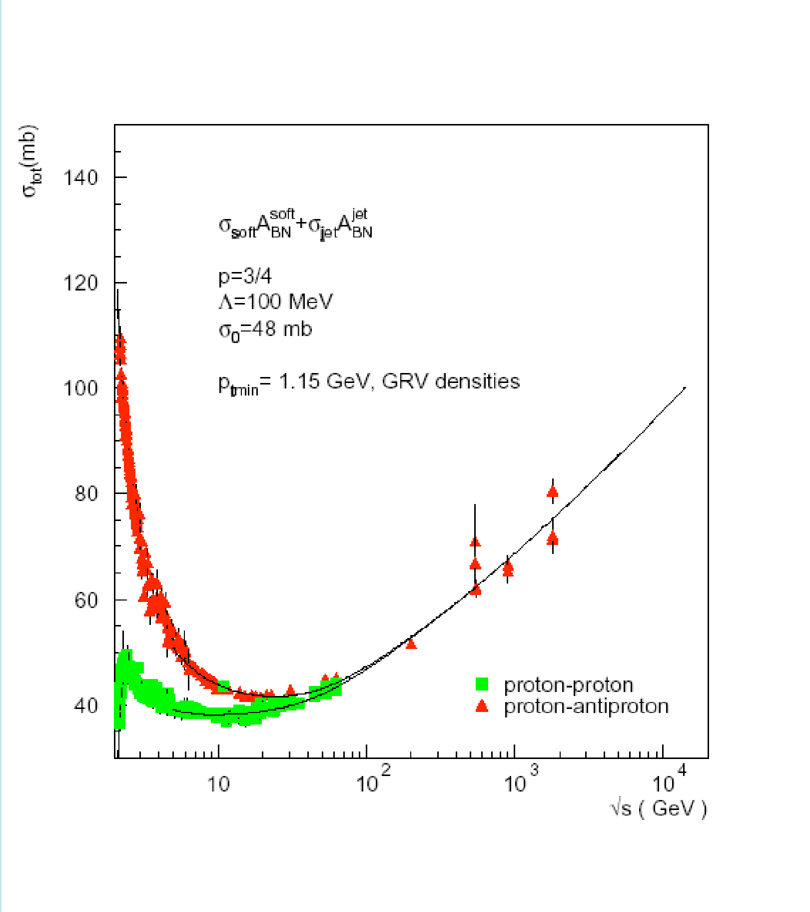
- At low energy minijets are negligible
- $n(b,s) \sim n(b,s)_{soft}$
- $n(b,s)_{soft} = A(b,s)□_{soft}$
- $A(b,s)$ only soft gluons with $q_{max} \sim 20\%\ p_{tmin}$

$□_{soft} = □_0$ for proton-proton

$□_{soft} = □_0\,[1 + 2/\mathrm{sqrt}(s)]$ for proton-anti proton

Present Phenomenology: the proton case

- Tevatron data allow for both log and log^2 and more than simple Regge + 1 Pomeron
- The EMM + BN model predicts **98 mb** at LHC with a specific choice of parameters
- Range of model parameters, like p_{tmin} and soft IR behaviour, needs to be determined



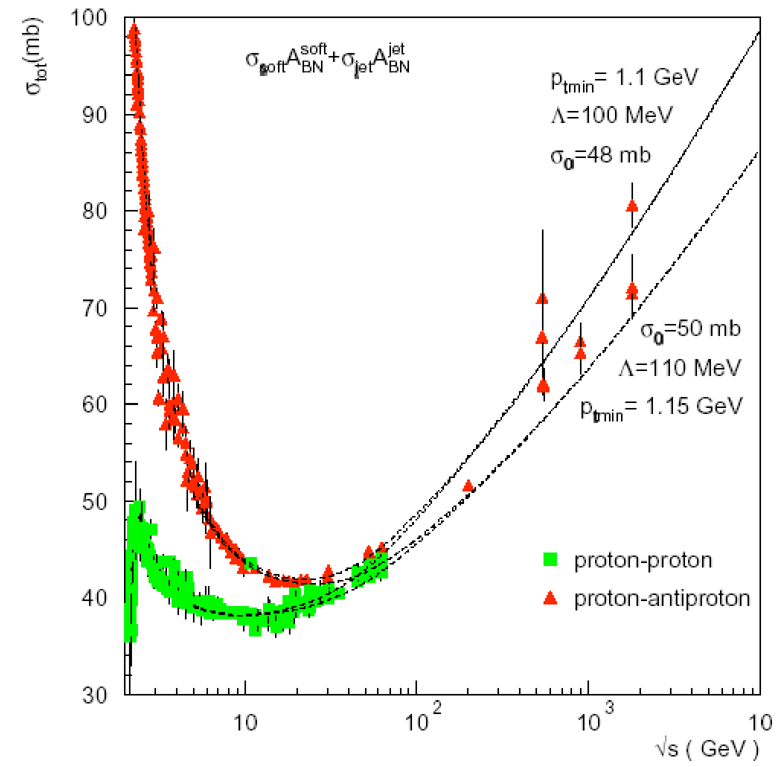
Dependence from various parameters

The fit depends upon

- P_{tmin}
- Parametrization of □ soft
- Lambda QCD
- Densities (we have not varied that yet)

and

- Singularity of □$_s$



The real question in any QCD approach to total cross-sections

- The real question in studying σ_{tot} with QCD is

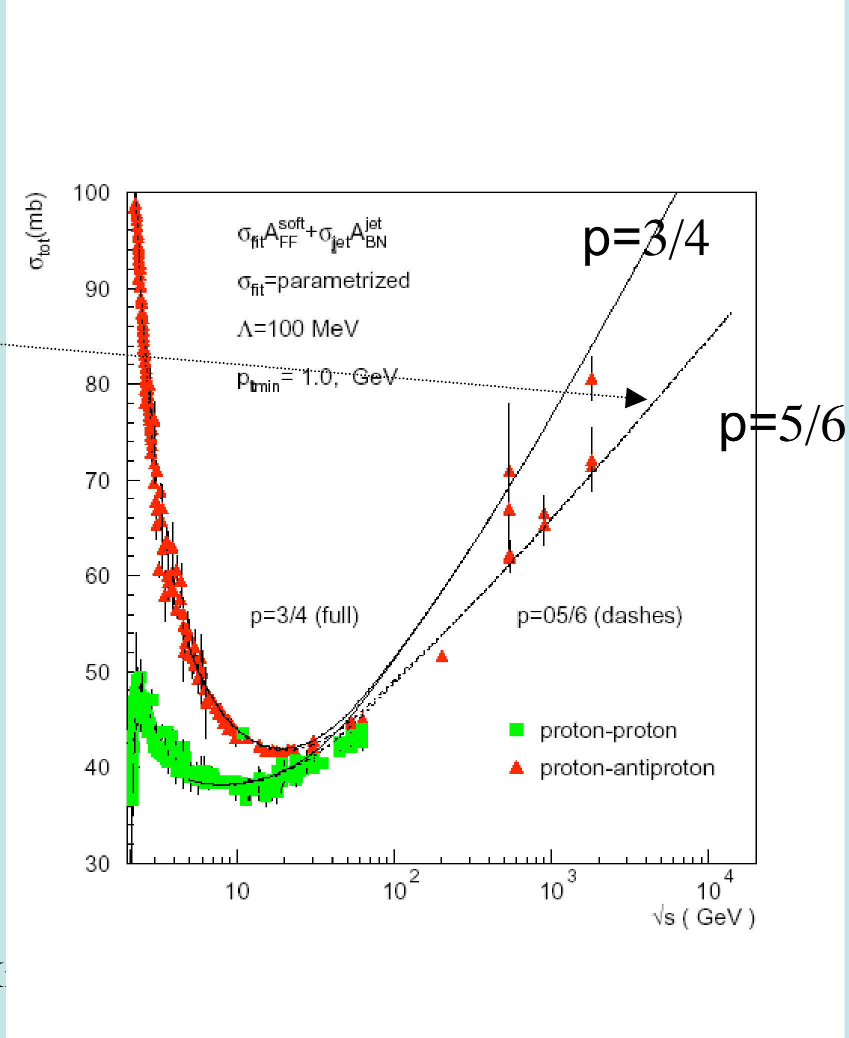
$$\alpha_s (k_t \to 0) \ ?$$

- Why?
- Because of soft physics
  - At low energy of course
  - At high energy as well because high energy parton-parton scattering needs soft gluon effects, treated with resummation, which means to integrate (there are many such soft photons) from $k_t = 0$ (they are soft!) to some kinematically determined maximum value

Dependence from how singular is α_s

- Singularity is regulated by
  1. Parameter p
  2. Lambda QCD
- The more singular α_s,

  - more taming,
  - more acollinearity
  - the slower the rise



Other Phenomenological Studies

- Completed □□and □p studies within the Eikonal minijet Model with Bloch-Nordsieck soft gluon resummation

for various photon densities

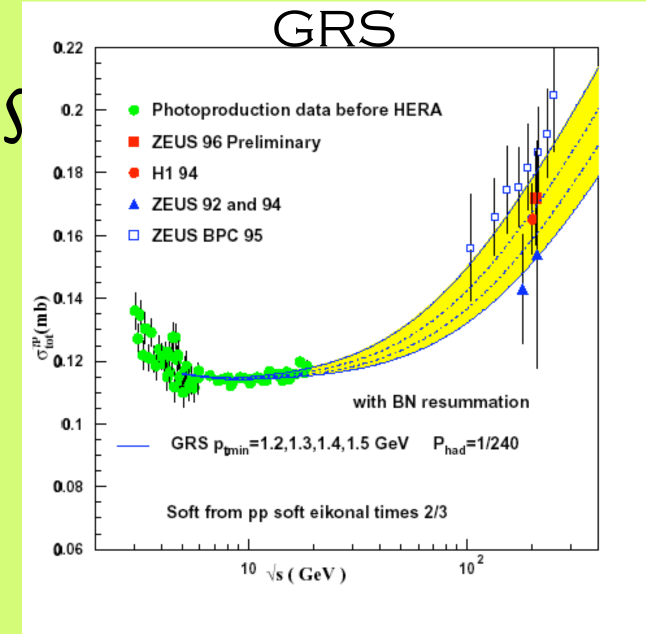
GRV	M.Gluck, E.Reya, and A.Vogt
GRS	M.Gluck, E.Reya and I.Schienbein
CJKL	F.Cornet, P. Jankowski, M.Krawczyk and A. Lorca

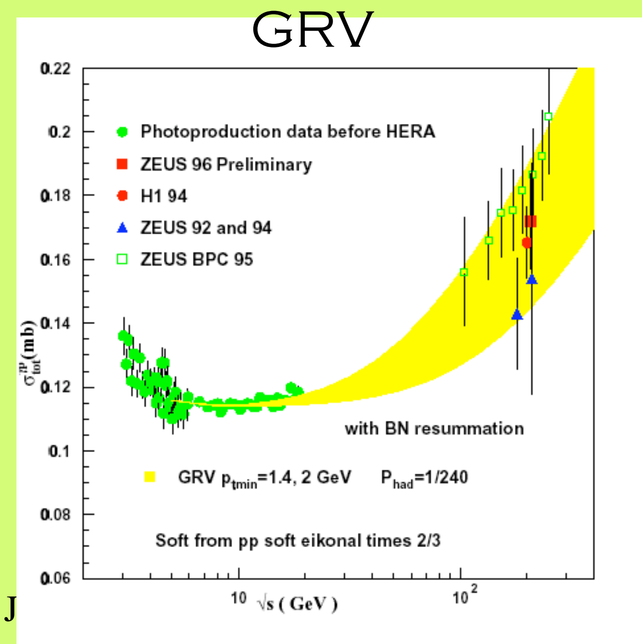
P_{tmin}=1.2 to 2 GeV

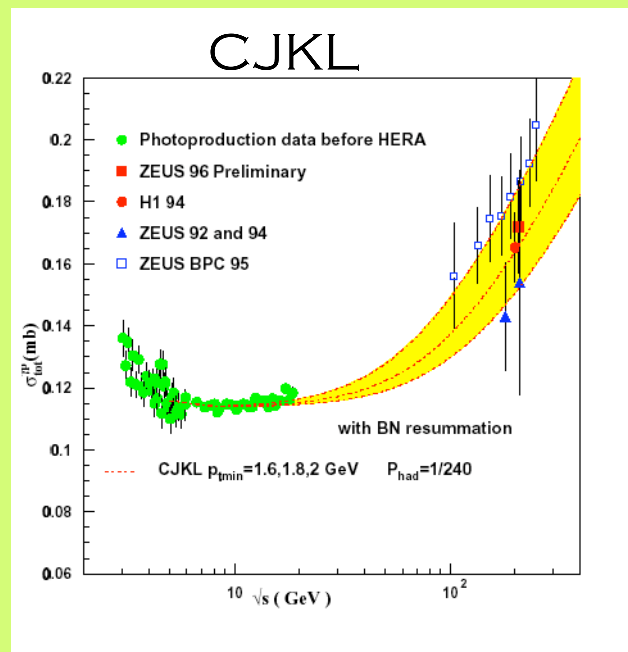
□p for various densities and P_{tmin}

GRV, GRS and CJKL Densities :

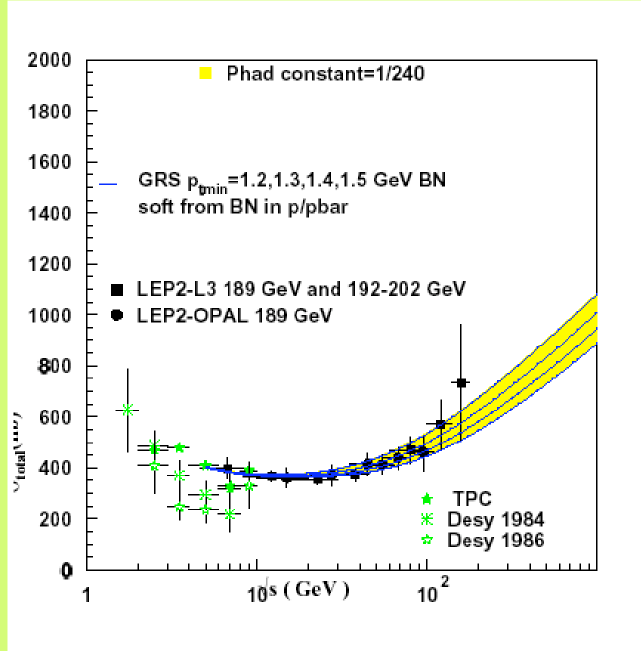
P_{tmin}=1.2 to 2 GeV







□□ For various densitie and P_{tmin}

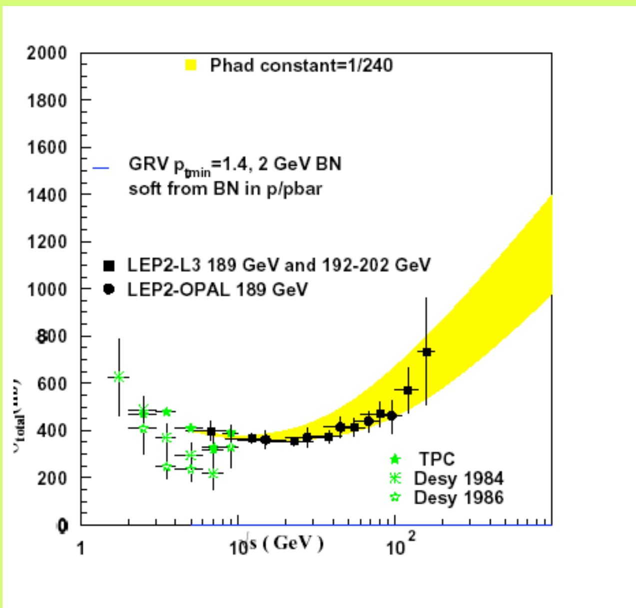


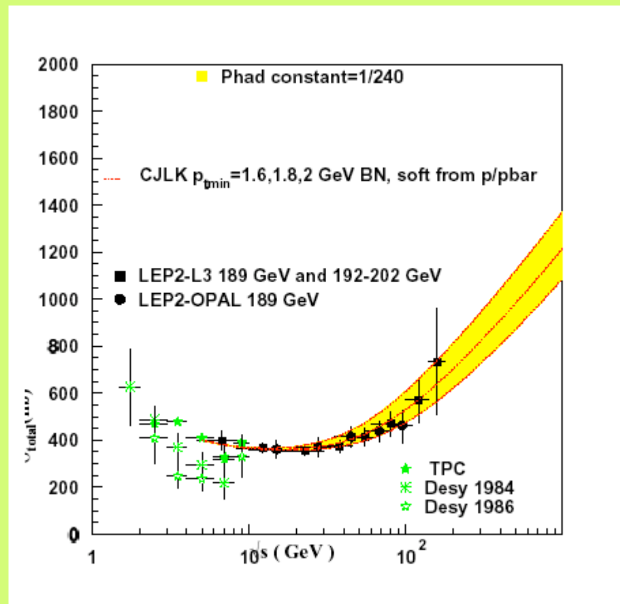
GRS

GRV, GRS and CJKL Densities：

P_{tmin}=1.2 to 2 GeV

GRV





CJKL

Conclusions and Workingprogram

- A work program to reach stable predictions for LC and learn about QCD contribution to σ_{total} needs LHC measurements and an understanding of how much parameters can vary.

- Need to vary parameters in models for α_s in the infrared region

- Include mass effects in the the Bloch-Nordsieck function h(b,s)
- Study virtual photon effects in the exact kinematics

  $$h(b,s) \rightarrow \rightarrow h(b,Q^2,s)$$

- From proton to photons : HERA data are crucial in order to constrain the photon parameters

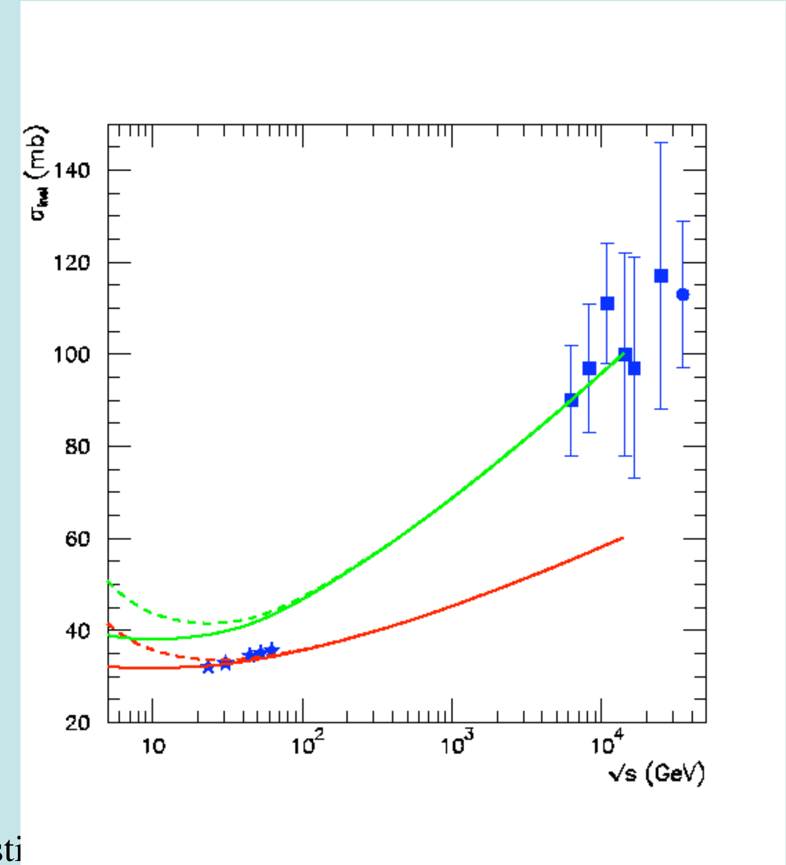
Extension to cosmic ray data

- The model fits nicely the cosmic ray data with
  - p_{tmin}=1.15 geV
  - GRV densities
  - p=3/4
  - Etc.

BUT

- It does not to do well with the inelastic cross-section
- Better understanding and more tuning of the parameters is necessary

Preliminary fit with L. Anchordoqui

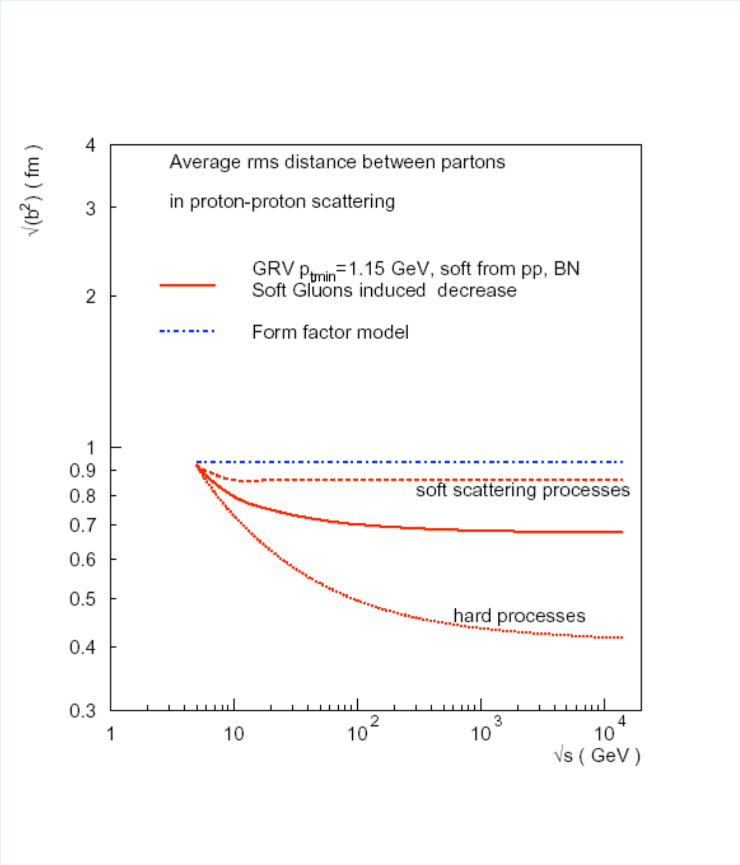


R.M.S. Distance between partons

- The model gives a decreasing b^2

with

- Faster decrease for hard scattering processes $p_t > p_{tmin}$
- Slower decrease for soft scattering $p_t < p_{tmin}$



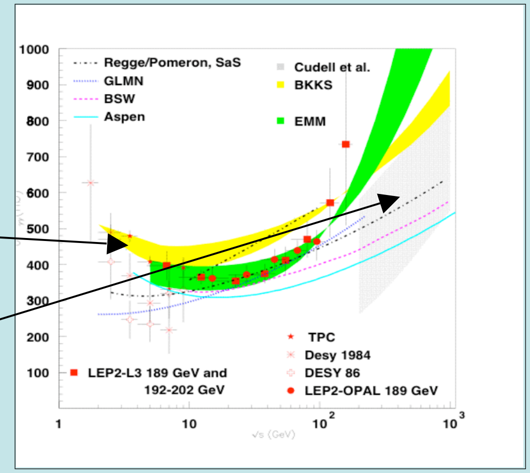
How can one make realistic predictions at Linear Collider?

□□ has uncertainties both in

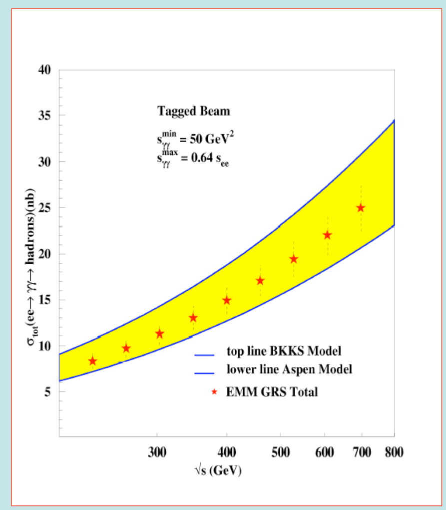
- the low energy region, (normalization)

and

- the high energy, i.e. how much □□ will rise in the 100-200 GeV c.m.

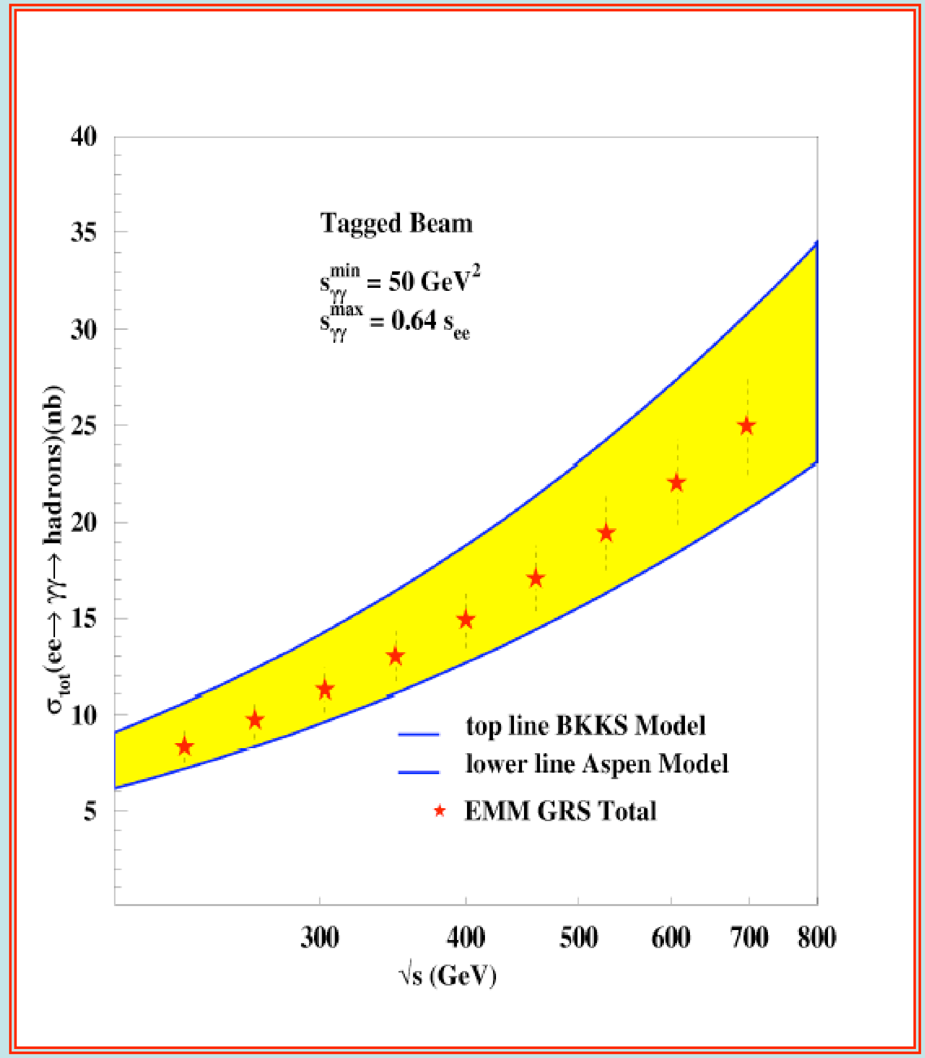


Predictions for e+e- at LC suffer from uncertainties in the □□ cross-section



LC and □□ Scattering

- Differences in predictions of total cross-sections in photon-photon collisions affect LC background studies



The Perturbative QCD Contribution

$$\frac{d^2\sigma^{AB}_{jet}(s,p_t)}{d^2\vec{p}_t} = \sum_{i,j,k,l} \int dx_1 dx_2 f_{i/a}(x_1) f_{j/b}(x_2) \frac{d^2\sigma^{ij\to lk}}{d^2\vec{p}_t}$$