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Anomalous operators for kaon decays

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Motivation

- *QCD* axial and trace anomalies

The invariance of $S^{cl}_{QCD}(m_q = 0)$ under

$\underline{U(1)}_A$

$$\psi(x) \rightarrow \exp(-i\gamma_5\alpha)\,\psi(x)$$

$\underline{dilatations}$

$$x^\mu \rightarrow \exp(\alpha)\,x^\mu$$
$$\psi(x) \rightarrow \exp(-3\alpha/2)\,\psi(x)$$
$$A_\mu(x) \rightarrow \exp(-\alpha)\,A_\mu(x)$$

is broken by quantum effects:

$$\partial_\mu j^\mu_A \sim \underbrace{G^a_{\mu\nu}\tilde{G}^{\mu\nu a}}$$

QCD axial anomaly

$$\partial_\mu j^\mu_D = \Theta^\mu_\mu \sim \underbrace{G^a_{\mu\nu}G^{\mu\nu a}}$$

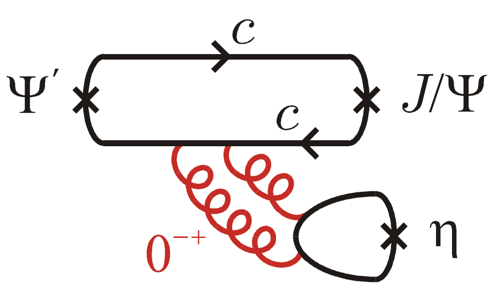
QCD trace anomaly

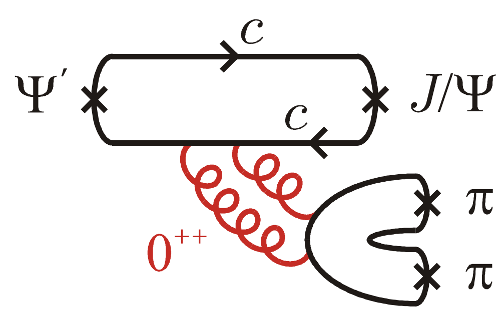
- This allows to estimate low energy matrix elements such as

$$\left\langle \eta_0 \middle| G^a_{\mu\nu}\tilde{G}^{\mu\nu a} \middle| 0 \right\rangle \qquad \left\langle \pi\pi \middle| G^a_{\mu\nu}G^{\mu\nu a} \middle| 0 \right\rangle$$

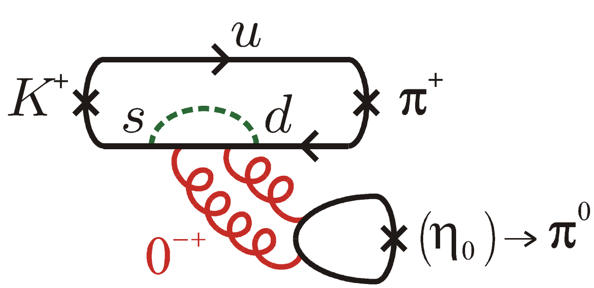
❖ Phenomenological implications

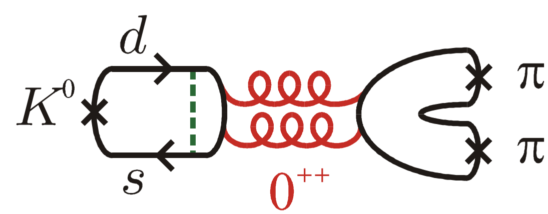
For example:

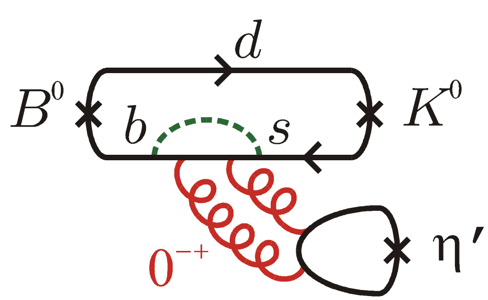




What about weak (kaon) decays?







possible contribution to the $\Delta I = 1/2$ rule

Outline

I Short distance: Heavy quark induced $\Delta S=1$ anomalous operators	II Long distance: Light quark induced $\Delta S=1$ anomalous operators
$Q_{AA} \sim \bar{d}_L s_R G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$	$Q_{AA} \sim U^{sd}\, Tr\left(\ln U - \ln U^\dagger\right)$
$Q_{TA} \sim \bar{d}_L s_R G^a_{\mu\nu} G^{\mu\nu a}$	$Q_{TA} \sim U^{sd}\, Tr\left(\partial_\mu U \partial^\mu U^\dagger\right)$
generated through penguinlike QCD corrections to	generated through chiral $(+1/N_C)$ corrections to
$Q = \left(\bar{d}\,\Gamma^A q_h\right)\left(\bar{q}_h \Gamma^B s\right)$	$Q = U^{sd}$

1-loop heavy quark integration → **Dimension 8 operators for c effects in K decays**

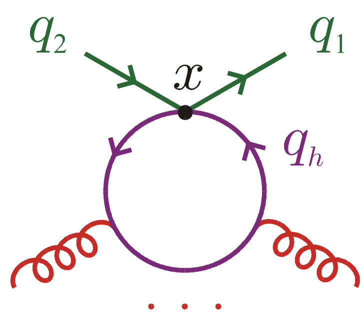
Part I: Heavy quark induced anomalous operators*

* J.-M. Gérard & S.T., Phys.Rev.D69, 113005 (2004)

1. Heavy quark integration (1-loop)

$$Q = \left(\bar{q}_1 \Gamma^A q_h\right)\ \left(\bar{q}_h \Gamma^B q_2\right)$$

$$\left(\frac{1}{\not{P} - M}\right)_{xx}$$



'penguinlike diagram'

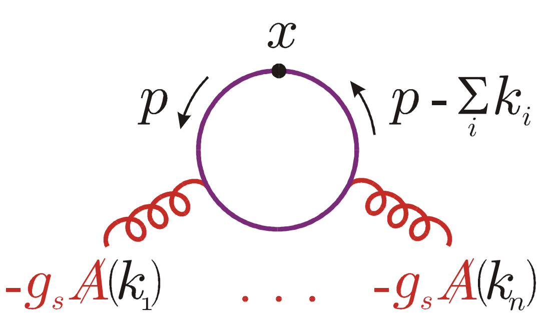
$$P_\mu = i\partial_\mu + g_s A_\mu$$

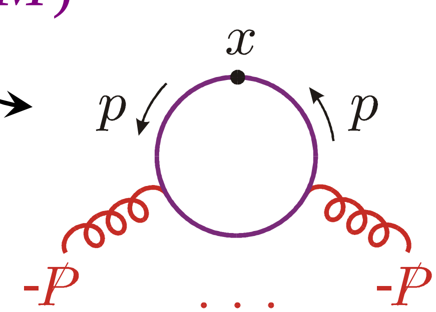
$$A_\mu = A_\mu^a \frac{\lambda^a}{2}$$

Central object = propagator of the heavy quark in external gluon fields:

$$\left(\frac{1}{\not{P} - M}\right)_{xx} = \sum T_n = \sum S_n$$

g_s^n ($\sum T_n$) $\qquad (1/M)^{n-3}$ ($\sum S_n$)





1. Heavy quark integration

$$S_n = \int \frac{d^d p\, \mu^\varepsilon}{(2\pi)^d} \frac{1}{\not p - M} \underbrace{(-\not P)\dots(-\not P)}_{n \text{ insertions}} \frac{1}{\not p - M}$$

$$S_2 = \frac{-ig_s}{2(4\pi)^2} \; M \; \left(\tfrac{2}{\varepsilon} + \ln 4\pi - \gamma - \ln\tfrac{M^2}{\mu^2}\right) G_{\mu\nu}\sigma^{\mu\nu}$$

$$S_3 = \frac{-ig_s}{3(4\pi)^2} \; 1 \; \left(\tfrac{2}{\varepsilon} + \ln 4\pi - \gamma - \ln\tfrac{M^2}{\mu^2}\right) D^\nu G_{\mu\nu}\gamma^\mu$$

$$S_4 = \frac{-ig_s^2}{6(4\pi)^2} \frac{1}{M} \left(G_{\mu\nu}G^{\mu\nu} + \tfrac{3}{2} i G_{\mu\nu}\tilde{G}^{\mu\nu}\gamma_5 - 3iG^\mu_\alpha G^{\nu\alpha}\sigma_{\mu\nu} - \tfrac{1}{2g_s} D_\alpha D^\alpha G_{\mu\nu}\sigma^{\mu\nu} \right)$$

$$S_5 = \frac{-ig_s^2}{8(4\pi)^2} \frac{1}{M^2} \Big(\tfrac{6}{5} i \left[D^\alpha G_{\mu\alpha}, G^{\mu\nu} \right]\gamma_\nu + \tfrac{2}{15} i \left[G_{\mu\alpha}, D^\alpha G^{\mu\nu} \right]\gamma_\nu$$

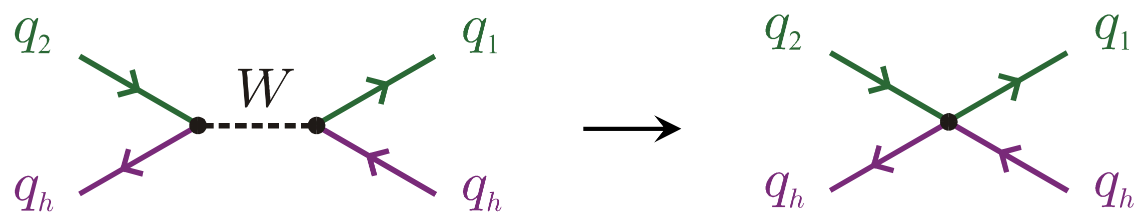
$$+ \tfrac{4}{3}\left\{ D^\alpha G_{\mu\alpha}, \tilde{G}^{\mu\nu} \right\}\gamma_\nu\gamma_5 + \tfrac{2}{3}\left\{ G_{\mu\alpha}, D^\alpha \tilde{G}^{\mu\nu} \right\}\gamma_\nu\gamma_5 - \tfrac{8}{15g_s} D^\alpha D_\alpha D^\nu G_{\mu\nu}\gamma^\mu \Big)$$

→ Effective Lagrangian:

$$L_{eff} = -ig_w \sum_{n=2}^{5} \bar{q}_1 \Gamma^A \underbrace{S_n} \Gamma^B q_2 + O(M^{-3})$$

2. Charm quark effects in K decays

W integration:



c integration:

$$Q = \bar{d}\gamma^{\mu}(1-\gamma_5)c \quad \bar{c}\gamma_{\mu}(1-\gamma_5)s$$

$$\left(\frac{1}{\not{P} - m_c}\right)_{xx}$$

$$= \not{S}_2 + S_3 + \not{S}_4 + S_5 + \ldots$$

$$\swarrow \qquad \searrow$$

$$L_{eff} = \not{L}_{5d} + L_{6d} + \not{L}_{7d} + L_{8d} + \ldots$$

Dimension 6 operators (S_3)

$$L_{6d} = -g_w \frac{g_s}{6\pi^2} \ln\frac{m_c^2}{\mu^2}\, \bar{d}_L D^\nu G_{\mu\nu} \gamma^\mu s_L$$

$\downarrow$ eom: $D^\nu G_{\mu\nu} = g_s \sum_q \left(\bar{q}\gamma_\mu \frac{\lambda^a}{2} q\right)\frac{\lambda^a}{2}$

+ Fierz identities,...

$$L_{6d} = -g_w \frac{\alpha_s}{4\pi} \ln\frac{m_c^2}{\mu^2} \sum_{k=3}^{6} c_k^{(6)} Q_k^{(6)}$$

Usual gluonic penguin operators

$$\begin{cases} Q_3^{(6)} = 4\sum_q \left(\bar{d}_L \gamma_\mu s_L\right)\left(\bar{q}_L \gamma^\mu q_L\right) \\ Q_4^{(6)} = 4\sum_q \left(\bar{d}_L \gamma_\mu q_L\right)\left(\bar{q}_L \gamma^\mu s_L\right) \\ Q_5^{(6)} = 4\sum_q \left(\bar{d}_L \gamma_\mu s_L\right)\left(\bar{q}_R \gamma^\mu q_R\right) \\ Q_6^{(6)} = -8\sum_q \left(\bar{d}_L q_R\right)\left(\bar{q}_R s_L\right) \end{cases}$$

$$q = u, d, s$$

$$q_R^L \equiv (1 \mp \gamma_5) q/2$$

$$c_3^{(6)} = c_5^{(6)} = -\tfrac{1}{9}, \qquad c_4^{(6)} = c_6^{(6)} = \tfrac{1}{3}$$

Dimension 8 operators (S_5)

$$L_{8d} = g_w \frac{\alpha_s}{4\pi} \frac{1}{m_c^2} \sum_{k=1}^{5} c_k^{(8)} Q_k^{(8)}$$

$$Q_1^{(8)} = i\,\bar{d}_L \left[D^\alpha G_{\mu\alpha}, G^{\mu\nu} \right] \gamma_\nu s_L \qquad c_1^{(8)} = \frac{6}{5}$$

$$Q_2^{(8)} = i\,\bar{d}_L \left[G_{\mu\alpha}, D^\alpha G^{\mu\nu} \right] \gamma_\nu s_L \qquad c_2^{(8)} = \frac{2}{15}$$

$$Q_3^{(8)} = \bar{d}_L \left\{ D^\alpha G_{\mu\alpha}, \widetilde{G}^{\mu\nu} \right\} \gamma_\nu s_L \qquad c_3^{(8)} = -\frac{4}{3}$$

$$Q_4^{(8)} = \bar{d}_L \left\{ G_{\mu\alpha}, D^\alpha \widetilde{G}^{\mu\nu} \right\} \gamma_\nu s_L \qquad c_4^{(8)} = -\frac{2}{3}$$

$$Q_5^{(8)} = \bar{d}_L D^\alpha D_\alpha D^\nu G_{\mu\nu} \gamma^\mu s_L \qquad c_5^{(8)} = -\frac{8}{15} g_s^{-1}$$

New set. Explicit CPS and gauge invariance.

Previous analysis:
Penin & Pivovarov, Phys.Rev.D49, 265 (1994)

Why these 5 operators?

Basis of dimension 8 operators

(Lorentz, CPS, gauge, Bianchi)

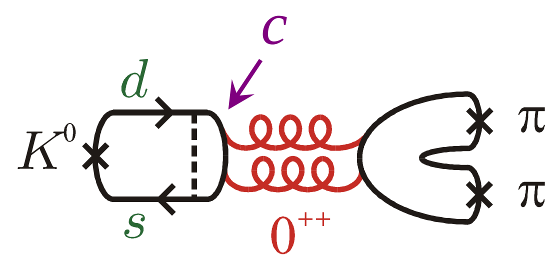
$$Q_1^{(8)} = i\,\bar{d}_L\left[D^\alpha G_{\mu\alpha}, G^{\mu\nu}\right]\gamma_\nu s_L$$
$$Q_2^{(8)} = i\,\bar{d}_L\left[G_{\mu\alpha}, D^\alpha G^{\mu\nu}\right]\gamma_\nu s_L$$
$$Q_3^{(8)} = \bar{d}_L\left\{D^\alpha G_{\mu\alpha}, \widetilde{G}^{\mu\nu}\right\}\gamma_\nu s_L$$
$$Q_4^{(8)} = \bar{d}_L\left\{G_{\mu\alpha}, D^\alpha \widetilde{G}^{\mu\nu}\right\}\gamma_\nu s_L$$
$$Q_5^{(8)} = \bar{d}_L D^\alpha D_\alpha D^\nu G_{\mu\nu}\gamma^\mu s_L$$
$$Q_6^{(8)} = \bar{d}_L\left\{\not{P}, G_{\mu\nu}G^{\mu\nu}\right\}s_L$$
$$Q_7^{(8)} = \bar{d}_L\left(P^\alpha G_{\mu\alpha}G^{\mu\nu} + G^{\mu\nu}G_{\mu\alpha}P^\alpha\right)\gamma_\nu s_L$$
$$Q_8^{(8)} = i\,\bar{d}_L\left[\not{P}, G_{\mu\nu}\widetilde{G}^{\mu\nu}\right]s_L$$
$$Q_9^{(8)} = i\,\bar{d}_L\left(P^\alpha G_{\mu\alpha}\widetilde{G}^{\mu\nu} - \widetilde{G}^{\mu\nu}G_{\mu\alpha}P^\alpha\right)\gamma_\nu s_L$$
$$Q_{10}^{(8)} = i\,\bar{d}_L\left(P^\alpha G^{\mu\nu}\widetilde{G}_{\mu\alpha} - \widetilde{G}_{\mu\alpha}G^{\mu\nu}P^\alpha\right)\gamma_\nu s_L$$
$$Q_{11}^{(8)} = \bar{d}_L P^\alpha D^\mu G_{\mu\nu} P_\alpha \gamma^\nu s_L$$
$$Q_{12}^{(8)} = \bar{d}_L\left(P^\alpha D_\alpha G_{\mu\nu}P^\mu + P^\mu D_\alpha G_{\mu\nu}P^\alpha\right)\gamma^\nu s_L$$
$$Q_{13}^{(8)} = \bar{d}_L\left(P^\alpha D^\mu G_{\mu\alpha}P_\nu + P_\nu D^\mu G_{\mu\alpha}P^\alpha\right)\gamma^\nu s_L$$
$$Q_{14}^{(8)} = \bar{d}_L\left\{P^\alpha,\left\{P_\alpha,\left\{P^\mu, \widetilde{G}_{\mu\nu}\right\}\right\}\right\}\gamma^\nu s_L$$
$$Q_{15}^{(8)} = i\,\bar{d}_L\left(P^\alpha D_\alpha \widetilde{G}_{\mu\nu}P^\mu - P^\mu D_\alpha \widetilde{G}_{\mu\nu}P^\alpha\right)\gamma^\nu s_L$$
$$Q_{16}^{(8)} = i\,\bar{d}_L\left(P_\alpha G_{\mu\beta}G_\gamma^\mu - G_\gamma^\mu G_{\mu\beta}P_\alpha\right)\varepsilon^{\alpha\beta\gamma\delta}\gamma_\delta s_L$$
$$Q_{17}^{(8)} = i\,\bar{d}_L P_\alpha D^\mu G_{\mu\beta} P_\gamma \varepsilon^{\alpha\beta\gamma\delta}\gamma_\delta s_L$$

To 1 loop: no derivative on $\bar{d}_L, s_L \;\rightarrow\; Q_1^{(8)} \ldots Q_5^{(8)}$

3. Anomalous operators

$$Q_{TA}^{(8)} \sim \left(m_s\, \bar{d}_L s_R + m_d\, \bar{d}_R s_L\right) G^a_{\mu\nu} G^{\mu\nu a} \in L_{8d}\ ?$$

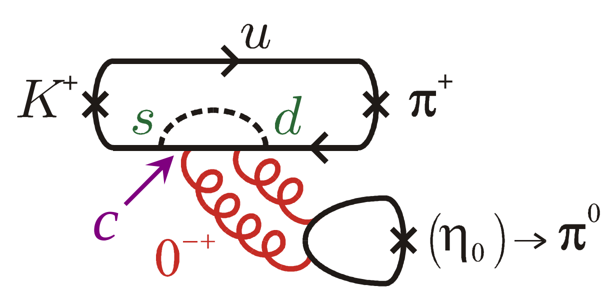
$$Q_1^{(8)} + Q_2^{(8)} = i\,\underbrace{\bar{d}_L\, D^\alpha \left[G_{\mu\alpha}, G^{\mu\nu}\right]\gamma_\nu s_L}$$



$\not\ni 1_C \quad \rightarrow$ no trace anomaly operator (CPS)

$$Q_{AA}^{(8)} \sim \left(m_s\, \bar{d}_L s_R - m_d\, \bar{d}_R s_L\right) G^a_{\mu\nu} \widetilde{G}^{\mu\nu a} \in L_{8d}\ ?$$

$$Q_3^{(8)} + Q_4^{(8)} = \underbrace{\bar{d}_L D^\alpha \left\{G_{\mu\alpha}, \widetilde{G}^{\mu\nu}\right\}\gamma_\nu s_L}$$



$\ni \frac{1}{12}\bar{d}_L \not\partial (G^a_{\mu\nu}\widetilde{G}^{\mu\nu a}) s_L \quad \rightarrow$ axial anomaly operator:

$$Q_{AA}^{(8)} = \tfrac{-i}{12}\left(m_s\, \bar{d}_L s_R - m_d\, \bar{d}_R s_L\right) G^a_{\mu\nu}\widetilde{G}^{\mu\nu a}$$

However: $\left\langle \pi^0 \middle| G^a_{\mu\nu}\widetilde{G}^{\mu\nu a} \middle| 0 \right\rangle \propto$ isospin breaking

4. Other applications

❖ Heavy quark contribution to light meson decay constants

ex: $$-if_{\eta'}^{(c)} p_{\eta'}^{\mu} \equiv \langle \eta' | \bar{c}\gamma^{\mu}\gamma_5 c | 0 \rangle = \langle \eta' | -iTr\left\{\gamma^{\mu}\gamma_5 \left(\frac{1}{\not{P}-m_c}\right)_{xx}\right\} | 0 \rangle$$

S_5

Alternative analysis:

Halperin & Zhitnitsky, Phys.Rev.D56, 7247 (1997)
Franz et al, Phys.Rev.D62, 074024 (2000)

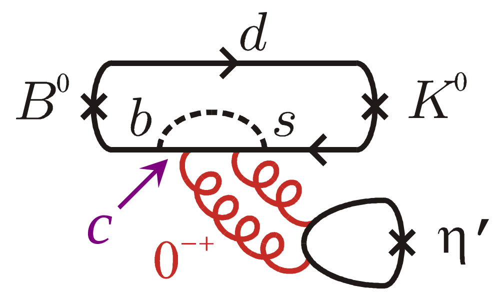
$$f_{\eta'}^{(c)} m_{\eta'}^2 = \langle \eta' | 2m_c i\bar{c}\gamma_5 c - \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu a} | 0 \rangle = \langle \eta' | 2m_c Tr\left\{\gamma_5 \left(\frac{1}{\not{P}-m_c}\right)_{xx}\right\} - \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu a} | 0 \rangle$$

S_6

« anomalous » contribution:

$$f_{\eta'}^{(c)AA} = \frac{-1}{32m_c^2} \langle \eta' | \frac{\alpha_s}{\pi} G_{\alpha\beta}^a \tilde{G}^{\alpha\beta a} | 0 \rangle \simeq -3MeV$$

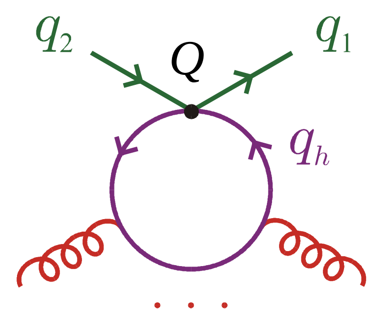
$\rightarrow$ only a few % of $A(B \rightarrow K\eta')_{\exp}$ in the factorization approximation



- Different chiral structure of the initial op. Q

ex: $Q = [\bar{d}(1+\gamma_5)c][\bar{c}(1+\gamma_5)s]$ (multi-higgs)

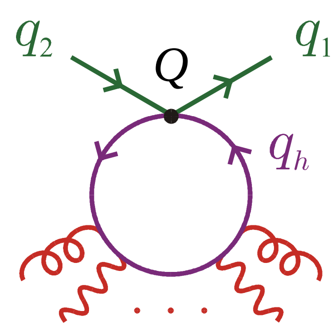
$$\rightarrow \; L_{eff} = -4ig_w \sum_{n=2}^{5} \bar{d}_L \underbrace{S_{2n}} s_R + O(M^{-3})$$



- Introduction of electromagnetism

$$P_\mu = i\partial_\mu + g_s A_\mu^a \frac{\lambda^a}{2} + eQA_\mu$$

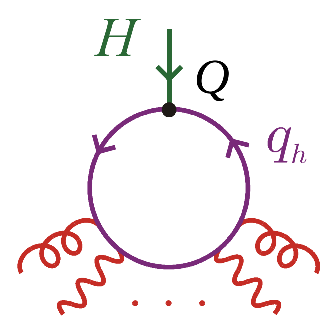
$$\begin{cases} g_s G_{\mu\nu} \rightarrow g_s G_{\mu\nu} + eQF_{\mu\nu} \\ g_s D_{\alpha_1} \ldots D_{\alpha_n} G_{\mu\nu} \rightarrow g_s D_{\alpha_1} \ldots D_{\alpha_n} G_{\mu\nu} + eQ\partial_{\alpha_1} \ldots \partial_{\alpha_n} F_{\mu\nu} \end{cases}$$



- Other type of initial operator Q

ex: $Q = H \, \bar{q}_h \, q_h$

$$\rightarrow \; L_{eff} = ig_H H \sum_{n=2}^{5} Tr\underbrace{\left(S_n\right)} + O(M^{-3})$$



Part II: Light quark induced anomalous operators

1. Nonlinear realizations

$$L_{NL} = \frac{F_\pi^2}{4} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_\pi^2}{4} r Tr\left(m U^\dagger + U m\right) + \frac{F_\pi^2}{16 N_C} m_0^2 \left(Tr \ln U - Tr \ln U^\dagger\right)^2$$

$$U = \exp\left(\frac{i\sqrt{2}}{F_\pi}\Phi\right) \qquad \Phi = \frac{\lambda^a}{\sqrt{2}}\varphi^a \qquad a = 0, \ldots, 8$$

Leading NL realizations of $G_{\mu\nu}^a G^{\mu\nu a}$ and $G_{\mu\nu}^a \widetilde{G}^{\mu\nu a}$ for $m_q \neq 0$?

Noether: $S = S' \iff \underbrace{\partial_\mu j_N^\mu + \left(\frac{\partial L}{\partial \phi} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi)}\right) i\Delta\phi}_{\equiv \rho} = 0 \implies \partial_\mu j_N^\mu \overset{eom}{=} 0$

without use of the eom!

$U(1)_A$	*dilatations*
$\rho_A^{QCD} = 2i\bar{\psi} m \gamma_5 \psi - \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \widetilde{G}^{\mu\nu a}$	$\rho_D^{QCD} = \bar{\psi} m (1 + \gamma_m^{(g_s)}) \psi + \frac{\beta(g_s)}{2 g_s} G_{\mu\nu}^a G^{\mu\nu a}$
$\rho_A^{NL} = \frac{iF_\pi^2}{2} r Tr(m(U - U^\dagger)) + \frac{iF_\pi^2}{2} m_0^2 Tr(\ln U - \ln U^\dagger)$	$\rho_D^{NL} = -F_\pi^2 r Tr(m(U + U^\dagger)) - \frac{F_\pi^2}{2} Tr(\partial_\mu U \partial^\mu U^\dagger) + (m_0^2)$

2. Anomalous operators

$$Q_{AA} \sim \left(mU^\dagger - Um\right)^{sd} m_0^2 Tr\left(\ln U - \ln U^\dagger\right)$$

$$Q_{TA} \sim \left(mU^\dagger + Um\right)^{sd} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right)$$

generated by 1-loop NL corrections to the density operator Q_D

$$Q_D = r\left(mU^\dagger + Um\right)^{sd} \quad \text{(coupling } G_D F_\pi^4\text{)}$$

$\downarrow NL$

B.F. method

$\Lambda(\varepsilon)$: UV (IR) cut-off

$$L_{eff} = G_D F_\pi^4 \left(1 - \frac{3\Lambda^2}{(4\pi F_\pi)^2} + \frac{m_0^2}{3(4\pi F_\pi)^2} \ln\left(\frac{\Lambda^2}{\varepsilon^2}\right)\right) Q_D + \frac{G_D F_\pi^2}{4(4\pi)^2} \ln\left(\frac{\Lambda^2}{\varepsilon^2}\right) \sum_{k=1}^{3} c_k^{(p^4)} Q_k^{(p^4)} + O\left(p^{(6)}\right)$$

$$Q_1^{(p^4)} = r\left(mU^\dagger + Um\right)^{sd} Tr\left[\partial_\mu U \partial^\mu U^\dagger + r\left(mU^\dagger + Um\right)\right] \qquad c_1^{(p^4)} = 1$$

$$Q_2^{(p^4)} = r\left(mU^\dagger - Um\right)^{sd} Tr\left[\partial_\mu \left(U \partial^\mu U^\dagger\right)\right] \qquad c_2^{(p^4)} = -2$$

$$Q_3^{(p^4)} = r Tr\left[\left(\lambda_+ mU^\dagger + Um\lambda_+\right)\left(\partial_\mu U \partial^\mu U^\dagger + r\left(mU^\dagger + Um\right)\right)\right] \qquad c_3^{(p^4)} = 3 \qquad \lambda_+^{ij} = \delta^{id}\delta^{js}$$

Effect in $K \to \pi\pi$?

• Effect of Q_{TA} in $K^0 \to \pi\pi$ (compared to other operators)

$$A(K^0 \to \pi^+\pi^-)_D = -\frac{iG_D}{8\sqrt{2}\pi^2 F_\pi}(m_K^2 - m_\pi^2)m_K^2 \,[C_1 + C_3]\, \ln\left(\frac{\Lambda^2}{\varepsilon^2}\right)$$

$$C_1 + C_3 = \underbrace{\left(1 - \frac{2m_\pi^2}{m_K^2}\right)}_{C_1^{TA}} + \underbrace{\left(\frac{2m_\pi^2}{m_K^2}\right)}_{C_1^m} + \underbrace{\left(\frac{3m_\pi^2}{2m_K^2}\right)}_{C_3} = 1 + \frac{3m_\pi^2}{2m_K^2}$$

TA is dominant: $\frac{C_1^{TA}}{C_3} \simeq 7.6,\quad \frac{C_1^{TA}}{C_1^m} \simeq 5.7,\quad \frac{C_1^{TA}}{C_1^m + C_3} \simeq 3.3$ → 15-20%?

• Effect of Q_{AA} in $K^+ \to \pi^+\pi^0$ (compared to other operators)

$$A(K^+ \to \pi^+\pi^0)_D = \frac{15iG_D}{64\pi^2 F_\pi}(m_K^2 - m_\pi^2)(m_{K^0}^2 - m_{K^+}^2 - \delta_{em}^2)\,[C_2 + C_3]\, \ln\left(\frac{\Lambda^2}{\varepsilon^2}\right)$$

$$C_2 + C_3 = \underbrace{\left(-\frac{4m_\pi^2}{5m_0^2}\right)}_{C_2} + \underbrace{\left(1 - \frac{6m_\pi^2}{5m_0^2}\right)}_{C_3} = 1 - \frac{2m_\pi^2}{m_0^2}$$

Small effect.

Conclusion

Anomalous operators for kaon decays

- Heavy quark induced (c): suppressed!
- Light quark induced (u,d,s): possible TA effect in $K\to\pi\pi$ (15-20%?)

\+ Leading nonlinear realizations of $G^a_{\mu\nu}G^{\mu\nu a}$ and $G^a_{\mu\nu}\tilde{G}^{\mu\nu a}$ for $m_q \neq 0$

Heavy quark propagator in QCD(QED) external fields

$$\left(\frac{1}{\not{P}-M}\right)_{xx} = S_2 + S_3 + S_4 + S_5 + O(M^{-3})$$

1-loop heavy quark integration

↙ ↘

Effective Lagrangians

Dimension 8 operators describing c quark effects in K decays (effect $\sim m_K^2/m_c^2 \sim$ 10%)

Matrix elements

Alternative computation of $f_{\eta'}^{(c)}$