Non-perturbative computation of Λ_{QCD} in the 2-flavour theory

Rainer Sommer, ALPHA Collaboration

DESY, Zeuthen

EURIDICE collaboration meeting, Frascati, February 2005

What do we want

$$\overbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_\mathrm{K} \\ m_\mathrm{D} \\ m_\mathrm{B} \end{bmatrix}}^{\text{Experiment}} \quad \underset{\Longrightarrow}{\mathcal{L}_\mathrm{QCD}(g_0, m_f)} \quad \overbrace{\begin{bmatrix} \Lambda_\mathrm{QCD} \\ \hat{M} = (M_\mathrm{u} + M_\mathrm{d})/2 \\ M_\mathrm{s} \\ M_\mathrm{c} \\ M_\mathrm{b} \end{bmatrix}}^{\text{QCD parameters (RGI)}} \quad + \quad \overbrace{\begin{bmatrix} \xi \\ F_B \\ B_B \\ \ldots \end{bmatrix}}^{\textbf{Predictions}}$$

The Collaboration

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In this talk

- Concentrate on the determination of the Λ parameter

Determinations of $\alpha_s(\mu) = \alpha_{\overline{\rm MS}}$

- involve various theoretical assumptions/approximations

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Determinations of $\alpha_s(\mu) = \alpha_{\overline{MS}}$

- involve various theoretical assumptions/approximations
- most notably: PT is always needed:
  – $\alpha_{\overline{MS}}(\mu)$ is only defined perturbatively;
  $\longrightarrow$ makes sense when it is small
- potential for determinations of $\alpha_s(\mu)$ from Lattice QCD
  - rather weak assumptions are necessary
  - potentially very precise results can be obtained
  - only way to compute $\alpha_{\overline{MS}}(M_Z)$ with experimental input from low energy QCD (hadron spectrum)
  - Comparison to $\alpha_{\overline{MS}}(M_Z)$ from LEP-data or HERA at high Q^2
    $\longrightarrow$ confirmation that QCD describes both weakly coupled quarks/gluons and hadrons
  - Continuous progress (+ sometimes a little jump)
    - on the technical problems
    - in computer technology

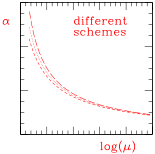
Renormalization group and Λ-parameter (mass–independent scheme)

RGE

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_{\rm f} \right)$$



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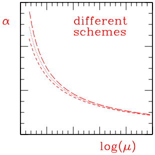

- *exact equation* for Λ $\qquad (\bar{g} \equiv \bar{g}(\mu))$

$$\Lambda = \mu \left(b_0 \bar{g}^2 \right)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0\bar{g}^2} \exp\left\{ - \int_0^{\bar{g}} \mathrm{d}g \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

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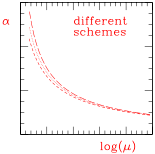

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- trivial scheme dependence:

$$\Lambda_a / \Lambda_b = \exp\{ c_{a,b} / (4\pi b_0) \} , \qquad \alpha_a = \alpha_b + c_{a,b}\, \alpha_b^2 + \mathrm{O}(\alpha_b^3)$$

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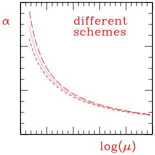

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- use a suitable physical coupling (scheme) and compute Λ

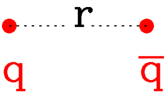
  Requires non-perturbative computation of $\beta(\bar{g})$

Physical couplings

– what is a non-perturbatively defined coupling

Example

$m_q \to \infty$



$$F(r) = \frac{4}{3}\frac{1}{r^2}\left\{\alpha_{\overline{\rm MS}}(\mu) + c_1[\alpha_{\overline{\rm MS}} bar(\mu)]^2 + \ldots\right\}, \quad \mu = 1/r$$

$$\equiv \frac{4}{3}\frac{1}{r^2}\,\alpha_{\rm qq}(\mu)$$

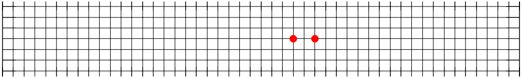
Physical couplings: their properties

- defined for all energies μ
- independent of the regularization procedure
  $\longrightarrow$ i.e. (on the lattice) the continuum limit can be taken
- any one of them defines a renormalization scheme
- the usual perturbative properties when α is small, e.g.

$$\alpha_a(\mu) = \alpha_b(\mu) + c_{a,b}\,[\alpha_b(\mu)]^2 + \ldots$$

(also for $a = \overline{\mathrm{MS}}$).

Problem in a lattice computation ($\alpha_{\rm qq}$ as an example)



$$L \gg \frac{1}{0.2\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a$$

L ↑ box size; $\frac{1}{0.2\mathrm{GeV}}$ ↑ confinement scale; a ↑ spacing

$$\Downarrow$$

$$L/a \gg 50$$

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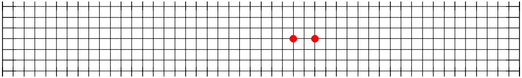

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Solution: $\boxed{L = 1/\mu}$ $\longrightarrow$ left with $L/a \gg 1$ [Wilson, ... , Lüscher, Weisz, Wolff]

Finite size effect as a physical observable; finite size scaling!

Strategy

finite volume coupling $\alpha_{\rm SF}(\mu), \mu = 1/L$
defined at zero quark mass

$L_{\rm max} = {\rm const.}/F_\pi = {\rm O}(\frac{1}{2}{\rm fm}) : \quad \longrightarrow \quad \alpha_{\rm SF}(\mu = 1/L_{\rm max})$

$\downarrow$

$\alpha_{\rm SF}(\mu = 2/L_{\rm max})$

$\downarrow$

•

always $a/L \ll 1$ •

•

$\downarrow$

$\alpha_{\rm SF}(\mu = 2^n/L_{\rm max} = 1/L_{\rm min})$

PT: $\downarrow$

$\Lambda_{\rm SF} L_{\rm max} = \#$

Result is a value for $\boxed{\Lambda_{\rm SF}/F_\pi = \#}$

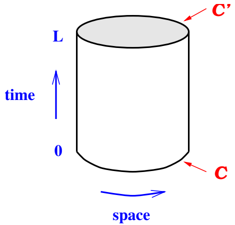
Schrödinger functional

[Lüscher, Narayanan, Weisz & Wolff; Sint]

practical finite volume renormalization scheme

- Boundary conditions at $x_0 = 0$:

$$
\begin{aligned}
k = 1,2,3: \quad A_k(x) &= C_k(\mathbf{x}) \\
\tfrac{1}{2}(1+\gamma_0)\psi(x) &= \rho(\mathbf{x}), \\
\overline{\psi}(x)\tfrac{1}{2}(1-\gamma_0) &= \bar{\rho}(\mathbf{x})
\end{aligned}
$$



(LxLxL box with periodic b.c.)

$$\text{Schrödinger functional} = \int_{\text{fields}} \mathrm{e}^{-S}$$

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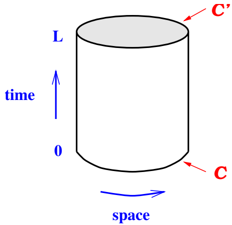

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$$\text{Schrödinger functional} = \int_{\text{fields}} \mathrm{e}^{-S}$$

$\bar{g}$ infinitessimal variation of boundary fields C, C'
cf. Casimir effect

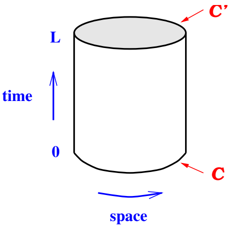
Schrödinger functional renormalization scheme, few details

- $$A_k(0,\mathbf{x}) = C(\eta)\,,\; A_k(T,\mathbf{x}) = C'(\eta)$$

$$\mathcal{Z}(\eta) = \mathrm{e}^{-\Gamma(\eta)} = \int_{\text{fields}} \mathrm{e}^{-S}$$

$$\Gamma' = \frac{\mathrm{d}\Gamma}{\mathrm{d}\eta} = \frac{1}{g_0^2}\Gamma_0' + \Gamma_1' + g_0^2\Gamma_2' + \ldots$$

$$\boxed{\bar{g}^2_{\mathrm{SF}}(L) = \frac{\Gamma_0'}{\Gamma'}}$$



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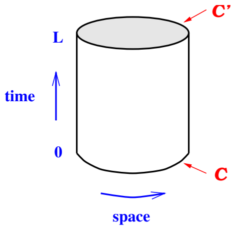

(LxLxL box with periodic b.c.)

- at zero quark mass:

$$m_{\mathrm{PCAC}} = \frac{\langle \mathcal{O}\; \partial_\mu\, \overline{\psi}\gamma_\mu\gamma_5\tau^a\psi\rangle}{\langle \mathcal{O}\; \overline{\psi}\gamma_5\tau^a\psi\rangle} = 0$$

Schrödinger functional

[Lüscher, Narayanan, Weisz & Wolff; Sint]

properties

- Dirac operator has spectral gap $\propto 1/L$ also at $m_{\rm quark} = 0$
- simulations with mass-less quarks possible and "easy"
- "easy" perturbation theory
  - 3-loop β-function known [Lüscher, Weisz; Christou & Panagopoulos; Bode, Weisz, Wolff]
  - 2-loop discretization errors known [Bode, Weisz, Wolff]

The step scaling function

- … is a discrete *beta* function:

$$\sigma(s, \bar{g}^2(L)) = \bar{g}^2(sL) \qquad \text{mostly } s = 2$$

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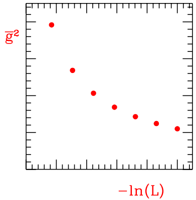

$$\begin{array}{rcl} u_0 & = & \bar{g}^2(L_{\max}) \\ & \downarrow & \\ \sigma(2, u_{k+1}) & = & u_k \\ & \downarrow & \\ u_k & = & \bar{g}^2(2^{-k} L_{\max}) \end{array}$$

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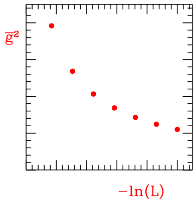


- can be determined on the lattice (spacing a)

$$\Sigma(s,u,a/L) = \sigma(s,u) + \mathrm{O}(a/L)$$

The step scaling function

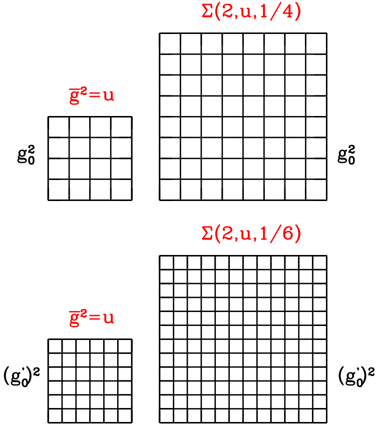
On the lattice:
additional dependence on the resolution a/L

g_0 fixed, L/a fixed:

$$\bar{g}^2(L) = u, \qquad \bar{g}^2(sL) = u' \,,$$
$$\Sigma(s,u,a/L) \;=\; u'$$

continuum limit:

$$\Sigma(s,u,a/L) = \sigma(s,u) + \mathrm{O}(a/L)$$



quark mass is set to zero everywhere

Results for the theory with $N_\mathrm{f} = 2$ and comparison to previous results for $N_\mathrm{f} = 0$

$N_\mathrm{f} = 2$: Phys.Lett.B515:49-56,2001 [Achim Bode, Roberto Frezzotti, Bernd Gehrmann, Martin Hasenbusch, Jochen Heitger, Karl Jansen, Stefan Kurth, Juri Rolf, Hubert Simma, Stefan Sint, R.S., Peter Weisz, Hartmut Wittig, Ulli Wolff]

hep-lat/0411025, to appear in NPB [Michele Della Morte, Roberto Frezzotti, Jochen Heitger, Juri Rolf, Rainer Sommer, Ulli Wolff]

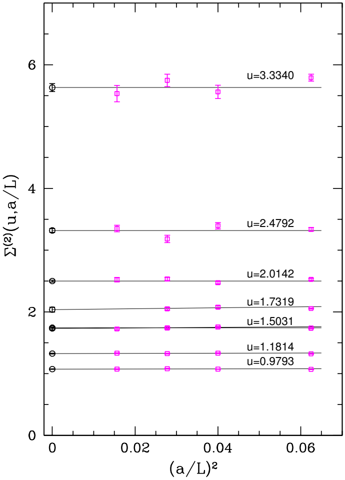
$N_\mathrm{f} = 0$: Nucl.Phys.B544:669-698,1999 [Stefano Capitani, Martin Lüscher, R.S., Hartmut Wittig]

Nucl.Phys.B413:481-502,1994 [Martin Lüscher, R.S., Peter Weisz, Ulli Wolff]

$N_\mathrm{f} = 2$: Numerical simulations on APE-computers at Zeuthen (APE-100, APE-1000)

Continuum limit of step scaling function, $N_\mathrm{f} = 2$

- $N_\mathrm{f} = 2$:
  continuum limit
  - various ansätze for $\mathrm{O}(a/L)$ terms
  - no statistically significant a-effects for $a/L < 1/4$
  - detailed study of a-effects in other quantities
    – other coupling
    – current quark mass

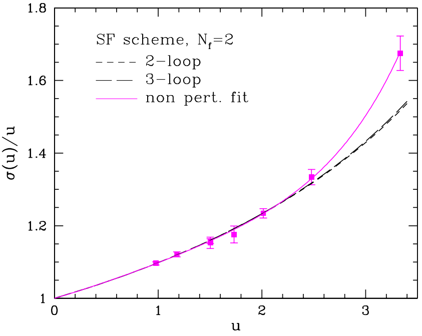


Step scaling function as function of $u = \bar{g}^2$

SSF $\sigma(u)$

comparison to PT

... and NP fit



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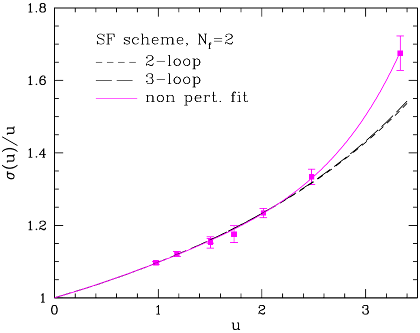
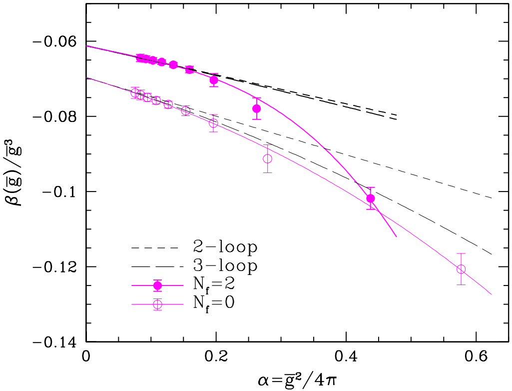
$$-2\log 2 = \int_u^{\sigma(u)} \frac{\mathrm{d}x}{\sqrt{x}\ \beta(\sqrt{x})}$$

determine also directly $\beta(\bar{g})$



β-function in SF scheme, $N_\mathrm{f} = 2$

comparison to PT and $N_\mathrm{f} = 0$ $\quad$ $\beta(\bar{g})/\bar{g}^3 = -b_0 - b_1\bar{g}^2 + \dots$

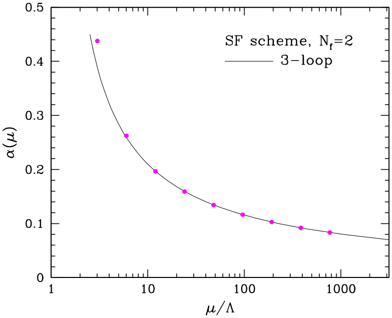


non-perturbative deviations from 3-loop β for $\alpha_{\mathrm{SF}} > 0.25$

Non-perturbative running of α, $N_\mathrm{f} = 2$

SF-scheme, NP, $N_\mathrm{f} = 2$

error bars are smaller than symbol size



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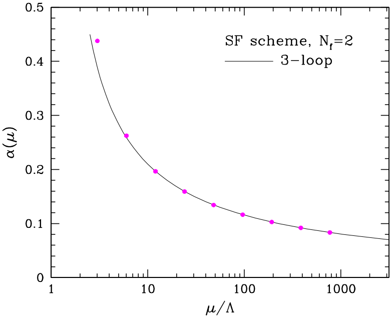
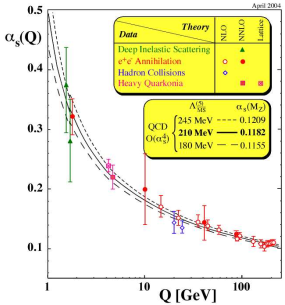
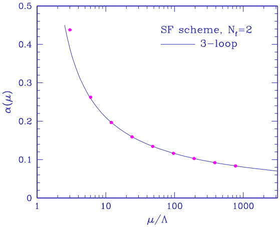


Experiment + PT
$\overline{\mathrm{MS}}$-scheme
[S. Bethke 2004]



The Lambda parameter

Define $L_{\max}$ by $\bar{g}^2(L_{\max}) = 5.5$
k steps with NP σ:
$\bar{g}^2(L_{\max}/2^k) = \sigma(\bar{g}^2(L_{\max}/2^{k+1}))$



$\Downarrow$

	global fit		const. fit, $L/a = 6, 8$		mixed cont. ext.	
i	u_i	$-\ln(\Lambda L_{\max})$	u_i	$-\ln(\Lambda L_{\max})$	u_i	$-\ln(\Lambda L_{\max})$
0	5.5	0.957	5.5	0.957	5.5	0.957
1	3.309(40)	1.070(26)	3.291(18)	1.081(12)	3.291(19)	1.081(12)
2	2.485(31)	1.089(37)	2.480(20)	1.096(23)	2.471(20)	1.106(24)
3	2.015(27)	1.085(49)	2.010(19)	1.093(35)	2.004(19)	1.103(35)
4	1.700(22)	1.077(58)	1.693(16)	1.094(43)	1.693(17)	1.095(44)
5	1.473(19)	1.071(65)	1.464(14)	1.101(49)	1.468(15)	1.088(53)
6	1.300(16)	1.066(73)	1.290(12)	1.111(56)	1.296(14)	1.081(63)
7	1.164(15)	1.063(83)	1.153(11)	1.124(63)	1.162(13)	1.077(75)
8	1.054(13)	1.062(94)	1.043(10)	1.138(71)	1.053(13)	1.074(88)

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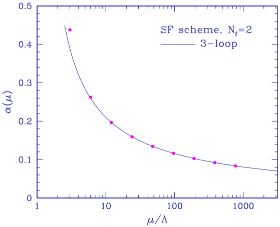

⇓

	global fit		const. fit, $L/a = 6, 8$		mixed cont. ext.	
i	u_i	$-\ln(\Lambda L_{\max})$	u_i	$-\ln(\Lambda L_{\max})$	u_i	$-\ln(\Lambda L_{\max})$
0	5.5	0.957	5.5	0.957	5.5	0.957
1	3.309(40)	1.070(26)	3.291(18)	1.081(12)	3.291(19)	1.081(12)
2	2.485(31)	1.089(37)	2.480(20)	1.096(23)	2.471(20)	1.106(24)
3	2.015(27)	1.085(49)	2.010(19)	1.093(35)	2.004(19)	1.103(35)
4	1.700(22)	1.077(58)	1.693(16)	1.094(43)	1.693(17)	1.095(44)
5	1.473(19)	1.071(65)	1.464(14)	1.101(49)	1.468(15)	1.088(53)
6	1.300(16)	1.066(73)	1.290(12)	1.111(56)	1.296(14)	1.081(63)
7	1.164(15)	1.063(83)	1.153(11)	1.124(63)	1.162(13)	1.077(75)
8	1.054(13)	1.062(94)	1.043(10)	1.138(71)	1.053(13)	1.074(88)

Result: $-\ln(\Lambda L_{\max}) = 1.07(7)$

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  r_0 defined from QQ-Force $F(r_0)r_0^2 = 1.65$ [R.S., 1994]

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- ... and r_0/a available only at three values of a [UKQCD; JLQCD+CPPACS]

		$u_{\max} = 3.65$		$u_{\max} = 4.61$	
β	r_0/a	$L_{\max}/a$	$\Lambda_{\overline{\rm MS}}\, r_0$	$L_{\max}/a$	$\Lambda_{\overline{\rm MS}}\, r_0$
5.20	5.45(5)(20)	4.00(6)	0.655(27)	6.00(8)	0.610(25)
5.29	6.01(4)(22)	4.67(6)	0.619(25)	6.57(6)	0.614(24)
5.40	7.01(5)(15)	5.43(9)	0.621(17)	7.73(10)	0.609(16)

$\Rightarrow \Lambda_{\overline{\rm MS}} = 0.62(4)(4)$ $\qquad \Rightarrow \Lambda_{\overline{\rm MS}}^{N_{\rm f}=2} = 245(16)(16)\,\mathrm{MeV}$

- errors due to: (r_0/a) $(\ln(\Lambda L_{\max}))$

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- looks like an irregular $N_{\rm f}$-dependence ... but relatively large errors
- Perturbatively: $N_{\rm f} = 4 \longrightarrow N_{\rm f} = 3$: Λ grows further, but

  Is a perturbatively determined $\Lambda_{\overline{\rm MS}}^{N_{\rm f}=3}$ accurate ?

  Need $\mu \ll m_{\rm beauty}$ where pert. theory is accurate.

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- Improvements of lattice results:
  needs continuum limit of $F_\pi \times L_{\mathrm{max}}$
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  $N_{\rm f} = 3$ is on the agenda of JLQCD and CPPACS, using the same methods.
- quark masses are in progress along the same lines
  $N_{\rm f} = 2$ simulations for the μ-dependence are finished
  final analysis to be done

Appendix: Quark masses, a (trivial?) question

- On the lattice (continuum limit ...)

$$\underbrace{M_{\rm ref}}_{\text{RGI}} = \underbrace{Z_{\rm M}}_{\text{SSF etc.}} \times \underbrace{m_{\rm ref}}_{\text{bare}}$$

with $(m_{\rm PS}(m_{\rm u}, m_{\rm d}, m_{\rm s})...)$

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(more assumptions/approx.'s)

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- At present the only lattice-method with results for $N_\mathrm{f} = 3$
- Take the lattice spacing from the computation of some hadronic quantity like the Υ spectrum:

$$a = \frac{(a\Delta m)_\mathrm{lattice}}{(\Delta m)_\mathrm{exp}}\,, \qquad a \leftrightarrow g_0$$

then $$\alpha_{\overline{\mathrm{MS}}}(s_0\, a^{-1}) = \alpha_0 + 4.45\alpha_0^3 + \mathrm{O}(\alpha_0^4) + \mathrm{O}(a)\,, \quad \alpha_0 = g_0^2/(4\pi)$$

  - $s_0 = 28.8$!
  - badly behaved expansion (for $s_0 = \mathrm{O}(1)$ it is even worse!)
  - α_0 is a bad expansion parameter

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- improved bare couplings: “tadpole improvement” [Parisi; Lepage,Mackenzie]
- idea: there is some approximately universal, large renormalization of α_0

  use some short distance observable such as

  $$P = \frac{1}{N}\langle \mathrm{tr}\, U(p)\rangle \qquad -\frac{1}{C_\mathrm{F}\pi}\ln(P) = \alpha_0 + 3.373\alpha_0^2 + 17.70\alpha_0^3 + \ldots$$

  to define an improved bare coupling

  $$\alpha_\Box \equiv -\frac{1}{C_\mathrm{F}\pi}\ln(\underbrace{P}_{\text{from MC!}})$$

  then

  $$\alpha_{\overline{\mathrm{MS}}}(s_0 a^{-1}) = \alpha_\Box + 0.614\alpha_\Box^3 + \mathrm{O}(\alpha_0^4) + \mathrm{O}(a)$$

properties of improved bare couplings

- only calculation of a hadronic scale is needed
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- can't reach high energy scales ($a^{-1} \approx 2\mathrm{GeV}$)
  must assume that perturbative running and matching is accurate
- results (not up to date)

  [Davies et al.] $\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.117(2)$

  [SESAM collaboration] $\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.112(2)$

  (average of PDG) $\alpha_{\overline{\mathrm{MS}}}(M_Z) = 0.115(3)$

  errors appear quite optimistic!