Euridice Meeting 2005

Long-distance effects in $K \to \pi\nu\bar{\nu}$

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Plan

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Introduction

- Experimental status

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) \overset{SM}{=} (7.8 \pm 1.2) \times 10^{-11}$$

$$Br(K_L \to \pi^0 \nu\bar{\nu}) \overset{SM}{=} (3.0 \pm 0.6) \times 10^{-11}$$

Experiment	*Place*	*Status*
KTeV	Fermilab	Now finished. Upper limit $Br(K_L \to \pi^0 \nu\bar{\nu}) < 5.9 \times 10^{-7}$
AGS-E787/E949	Brookhaven	Observed 3 events: $Br(K^+ \to \pi^+ \nu\bar{\nu}) = 14.7^{+13.0}_{-8.9} \times 10^{-11}$ Expects around 10 events in all.
E391a	KEK	Now running. Expect sensitivity to 10^{-10} for $K_L \to \pi^0 \nu\bar{\nu}$
KOPIO	Brookhaven	Start construction 2005? $K_L \to \pi^0 \nu\bar{\nu}$ (60 events)
J-PARC	KEK	Construction ends 2008? $K_L \to \pi^0 \nu\bar{\nu}$ (1000 events)
NA48 /3 /4 /5...	CERN	$K^+ \to \pi^+ \nu\bar{\nu}$ Letter of intent (100 events by 2008) $K_L \to \pi^0 \ell^+ \ell^-$ Proposals... running in 2008-2010? $K_L \to \pi^0 \nu\bar{\nu}$
???	Fermilab	???

- Theoretical aspects of $K \to \pi\nu\bar{\nu}$

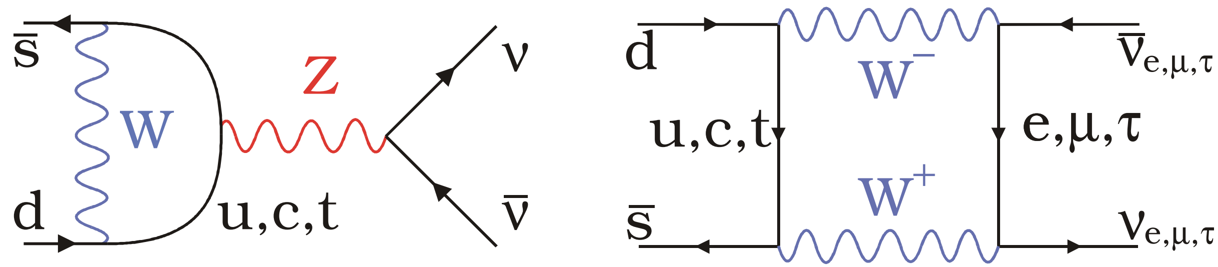
1- Properties: Dominated by short-distance physics.
Probe the SM at the quantum level (FCNC).
Clean theoretical predictions.
High sensitivity to New Physics.

$$H_{eff}(\bar{s}d \to \nu\bar{\nu}) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \left[V_{ts}^* V_{td} X(x_t) + V_{cs}^* V_{cd} X(x_c) \right] (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

2- *FCNC process*: $X_0(x_t) = C_0(x_t) - 4B_0(x_t) \approx x_t \quad when \ x_t \to \infty$

(short-distance)

$$x_q = m_q^2 / M_W^2$$



3- *Matrix elements* are known from the well-measured $K^+ \to \pi^0 \ell^+ \nu_\ell \ (K_{\ell 3})$ using isospin symmetry:

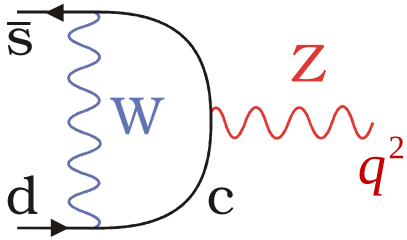
$$\langle \pi^0 | (\bar{s}u)_V | K^+ \rangle \sim \langle \pi^0 | (\bar{s}d)_V | K^0 \rangle \sim \langle \pi^+ | (\bar{s}d)_V | K^+ \rangle$$

- Our goals:

(Gino Isidori, Federico Mescia, C.S.)

Improving the theoretical control over subleading contributions:

1- Identification of *dimension eight operators*



$$= \text{“}C_0(x_c, q^2)\text{”} = C_0(x_c)\left(1 + \frac{q^2}{m_c^2}\log x_c + \ldots\right)$$

generates new local operator of the kind $(\bar{s}d)_{V-A}\partial^2(\bar{\nu}\nu)_{V-A}$.

Typically, $q^2 \approx m_K^2 \Rightarrow$ expected corrections to $X_0(x_c)$ of the order of $\approx 15\%$.

2- Residual non-local, *long-distance u-quark contributions*

Estimation from Chiral Perturbation Theory including, contrary to previous works, bosonization of all the relevant FCNC local interactions.

Effects of the order of $\Lambda_{QCD}^2 / m_c^2 \approx 10\%$ possible.

3- Improved estimation of *matrix elements of c-quark dim-8 operators*

by matching with the LD, u-quark amplitude.

Dim. 6 **Dim. 8** **Dim. 10**

$$A_{eff}\left(q^2 \to 0\right) \approx \left(\bar{s}d\right)_{V-A} \lambda_t \left[X_0(x_t) + x_t \frac{q^2}{m_t^2}\left(\log x_t + \frac{q^2}{m_t^2} + \ldots\right)\right]\left(\bar{\nu}\nu\right)_{V-A}$$

$$+\left(\bar{s}d\right)_{V-A} \lambda_c \left[X_0(x_c) + x_c \frac{q^2}{m_c^2}\left(\log x_c + \frac{q^2}{m_c^2} + \ldots\right)\right]\left(\bar{\nu}\nu\right)_{V-A}$$

$$+\left(\bar{s}d\right)_{V-A} \lambda_u \left[X_0(x_u) + x_u \frac{q^2}{m_u^2}\left(\log x_u + \frac{q^2}{m_u^2} + \ldots\right)\right]\left(\bar{\nu}\nu\right)_{V-A}$$

- High-virtuality u-quarks are removed using CKM unitarity ($\mu_{IR} = \mu_{UV} \approx$ 1 GeV)

$$\lambda_c \frac{q^2}{M_W^2}\left(\log x_c - \log x_u\right) \to \lambda_c \frac{q^2}{M_W^2}\left(\log \frac{m_c}{\mu_{IR}} - \log \frac{m_u}{\mu_{UV}}\right)$$

- The remaining u-quark effects are non-local, since $q^2 \approx m_K^2$.
  $\Rightarrow$ Compute the one-loop amplitude in ChPT (matrix element).

- The ChPT amplitude UV-divergence is (approximately) matched with the c-quark dimension-eight operator matrix elements such that $\mu_{IR} = \mu_{UV}$ cancel out.

OPE at the Charm Scale

- Down from the M_W scale

Integrating out the W, Z and t, but keeping c and u as dynamical:

$$H_{eff}(M_W) \sim \left[X(x_t) Q_{\nu\bar{\nu}}^{\bar{s}d} \right]_t + \left[C_{\nu\bar{\nu}} Q_{\nu\bar{\nu}}^{c\bar{c}} + C_{\ell\nu} Q_{\nu\ell}^{\bar{c}s,\bar{c}d} + C_1 Q_1^c + C_2 Q_2^c + C_3 Q_{\nu\bar{\nu}}^{\bar{s}d} \right]_c$$

$$+ \left[C_{\nu\bar{\nu}} Q_{\nu\bar{\nu}}^{u\bar{u}} + C_{\ell\nu} Q_{\nu\ell}^{\bar{u}s,\bar{u}d} + C_1 Q_1^u + C_2 Q_2^u + C_3 Q_{\nu\bar{\nu}}^{\bar{s}d} \right]_u$$

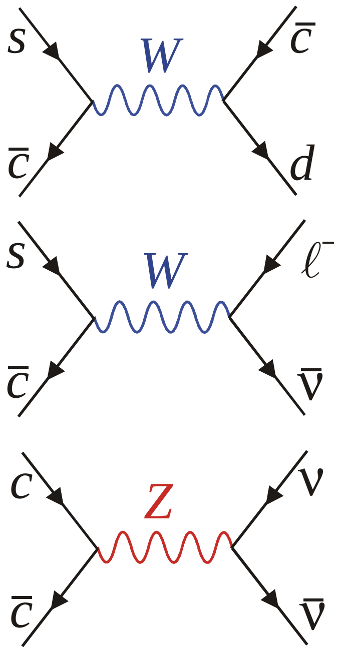
With the operators:

CC: $Q_1^c = (\bar{s}d)_{V-A} \otimes (\bar{c}c)_{V-A}, Q_2^c = (\bar{s}c)_{V-A} \otimes (\bar{c}d)_{V-A}$

CC: $Q_{\bar{\nu}\ell}^{\bar{c}s} = (\bar{c}s)_{V-A} \otimes (\bar{\nu}\ell)_{V-A}$ ($\Rightarrow$ W boxes)

NC: $Q_{\bar{\nu}\nu}^{\bar{c}c} = (\bar{c}\gamma_\mu(v_f - a_f\gamma_5)c) \otimes (\bar{\nu}\nu)_{V-A}$ ($\Rightarrow$ Z penguins)

FCNC: $Q_{\bar{\nu}\nu}^{\bar{s}d} = (\bar{s}d)_{V-A} \otimes (\bar{\nu}\nu)_{V-A}$



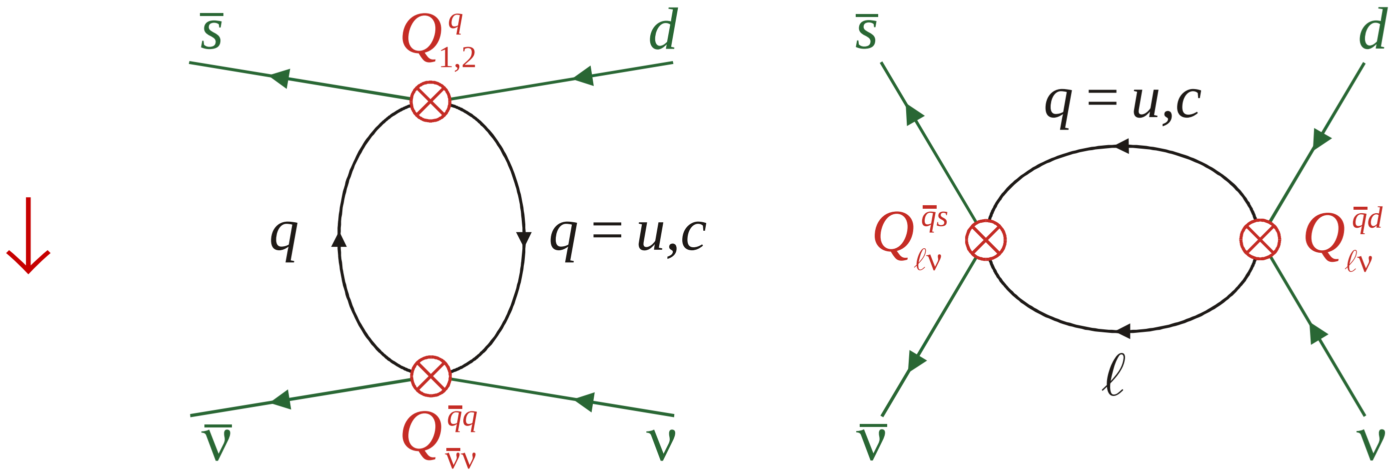
And initial conditions: $C_{\nu\bar{\nu}}(M_W) = C_{\ell\nu}(M_W) = 1,$

$C_1(M_W) = 0, C_2(M_W) = 1, C_3(M_W) = 0$

- Integrating out the charm

The Wilson coefficients get modified by QCD effects through the running from M_W to m_c, at which point the c is removed from the degrees of freedom:

$$H_{eff}(>m_c) \sim \left[C_{\nu\bar{\nu}} Q_{\nu\bar{\nu}}^{c\bar{c}} + C_{\ell\nu} Q_{\nu\ell}^{\bar{c}s,\bar{c}d} + C_1 Q_1^c + C_2 Q_2^c + C_3 Q_{\nu\bar{\nu}}^{\bar{s}d} \right]_c \quad (C_i = C_i(m_c))$$



$$H_{eff}(<m_c) \sim \lambda_c \left[C_3' Q_{\nu\bar{\nu}}^{\bar{s}d} + \sum C_i^{(8)} Q_i^{(8)} + \dots \right]_c$$

- Dimension eight operators

Concerning the dimension six (C_3'): known at NLL (Buchalla and Buras, 1993)

Three dimension-eight operators are generated (Falk, Lewandowski, Petrov, 2000)

$$Q_1^{(8)} = (\bar{s}d)_{V-A} \otimes \partial^2 (\bar{\nu}\nu)_{V-A}$$

$$Q_2^{(8)} = (\bar{s}\overleftarrow{D}_\alpha)\gamma_\mu(1-\gamma_5)(\vec{D}^\alpha d) \otimes (\bar{\nu}\nu)_{V-A}$$

$$Q_3^{(8)} = (\bar{s}\overleftarrow{D}^\alpha)\gamma_\mu(1-\gamma_5)d \otimes (\bar{\nu}[\overleftarrow{\partial}_\alpha - \vec{\partial}_\alpha])\gamma_\mu(1-\gamma_5)\nu$$

with weigths: $$C_1^Z(\mu_{IR}) = \frac{1}{6}\left(1-\frac{4}{3}\sin^2\theta_W\right)\log(m_c/\mu_{IR})\left[3C_1(m_c)+C_2(m_c)\right]$$

$$C_2^{e,\mu}(\mu_{IR}) = -C_3^{e,\mu}(\mu_{IR}) = \log(m_c/\mu_{IR})\, C_{\ell\nu}(m_c)$$

$$C_2^{\tau}(\mu_{IR}) = -C_3^{\tau}(\mu_{IR}) = -f(m_c^2/m_\tau^2)\, C_{\ell\nu}(m_c)$$

The CC Wilson coeff. $C_1(m_c), C_2(m_c)$ are known, while $C_{\ell\nu}(m_c) = 1$.

The τ lepton mass is not neglected w.r.t. the c one:

$$f(x) = \frac{1}{4}\left(\frac{6x-2}{(x-1)^2} - 2\right)\log x - \frac{x}{(x-1)^2}$$

Chiral Perturbation Theory

- Generalities

Strong interactions are represented as

$$L_{strong}^{(2)} = \frac{F_\pi^2}{4}\left\langle D_\mu U D^\mu U^\dagger + \chi^\dagger U + U^\dagger \chi \right\rangle$$

The current-current hadronic $\Delta S = 1$ operators as

$$L_{|\Delta S|=1}^{(2)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \left\langle \lambda_6 L_\mu L^\mu \right\rangle + g_{27} \left\langle \ldots \right\rangle \right\} \qquad L_\mu = i F_\pi^2 U D_\mu U^\dagger$$

The Z boson is introduced as an external source through the covariant derivative:

$$D_\mu U = \partial_\mu U - i \frac{g}{\cos\theta_W} Z_\mu \left(\sin^2\theta_W \left[Q, U \right] + U T_3 \right) \qquad T_3 = a_8 Q - a_1 \frac{1}{6}$$

The external W boson is introduced similarly (for the box diagram).

- GIM mechanism and FCNC

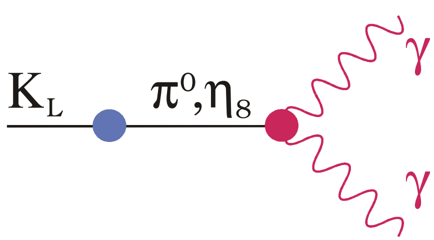
The G_F^2 Lagrangian contains at tree-level (with or without weak rotation)

$$L_{strong}^{(2)} + L_{|\Delta S|=1}^{(2)} = 4F_\pi^3 G_8 \frac{g}{\cos\theta_W}\left(\frac{a_1}{6}+\frac{a_8}{3}\right) Z^\mu \partial_\mu K_L - F_\pi \frac{g}{\cos\theta_W}\left(\frac{a_8}{2}\right) Z^\mu \left(\partial_\mu \pi^0 + \frac{\partial_\mu \eta_8}{\sqrt{3}}\right)$$

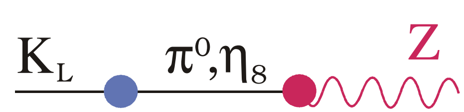
GIM mechanism forbids such a direct K_L-Z coupling; it is already included in dimension 6 operators $\Rightarrow$ remove from $L_{|\Delta S|=1}^{(2)}$ the part corresponding to the structure $(\bar{s}_L \gamma_\mu d_L)\ Z^\mu$:

$$L_{|\Delta S|=1}^{(2)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \left\{ \langle \lambda_6 L_\mu L^\mu \rangle - \frac{2ig}{\cos\theta_W} F_\pi^2 \langle \lambda_6 L_\mu T_3 \rangle Z^\mu \right\}$$

With this new piece, the suppression appears in the same way for both:



$\overset{GMO}{=} 0$



$\overset{GMO}{=} 0$

- Singlet part of the source

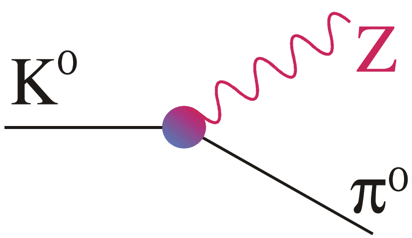
The fact that the T_3 generator is not traceless $\Rightarrow \langle L_\mu \rangle \sim \langle U D_\mu U^\dagger \rangle \sim a_1 Z_\mu$

With the previous modification, the a_1 piece decouples at $O(p^2)$ in SU(3):

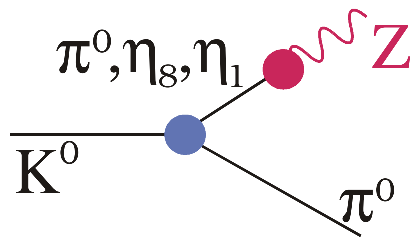
$$L^{(2)}_{strong} \sim \langle L_\mu \rangle \langle L^\mu \rangle \sim Z_\mu Z^\mu$$ $\Rightarrow$ decoupled, but in U(3): $L^{(2)}_{strong} \sim a_1 \partial_\mu \eta_1 Z^\mu$

$$L^{(2)}_{|\Delta S|=1} \sim \langle \lambda_6 L_\mu \rangle \langle L^\mu \rangle \sim a_1 \partial_\mu K_L Z^\mu$$ $\Rightarrow$ forced to vanish.

The only sensitivity to a_1 is through the singlet field pole diagrams:



$= 0$



$$\partial_\mu \eta_1 Z^\mu \rightarrow q_\mu \left(\bar{\nu}_L \gamma^\mu \nu_L \right) = 0$$

$$\mathfrak{M}(K^0 \rightarrow \pi^0 Z^\mu)_{tree} = 0, \quad \mathfrak{M}(K^+ \rightarrow \pi^+ Z^\mu)_{tree} = i \frac{G_8 g}{\cos\theta_W} F_\pi^2 p_K^\mu \left(2a_8\right)$$

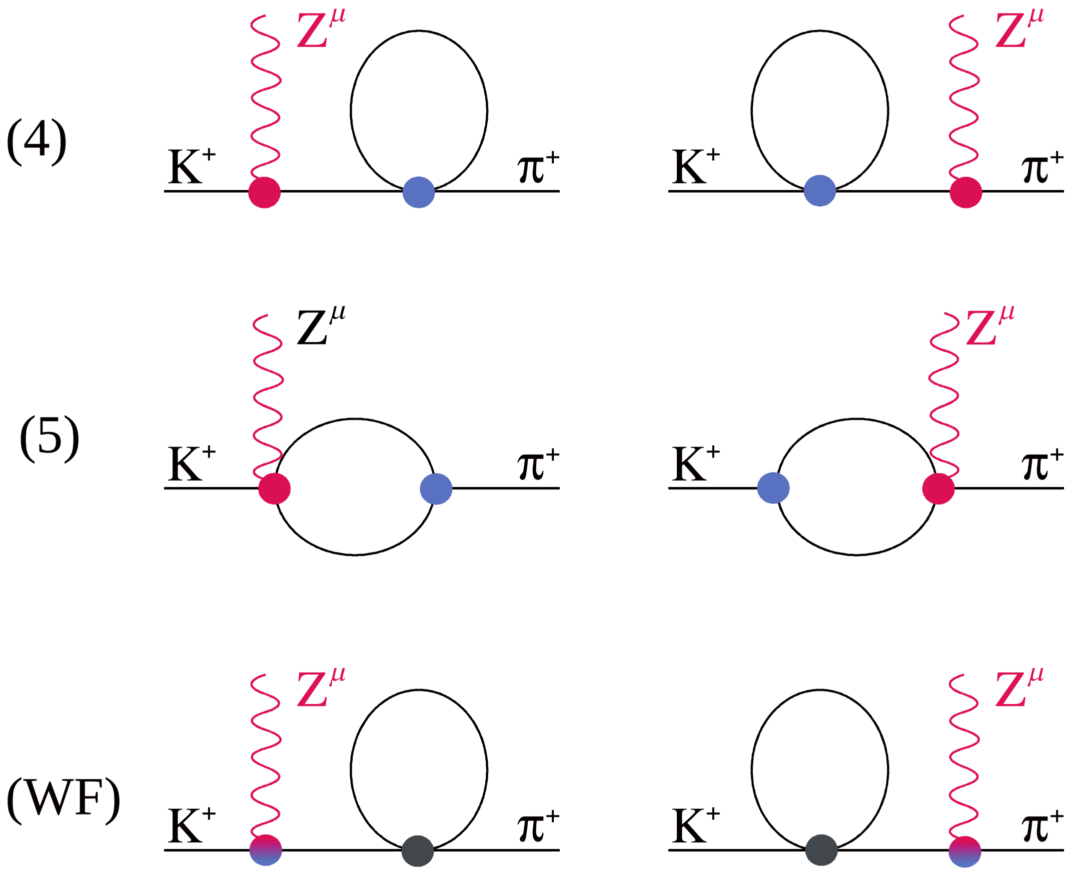
True also at loop level: no contribution from a_1, and no special sensitivity to η'.

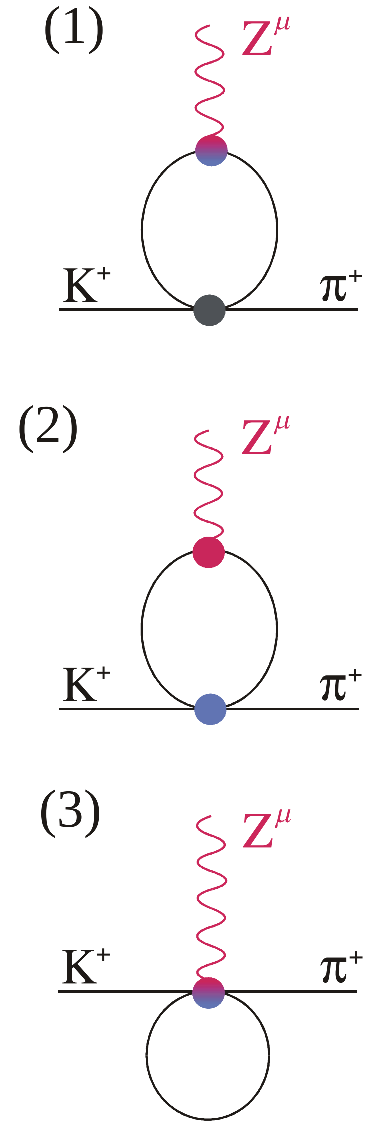
At $O(p^4)$, a lot of counterterms occur, but for a_1, only in a finite combination.

- Loop processes

  (with weak rotation)

● QCD
● $Z\ (QCD)$
● $\Delta S=1$
● $Z\ (\Delta S=1)$





- Amplitude

The general structure is

$$\mathfrak{M}(K^+ \to \pi^+ Z^\mu(q)) = i\,\frac{G_8 g}{\cos\theta_W} F_\pi^2 p_K^\mu \left[2a_8 + \frac{\mathfrak{M}_{loops}}{(4\pi F_\pi)^2} \right]$$

$$\mathfrak{M}_{loops} = a_8 \mathbf{M}_A(q^2) + \left(a_8 - \tfrac{4}{3}\sin^2\theta_W \right) \mathbf{M}_B(q^2)$$

The leading part of $\mathbf{M}_A$ is proportional to m_K^2, m_π^2, and corresponds to dimension 6, i.e. $X(m_u)$. Terms proportional to q^2 are small. In addition, $\mathbf{M}_A|_{SU(3)} = 0$.

The leading part of $\mathbf{M}_B$ is proportional to q^2, and corresponds to dimension 8 operators. This function is exactly the one occuring in $K^+ \to \pi^+\gamma^*$.

The divergence is fully contained in $\mathbf{M}_B$, and appears with the same coefficient as in the short-distance analysis:

$$C_1^Z(\mu_{IR}) = \frac{1}{6}\left(1 - \tfrac{4}{3}\sin^2\theta_W\right)\log(m_c/\mu_{IR})\left[3C_1(m_c) + C_2(m_c)\right]$$

The removal of the FCNC piece is crucial: only then the q^2 piece has this coefficient

Phenomenology

- Final parametrization

$$Br(K^+ \to \pi^0 \nu\bar{\nu}) = \kappa^+ \left[\left| \mathrm{Im}\,\lambda_t X(x_t) \right|^2 + \left| \mathrm{Re}\,\lambda_t X(x_t) + \mathrm{Re}\,\lambda_c P_{LQ} \right|^2 \right]$$

$$\approx \kappa^+ \left[\left| A^2\lambda^5 \eta X(x_t) \right|^2 + \left| A^2\lambda^5 (1-\rho) X(x_t) + \lambda P_{LQ} \right|^2 \right]$$

$$\approx 4.1\times 10^{-11} A^4 X^2(x_t) \left[\eta^2 + (\rho_0 - \rho)^2 \right]$$

with

$$\kappa^+ = \frac{3\alpha^2 Br(K^+ \to \pi^0 e^+ \nu)}{2\pi\lambda^2 \sin^4\theta_W}, \quad \rho_0 = 1 + \frac{P_{LQ}}{A^2\lambda^4 X(x_t)}, \quad \eta \approx \frac{1}{3}, \rho \approx \lambda \approx \frac{1}{5}, A \approx \frac{4}{5}$$

The *matrix element* of the leading dimension 6 operator is hidden in κ^+.

The *top quark effects* are included in the function $X(x_t) = 1.529 \pm 0.042$

All the *light quarks (c and u) effects* are contained in P_{LQ}:

$$P_{LQ} = P_c + P_c^{(8)} + \mathrm{Re}\,P_{LD} \qquad P_c = \frac{2}{3} X_{NL}^e + \frac{1}{3} X_{NL}^\tau \approx \lambda^4 (0.39 \pm 0.07)$$

Conclusion

- For c-quark, dimension-eight operators:

We confirm both the basis and Wilson coefficients obtained by Falk, Lewandowski and Petrov in 2000.

- For u-quark, long-distance effects:

We have motified the basis of operators to be used in ChPT at $O(G_F^2 p^2)$ by including a FCNC local operator, in order to satisfy GIM mechanism.

Contrary to earlier wor ks: 1/ No sensitivity to the singlet part of the Z current.
2/ At loop level, a structure that matches SD arises.

- For the final estimation of the impact on P_c :

For both $P_c^{(8)}$ and $\mathrm{Re}\,P_{LD}$, everything is in place, just a little more work needed. Completion expected very soon...