Isospin violation in semileptonic kaon decays

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$\boxed{\text{Isospin violation}}$

$\boxed{m_u \neq m_d}$

fully contained in

$$\mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \mathcal{L}_{p^6} + \ldots$$

through external source term

$$\chi \to 2B\mathcal{M}_{\text{quark}}, \quad \mathcal{M}_{\text{quark}} = \text{diag}(m_u, m_d, m_s)$$

$\boxed{\text{Electromagnetism}}$

- photon and light leptons as additional dynamical degrees of freedom
- extension of chiral Lagrangian:

$$\mathcal{L}_{e^2p^0} + \mathcal{L}_{e^2p^2} + \mathcal{L}_{\text{lept}} + \mathcal{L}_\gamma$$

Parameters of the effective Lagrangian

lowest order

$\mathcal{L}_{p^2}$ F, B, e, G_F

$\mathcal{L}_{e^2p^0}$ Z

next-to-leading order

$\mathcal{L}_{p^4}$ $L_1, L_2, \ldots L_{12}$ Gasser, Leutwyler

$\mathcal{L}_{e^2p^2}$ $K_1, K_2, \ldots K_{14}$ Urech

$\mathcal{L}_{\text{lept}}, \mathcal{L}_\gamma$ Knecht, N., Rupertsberger, Talavera

"leptonic" couplings $X_1, X_2, \ldots X_8$

(only X_1, X_2, X_3, X_6 appear in realistic observables)

$\pi_{\ell 2(\gamma)}$ decay rate

$$\Gamma(\pi \to \ell\nu_\ell(\gamma)) = \frac{G_F^2 S_{\mathrm{EW}} |V_{ud}|^2 F^2 m_\ell^2 M_{\pi^\pm}}{4\pi} (1 - z_{\pi\ell})^2$$

$$\Bigg\{ 1 + \frac{8}{F^2} \left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu) M_\pi^2 \right]$$

$$- \frac{1}{2(4\pi)^2 F^2} \Bigg[2M_{\pi^\pm}^2 \ln \frac{M_{\pi^\pm}^2}{\mu^2} + 2M_{\pi^0}^2 \ln \frac{M_{\pi^0}^2}{\mu^2}$$

$$+ M_{K^\pm}^2 \ln \frac{M_{K^\pm}^2}{\mu^2} + M_{K^0}^2 \ln \frac{M_{K^0}^2}{\mu^2} \Bigg]$$

$$+ e^2 E^r(\mu) + \frac{e^2}{(4\pi)^2} \left[3 \ln \frac{M_\pi^2}{\mu^2} + H(z_{\pi\ell}) \right] \Bigg\}$$

$$z_{\pi\ell} = m_\ell^2 / M_{\pi^\pm}^2$$

$$E^r = \left(\frac{8}{3} K_1 + \frac{8}{3} K_2 + \frac{20}{9} K_5 + \frac{20}{9} K_6 + 4K_{12} - \frac{4}{3} X_1 - 4X_2 + 4X_3 - \widetilde{X}_6\right)^r$$

Marciano, Sirlin; Knecht, N., Rupertsberger, Talavera

$K_{\ell 2(\gamma)}$ decay rate

$$
\begin{aligned}
\Gamma(K \to \ell \nu_\ell(\gamma)) = {} & \frac{G_F^2 S_{\mathrm{EW}} |V_{us}|^2 F^2 m_\ell^2 M_{K^\pm}}{4\pi} (1 - z_{K\ell})^2 \\
& \Bigg\{ 1 + \frac{8}{F^2} \left[L_4^r(\mu)(M_\pi^2 + 2M_K^2) + L_5^r(\mu) M_K^2 \right] \\
& - \frac{1}{4(4\pi)^2 F^2} \Bigg[2M_{\pi^\pm}^2 \ln \frac{M_{\pi^\pm}^2}{\mu^2} + M_{\pi^0}^2 \ln \frac{M_{\pi^0}^2}{\mu^2} \\
& + 4M_{K^\pm}^2 \ln \frac{M_{K^\pm}^2}{\mu^2} + 2M_{K^0}^2 \ln \frac{M_{K^0}^2}{\mu^2} + 3M_\eta^2 \ln \frac{M_\eta^2}{\mu^2} \Bigg] \\
& + \frac{16\sqrt{3}\, \varepsilon^{(2)}}{3F^2} L_5^r(\mu)(M_\pi^2 - M_K^2) \\
& - \frac{\sqrt{3}\, \varepsilon^{(2)}}{2(4\pi)^2 F^2} \left[M_\pi^2 \ln \frac{M_\pi^2}{\mu^2} - M_\eta^2 \ln \frac{M_\eta^2}{\mu^2} \right] \\
& + e^2 E^r(\mu) + \frac{e^2}{(4\pi)^2} \left[3 \ln \frac{M_K^2}{\mu^2} + H(z_{K\ell}) \right] \Bigg\}
\end{aligned}
$$

$$
z_{K\ell} = m_\ell^2 / M_{K^\pm}^2
$$

leading order π^0–η mixing angle:

$$
\varepsilon^{(2)} = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \widehat{m}}, \quad \widehat{m} = (m_u + m_d)/2
$$

Decay constants

$$
\begin{aligned}
F_\pi &= F\Bigg\{1+\frac{4}{F^2}\left[L_4^r(\mu)(M_\pi^2+2M_K^2)+L_5^r(\mu)M_\pi^2\right] \\
&\quad -\frac{1}{2(4\pi)^2F^2}\left[2M_\pi^2\ln\frac{M_\pi^2}{\mu^2}+M_K^2\ln\frac{M_K^2}{\mu^2}\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
F_{K^\pm} &= F\Bigg\{1+\frac{4}{F^2}\left[L_4^r(\mu)(M_\pi^2+2M_K^2)+L_5^r(\mu)M_K^2\right] \\
&\quad -\frac{1}{8(4\pi)^2F^2}\Bigg[3M_\pi^2\ln\frac{M_\pi^2}{\mu^2}+6M_K^2\ln\frac{M_K^2}{\mu^2} \\
&\quad +3M_\eta^2\ln\frac{M_\eta^2}{\mu^2}\Bigg] \\
&\quad -\frac{8\sqrt{3}\,\varepsilon^{(2)}}{3F^2}L_5^r(\mu)(M_K^2-M_\pi^2) \\
&\quad -\frac{\sqrt{3}\,\varepsilon^{(2)}}{4(4\pi)^2F^2}\Bigg[M_\pi^2\ln\frac{M_\pi^2}{\mu^2}-M_\eta^2\ln\frac{M_\eta^2}{\mu^2} \\
&\quad -\frac{2}{3}(M_K^2-M_\pi^2)\left(\ln\frac{M_K^2}{\mu^2}+1\right)\Bigg]\Bigg\}
\end{aligned}
$$

The ratio $\Gamma(K_{\ell 2})/\Gamma(\pi_{\ell 2})$

$$
\begin{aligned}
\frac{\Gamma(K \to \ell\nu_\ell(\gamma))}{\Gamma(\pi \to \ell\nu_\ell(\gamma))} \quad &= \quad \frac{|V_{us}|^2 F_K^2 M_K (1 - z_{K\ell})^2}{|V_{ud}|^2 F_\pi^2 M_\pi (1 - z_{\pi\ell})^2} \\
&\times \quad \left\{ 1 + \frac{e^2}{(4\pi)^2} \left[(3 - Z) \ln \frac{M_K^2}{M_\pi^2} \right.\right. \\
&\qquad \left.\left. + H(z_{K\ell}) - H(z_{\pi\ell}) \right] \right\}
\end{aligned}
$$

$$2e^2 Z F^2 = M_{\pi^\pm}^2 - M_{\pi^0}^2$$

independent of unknown electromagnetic LECs

determine V_{us} using lattice data for F_K/F_π
Marciano

Lattice result for F_K/F_π

$1.210 \pm 0.004(\text{stat.}) \pm 0.013(\text{syst.})$ MILC

Determination of V_{us}/V_{ud}

$\rightarrow |V_{us}/V_{ud}| = 0.2278$

$\pm 0.0004(\text{exp.}) \pm 0.0026(\text{lattice}) \pm 0.00001(\text{elm.})$

PDG value for $\Gamma(K_{\mu 2})$ based on very old data :

$\Gamma(K_{\mu 2}) = (51.22 \pm 0.17) \times 10^6 s^{-1}$ (PDG fit),

$(51.2 \pm 0.8) \times 10^6 s^{-1}$ (W.T. Ford et al., 1967)

Determination of V_{us}

using $|V_{ud}| = 0.9740 \pm 0.0005$ SAF $0^+ \rightarrow 0^+$

$\rightarrow |V_{us}| = 0.2219 \pm 0.0026$

to be compared with:

$|V_{us}^{\text{unit.}}| = \sqrt{1 - |V_{ud}|^2} = 0.2265 \pm 0.0021$

Determination of V_{us} ctd.

using $|V_{ud}| = 0.9757 \pm 0.0012$ $n\beta$ Serebrov et al.

$\tau_n = 885.7 \pm 0.8s$ (PDG 04) $\rightarrow$
$\tau_n = 878 \pm 0.7_{\text{stat.}} \pm 0.3_{\text{syst.}}$

(6.5 σ !)

$\rightarrow |V_{us}| = 0.2223 \pm 0.0026$

to be compared with

$$|V_{us}^{\text{unit.}}| = \sqrt{1 - |V_{ud}|^2} = 0.2191 \pm 0.0053$$

$\boxed{K_{e3} \text{ decays}}$

Most of the relevant data given by PDG 04 already superseded by more recent (partly preliminary) experimental results:

$$\boxed{\mathrm{BR}(K^+ \to \pi e \nu(\gamma))}$$

$(4.87 \pm 0.06)\%$ (PDG 04) $\to$

$(5.137 \pm 0.057)\%$ (E865, NA48)

$$\boxed{\mathrm{BR}(K_L^0 \to \pi e \nu(\gamma))}$$

$(38.81 \pm 0.27)\%$ (PDG 04) $\to$

$(40.56 \pm 0.17)\%$ (KTeV, KLOE, NA48)

$$\boxed{K_L^0 \text{ lifetime}}$$

$(5.18 \pm 0.04) \times 10^{-8} s$ (PDG 04) $\to$

$(5.135 \pm 0.02) \times 10^{-8} s$ (KLOE)

$K^0_S \to \pi e \nu(\gamma)$

$\Gamma(K^0_S \to \pi e \nu(\gamma)) = (7.92 \pm 0.12) \times 10^6 s^{-1}$ KLOE

Slope and curvature of $f_+(t)$

$$f_+(t) = f_+(0) \left(1 + \lambda_+ \frac{t}{M^2_{\pi^\pm}} + \lambda'_+ \frac{t^2}{M^4_{\pi^\pm}} \right)$$

linear fit

$\lambda_+(K^+_{e3}) = 0.0277 \pm 0.0005$ PDG 04

$\lambda_+(K^0_{e3}) = 0.0285 \pm 0.0007$ KTeV, NA48, ...

quadratic fit

$\lambda_+(K^+_{e3}) = 0.02324 \pm 0.00155$

$\lambda'_+(K^+_{e3}) = 0.00084 \pm 0.00041$ ISTRA

$\lambda_+(K^0_{e3}) = 0.02167 \pm 0.00199$

$\lambda'_+(K^0_{e3}) = 0.00144 \pm 0.00039$ KTeV

$K_{e3(\gamma)}$ decay distribution

$$\rho(y,z) = \mathcal{N}\, S_{\mathrm{EW}}\, A_1(y,z)\, |f_+(t)|^2$$

$$z = \frac{2p_\pi \cdot p_K}{M_K^2} = \frac{2E_\pi}{M_K}, \qquad y = \frac{2p_K \cdot p_e}{M_K^2} = \frac{2E_e}{M_K}$$

$$\mathcal{N} = C^2 \frac{G_{\mathrm{F}}^2\, |V_{us}|^2 M_K^5}{128\pi^3}, \qquad \Gamma = \int_{\mathcal{D}} dy\, dz\; \rho(y,z)$$

$$C = \begin{cases} 1 & \text{for } K_{e3}^0 \\ \frac{1}{\sqrt{2}} & \text{for } K_{e3}^+ \end{cases}$$

$$t = (p_K - p_\pi)^2$$

Phase space integral

expansion of the effective form factor:

$$f_+(t) = f_+(0) \left(1 + \frac{t}{M_{\pi^\pm}^2}\lambda_+ + \frac{t^2}{M_{\pi^\pm}^4}\lambda'_+\right)$$

$$\Gamma(K_{e3(\gamma)}) = \mathcal{N}\, S_{\rm EW}\, |f_+(0)|^2\, I_K$$

$$I_K = \int_{\mathcal{D}} dy\, dz\, A_1(y,z) \left(1 + \frac{t}{M_{\pi^\pm}^2}\lambda_+ + \frac{t^2}{M_{\pi^\pm}^4}\lambda'_+\right)^2$$

$$\alpha \neq 0 \Rightarrow I_{K^+} \searrow 1.27\%$$

$$\alpha \neq 0 \Rightarrow I_{K^0} \searrow 0.32\%$$

K_{e3} form factors

p^4	Gasser, Leutwyler
e^2p^2	Cirigliano, Knecht, Neufeld, Pichl, Rupertsberger
p^6	Leutwyler, Roos; Post, Schilcher; Bijnens, Talavera

$$f_+(t) = \widetilde{f}_+(t) \, + \, \widehat{f}_+$$

K^0_{e3}

$$\begin{aligned} \widetilde{f}_+^{K^0\pi^-}(t) &= 1 \\ &+ \frac{1}{2} H_{K^+\pi^0}(t) + \frac{3}{2} H_{K^+\eta}(t) + H_{K^0\pi^-}(t) \\ &+ \sqrt{3}\,\varepsilon^{(2)} \left[H_{K\pi}(t) - H_{K\eta}(t) \right] + \ldots \end{aligned}$$

$$\widetilde{f}_+^{K^0\pi^-}(0) = 0.97698 \pm 0.00002$$

$$\begin{aligned} \widehat{f}_+^{K^0\pi^-} &= 4\pi\alpha \Bigg[2K_{12}^r(\mu) \, + \, \frac{4}{3} X_1 - \frac{1}{2} \widetilde{X}_6^r(\mu) \\ &- \frac{1}{32\pi^2} \left(3 + \log \frac{m_e^2}{M_{\pi^\pm}^2} + 3 \log \frac{M_{\pi^\pm}^2}{\mu^2} \right) \Bigg] \\ &= 0.0046 \pm 0.0008 \end{aligned}$$

$$\boxed{K^+_{e3}}$$

$$\begin{aligned}
\widetilde{f}_+^{K^+\pi^0}(t) \quad = \quad & 1 + \sqrt{3}\left(\varepsilon^{(2)} + \varepsilon_{\rm S}^{(4)} + \varepsilon_{\rm EM}^{(4)}\right) \\
& + \frac{1}{2} H_{K^+\pi^0}(t) + \frac{3}{2} H_{K^+\eta}(t) + H_{K^0\pi^-}(t) \\
& + \sqrt{3}\,\varepsilon^{(2)} \left[\frac{5}{2} H_{K\pi}(t) + \frac{1}{2} H_{K\eta}(t)\right] + \ldots
\end{aligned}$$

$$\begin{aligned}
\varepsilon_{\rm S}^{(4)} = & -\frac{2\,\varepsilon^{(2)}}{3(4\pi F_0)^2 (M_\eta^2 - M_\pi^2)} \\
\times \quad & \Bigg\{ (4\pi)^2\, 64\, [3L_7 + L_8^r(\mu)]\, (M_K^2 - M_\pi^2)^2 \\
& - M_\eta^2 (M_K^2 - M_\pi^2) \log \frac{M_\eta^2}{\mu^2} \\
& + M_\pi^2 (M_K^2 - 3M_\pi^2) \log \frac{M_\pi^2}{\mu^2} \\
& - 2M_K^2 (M_K^2 - 2M_\pi^2) \log \frac{M_K^2}{\mu^2} \\
& - 2M_K^2 (M_K^2 - M_\pi^2) \Bigg\}
\end{aligned}$$

$$
\begin{aligned}
\varepsilon^{(4)}_{\mathrm{EM}} = {} & \frac{2\sqrt{3}\,\alpha\, M_K^2}{108\,\pi\,(M_\eta^2 - M_\pi^2)} \\
\times \quad & \Bigg\{ 2(4\pi)^2 \Big[-6K_3^r(\mu) + 3K_4^r(\mu) + 2K_5^r(\mu) + 2K_6^r(\mu) \Big] \\
& - 9Z \left(\log \frac{M_K^2}{\mu^2} + 1 \right) \Bigg\}
\end{aligned}
$$

$$
\widetilde{f}_+^{K^+\pi^0}(0) = 1.0005 \pm 0.0022
$$

numerical values for $K_i^r(M_\rho)$: Moussallam

local electromagnetic contribution:

$$
\begin{aligned}
\widehat{f}_+^{K^+\pi^0} &= 4\pi\alpha \Bigg[2K_{12}^r(\mu) - \frac{8}{3}X_1 - \frac{1}{2}\widetilde{X}_6^r(\mu) \\
&- \frac{1}{32\pi^2}\left(3 + \log \frac{m_e^2}{M_{K^\pm}^2} + 3\log \frac{M_{K^\pm}^2}{\mu^2} \right) \Bigg] \\
&= 0.0032 \pm 0.0016
\end{aligned}
$$

contribution of order p^6

$$f_+(0)\Big|_{p^6} = f_+^{2-\mathrm{loops}}(0) + f_+^{\mathrm{L_i}\times\mathrm{loop}}(0) + f_+^{\mathrm{tree}}(0)\Big|_{p^6}$$

at $\mu = M_\rho$:

$$f_+^{2-\mathrm{loops}}(0) = 0.0113$$

$$f_+^{\mathrm{L_i}\times\mathrm{loop}}(0) = -0.0020 \pm 0.0005$$

Bijnens, Talavera

$$f_+^{\mathrm{tree}}(0)\Big|_{p^6} = -0.002 \pm 0.006_{1/N_C} \pm 0.002_{M_S}$$

Cirigliano, Ecker, Eidemüller, Kaiser, Pich

Portolés

$$\rightarrow f_+(0)\Big|_{p^6} = 0.007 \pm 0.010$$

final values at $\mathcal{O}(p^6, (m_d - m_u)p^2, e^2p^2)$:

$$f_+^{K^0\pi^-}(0) = 0.989 \pm 0.010$$

$$f_+^{K^+\pi^0}(0) = 1.011 \pm 0.010$$

theoretical errors dominated by p^6 uncertainty

Combined analysis of K^0_{e3} and K^+_{e3} data

ratio of form factors at $t = 0$

$$r_{+0} := f_+^{K^+\pi^0}(0) \Big/ f_+^{K^0\pi^-}(0)$$

theoretical prediction:

$$\begin{aligned} r_{+0}^{\mathrm{th}} &= 1 + \sqrt{3}\left(\varepsilon^{(2)} + \varepsilon_{\mathrm{S}}^{(4)} + \varepsilon_{\mathrm{EM}}^{(4)}\right) \\ &\quad - \frac{\alpha}{4\pi} \log \frac{M^2_{K^\pm}}{M^2_{\pi^\pm}} - 16\pi\alpha X_1 + \ldots \\ &= 1.022 \pm 0.003 - 16\pi\alpha X_1 \end{aligned}$$

independent of $f_+|_{p^6}$, X_6, K_{12}

dimensional analysis: $X_1 \simeq 1/(4\pi)^2$

$\Rightarrow$ theoretical prediction for r_{+0}:

$$1.017 \le r_{+0}^{\mathrm{th}} \le 1.027$$

corresponding observable quantity:

$$r_{+0}^{\exp} = \left(\frac{2\,\Gamma(K_{e3(\gamma)}^{+})\, M_{K^0}^5\, I_{K^0}}{\Gamma(K_{e3(\gamma)}^{0})\, M_{K^+}^5\, I_{K^+}} \right)^{1/2}$$

using K_L width and linear form factor fits:

$$\begin{aligned} r_{+0}^{\exp} &= 1.038 \pm 0.006 \pm 0.003 \pm 0.001 \pm 0.001 \\ &= 1.038 \pm 0.007 \end{aligned}$$

errors from $\Gamma(K_{e3(\gamma)}^{+})$, $\Gamma(K_{e3(\gamma)}^{0})$, I_{K^+}, I_{K^0}

using quadratic form factor fits:

$$\begin{aligned} r_{+0}^{\exp} &= 1.036 \pm 0.006 \pm 0.003 \pm 0.003 \pm 0.004 \\ &= 1.036 \pm 0.008 \end{aligned}$$

using K_S width and linear form factor fits:

$$r_{+0}^{\exp} = 1.036 \pm 0.010$$

using quadratic form factor fits:

$$r_{+0}^{\exp} = 1.035 \pm 0.011$$

inconsistency of K_{e3}^{+} and K_{e3}^{0} data?

K^{+}, K_L lifetimes?

X_1 abnormally large?

Determination of V_{us} from K_{e3} data

K_{e3}^{+}

linear fit:

$$|V_{us}| = 0.2221 \pm 0.0026$$

quadratic fit:

$$|V_{us}| = 0.2229 \pm 0.0027$$

remember:

$$0^+ \to 0^+ \quad \to \quad |V_{us}^{\mathrm{unit.}}| = 0.2265 \pm 0.0021$$

$$n\beta \quad \to \quad |V_{us}^{\mathrm{unit.}}| = 0.2191 \pm 0.0053$$

$\boxed{K^0_{L\,e3}}$

linear fit:

$$|V_{us}| = 0.2188 \pm 0.0023$$

quadratic fit:

$$|V_{us}| = 0.2199 \pm 0.0025$$

$\boxed{K^0_{S\,e3}}$

linear fit:

$$|V_{us}| = 0.2191 \pm 0.0028$$

quadratic fit:

$$|V_{us}| = 0.2201 \pm 0.0029$$

Pionic beta decay

information on $|V_{ud}|$ from

- SAF $0^+ \to 0^+$: $|V_{ud}| = 0.9740 \pm 0.0005$
  nuclear structure effects?
- neutron beta decay: $|V_{ud}| = 0.9757 \pm 0.0012$
  g_A/g_V
- pion beta decay: PIBETA experiment (PSI)
  theoretically very clean
  same methods as for $K_{\ell 3}$ but BR $\simeq 10^{-8}$

form factor

$$\widetilde{f}_+(t) = 1 + 2H_{\pi^+\pi^0}(t) + H_{K^+K^0}(t) + \ldots$$

$$\widetilde{f}_+(0) = 1 - 7 \times 10^{-6}$$

$$\begin{aligned} \widehat{f}_+ &= 4\pi\alpha\Big[2K_{12}^r(\mu) - \frac{2}{3}X_1 - \frac{1}{2}\widetilde{X}_6^r(\mu) \\ &- \frac{1}{32\pi^2}\Big(3 + \log\frac{m_e^2}{M_{\pi^+}^2} + 3\log\frac{M_{\pi^+}^2}{\mu^2}\Big)\Big] \\ &= (4.6 \pm 0.5) \times 10^{-3} \end{aligned}$$

Cirigliano, Knecht, Neufeld, Pichl

final result:

$$f_+(0) = 1.0046 \pm 0.0005$$

phantastic theoretical precision!

PIBETA experiment (PSI)

present result:

$$\mathrm{BR} = (1.036 \pm 0.006) \times 10^{-8}$$

$\Rightarrow$

$$\begin{aligned} |V_{ud}| &= 0.9728 \pm 0.0030(\mathrm{exp.}) \pm 0.0005(\mathrm{theor.}) \\ &= 0.9728 \pm 0.0030 \end{aligned}$$

Conclusions

- theoretical framework for treatment of all isospin-violating contributions in semileptonic decays $\surd$
- form factors for $K_{\ell 3}$, π_{e3} including electromagnetic contributions to $\mathcal{O}(e^2p^2)$ $\surd$
- K_{e3}, π_{e3} numerics $\mathcal{O}(p^6,(m_d-m_u)p^2,e^2p^2)$ $\surd$
- PDG 04 data for K_{e3} already outdated by new experiments
- correct treatment of electromagnetic corrections mandatory
- inconsistency between K^+_{e3} and K^0_{e3} data?
- update/check relevant experimental input!
- theoretical determination of X_1 welcome!