# Testing a topology conserving action in lattice QCD

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hep-lat/0412017, hep-lat/0409073

Euridice Network EURIDICE Collaboration Meeting
8-13 February 2004, Frascati

# Motivations

$\epsilon$- expansion of QCD:

$m_\pi \sim \frac{1}{L^2} \sim \mathcal{O}(\epsilon^2)$

$m_\pi^{-1} \gg L \qquad\qquad \left(L \gg \frac{1}{2F_\pi}\right)$

*Gasser, Leutwyler (1987)*

- within $\chi$PT one can account for finite size effects in a systematic manner $\rightarrow$ the low energy constants of $\chi$PT take the same value as in infinite volume

- The topology is relevant: predictions for observables defined in fixed topological sectors

*Leutwyler, Smilga (1992)*

Constraints for lattice calculations:

- The physical volume must be $\gtrsim 1.1\mathrm{fm}$
- Chiral properties must be preserved as much as possible, to reach small pion masses
- A definition of the topological charge is needed

$\rightarrow$ Ginsparg-Wilson fermions

*Ginsparg, Wilson (1982), Neuberger (1997)*

- exact, lattice modified chiral symmetry
- no additive mass renormalization
- exact zero modes with definite chirality

topological charge: $\nu = n_+ - n_-$

*P. Hasenfratz et al.(1998), Lüscher (1998)*

Recent results in the $\epsilon$-regime with quenched Ginsparg-Wilson fermions:

- Dirac spectrum (comparison with RMT)

*Bietenholz et al. (2003), Giusti et al. (2003), Galletly et al. (2004)*

- Axial correlation functions

*Bietenholz et al. (2004), Giusti et al. (2003,2004)*

results compared with quenched $\chi$PT

*Damgaard et al (2002,2003), Bijnens et al (2004)*

- Low-energy constant of the weak $\chi$ Lagrangian for non-leptonic K-decays *Giusti et al (2004)*

Lattice gauge action:

$S = \beta \sum_P S_P$ $\qquad S_P = 1 - \frac{1}{3}\mathrm{ReTr} U_P$ $\qquad$ plaquette variable

The overlap Dirac operator is local (with exponentially decaying tails) if the gauge field is assumed to satisfy the smoothness condition

$\|S_P\| < \varepsilon$, for all plaquettes $P$, $\qquad \varepsilon < \frac{1}{6(2+\sqrt{2})} \sim \frac{1}{20.45}$

*Lüscher (1999), Hernández et al (1999), Neuberger (2000)*

Gauge actions that suppress the occurrence of small plaquette values:

-improves the locality of $D_{ov}$
-could speed up the dynamical simulations
-suppresses the change of $\nu$
$\rightarrow$ the continuum property of stable topologies is reproduced
$\rightarrow$ collect large number of configurations at fixed topological charge
($|\nu| = 1, 2$ are the most useful; $\nu = 0$ has large fluctuations; for $|\nu| > 2$ quenched $\chi$PT fails for $L \lesssim 1.5\mathrm{fm}$ )

Possible gauge actions:

(1) $$\beta S_{\varepsilon,n}(U_P) = \beta \frac{S_P}{(1-\varepsilon^{-1}S_P)^\alpha} \text{ if } \mathrm{S_P} < \varepsilon, \text{ and } +\infty \text{ otherwise}$$

(2) $$\beta S_{\varepsilon,n}(U_P) = \beta S_P + \varepsilon^{-1} S_P^n$$

(3) $$\beta S_{\varepsilon,n}(U_P) = \beta S_P \exp\left[\varepsilon^{-1} S_P^n\right]$$

still in the same universality class as Wilson plaquette action

(1) introduced by Lüscher (1999) ($\alpha = 1$); already applied for the Schwinger model

*Fukaya, Onogi (2003,2004)*

Update with local hybrid Monte Carlo algorithm.
Aim: conciliate advantages with reasonable lattice spacings and lattice sizes, with moderate lattice artefacts and reasonably decorrelated sampling of observables

# *Results with action (1)*

| $\varepsilon^{-1}$ | $\beta$ | $r_0/a$ | $\beta_W$ | $\tau^{\rm plaq}$ | $f_J$ | $dt$ | Acceptance |
|---|---|---|---|---|---|---|---|
| 0 | 6.18 | 7.14(3) | 6.18 | 7(1) | *0.015* | 0.1 | $> 99\%$ |
| 1.00 | 1.5 | 6.6(2) | 6.13(2) | 2.0(1) | 0.0027 | 0.05 | $> 99\%$ |
| 1.18 | 1.0 | 7.2(2) | 6.18(2) | 1.3(1) | 0.0014 | 0.02 - 0.01 | $> 99\%$ |
| 1.25 | 0.8 | 7.0(1) | 6.17(1) | 1.1(1) | 0.0025 | 0.1 | $> 99\%$ |
| 1.52 | 0.3 | 7.3(4) | 6.19(4) | 0.8(1) | 0.0008 | 0.1 | $\sim 95\%$ |
| 1.64 | 0.1 | 6.8(3) | 6.15(3) | 1.0(1) | 0.0007 | 0.1 | $\sim 65\%$ |

$V = 16^4$

topological charge estimated through *cooling* technique

*Ilgenfritz et al (1986)*

$dt$: discretization of the HMC trajectories (trajectory length =1)

$f_J$: (# of jumps of $\nu$)/(# trajectories in the full history)

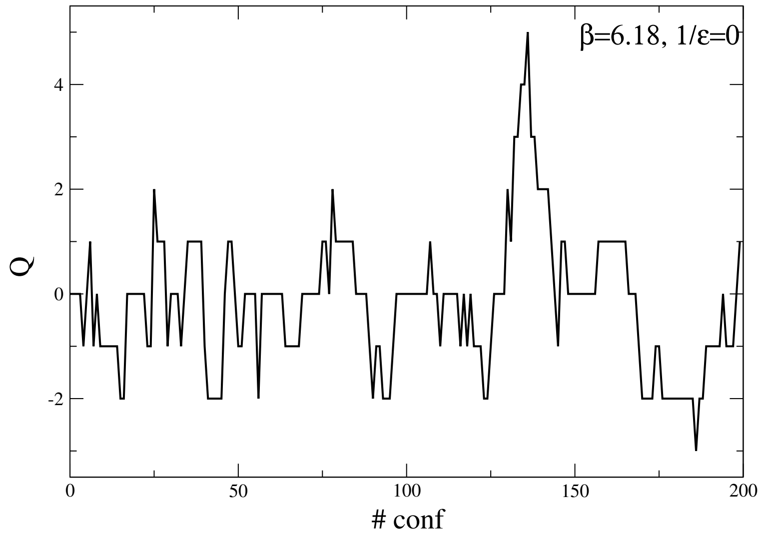
$\tau_{plaq}$: autocorrelation of the plaquette

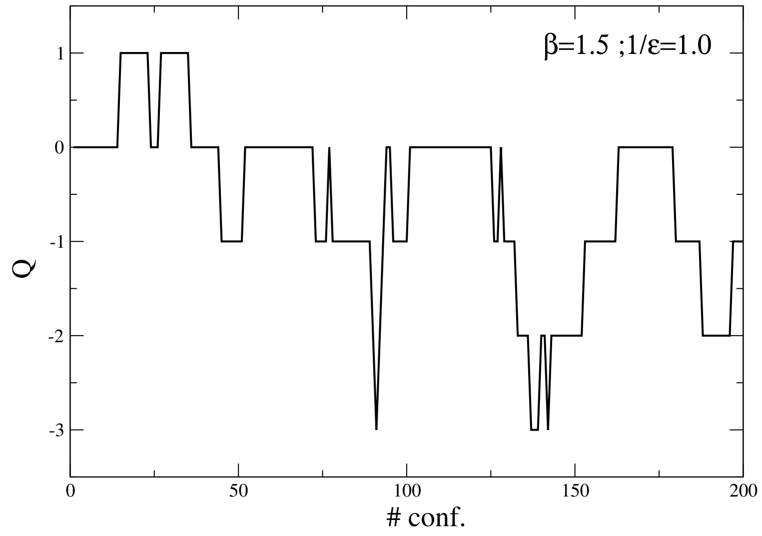
$r_0/a$: Sommer scale ($r_0 = 0.5$ fm)

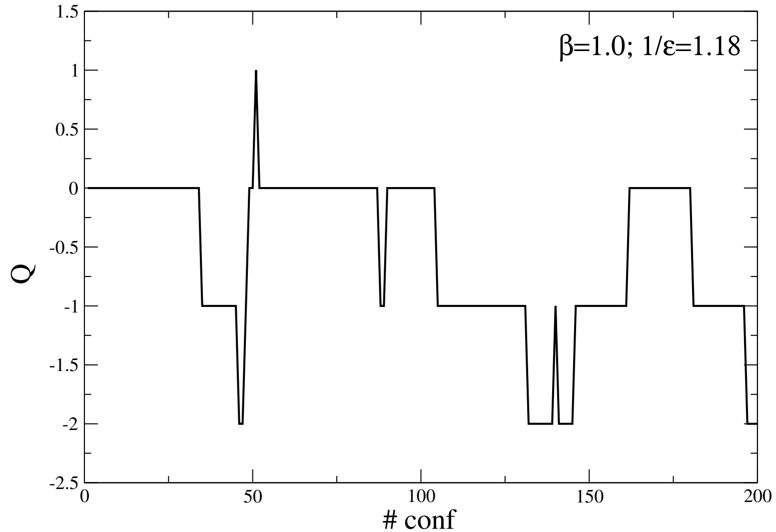
$\beta_W$: Wilson coupling corresponding to the same physical scale

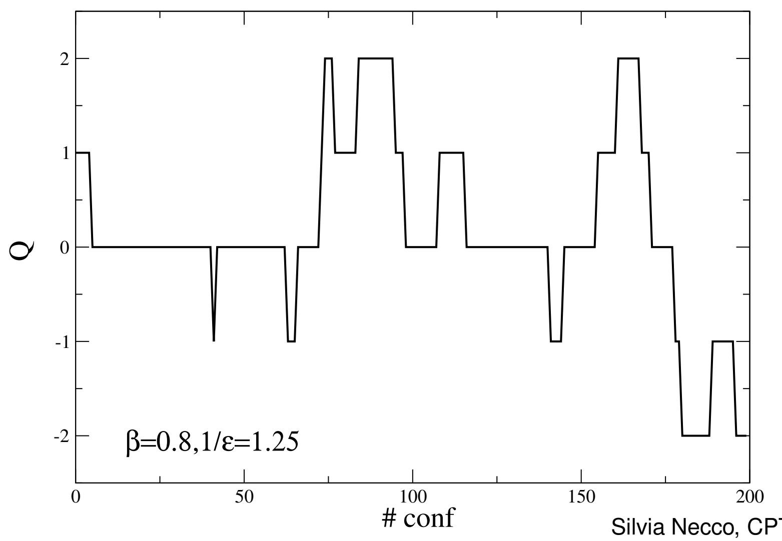
$\rightarrow$ the topological stability improves by decreasing $\varepsilon$ along lines of approximately constant physics ($\varepsilon$ still $>>$ theoretical bound)

# MC history of the topological charge/I

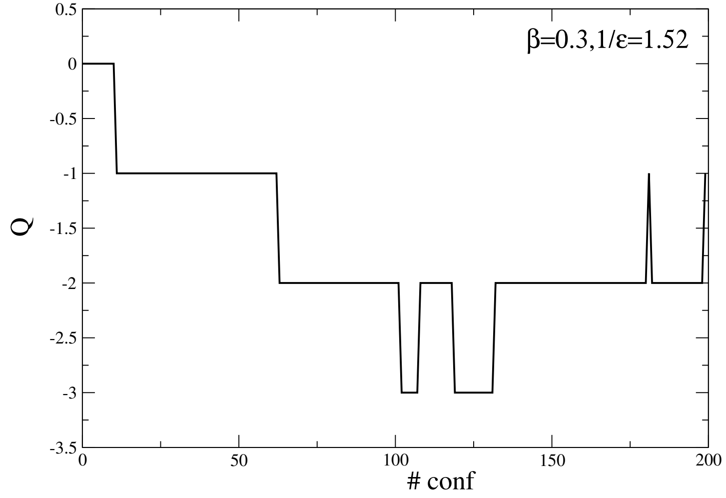


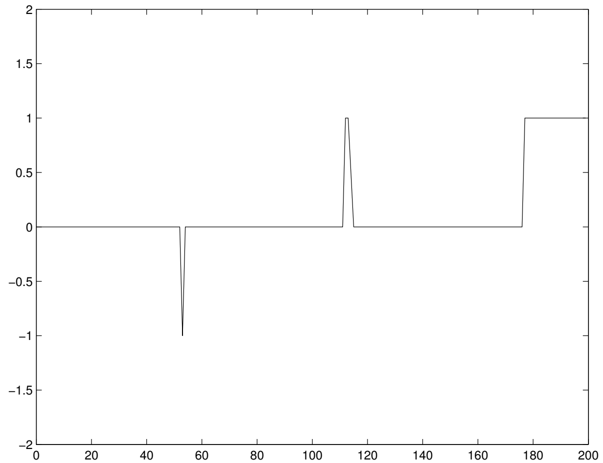



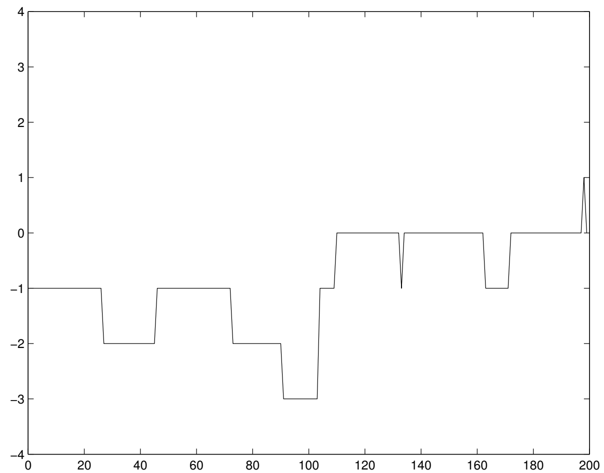



# MC history of the topological charge/II





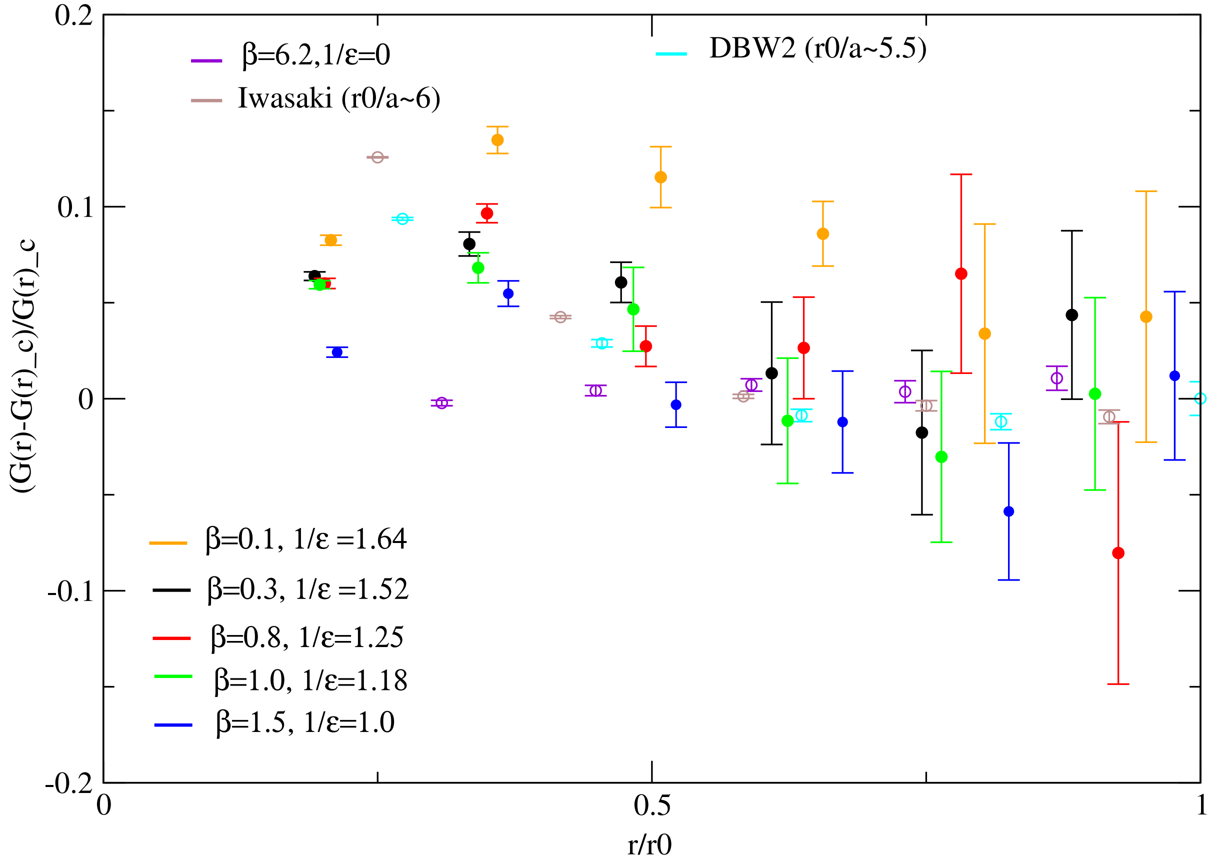
$\beta = 0.1, 1/\varepsilon = 1.64$

# *Lattice artefacts*

Comparison with continuum results

$G(r) = r^2 F(r)$

*S.N., R.Sommer (2001)*




Lattice artefacts $\lesssim 10\%$ (comparable with Iwasaki, DBW2)

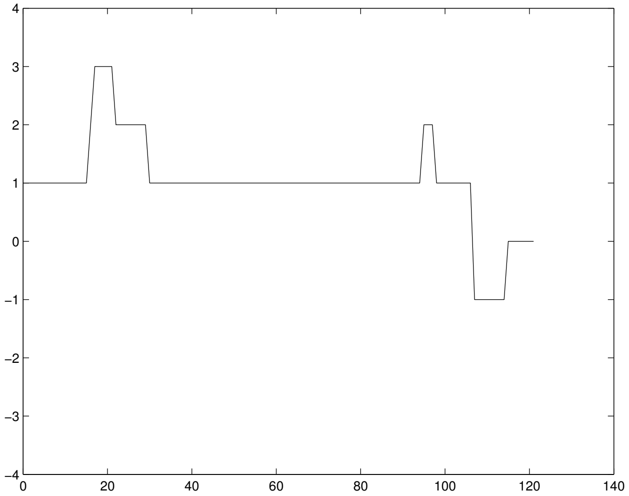
# *Action (3)*

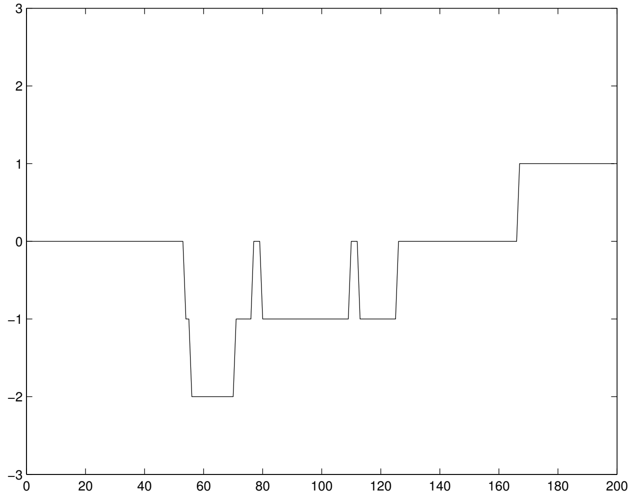
Possible problems of actions (1):
- do not allow for the existence of a positive definite transfer matrix?

*Creutz (2004)*

- for *global* HMC updatings: *local* rejection due to the violation of the constraint $\rightarrow$ rejection of the complete configuration $\rightarrow$ small acceptance rate

$\rightarrow$ use an action with same behavior at small values of the plaquette, but no infinite wall.
Possible candidates:
$S'(U_P) = S_P \; e^{90 \; S_P^6}$; $\qquad S'(U_P) = S_P \; e^{30 \; S_P^4}$





# *Conclusions*

Topology conserving gauge actions could be highly profitable in QCD simulations
$\rightarrow$ suppression of small plaquette values may speed up dynamical simulations
$\rightarrow$ a stable $\nu$ could be useful in the $\epsilon$-regime

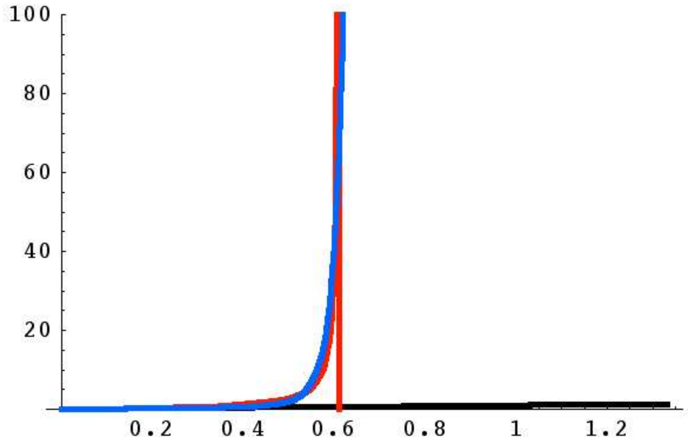
Work in progress, a completely satisfactory formulation for topology-conserving SU(3) gauge actions is still under construction

Next steps:
- check whether small eigenvalues of $AA^{\dagger}$ are repelled/suppressed
- try other formulations
- check sampling of correct topological charge distribution in the dynamical case

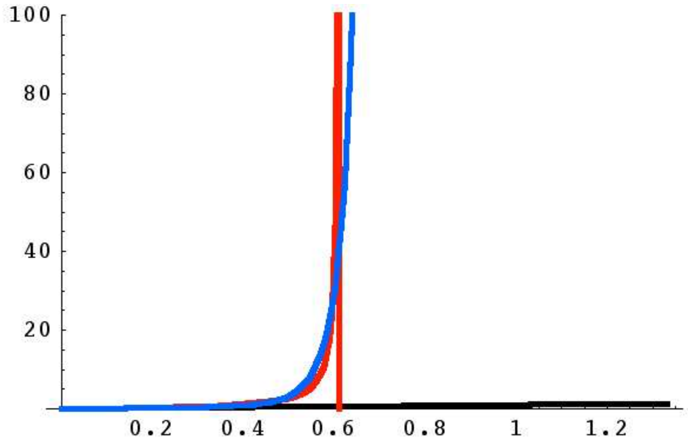
# Action (3)

$$S_P(U_P) \to S'(U_P) = S_P(U_P)\exp(\gamma {S_P}^{\delta}(U_P))$$

Repelling force: $F_{x,\alpha} = \frac{\partial S_P(U_P)}{\partial U_{x,\mu}} \exp(\gamma {S_P}^{\delta}(U_P)) \left(1 + \gamma\delta {S_P}^{\delta}(U_P)\right)$




$$S'(U_P) = S_P\ e^{90\ S_P^6}$$




$$S'(U_P) = S_P\ e^{30\ S_P^4}$$

# Lattice artefacts