Meson Masses and Decay Constants at Two Loops in Partially Quenched ChPT

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Overview of Objectives

- Calculation of pseudoscalar meson masses and decay constants to NNLO, or $\mathcal{O}(p^6)$ in Partially Quenched Chiral Perturbation Theory (PQχPT).

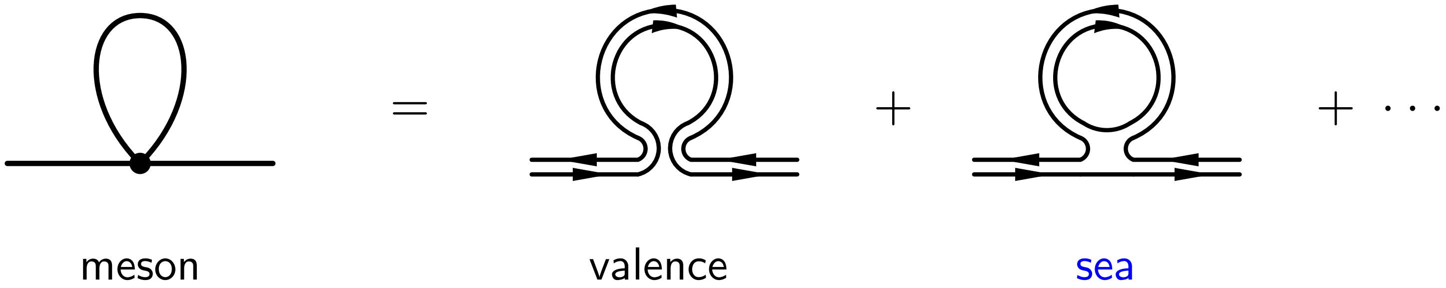
- Calculation of electromagnetic form factors of pseudoscalar mesons to NNLO in PQχPT are also planned.

- Expressions in NNLO PQχPT are very large; Numerical implementation of the results for the benefit of the Lattice QCD community will be made available from the authors websites.

- Collaboration at Lund University with Dr. Johan Bijnens and Niclas Danielsson (Graduate Student).

Valence and Sea Quarks in QCD

- Valence quark and sea quark contributions can be treated separately in Lattice QCD - no such distinction in standard χPT.

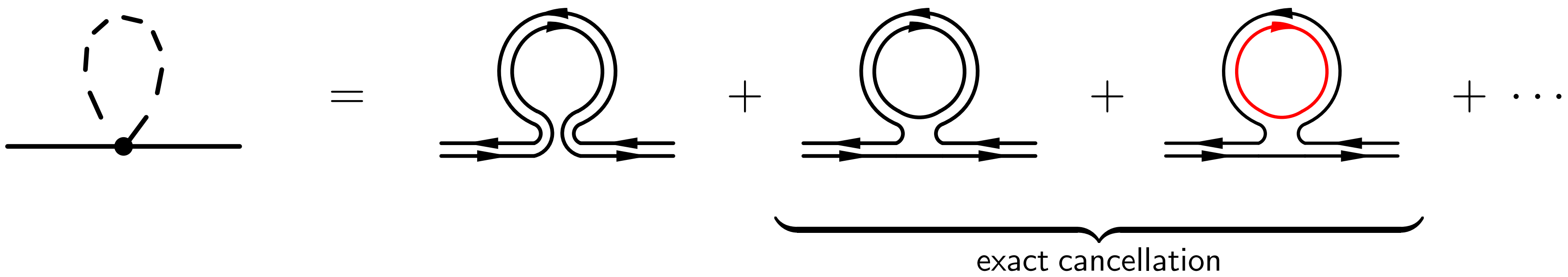


- The valence quark loops are relatively easy to treat in Lattice QCD, but sea quarks are very difficult.

- Lattice QCD simulations with physical u, d sea quark masses are not (yet?) possible $\longrightarrow$ Partially Quenched χPT required !

Quenching of Meson Loops

- A first step toward Partially Quenched theories is to remove the contributions with disconnected quark lines from the meson loops:



- The cancellation of loops with disconnected quark lines by means of bosonic ghost quarks is referred to as the Supersymmetric formulation[1] of quenched χPT.

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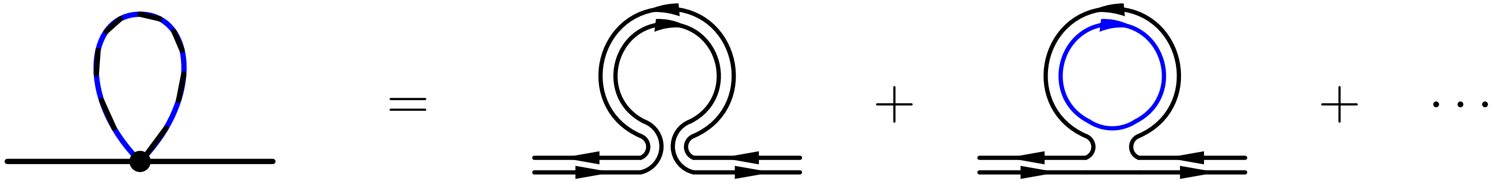
[1] C.W. Bernard and M.F.L. Golterman, Phys.Rev. **D49** (1994) 486

Partially Quenched Theories

- Partially Quenched χPT is obtained by addition of explicit sea quarks $q, \bar{q}$, whose masses are free parameters, to the quenched valence quark loops. Thus the valence and sea sectors are now treated independently.



- PQχPT is a systematic Lagrangian framework[2] where all necessary cancellations are carried out automatically, for any given set of Feynman diagrams.

[2] S. Sharpe and N. Shoresh, Phys.Rev. **D64** (2001) 114510, Phys.Rev. **D62** (2000) 094503

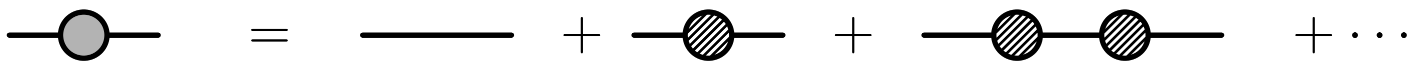
- The Lagrangians of PQχPT may be obtained from those of unquenched $SU(n)$ χPT by replacement of traces with supertraces.
- The low-energy constants of PQχPT are simply related to those of unquenched $SU(n)$ χPT $\longrightarrow$ PQ simulations may yield direct information about unquenched QCD!
- The mesons and quark masses of PQχPT are arranged into the meson matrix ϕ_{ij} and the mass matrix χ, where $\chi_i = 2B_0\, m_i$:

$$\phi_{ij} = \left(\begin{array}{c|c|c} q\bar{q} & q\bar{q} & q\bar{q} \\ \hline q\bar{q} & q\bar{q} & q\bar{q} \\ \hline q\bar{q} & q\bar{q} & q\bar{q} \end{array} \right) \quad \chi_i = \left(\begin{array}{c|c|c} \chi_v & & \\ \hline & \chi_s & \\ \hline & & \chi_g \end{array} \right)$$

- For the case of 3 valence and 3 sea quarks ϕ is a 9×9 matrix, consequently 81 fields as compared to 9 in $SU(3)$!

The Self-Energy $-i\Sigma$ at $\mathcal{O}(p^6)$

- The contributions from irreducible self-energy insertions $-i\Sigma$ to the propagator of a pseudoscalar field may be summed by means of a geometric series:



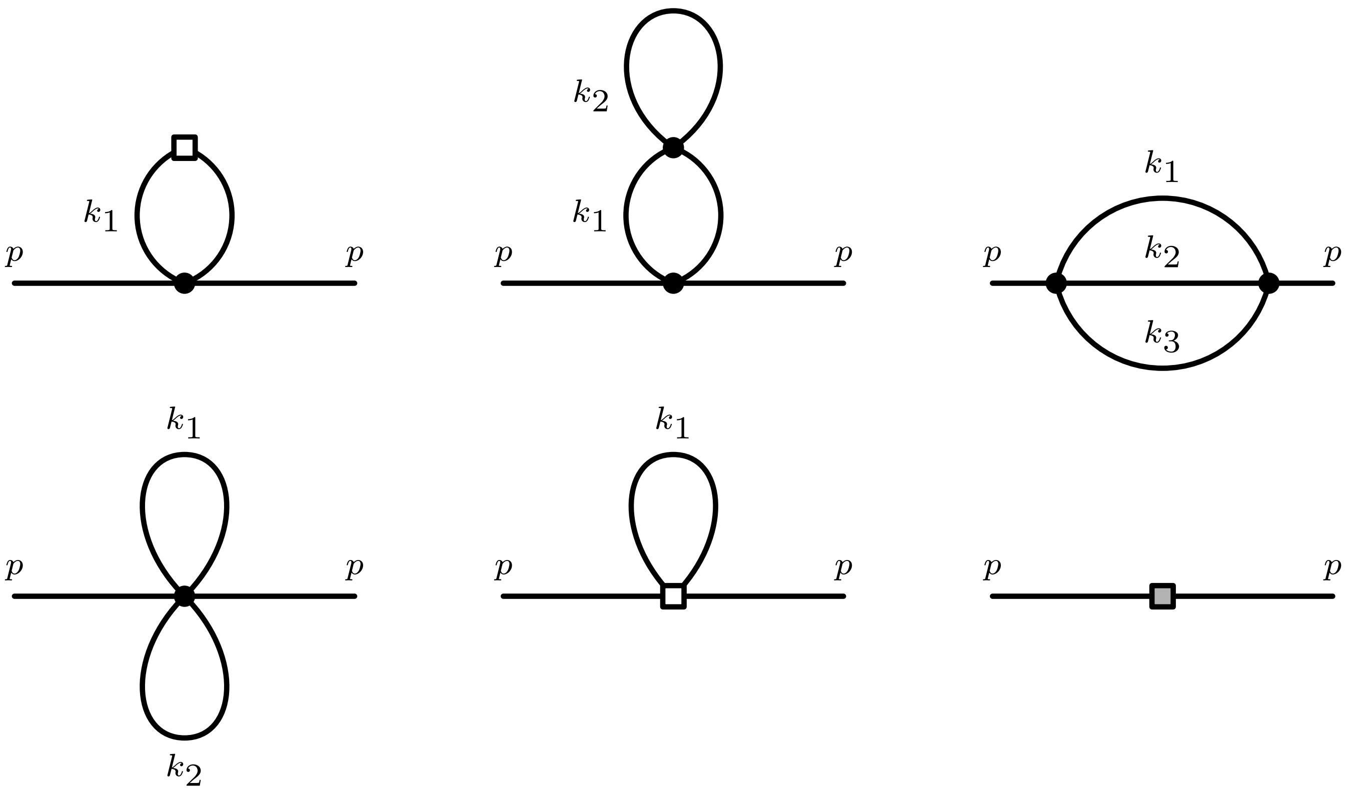
$$
\begin{aligned}
i\Delta(p) &= i\Delta_0 + i\Delta_0[-i\Sigma]i\Delta_0 + i\Delta_0[-i\Sigma]i\Delta_0[-i\Sigma]i\Delta_0 + \cdots \\
&= \frac{i}{p^2 - M_0^2 - \Sigma(p^2) + i\varepsilon}.
\end{aligned}
$$

- The physical meson mass M^2 is then defined in terms of the pole position

$$M^2 = M_0^2 + \Sigma(M^2),$$

where M_0^2 denotes the lowest order Goldstone boson mass.

- At NNLO, the following diagrams contribute to $-i\Sigma$:



- Black dots, open squares and shaded squares denote vertices of $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$, respectively.

Meson Masses to NNLO

- The physical squared masses $M^2_{\rm phys} = M^2$ are calculated from the pole position of the summed propagator,

$$M^2 = M_0^2 + \Sigma(M^2).$$

- Up to NNLO, in terms of the lowest order mass M_0 and the self-energy contribution Σ, the masses of the pseudoscalar mesons are given by

$$M^2_{\rm phys} \quad = \quad M_0^2 + \Sigma_4(M_0^2) + \underbrace{\Sigma_4(M_0^2)\,\frac{\partial \Sigma_4(p^2)}{\partial p^2}\bigg|_{M_0^2} + \Sigma_6(M_0^2)}_{\mathcal{O}(p^6)\text{ contribution}} + \mathcal{O}(p^8).$$

- The self-energy Σ_4 represents the NLO (one-loop) mass shift.

Decay Constants to NNLO

- The decay constants F^a are calculated from the matrix element of the axial current operator,
$$\langle 0|A^a_\mu(0)|\phi^a(p)\rangle = i\sqrt{2}\,p_\mu\,F^a,$$

- Up to NNLO, in terms of the lowest order result $F_2 = F_0$ and the self-energy contributions Σ from the wavefunction renormalization, one obtains

$$F_{\rm phys} = \underbrace{F_0 + F_4(M_0^2) + F_0\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2}}_{\mathcal{O}(p^4)\text{ contribution}} + F_0\,\frac{3}{8}\left(\frac{\partial\Sigma_4(p^2)}{\partial p^2}\bigg|_{M_0^2}\right)^2$$
$$+ \quad F_0\,\frac{\partial\Sigma_6(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_4(M_0^2)\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_6(M_0^2) + \mathcal{O}(p^8).$$

Technical Issues and Numerical Results

- The main technical problem in NNLO PQχPT calculations is the size of the expressions $\longrightarrow$ Heavy use of `FORM`[3].

- The results obtained in `FORM` are highly redundant in the number of terms and require simplification $\longrightarrow$ End results have been compressed by an order of magnitude with `MAPLE`

- The simplification is a difficult and tedious task, since most of the possible simplifications are not easily apparent. Most problems are now solved, but publication delayed by ~ 6 months.

[3] J. Vermaseren, `http://www.nikhef.nl/~form`

- In the most general case, the masses and decay constants of the charged pseudoscalar mesons are functions of the valence quark masses χ_1, χ_3 and the sea quark masses χ_4, χ_5, χ_6.

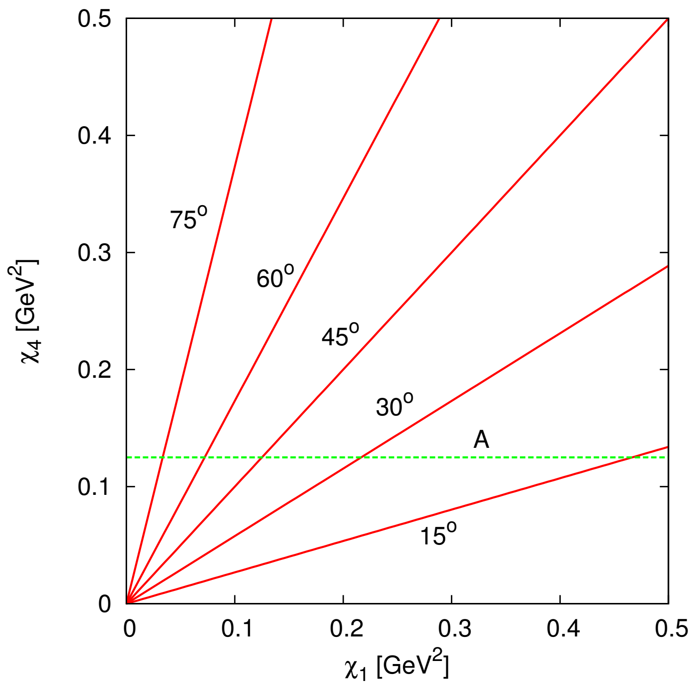
- All results are calculated for three flavors of valence and sea quarks, and are characterized by:
  - Number of nondegenerate valence quarks, $d_{\rm val}$
  - Number of nondegenerate sea quarks, $d_{\rm sea}$

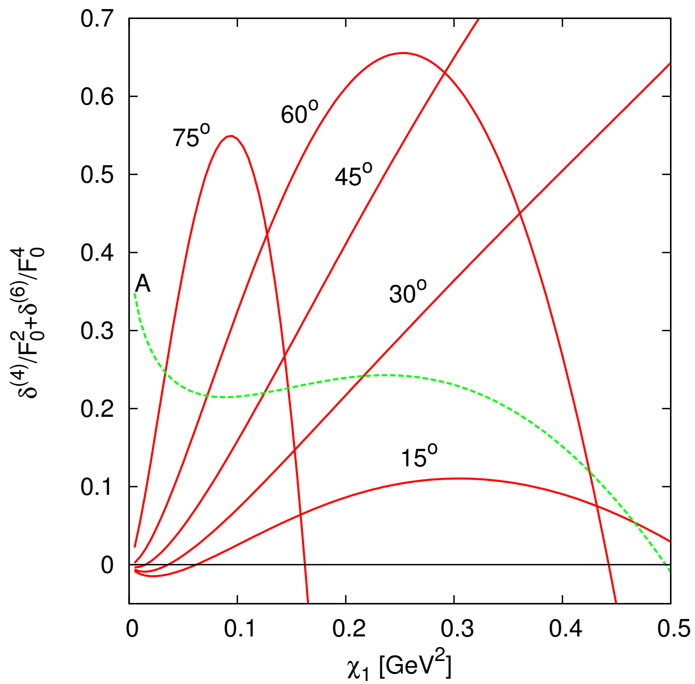
- The results are plotted along different curves in the χ_1 - χ_4 plane, for the cases of:
  - Constant ratio between sea and the valence masses, $\chi_4/\chi_1 = \tan(\theta)$
  - Constant sea quark mass, $\chi_4 = C$

- Unquenched NNLO χPT is thus recovered for $\theta = 45^\circ$.

- Numerical results for M^2 to NNLO in PQχPT, for $d_{\rm val} = 1$ and $d_{\rm sea} = 1$. The quantity plotted, Δ, is defined as $M^2 = M_0^2(1+\Delta)$.

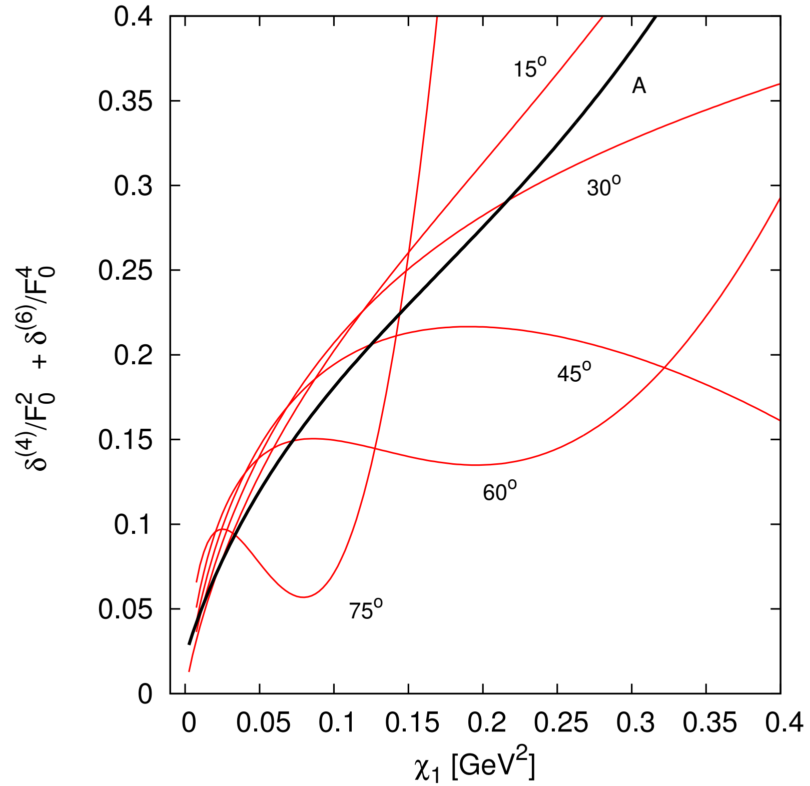
- The NNLO expression for M^2 depends on 9 $\mathcal{O}(p^4)$ and 12 $\mathcal{O}(p^6)$ LEC:s.

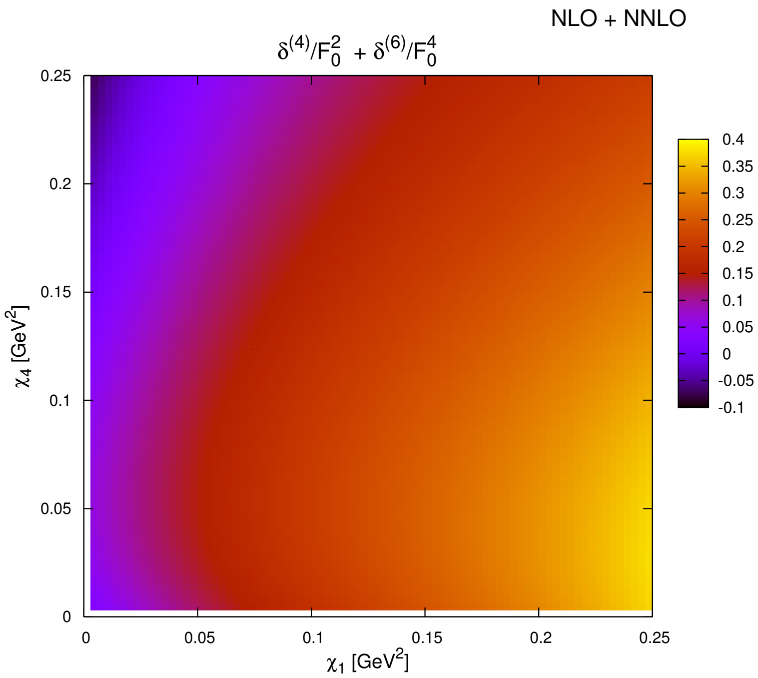




- Numerical results for F_a to NNLO in PQχPT, for $d_{\rm val} = 1$ and $d_{\rm sea} = 1$. The quantity plotted, Δ, is defined as $F = F_0(1 + \Delta)$.

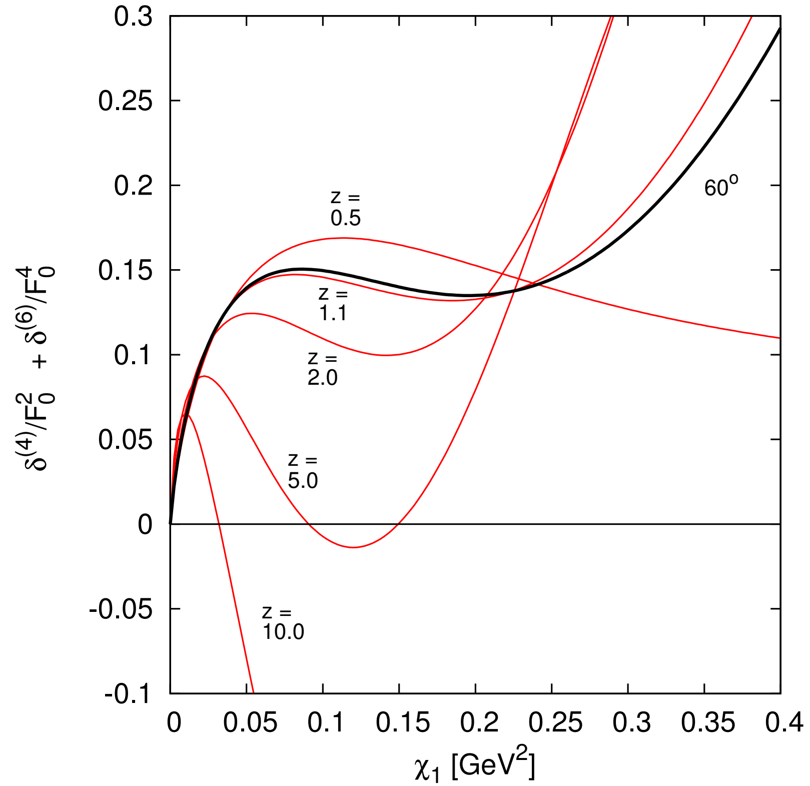
- The NNLO expression for F_a depends on 9 $\mathcal{O}(p^4)$ and 5 $\mathcal{O}(p^6)$ LEC:s.

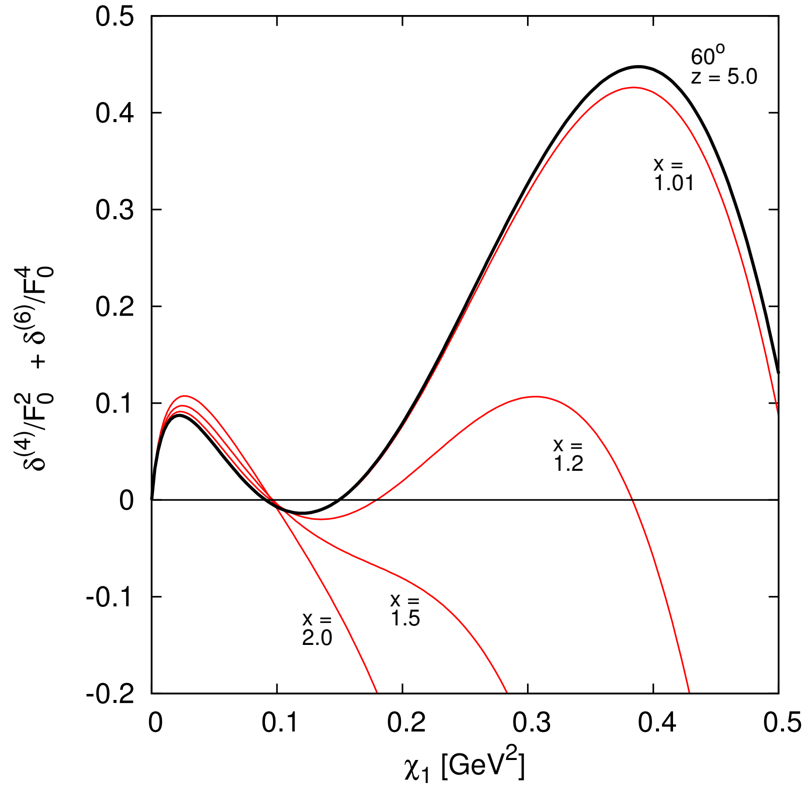




- Left-hand plot: F_a for $d_{\mathrm{val}} = 1$, $d_{\mathrm{sea}} = 2$, and $z = \chi_6/\chi_4$.

- Right-hand plot: F_a for $d_{\mathrm{val}} = 2$, $d_{\mathrm{sea}} = 2$, and $x = \chi_3/\chi_1$.





The following NNLO calculations in PQχPT are completed and published:

- The mass of a pseudoscalar meson for $d_{\mathrm{val}} = 1$, $d_{\mathrm{sea}} = 1$ for $n_{\mathrm{fl}} = 3$; $\longrightarrow$ Phys.Rev.**D70**:111503 (2004), hep-lat/0406017
- The decay constants of charged pseudoscalar mesons for $d_{\mathrm{val}} = 2$, $d_{\mathrm{sea}} = 2$ and $d_{\mathrm{val}} = 1$, $d_{\mathrm{sea}} = 3$ for $n_{\mathrm{fl}} = 3$; $\longrightarrow$ hep-lat/0501014

The following calculations will appear shortly:

- Meson masses and decay constants in two-flavor PQχPT ($n_{\mathrm{fl}} = 2$)
- Full results for charged meson masses in three-flavor PQχPT ($n_{\mathrm{fl}} = 3$)

The following calculations are planned:

- Pion and kaon electromagnetic form factors in PQχPT
- Masses and decay constants of neutral pseudoscalars