Radiative K_{e3} decays revisited

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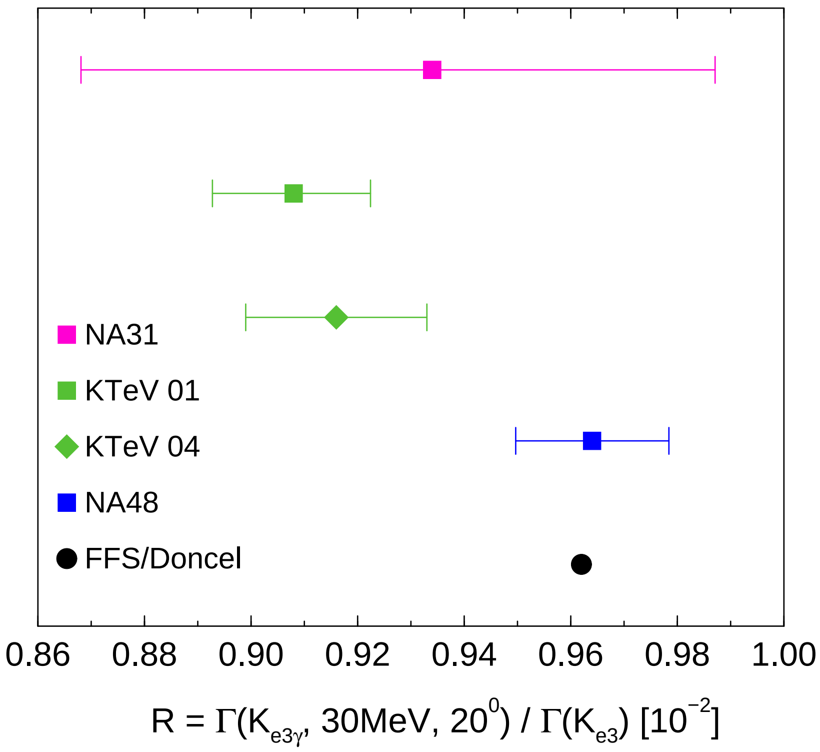
EURIDICE Collaboration Meeting

INFN Frascati, 9/2/2005

$K_L \to \pi^{\mp} e^{\pm} \nu_e \gamma \quad [K_{e3\gamma}]$

- Experimental status
- Introduction : ChPT and Low's theorem
- Formalism : K_{e3}, $K_{e3\gamma}$ amplitudes
- Observables : decay widths, the ratio R
- Electromagnetic corrections
- Structure dependent terms from differential rates
- Summary

Experimental status



KTeV 2001:

$\Rightarrow$ "…significantly lower than all published theoretical predictions."

KTeV, Phys. Rev. D64 (2001) 112004

latest results confirm this…

KTeV, Phys. Rev. D71 (2005) 012001

…or don't:

NA48, Phys. Lett. B605 (2005) 247

Introduction

Chiral Perturbation Theory

- expansion in small masses (M_π, M_K) and momenta
- equivalent to loop expansion; $K \to \pi\, \ell \nu_\ell\, \gamma$ $[K_{\ell 3\gamma}]$ up to one loop:

J. Bijnens, G. Ecker, J. Gasser, Nucl. Phys. B396 (1993) 81

Low's Theorem

- radiative processes $X \to Y\gamma$ involving charged particles are infrared divergent for photon momentum $q \to 0$:

$$T(X \to Y\gamma) \quad = \quad \underbrace{\frac{T_{-1}}{q} + T_0}_{\text{inner bremsstrahlung (IB)}} \quad + \quad \underbrace{T_1\, q \;+\; \ldots}_{\text{structure dependent (SD)}}$$

- Low's theorem : T_{-1}, T_0 given in terms of the non-radiative amplitude (and derivatives thereof):

$$T_{-1},\ T_0\ \propto\ T(X \to Y),\ \partial T(X \to Y)$$

F.E. Low, Phys. Rev. 110 (1958) 974

Strategy: ChPT + Low's Theorem

- chiral representation fulfils Low's theorem
- expect bremsstrahlung to dominate due to infrared singularity
  $\Rightarrow$ improve on chiral representation by using
  phenomenological amplitudes for T_{-1}, T_0
- ChPT for structure dependent contributions

Formalism

- amplitude for K_{e3} described in terms of single form factor $f_+(t)$, parametrisation $f_+(t) = f_+(0)\left\{1 + \lambda_+ \frac{t}{M_\pi^2}\left(+\lambda''_+ \frac{t^2}{M_\pi^4}\right)\right\}$
- amplitude for $K^0_{e3\gamma}$ $\left(K^0(p) \to \pi^-(p')\, e^+(p_e)\, \nu_e(p_\nu)\, \gamma(q)\right)$:

$$T(K^0_{e3\gamma}) = e\, G_F\, V^*_{us}\, \epsilon^\mu(q)^* \Bigg[\left(V_{\mu\nu} - A_{\mu\nu}\right) \underbrace{\bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, v(p_e)}_{\text{leptonic weak current}}$$

$$+ \underbrace{f_+(t) \frac{(p+p')_\nu}{2 p_e q}\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, (m_e - \not{p}_e - \not{q})\, \gamma_\mu\, v(p_e)}_{\text{photon radiation from positron}} \Bigg]$$

Characterisation of $V_{\mu\nu},\ A_{\mu\nu}$:

- decomposition $V_{\mu\nu} = V_{\mu\nu}^{\mathsf{IB}} + V_{\mu\nu}^{\mathsf{SD}}$,
  $V_{\mu\nu}^{\mathsf{IB}} \simeq$ photon radiation from pion, depends *only* on f_+
  H.W. Fearing, E. Fischbach, J. Smith, Phys. Rev. D2 (1970) 542

- Ward identities:

$$q^\mu\, V_{\mu\nu}^{\mathsf{IB}} = f_+(t)\,(p+p')_\nu \quad , \qquad q^\mu\, V_{\mu\nu}^{\mathsf{SD}} = q^\mu\, A_{\mu\nu} = 0$$

- $V_{\mu\nu}^{\mathsf{SD}}$, $A_{\mu\nu}$ decomposed in terms of 8 functions $V_{1-4}\ ,\ A_{1-4}$:
  $\Rightarrow$ V_3 , A_3 suppressed by $m_e^2/M_K^2 \approx 10^{-6}$, not observable in $K_{e3\gamma}$
  $\Rightarrow$ V_4 , A_4 suppressed by two orders in the chiral expansion
  $\Rightarrow$ (essentially) 4 functions $\boxed{V_{1/2}\,,\ A_{1/2}}$ for SD contribution

Chiral one-loop prediction for structure dependent part:

$$
\begin{aligned}
V_1 &= -\frac{8}{F_\pi F_K} L_9 + (\pi,\, K,\, \eta)\text{−loops} \\
V_2 &= \frac{4}{F_\pi F_K}\big(L_9 + L_{10}\big) + (\pi,\, K,\, \eta)\text{−loops} \\
A_1 &= 0 \\
A_2 &= -\frac{1}{8\pi^2 F_\pi F_K}
\end{aligned}
$$

- low-energy constants: $L_9 \leftrightarrow \langle r^2\rangle_\pi^V$ or λ_+, $\quad L_{10} \leftrightarrow (\pi \to e\nu\gamma)$
- axial form factors A_i given in terms of chiral anomaly

J. Wess, B. Zumino, Phys. Lett. B37 (1971) 95

E. Witten, Nucl. Phys. B223 (1983) 422

Observables: decay widths

- for the non-radiative and radiative widths:

$$\Gamma\,(K_{e3}) = \frac{M_K^5 G_F^2 |V_{us}|^2}{128\pi^3} f_+(0)^2 \;\times\; I$$

$$\Gamma(K_{e3\gamma}) = \frac{\alpha M_K^5 G_F^2 |V_{us}|^2}{16\pi^7} f_+(0)^2 \;\times\; I^\gamma(E_\gamma^{\text{cut}}, \theta_{e\gamma}^{\text{cut}})$$

in the following: "standard cuts" $E_\gamma^{\text{cut}} = 30$ MeV , $\theta_{e\gamma}^{\text{cut}} = 20^\circ$

- accessible in experiments:

$$R = \frac{\Gamma(K_{e3\gamma})}{\Gamma(K_{e3})} = \frac{8\alpha}{\pi^4} \frac{I^\gamma}{I}$$

$\Rightarrow$ all sorts of constants cancel in the ratio

- I, I^γ depend on λ_+: $\quad I^{(\gamma)} = a_0^{(\gamma)} + a_1^{(\gamma)}\,\lambda_+ + a_2^{(\gamma)}\,\lambda_+^2 \quad ,$

$$\frac{I^{(\gamma)}\left(\lambda_+ = \lambda_+^{\text{exp}}\right)}{I^{(\gamma)}\left(\lambda_+ = 0\right)} - 1 \approx 0.1 \quad \text{but} \quad \frac{R\left(\lambda_+ = \lambda_+^{\text{exp}}\right)}{R\left(\lambda_+ = 0\right)} - 1 = \mathcal{O}\left(10^{-4}\right)$$

form factor dependence cancels completely in R!

- in ChPT: only visible one-loop effect from structure dependent contributions:

$$\frac{R\left(\text{IB} + \text{SD}\right)}{R\left(\text{IB}\right)} - 1 \approx -0.01$$

$\Rightarrow$ inner bremsstrahlung completely dominant!

tree prediction that is accurate at the 1%-level!

Electromagnetic corrections

Hadronic prediction of $\sim 1\%$ accuracy – radiative corrections?

$$R = \frac{\Gamma_{\text{incl}}(K_{e3\gamma}, E^*_\gamma > E^{\text{cut}}_\gamma, \theta^*_{e\gamma} > \theta^{\text{cut}}_{e\gamma})}{\Gamma_{\text{incl}}(K_{e3})}$$

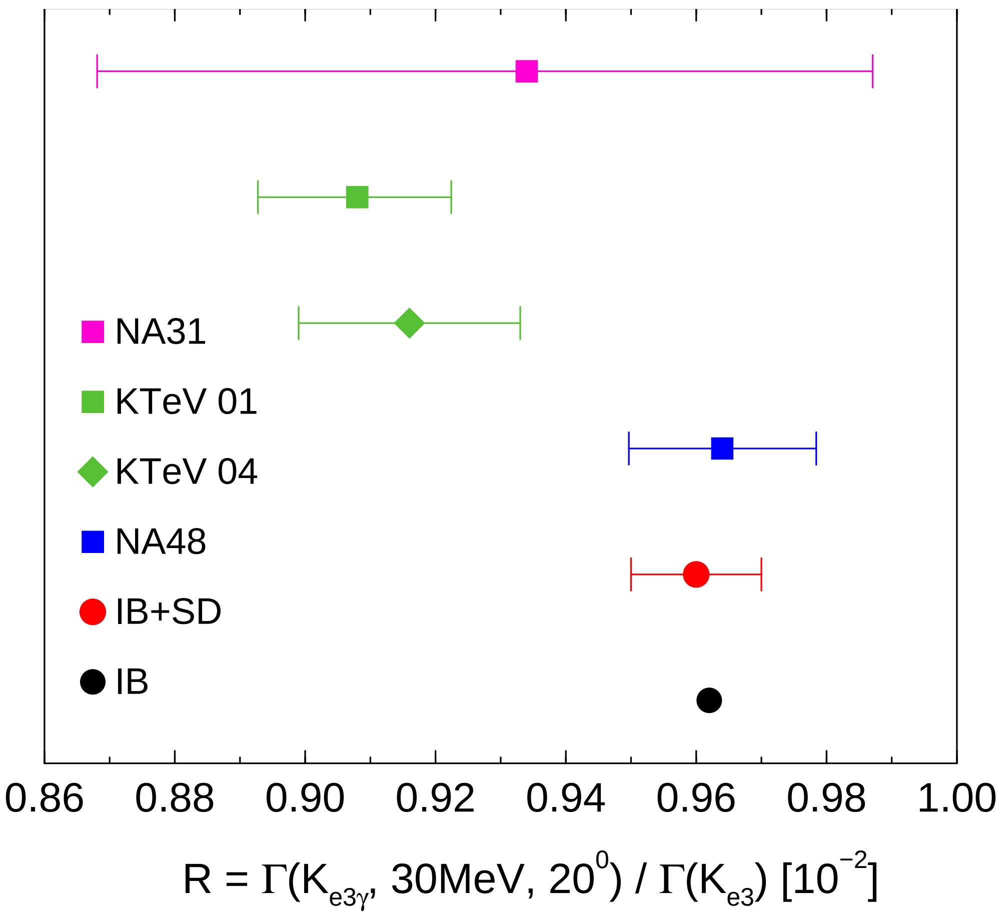
- denominator: see H. Neufeld's talk, net effect: I increased by 0.2%
- numerator: potential large logarithms $\log \frac{m_e}{M_K}$ known (KLN theorem) largely absorbed in $f_+(0)$, remaining effect also 0.2% only

$\Rightarrow$ assume non-enhanced corrections to be small, $\pm 1.0\%$ uncertainty

$\Rightarrow$ our estimate Low + ChPT:

$$\boxed{R = (0.96 \pm 0.01) \times 10^{-2}}$$

Comparison experiment/theory:



A second look at structure dependent terms

- Idea: access information independent on branching ratios by studying (unnormalised) differential rates $\Rightarrow$ $V_{1/2}$, $A_{1/2}$
- $d\Gamma/dE_\gamma^*$ most promising due to special role of E_γ^* in IB vs. SD
- KTeV : study $d\Gamma/dE_\gamma^*$ with
  1. real and constant structure functions
  2. two SD terms, neglect other two in "soft kaon approximation"

$\Rightarrow$ ChPT : first assumption reasonable?

$\Rightarrow$ check validity of the second assumption in distributions

$\Rightarrow$ study also other distributions

1. Real & constant? – Analytic structure of V_i, A_i

- V_i, A_i functions of three kinematical variables, e.g.

$$s = (p' + q)^2 \quad , \qquad t = (p - p')^2 \quad , \qquad u = (p - q)^2$$

$\Rightarrow$ three-body decay with variable "mass" $W^2 = (p_e + p_\nu)^2$

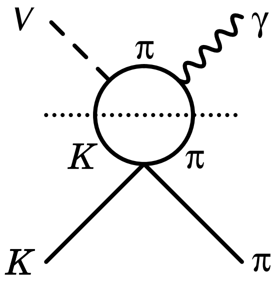
- Reminder: V_i at $\mathcal{O}(p^4)$

$$V_1 \; = \; \underbrace{-\frac{8L_9}{F_\pi F_K}}_{\text{constant}} + \underbrace{\text{loops}}_{\text{non-trivial}} \quad , \qquad V_2 \; = \; \underbrace{\frac{4\big(L_9 + L_{10}\big)}{F_\pi F_K}}_{\text{constant}} + \underbrace{\text{loops}}_{\text{non-trivial}}$$

V_i at $\mathcal{O}(p^6)$: linear terms in s, t, u, M_K^2, M_π^2 + two-loop diagrams

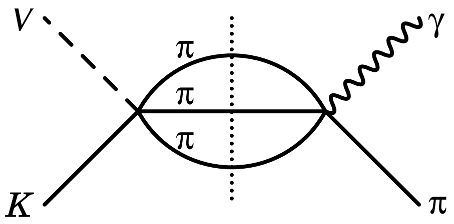
- WZW–anomaly $\mathcal{O}(p^4) \Rightarrow$ constant A_i
  at $\mathcal{O}(p^6)$ linear terms in s, t, u, M_K^2, M_π^2 + one-loop diagrams

Diagrams corresponding to the most important cuts:



$\mathcal{O}(p^4)$: only t-channel cut in V_i

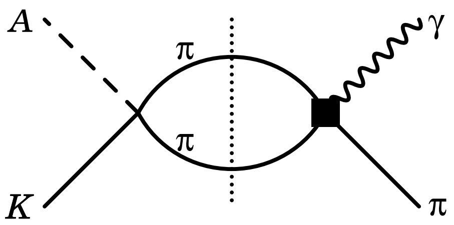
far outside physical region $\Rightarrow$ smooth & flat



$\mathcal{O}(p^6)$: "physical" s-channel cut in V_i

$\hat{=}$ intermediate 3π state $\Rightarrow$ two–loop

kinematically suppressed (phase space!)



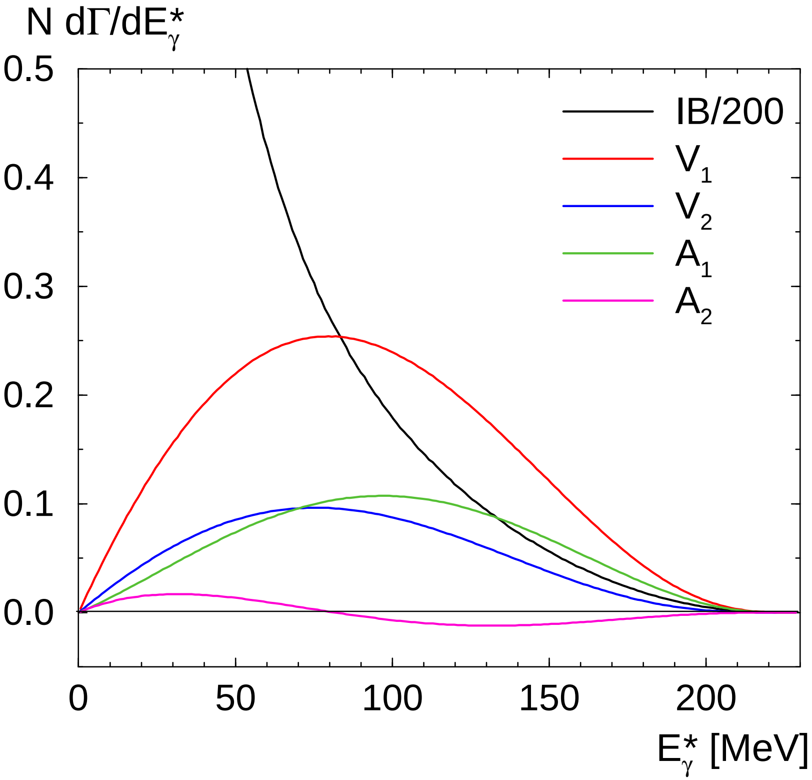
$\mathcal{O}(p^6)$: "physical" s-channel cut in A_i

$\hat{=}$ intermediate 2π state $\Rightarrow$ one–loop $+$

anomalous rescattering $\longrightarrow$ calculated $\Rightarrow$ small!

dominant $\mathcal{O}(p^6)$ are $\mathcal{O}(M_K^2)$ corrections $\Rightarrow$ still smooth & flat!

2. "Soft kaon"? – structure dependent terms in $d\Gamma/dE_\gamma^*$



$d\Gamma/dE_\gamma^*$ is essentially sensitive to *one* linear combination:

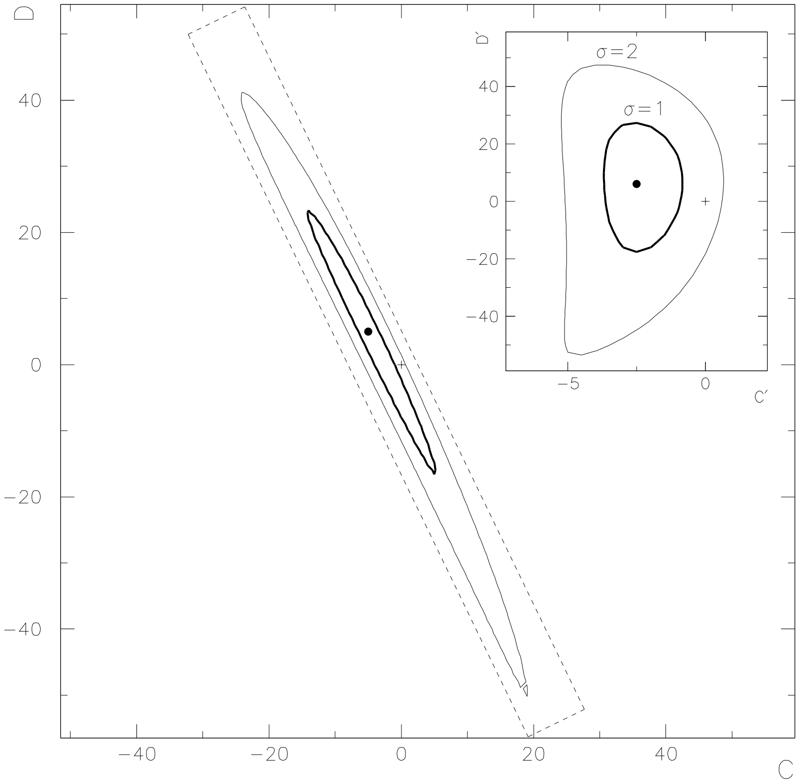
$$C' = 0.9\,V_1 + 0.4\,V_2 + 0.4\,A_1$$

$$= 0.9(V_1 - V_2) + \underbrace{1.3\,V_2}_{\text{neglected?!}}$$

$$+ 0.4(A_1 + A_2) - \underbrace{0.4\,A_2}_{\text{neglected?!}}$$

$\Rightarrow$ "soft kaon" not a good approximation!

Compare KTeV result to ChPT prediction:



KTeV, Phys. Rev. D64 (2001) 112004

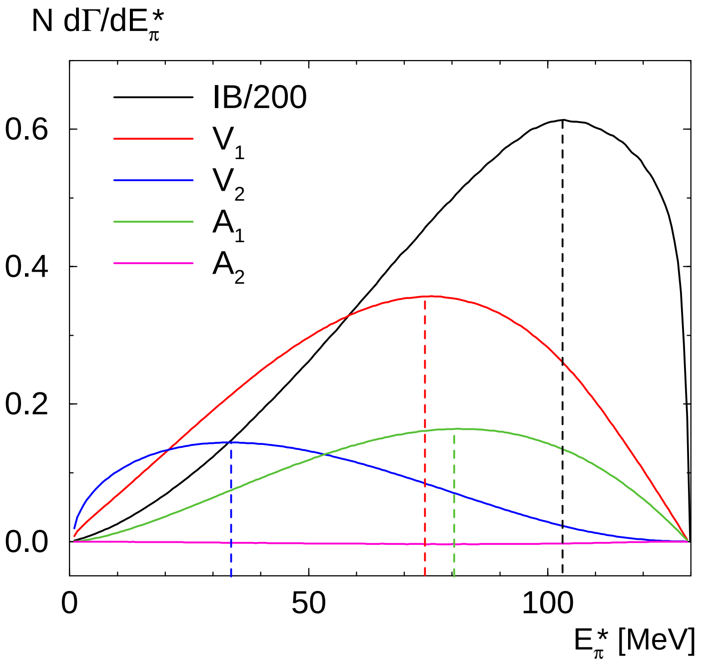
- KTeV: $C' = -2.5^{+1.5}_{-1.0} \pm 1.5$
- ChPT: $C' = -1.6 \pm 0.4$

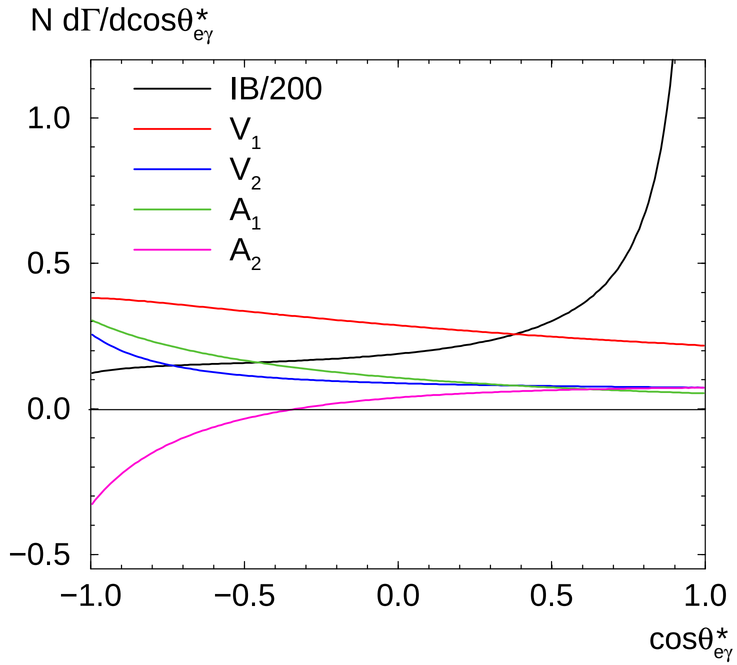
$\Rightarrow$ good numerical agreement within 1-σ error

$\Rightarrow$ interpretation of C' in terms of structure functions different

$\Rightarrow$ serious constraints on SD terms feasible !

Other distributions: $d\Gamma/dE_\pi^*$, $d\Gamma/d\cos\theta_{e\gamma}^*$





⇒ potentially sensitive to V_2 : peaked at low pion energies

⇒ potentially sensitive to A_2 : strong backward variation

Summary (1)

Low's theorem + ChPT allow for a precise prediction of $\Gamma(K_{e3\gamma})/\Gamma(K_{e3})$

- completely insensitive to details of K_{e3} form factor
- structure dependent terms very small
- constants like G_F or $|V_{us}|$ cancel

Precision limited by radiative corrections :

$$R = \Gamma(K_{e3\gamma})/\Gamma(K_{e3}) = (0.96 \pm 0.01) \times 10^{-2}$$

vs.

$$R = (0.908 \pm 0.008_{\text{stat}} \, {}^{+\,0.013}_{-\,0.012}{}_{\text{syst}}) \times 10^{-2} \quad \text{(KTeV 2001)}$$

$$R = (0.916 \pm 0.017_{\text{stat+syst}}) \times 10^{-2} \quad \text{(KTeV 2004)}$$

$$R = (0.964 \pm 0.008_{\text{stat}} \, {}^{+\,0.011}_{-\,0.009}{}_{\text{syst}}) \times 10^{-2} \quad \text{(NA48 2004)}$$

Summary (2)

Experimental extraction of structure dependent terms from $d\Gamma/dE_\gamma^*$ shown to be feasible by KTeV collaboration:

$$C' = -2.5\,^{+1.5}_{-1.0}\,_{\text{stat}} \pm 1.5_{\text{syst}} \quad \text{(KTeV)} \quad \text{vs.} \quad C' = -1.6 \pm 0.4 \quad \text{(ChPT)}$$

ChPT: these structure functions are fairly smooth
imaginary parts and momentum dependence suppressed

Tentative ideas for experimentalists and theorists:

- extract V_2 from $d\Gamma/dE_\pi^*$ in order to disentangle V_1 and V_2 in C'
- chiral anomaly in A_2 at most accessible in $d\Gamma/d\cos\theta_{e\gamma}^*$
- perform same study for other $K_{\ell 3\gamma}$ channels !