Isospin Breaking in $K \to 3\pi$ Decays

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Overview

- $K \to 3\pi$ Decays in Chiral Perturbation Theory, J. Bijnens, P. Dhonte and F. Persson, hep-ph/0205341, Nucl. Phys. B648 (2003) 317-344.
- Isospin Breaking in $K \to 3\pi$ Decays I: Strong Isospin Breaking, J. Bijnens and F. Borg, hep-ph/0405025, Nuclear Physics B697 (2004) 319-342.
- Isospin Breaking in $K \to 3\pi$ Decays II: Radiative Corrections, J. Bijnens and F. Borg, hep-ph/0410333, accepted in Eur. Phys. J. C.
- Isospin Breaking in $K \to 3\pi$ Decays III: Bremsstrahlung and Fit to Experiment, J. Bijnens and F. Borg, hep-ph/0501163.

Note: Fredrik Borg = Fredrik Persson, obtained PhD 28/1/2005

Overview

- ChPT in the nonleptonic mesonic sector
- Lagrangians
- $K \to 3\pi$ kinematics and isospin
- Overview of calculations and results
- Data and Fits
- Conclusions

ChPT in the nonleptonic sector

- some earlier work: espescially on decays with photons
- Kambor, Missimer, Wyler (KMW) : Constructed $\mathcal{L}$ and ∞ 1990
- G. Esposito-Farese: Checked $\mathcal{L}$ and ∞ 1991
- KMW : Calculated $K \to 2\pi$ and $K \to 3\pi$ 1991
- Donoghue + Holstein + KMW : clarified the relations between observables 1992
- BUT: explicit formulas lost (US Mail)
- Kambor, Ecker, Wyler : simplified octet $\mathcal{L}$ 1993
- $K \to 2\pi$ redone: Bijnens, Pallante, Prades 1998

ChPT in the nonleptonic sector

- First paper: redo $K \to 3\pi$
- Ecker Isidori Muller Neufeld Pich: Electromagnetic octet $\mathcal{L}$ Lagrangian plus ∞ 2000
- applications to $K \to 2\pi$: Several papers
- Isospin breaking in $K \to 3\pi$: remaining papers $\mathcal{O}\,(p^4, p^2(m_u - m_d), e^2p^2)$.

Lagrangians: p^2 and e^2

$$\mathcal{L}_2 = \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2}$$

$$\mathcal{L}_{S2} = \frac{F_0^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

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$$u_\mu = i u^\dagger D_\mu U\, u^\dagger = u_\mu^\dagger\,, \quad u^2 = U\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,,$$

U contains the Goldstone boson fields

$$U = \exp\left(\frac{i\sqrt{2}}{F_0} M\right)\,, \qquad M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{-2}{\sqrt{6}}\eta_8 \end{pmatrix}.$$

Here $\chi = 2B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$ and $D_\mu U = \partial_\mu U - ie\,[Q, U]\,.$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C F_0^4 \Big[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \, \langle \Delta_{ij} \; u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \Big]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia} \, \delta_{jb} \,, \quad t^{21,13} = t^{13,21} = 1/3, \ldots$$

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$C = -\frac{3}{5}\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*$: chiral and large N_c limits $G_8 = G_{27} = 1$

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Electromagnetic: $\mathcal{L}_{E2} = e^2 F_0^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle + \left(e^2 C F_0^6 G_E \langle \Delta_{32} \mathcal{Q}_R \rangle \right)$

$$\mathcal{Q}_L = u Q u^\dagger\,, \quad \mathcal{Q}_R = u^\dagger Q u \ \ \text{with} \ \ Q = \text{diag}\,(2/3, -1/3, -1/3)$$

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Note: F_0 not well known, fit $CF_0^4 G_8$,. . .

use *numerically* $F_0 = F_\pi$ to quote G_8,. . .

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$\mathcal{L}_{S4}$ values of L_i^r use Amoros, Bijnens, Talavera p^4 fit

$\mathcal{L}_{W4}$ thirteen N_i^r (octet) and twelve D_i^r might contribute (two N_i^r and two D_i^r extra for photon-reducible)

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Done here :

Isospin conserved: 11 combinations of N_i^r, D_i^r relevant: $\tilde{K}_i$
7 coefficients order m_K^4, 4 order $M_K^2 m_\pi^2$
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Isospin Broken: 30 combinations of N_i^r, D_i^r, Z_i^r relevant

Results from isospin breaking with $K_i^r = Z_i^r = 0$

$K \to 3\pi$ Kinematics and Isospin

$$\begin{aligned}
K_L(k) &\to \pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\,, & [A^L_{000}]\,,\\
K_L(k) &\to \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, & [A^L_{+-0}]\,,\\
K_S(k) &\to \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, & [A^S_{+-0}]\,,\\
K^+(k) &\to \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\,, & [A_{00+}]\,,\\
K^+(k) &\to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\,, & [A_{++-}]\,,
\end{aligned}$$

plus charge conjugate ones

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Dalitz plot variables:

$$
y = (s_3 - s_0)/m^2_{\pi^+}\,,\ x = (s_2 - s_1)/m^2_{\pi^+}\,,\ s_0 = (s_1+s_2+s_3)/3\,.
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Bremsstrahlung problem: s_i from E_i or $m^2_{\pi\pi}$ different

Kinematics and Isospin

Valid also at p^6

$$
\begin{aligned}
A^L_{000} &= M_0(s_1)+M_0(s_2)+M_0(s_3)\,,\\
A^L_{+-0} &= M_1(s_3)+M_2(s_1)+M_2(s_2)+M_3(s_1)(s_2-s_3)+M_3(s_2)(s_1-s_3)\,,\\
A^S_{+-0} &= M_4(s_1)-M_4(s_2)+M_5(s_1)(s_2-s_3)-M_5(s_2)(s_1-s_3)+M_6(s_3)(s_1-s_2)\,,\\
A_{00+} &= M_7(s_3)+M_8(s_1)+M_8(s_2)+M_9(s_1)(s_2-s_3)+M_9(s_2)(s_1-s_3)\,,\\
A_{++-} &= M_{10}(s_3)+M_{11}(s_1)+M_{11}(s_2)+M_{12}(s_1)(s_2-s_3)+M_{12}(s_2)(s_1-s_3)\,.
\end{aligned}
$$

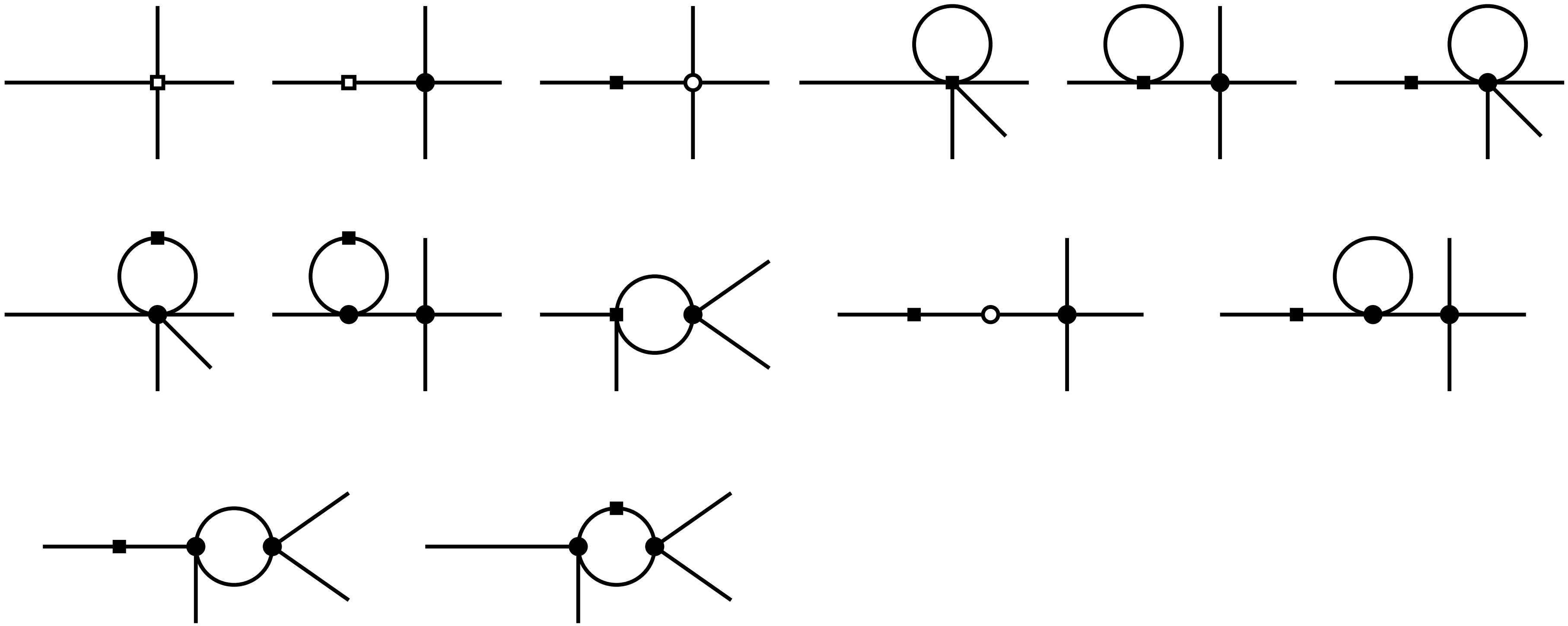
polynomial ambiguity from $s_1 + s_2 + s_3 = \sum_i m_i^2$

Isospin:

$$
\begin{aligned}
M_0(s) &= M_1(s) + 2M_2(s)\,,\\
2M_7(s) + 4M_8(s) &= M_{10}(s) + 2M_{11}(s)\,,\\
M_4(s) &= \frac{1}{3}\left(M_7(s) - M_8(s) + M_{10}(s) - M_{11}(s)\right)\\
M_5(s) - M_6(s) &= M_9(s) + M_{12}(s)\,.
\end{aligned}
$$

Calculations: isospin limit

The diagrams of order p^4



Expressions for the $M_i(s)$: see first paper

Confirmed by Gamiz, Prades, Scimemi, hep-ph/0305164
Lashin, hep-ph/0308200 (how not clear)

Calculations: I: Strong Isospin Breaking

Includes:

- $m_u - m_d$ and the effects of $\mathcal{L}_{E2}$, $\mathcal{L}_{E2S2}$ and $\mathcal{L}_{W2E2}(G_8)$
- π^0-η mixing (η' via LECs)
- Same diagrams as before
- Formulas immediately a lot longer:
  see `http://www.thep.lu.se/~bijnens/chpt.html`

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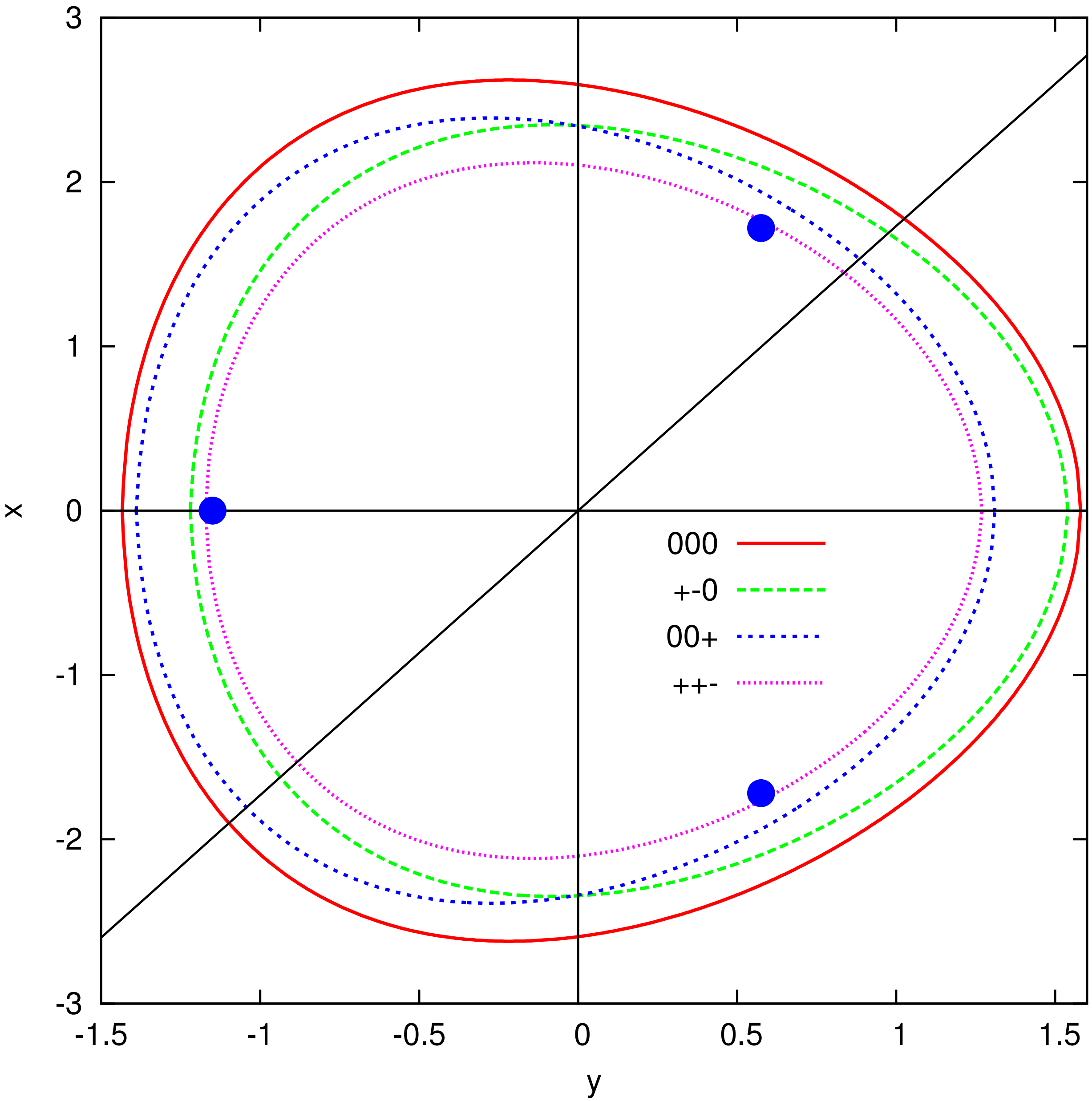
Effects:

- F_{π^+}, F_{K^+} overall factor
- Mass difference π^+-π^0
- Others typically a few %

Phasespace



Curves:

Phasespace boundaries

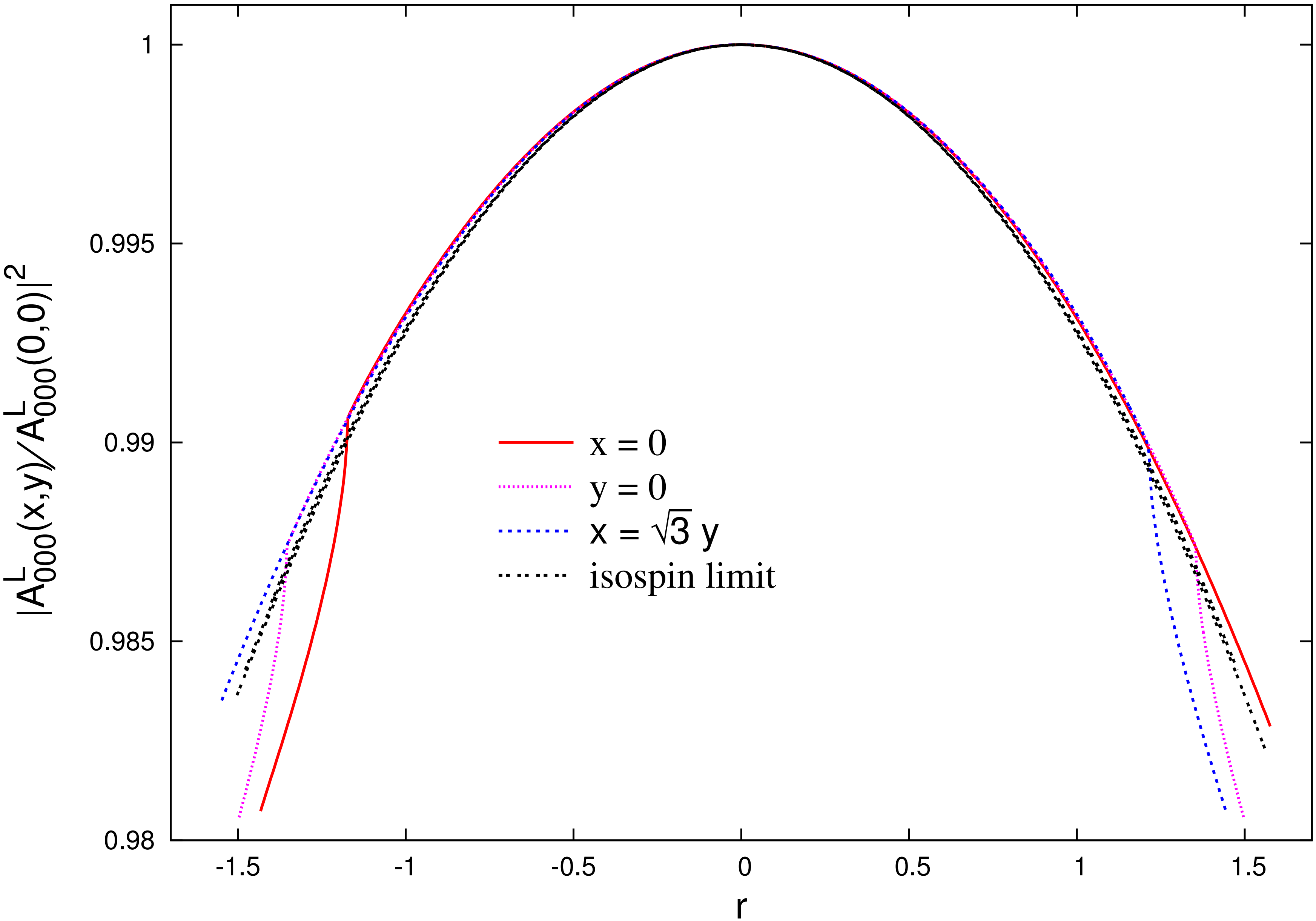
Dots:

$\pi\pi$ pair at rest

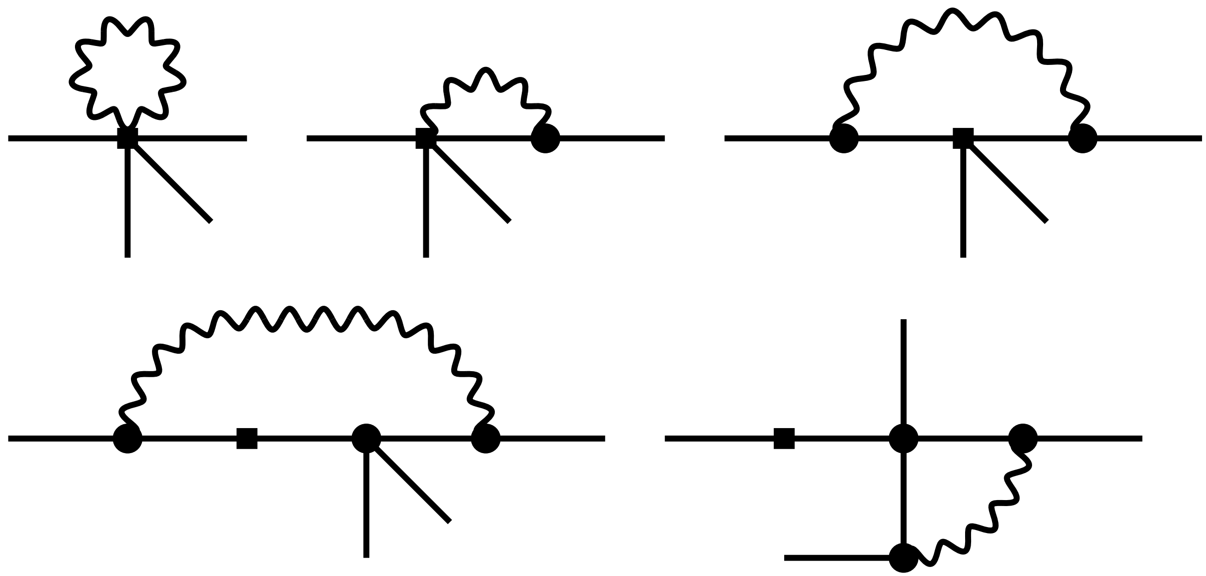
Lines:

Show $|A|^2$ along these

$K_L \to \pi^0\pi^0\pi^0$

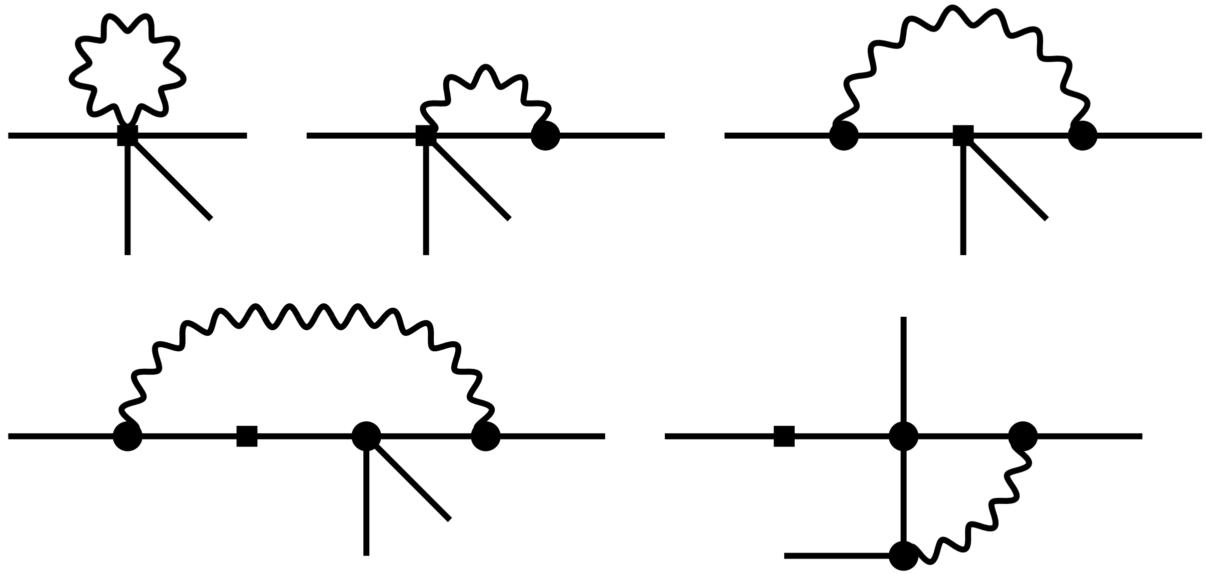


- Photon Loops

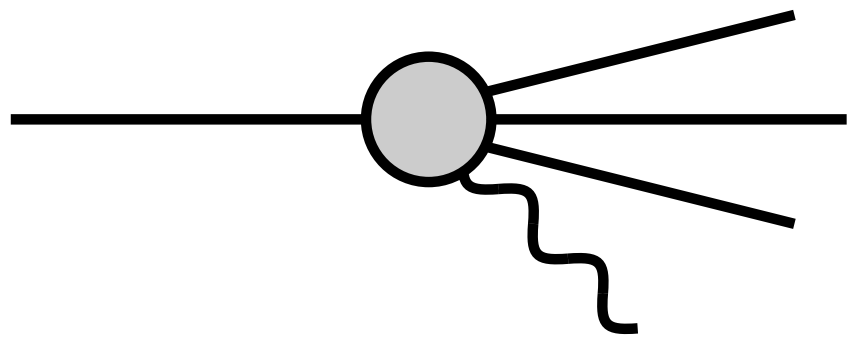


Calculations: II: Radiative Corrections

- Photon Loops

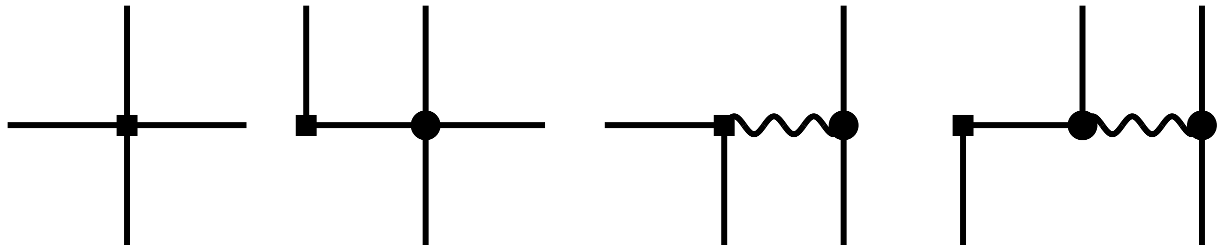


- Soft Bremsstrahlung



Calculations: II: Radiative Corrections

- Photon reducible diagrams

- Tree Level: 

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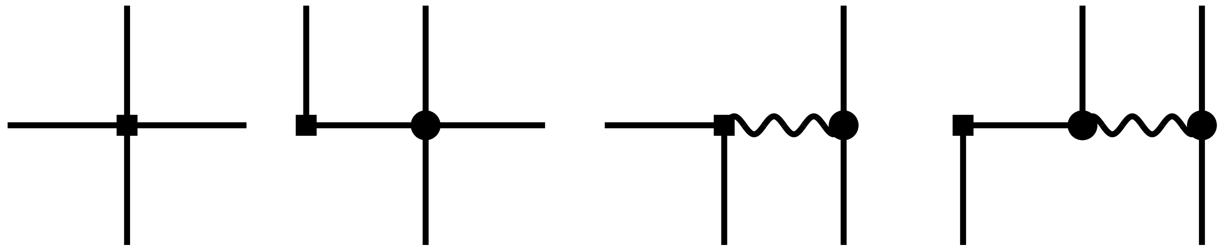

Extra vanish exactly

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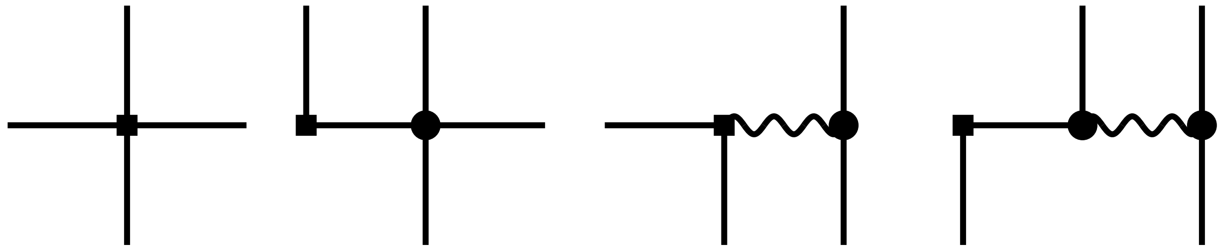

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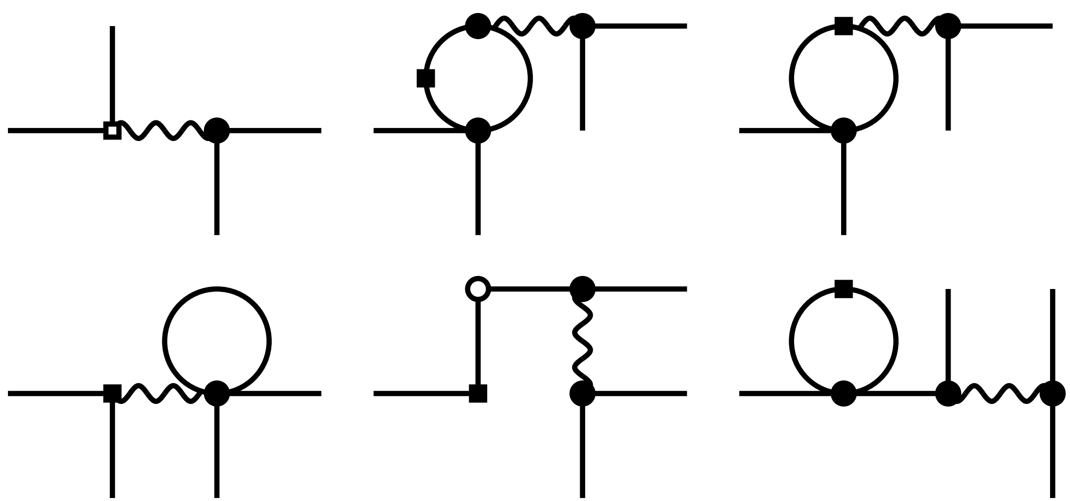
- One-loop Level:



Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

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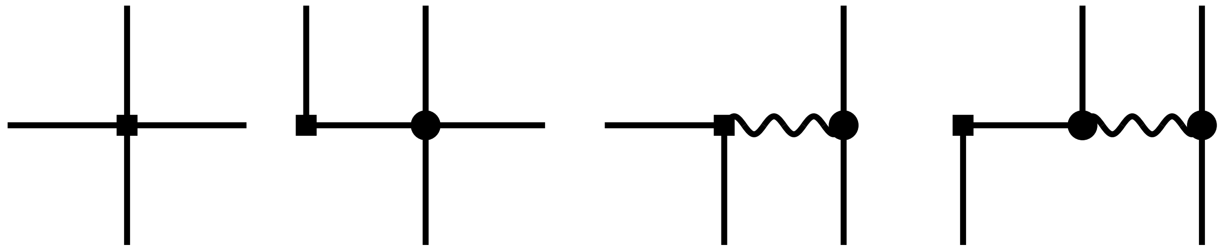

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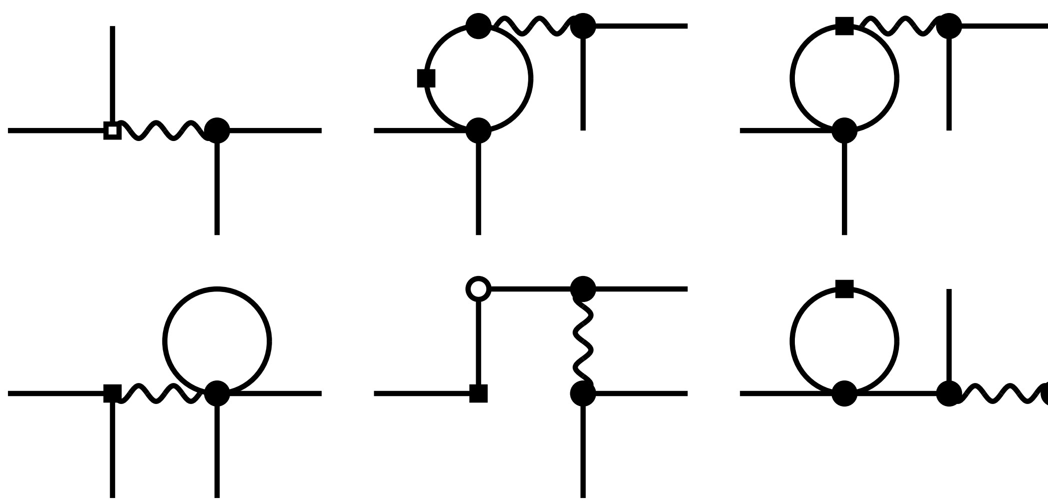
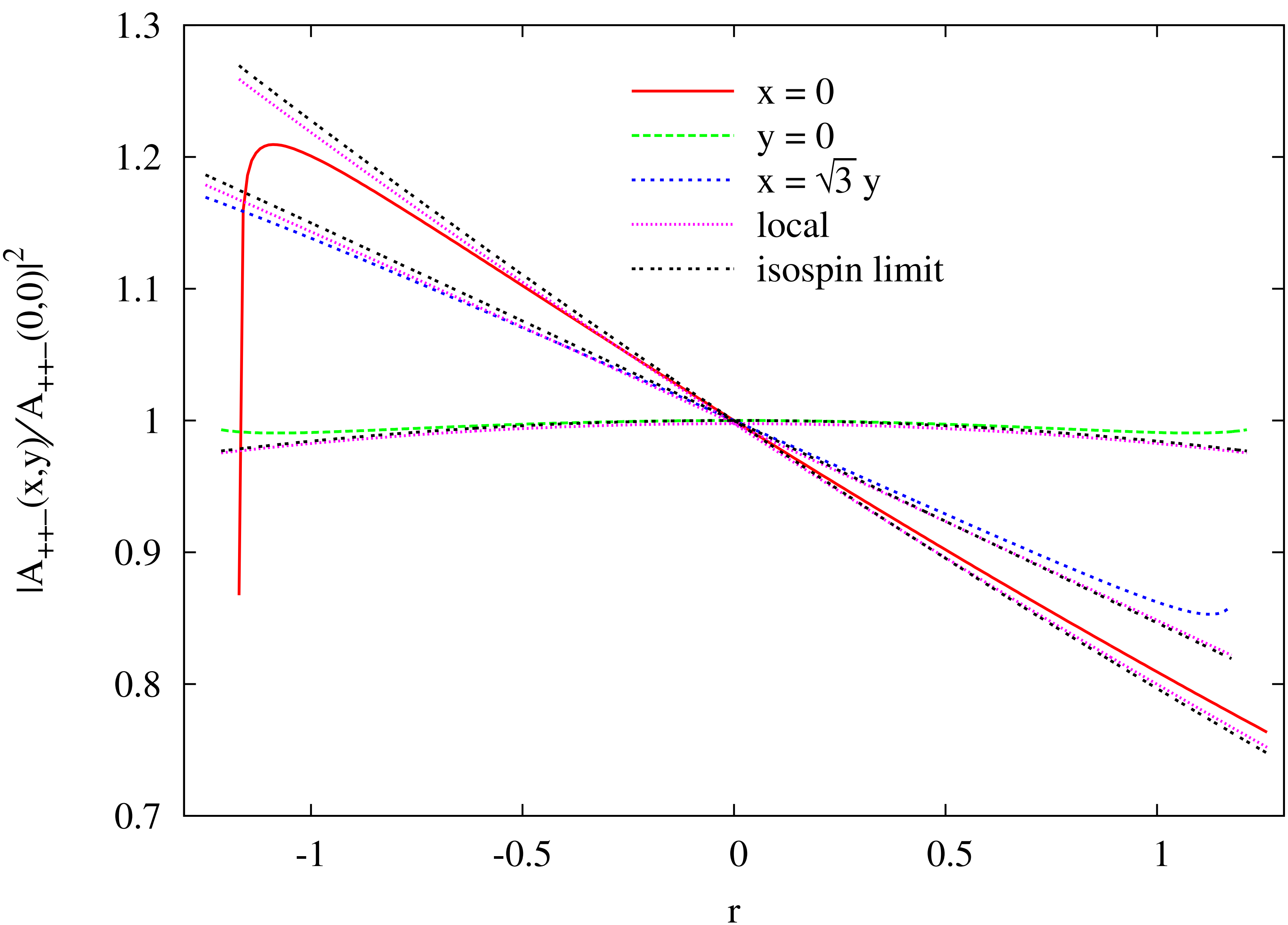


Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

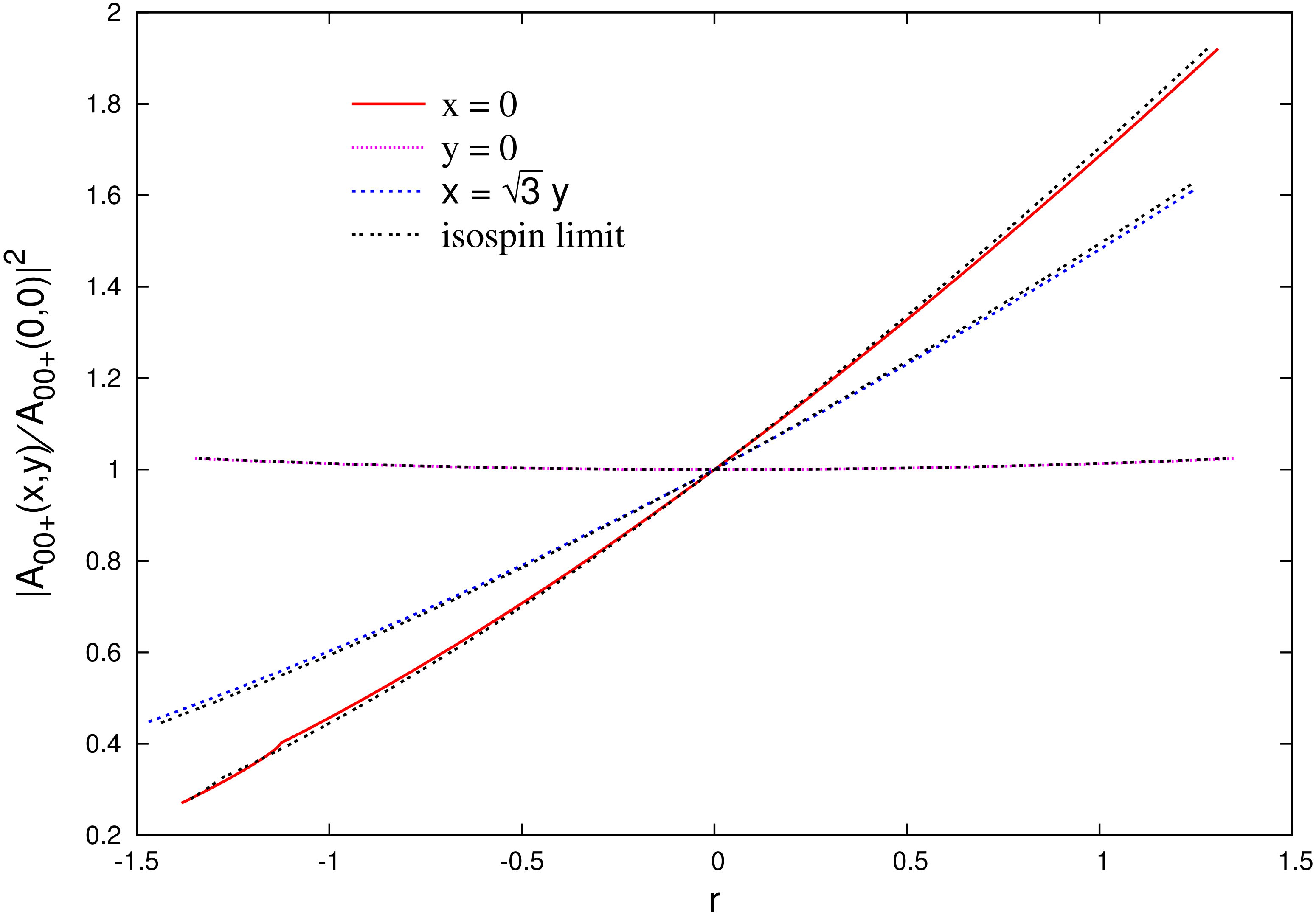
Numerically Negligible

$K^+ \to \pi^+\pi^+\pi^-$



1.3
1.2
1.1
1
0.9
0.8
0.7
-1
-0.5
0
0.5
1
r
|A++−(x,y)/A++−(0,0)|²
x = 0
y = 0
x = √3 y
local
isospin limit

$K^+ \to \pi^0\pi^0\pi^+$



Note: Disagree numerically with Nehme, hep-ph/0406209

Calculations: III: Hard Bremsstrahlung

- Include hard photon Bremsstrahlung to lowest order
  Full amplitude from Low's theorem
- Do everything also for $K \to 2\pi$ to have same treatment
  Note: some new pieces, e.g., $e^2 p^2 G_{27}$

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Checks:
- ⊛ Everything done twice indepently
- ⊛ UV infinities cancel analytically: no $1/(d-4)$
- ⊛ IR infinities cancel analytically: no m_γ
- ⊛ soft-hard Bremsstrahlung match: no E_γ
  (except for F_π, F_K parts)

Experimental Inputs

- Decay Rates
- Dalitz Plot Parameters

$$\left|\frac{A(s_1,s_2,s_3)}{A(s_0,s_0,s_0)}\right|^2 = 1 + gy + hy^2 + kx^2$$

g, h, k for $K_L \to \pi^+\pi^-\pi^0$,
$K^+ \to \pi^0\pi^0\pi^+$,
$K^+ \to \pi^+\pi^+\pi^-$

h for $K_L \to \pi^0\pi^0\pi^0$
($g = 0$ and $k = h/3$)

$A^S_{+-0} = \gamma_S x - \xi_S xy$ for $K_S \to \pi^+\pi^-\pi^0$

- All input taken from PDG2004

Treatment of Bremsstrahlung

Old experiments: what to do?

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We adopted:

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- From remainder get $|A(s_0,s_0,s_0)|^2$ using experimental g,h,k
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Advantages:
- Mimicks experimental determination (well, sort of)
- Coulomb singularity avoided

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We adopted:
- Subtract hard Bremsstrahlung from Decay Rate
- From remainder get $|A(s_0, s_0, s_0)|^2$ using experimental g, h, k
- Fit $|A(s_0, s_0, s_0)|^2$, g, h, k with soft Bremsstrahlung and loops, ...

Advantages:
- Mimicks experimental determination (well, sort of)
- Coulomb singularity avoided

We really need to know what Phasespace/Photons covered

Results

$K \to \pi\pi$ only: essentially same as Ecker et al.

Tree level: $G_8 = 10.36$ $G_{27} = 0.550$

Full: $G_8 = 5.39$ $G_{27} = 0.359$

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Full fit:

μ	0.77 GeV	1.0 GeV	0.6 GeV	0.77 GeV
G_8	5.39(1)	4.60(1)	6.43(1)	5.39(1)
G_{27}	0.359(2)	0.301(1)	0.438(2)	0.359(2)
$\delta_2 - \delta_0$	$-57.9(1.5)^o$	$-57.3(1.4)^o$	$-58.9(1.4)^o$	$-57.9(1.4)^o$
$10^3 \tilde{K}_1/G_8$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_2/G_8$	48.5(2.4)	56.5(2.4)	41.2(1.9)	46.6(1.6)
$10^3 \tilde{K}_3/G_8$	2.6(1.2)	−1.7(1.1)	6.7(1.0)	3.5(0.8)
$10^3 \tilde{K}_4/G_{27}$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_5/G_{27}$	−41.2(16.9)	−52.0(17.7)	−31.1(12.0)	−27.0(8.3)
$10^3 \tilde{K}_6/G_{27}$	−102(105)	−114(105)	$-93(76)$	$\equiv 0$
$10^3 \tilde{K}_7/G_{27}$	78.6(33)	78.0(33.5)	79.6(22.7)	50.0(13.0)
χ^2/DOF	29.3/10	27.2/10	33.0/10	30.5/11

vary $\tilde{K}_1$, $\tilde{K}_4$ see first paper

Octet and μ-variation

μ	octet 0.77 GeV	μ variation 1.0 GeV	μ variation 0.6 GeV
G_8	4.84(1)	–	–
G_{27}	0.430(1)	–	–
$\delta_2 - \delta_0$	$-57.9(0.2)^o$	–	–
$10^3\tilde{K}_1/G_8$	2.0(1)	−5.88	5.61
$10^3\tilde{K}_2/G_8$	63.0(1.5)	−2.69	2.57
$10^3\tilde{K}_3/G_8$	−6.0(7)	0.159	−0.152
$10^3\tilde{K}_4/G_{27}$	$\equiv 0$	−9.93	9.48
$10^3\tilde{K}_5/G_{27}$	$\equiv 0$	0	0
$10^3\tilde{K}_6/G_{27}$	$\equiv 0$	27.0	−25.8
$10^3\tilde{K}_7/G_{27}$	$\equiv 0$	−21.5	20.5
$10^3\tilde{K}_8/G_8$	20.4(1)	−0.546	0.521
$10^3\tilde{K}_9/G_8$	9.1(1)	−2.92	2.79
$10^3\tilde{K}_{10}/G_8$	$\equiv 0$	11.6	−11.1
$10^3\tilde{K}_{11}/G_8$	$\equiv 0$	−1.66	1.58
χ^2/DOF	33.3/10	–	–

Octet: $G_{27} = 0$
keep also $\tilde{K}_i$
with $m_K^2 m_\pi^2$ factors

μ variation
$\tilde{K}_i(\mu) - \tilde{K}_i(0.77\ GeV)$

Fit about same as before, some experiments fit better, others smaller error

Models

See Ecker, Kambor, Wyler 1993 for explanations

- Vector octet dominance $\Longrightarrow \tilde{K}_3 = -\frac{1}{2}\tilde{K}_2$: Not at all

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\begin{aligned}
N_1^r &= 2k_f(32/3\,L_1^r + 4\,L_3^r + 2/3\,L_9^r)\,,\\
N_2^r &= 2k_f(16/3L_1^r + 4L_3^r + 10/3\,L_9^r)\,,\\
N_3^r &= 2k_f(8L_2^r - 2L_9^r)\,,\\
N_4^r &= 2k_f(-16/3\,L_1^r - 8/3\,L_3^r - 4/3\,L_9^r)\,,\\
N_5^r &= 2k_f(-L_5^r)\,,\\
N_6^r &= 2k_f(2/3\,L_5^r)\,,\\
N_7^r &= 2k_f(L_5^r)\,,\\
N_8^r &= 2k_f(4\,L_4^r + 2\,L_5^r)\,,\\
N_9^r &= N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
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\end{aligned}
$$

μ	0.77 GeV	0.9 GeV	0.842 GeV
G_8	4.18(1)	4.42	4.22(1)
G_{27}	0.360(2)	0.326(10)	0.339(10)
k_F	2.61(1)	4.94(2)	3.60(5)
χ^2/DOF	109/14	182/14	60.4/13

WDM or $k_f = 1/2$ No

Factorization: surprisingly OK

But k_f not naive one

Data and Fits

Decay	Width [GeV]	ChPT [GeV]	Fact. [GeV]
$K^+ \to \pi^+\pi^0$	$(1.1231 \pm 0.0078) \cdot 10^{-17}$	$1.123 \cdot 10^{-17}$	$1.127 \cdot 10^{-17}$
$K_S \to \pi^0\pi^0$	$(2.2828 \pm 0.0104) \cdot 10^{-15}$	$2.282 \cdot 10^{-15}$	$2.283 \cdot 10^{-15}$
$K_S \to \pi^+\pi^-$	$(5.0691 \pm 0.0108) \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$
$K_L \to \pi^0\pi^0\pi^0$	$(2.6748 \pm 0.0358) \cdot 10^{-18}$	$2.618 \cdot 10^{-18}$	$2.698 \cdot 10^{-18}$
$K_L \to \pi^+\pi^-\pi^0$	$(1.5998 \pm 0.0271) \cdot 10^{-18}$	$1.658 \cdot 10^{-18}$	$1.711 \cdot 10^{-18}$
$K^+ \to \pi^0\pi^0\pi^+$	$(9.195 \pm 0.0213) \cdot 10^{-19}$	$8.934 \cdot 10^{-19}$	$8.816 \cdot 10^{-19}$
$K^+ \to \pi^+\pi^+\pi^-$	$(2.9737 \pm 0.0174) \cdot 10^{-18}$	$2.971 \cdot 10^{-18}$	$2.933 \cdot 10^{-18}$

Decay	Quantity	Experiment	ChPT	Fact.
$K_L \to \pi^0\pi^0\pi^0$	h	-0.0050 ± 0.0014	-0.0062	-0.0025
$K_L \to \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	0.678	0.654
	h	0.076 ± 0.006	0.088	0.083
	k	0.0099 ± 0.0015	0.0057	0.0068
$K_S \to \pi^+\pi^-\pi^0$	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$	$2.9 \cdot 10^{-8}$
$K^\pm \to \pi^0\pi^0\pi^\pm$	g	0.638 ± 0.020	0.636	0.648
	h	0.051 ± 0.013	0.077	0.080
	k	0.004 ± 0.007	0.0047	0.0069
$K^+ \to \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	-0.215	-0.226
	h	0.012 ± 0.008	0.012	0.019
	k	-0.0101 ± 0.0034	-0.0034	-0.0033

Conclusions

- Calculated $K \to 3\pi$ to first nontrivial order in isospin breaking fully
- New fit done and tested a few models
- Fit with/without isospin breaking seems similar
- Need Photon/Phasespace information from experiment
- CP violation: not started yet
- Corrections for previous talk: higher order than those included here