πK scattering lengths

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Outline

- Motivation
- Isospin odd scattering length
- Renormalization group equations
- Double chiral logs and log $\times L_i^r$ terms
- Numerics
- Outlook

Motivation

- πK atoms decay predominantly into $\pi^0 K^0$
- Decay width of the ground state

$$\Gamma = 8\alpha^3 m_r^2 p^* (a_0^-)^2 (1+\delta), \quad \delta = (4.0 \pm 2.2) \times 10^{-2}$$

Bilenky, Nguyen, Nemenov and Tkebuchava 69
Nehme, Talavera 02; Kubis, Meissner 02; J.S. 04

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  - p^* 3-momentum of the outgoing $\pi^0 K^0$ pair
  - δ isospin symmetry breaking corrections, of order α and $m_u - m_d$

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- Isospin odd scattering length a_0^- is defined at $m_u = m_d$, $\alpha = 0$ and $M_\pi \doteq M_{\pi^+}$, $M_K \doteq M_{K^+}$
- $\pi^- K^+ \to \pi^0 K^0$ amplitude at threshold

$$\mathrm{Re}\, A_{\mathsf{thr}}(\pi^- K^+ \to \pi^0 K^0)\,|_{m_u=m_d,\alpha=0} \propto \; a_0^-$$

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Isospin odd scattering length

- Current algebra result

$$a_0^- = \frac{M_K M_\pi}{8\pi F_\pi^2 (M_\pi + M_K)} = 0.071 M_\pi^{-1}$$

Weinberg, Phys. Rev. Lett. 17 (1966) 616

- Low energy theorem (Roessl)

$$a_0^- = \frac{M_K M_\pi}{8\pi F_\pi^2 (M_\pi + M_K)} \left\{ 1 + \frac{M_\pi^2}{F_\pi^2} c_0 + \mathcal{O}(M_\pi^4) \right\}$$

SU(2) χPT : $a_0^- = (0.077 \pm 0.003) M_\pi^{-1}$

Roessl, Nucl. Phys. B 555 (1999) 507

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Isospin odd scattering length

- Roy-Steiner evaluation

$$a_0^- = (0.090 \pm 0.005) M_\pi^{-1}$$

Büttiker, Descotes-Genon, Moussallam Eur. Phys. J. C 33 (2004) 409

- SU(3) χPT prediction
  - at order p^4

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Bernard, Kaiser, Meissner, Nucl. Phys. B 357 (1991) 129
Kubis, Meissner Phys. Lett. B 529 (2002) 69

  - at order p^6

$$a_0^- = 0.089 M_\pi^{-1}$$

Bijnens, Dhonte, Talavera, JHEP 0405 (2004) 036

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- SU(3) χPT prediction

$$a_0^- = 0.071(1 + 0.11 + 0.14 + \cdots) M_\pi^{-1}$$

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$\mathcal{O}(p^4)$

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- SU(3) χPT prediction

$$a_0^- = 0.071(1 + 0.11 + 0.14 + \cdots) M_\pi^{-1}$$

$\mathcal{O}(p^4)$ → 0.11 $\mathcal{O}(p^6)$ → 0.14

goal: to understand the rather large $\mathcal{O}(p^6)$ correction

Renormalization group equations

- πK amplitude at two loop

$$A_{\text{ph}} = A(\mu) + a_i C_i^r(\mu) + b_{ij} L_i^r(\mu) L_j^r(\mu)$$

Bijnens, Dhonte, Talavera, JHEP 0405 (2004) 036

$\Rightarrow$ aim: to extract double logs and log $\times L_i^r$

- Renormalization group equations (RGE)

$$\mu \frac{d}{d\mu} L_i^r(\mu) = -\frac{1}{(4\pi)^2}\Gamma_i, \quad \mu \frac{d}{d\mu} C_i^r(\mu) = \frac{1}{(4\pi)^2}\left[2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu)\right]$$

Bijnens, Colangelo, Ecker, Annals Phys. 280 (2000) 100

  - $\Gamma_i^{(L)}$ are linear combinations of L_i^r: $\mu \frac{d}{d\mu}\Gamma_i^{(L)}(\mu) = -\frac{\Gamma_i^{(2)}}{8\pi^2}$
  - Γ_i, $\Gamma_i^{(1)}$ and $\Gamma_i^{(2)}$ are pure numbers

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Renormalization group equations

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$$A_{\rm ph} = A(\mu) + a_i C_i^r(\mu) + b_{ij} L_i^r(\mu) L_j^r(\mu)$$

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$\Rightarrow$ aim: to extract double logs and log $\times L_i^r$

- Solutions of the RGE

$$\begin{aligned} L_i^r(\mu) &= L_i^r(\mu_0) - \frac{\Gamma_i}{(4\pi)^2} \ln \frac{\mu}{\mu_0} \\ C_i^r(\mu) &= C_i^r(\mu_0) - \frac{1}{(4\pi)^4} \Gamma_i^{(2)} \ln^2 \frac{\mu}{\mu_0} \\ &\quad + \frac{1}{(4\pi)^2} \left[2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_0) \right] \ln \frac{\mu}{\mu_0} \end{aligned}$$

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Double chiral logs and log $\times L_i^r$

- Expansion in powers of M_π^2

$$a_0^- = \frac{M_K M_\pi}{8\pi F_\pi^2 (M_\pi + M_K)} \left\{ 1 + \frac{M_\pi^2}{F_\pi^2} c_0 + \mathcal{O}(M_\pi^4) \right\}$$

  - we neglect $\mathcal{O}(M_\pi^4)$ terms in $\{\dots\}$
  - M_π, M_K and F_π physical masses and decay constant
  - c_0 starts at order p^0

$$c_0 = c_0^{(0)} + c_0^{(2)} + \cdots$$

- $c_0^{(0)}$ depends on L_5^r only

$$c_0^{(0)} = 8L_5^r(\mu) + \chi\text{logs}$$

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Double chiral logs and log $\times L_i^r$

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  - we neglect $\mathcal{O}(M_\pi^4)$ terms in $\{\dots\}$
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$$c_0 = c_0^{(0)} + c_0^{(2)} + \cdots$$

- Partial 2-loop corrections

$$c_0^{(2)} = c_0 \left.\right|_{L_i \times L_j} + c_0 \left.\right|_{C_i} + c_0 \left.\right|_{\text{log}^2} + c_0 \left.\right|_{\text{log} \times L_i} + \cdots$$

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Double chiral logs and log $\times L_i^r$

- $\mathcal{O}(p^6)$ coupling constants: $c_0\,|_{C_i} = -32M_K^2 C_1^r(\mu) + \cdots$
- Products of two p^4 constants

$$c_0\,|_{L_i\times L_j} = \frac{64M_K^2}{F_\pi^2} L_5^r(\mu)\left[2(L_4^r(\mu) - 2L_6^r(\mu)) - L_5^r(\mu)\right]$$

- Double chiral logs

$$c_0\,|_{\mathrm{log}^2} = \frac{37}{2}\frac{M_K^2}{(4\pi)^4F_\pi^2}\ln^2\frac{M_\chi}{\mu}$$

M_χ characteristic meson mass

- Log $\times L_i$ terms contain $L_1^r, \ldots, L_8^r$

$$c_0\,|_{\mathrm{log}\times L_i} = -112\frac{M_K^2}{(4\pi)^2F_\pi^2}\ln\frac{M_\chi}{\mu}L_1^r(\mu) + \cdots$$

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Numerics

- Preliminary results for the partial p^6 corrections

	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	log^2	log$\times L_i$	$L_i \times L_j$	C_i
$\frac{M_\pi^2}{F_\pi^2} c_0$	0.11	0.14	0.01	0.04	-0.02	0.07
$\frac{M_\pi^2}{F_\pi^2} c_0 \,\big\vert_{C_i = L_i = 0}$	0.10	0.05	0.01			

- full $\mathcal{O}(p^6)$ expressions
  Bijnens, Dhonte, Talavera, JHEP 0405 (2004) 036
- characteristic meson mass $M_\chi = M_K$
- $L_i^r(\mu)$ at $\mu = 0.77$ GeV are taken from fit 10
  Amoros, Bijnens and Talavera, Nucl. Phys. B 602 (2001) 87

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	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	$\log^2$	$\log\times L_i$	$L_i \times L_j$	C_i
$\frac{M_\pi^2}{F_\pi^2} c_0$	0.11	0.14	0.01	0.04	-0.02	0.07
$\frac{M_\pi^2}{F_\pi^2} c_0 \mid_{C_i=L_i=0}$	0.10	0.05	0.01		0.09 (sum of $\log\times L_i$, $L_i \times L_j$, C_i)	

- full $\mathcal{O}(p^6)$ expressions
  Bijnens, Dhonte, Talavera, JHEP 0405 (2004) 036
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Outlook

- Today, the πK amplitude is available at two loop order in the chiral expansion

  Bijnens, Dhonte, Talavera, JHEP 0405 (2004) 036

- So far, we extracted the 'low cost' terms for the isospin odd πK scattering length a_0^-
  - C_i^r terms
  - $L_i^r \times L_j^r$ terms
  - double chiral logs
  - log $\times L_i^r$ terms
- What remains to be done:
  - numerical analysis of the remaining part
  - ultimate goal: full $\mathcal{O}(p^6)$ contributions to c_0?

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