The A_5 and the pion field

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Outline of the talk

- Motivation: what is this about ?

- 5D Yang-Mills theory on an interval
- KK decomposition: selecting zero modes by BCs

- Pion field and chiral symmetry
- Pion decay constant
- L_i's at tree level

- Spectrum of vector and axial resonances
- Recovering the $\log$'s of pQCD: the metric

- Outlook: scalar fields in the bulk

Motivation

- EFTs/models of GBs + spin-1 resonances Ecker et al. '89
  - Constraint: reproduce soft HE behavior of QCD
- At tree level: automatic if resonances introduced as Ecker et al. '89
  - Antisymmetric tensors
  - Gauge fields getting mass via Higgs mechanism
    - In this case, should remain true at loop level Cornwall et al. '74
- Expansion: $1/N_c \implies$ want ∞ number of such resonances
  - Moose models (discretized 5th D) Son, Stephanov '03
  - 5D models: locality Pomarol et al. '03

- 5D Yang-Mills A_M on an interval tree level
  - Resonances = KKs of A_μ: 'ate' A_5 KKs via Higgs mechanisms
  - Choose BCs to get QCD-like spectrum: zero modes
    - One zero mode from the $A_5 \implies$ pion
    - No zero modes for vector fields

$SU(N_f) \times SU(N_f)$ Yang-Mills on an interval

Work in the chiral limit: do not consider (pseudo)-scalar densities

- Start with terms $\leqslant 2$ derivatives. Symmetric under $L \leftrightarrow R$

$$\mathcal{L} = -\frac{1}{2\,g_5^2} \int \mathrm{d}^4x \int_{L_0}^{L_1} \mathrm{d}z \, \sqrt{|\det g|}\, g^{MN} g^{RS} \langle L_{MR} L_{NS} + R_{MR} R_{NS} \rangle$$

Work in conformally flat coordinates $\mathrm{d}s^2 = w^2(z) \left(\eta_{\mu\nu}\, \mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right)$
With $w(L_0) > w(L_1)$, $z = L_0$ is the UV brane, $z = L_1$ the IR brane

<u>IR BCs</u>: symmetry-breaking

- $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_{L+R}$ à la 'Higgsless' Csáki et al. '03

$$(R_\mu - L_\mu)|_{L_1} = 0 \qquad \text{IR brane}$$

- Get rid of V_5 zero mode

$$(R_5 + L_5)|_{L_1} = 0 \qquad \text{IR brane}$$

<u>UV BCs</u>: get rid of massless vectors

$$R_\mu|_{L_0} = L_\mu|_{L_0} = 0 \qquad \text{UV brane}$$

SχSB: the pion field

- Because of UV BCs, transformations at $z = L_0$ must be x^μ-independent

$$\partial_\mu R(x, z = L_0) \;=\; \partial_\mu L(x, z = L_0) \;=\; 0$$

  - This is the global $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ chiral symmetry

$$g_R \;\equiv\; R(z = L_0)\,, \qquad g_L \;\equiv\; L(z = L_0)$$

- At $z = L_1$, x^μ-dependent vector transformations

$$h(x) \;\equiv\; R(x, z = L_1) \;=\; L(x, z = L_1)$$

still allowed if, in addition to previous BCs

$$\partial_5 (R_\mu + L_\mu)|_{L_1} \;=\; 0 \qquad\qquad \text{IR brane}$$

Construct $U \;\longmapsto\; g_R\, U\, g_L^\dagger$ using the Wilson line $\Gamma^R_{z_2, z_1}(x) \;\equiv\; \mathrm{P}\left\{ \mathrm{e}^{\mathrm{i}\int_{z_1}^{z_2} \mathrm{d}z\, R_5(x,z)} \right\}$

$$\xi_R \;\equiv\; \Gamma^R_{L_0, L_1} \;\longmapsto\; g_R\, \xi_R\, h^\dagger \qquad \xi_L \;\equiv\; \Gamma^L_{L_0, L_1} \;\longmapsto\; g_L\, \xi_L\, h^\dagger$$
$$U \;\equiv\; \xi_R\, \xi_L^\dagger$$

- Vacuum ($U = \mathbb{1}$) invariant under $g_R = g_L$ only: SχSB

BCs and sources

Local g_R, g_L transformations

$\Longrightarrow$ Gauge-invariant BCs at the UV brane $R_{5\mu}|_{L_0} = L_{5\mu}|_{L_0} = 0$

- With also $R_\mu(L_0)$ & $L_\mu(L_0) \neq 0$: sources!

$$\begin{aligned} \partial_5 R_\mu|_{L_0} &= \partial_5 L_\mu|_{L_0} = 0 \\ R_5|_{L_0} &= L_5|_{L_0} = 0 \end{aligned}$$

- Explicitly separate fields and sources using field redefinitions

$$R_\mu(z), L_\mu(z) \longmapsto R_\mu(L_0), L_\mu(L_0) \quad \& \quad V_\mu(z), A_\mu(z)$$

  - Where V_μ, A_μ obey original BCs (remember selection of modes)
  - Also trade R_5, L_5 for Wilson lines $\xi_R, \xi_L \Longrightarrow$ set $V_5(z) = A_5(z) = 0$

$$\begin{aligned} 2\,V_M(z) &\equiv \xi_R^\dagger \Big\{ \Gamma^R_{L_0,z} \{ \mathrm{i}\,\partial_M + R_M(z) \} \Gamma^R_{z,L_0} - R_M(L_0) \Big\} \xi_R + \{R \to L\} \\ 2\,A_M(z) &\equiv \xi_R^\dagger \Big\{ \Gamma^R_{L_0,z} \{ \mathrm{i}\,\partial_M + R_M(z) \} \Gamma^R_{z,L_0} - R_M(L_0) \Big\} \xi_R - \{R \to L\} \\ &+ \alpha(z)\, u_M \qquad \text{with } u_\mu = \mathrm{i} \left(\xi_R^\dagger D_\mu \xi_R - \xi_L^\dagger D_\mu \xi_L \right), u_5 = 0 \end{aligned}$$

$$V_M(x,z) \longmapsto h(x)\, V_M(x,z)\, h(x)^\dagger$$

- A_μ indeed vanishes on the UV & IR branes provided

$$\alpha(L_0) = 0 \qquad \alpha(L_1) = 1$$

The function $\alpha(z)$

- $\langle R_{5\mu} R_{5\nu} + L_{5\mu} L_{5\nu} \rangle \implies \partial_\mu \pi - A_\mu$ mixing
  - Vanishes provided we choose $\alpha(z)$ properly

$$\partial_5 \left(\sqrt{g}\, g^{\mu\nu} g^{55} \partial_5\right) \alpha \;=\; 0 \qquad \text{(will correspond to massless KK)}$$

- The same $\langle R_{5\mu} R_{5\nu} + L_{5\mu} L_{5\nu} \rangle$ term yields the GB $\mathcal{O}(p^2)$ term, with

$$\eta^{\mu\nu} \frac{f_0^2}{4} \langle u_\mu u_\nu \rangle \;=\; -\frac{2}{g_5^2} \langle u_\mu u_\nu \rangle \int_{L_0}^{L_1} \mathrm{d}z \sqrt{g}\, g^{\mu\nu} g^{55} \left(\partial_5 \alpha\right)^2$$

- To determine the tree-level contribution of resonances to $\mathcal{O}(p^4)$
  - Set $V_\mu(z) = A_\mu(z) = 0$ but maintain sources $R_\mu(L_0)\,\&\,L_\mu(L_0) \neq 0$
  - Find, for L_{10}
    (Analogue for moose in
    Hirn, Stern '04; Casalbuoni et al. '04; Georgi '04)

$$\eta^{\mu\nu} \eta^{\rho\sigma} L_{10} \;=\; -\frac{1}{g_5^2} \int_{L_0}^{L_1} \mathrm{d}z \sqrt{g}\, g^{\mu\nu} g^{\rho\sigma} \alpha \left(1 - \alpha/2\right)$$

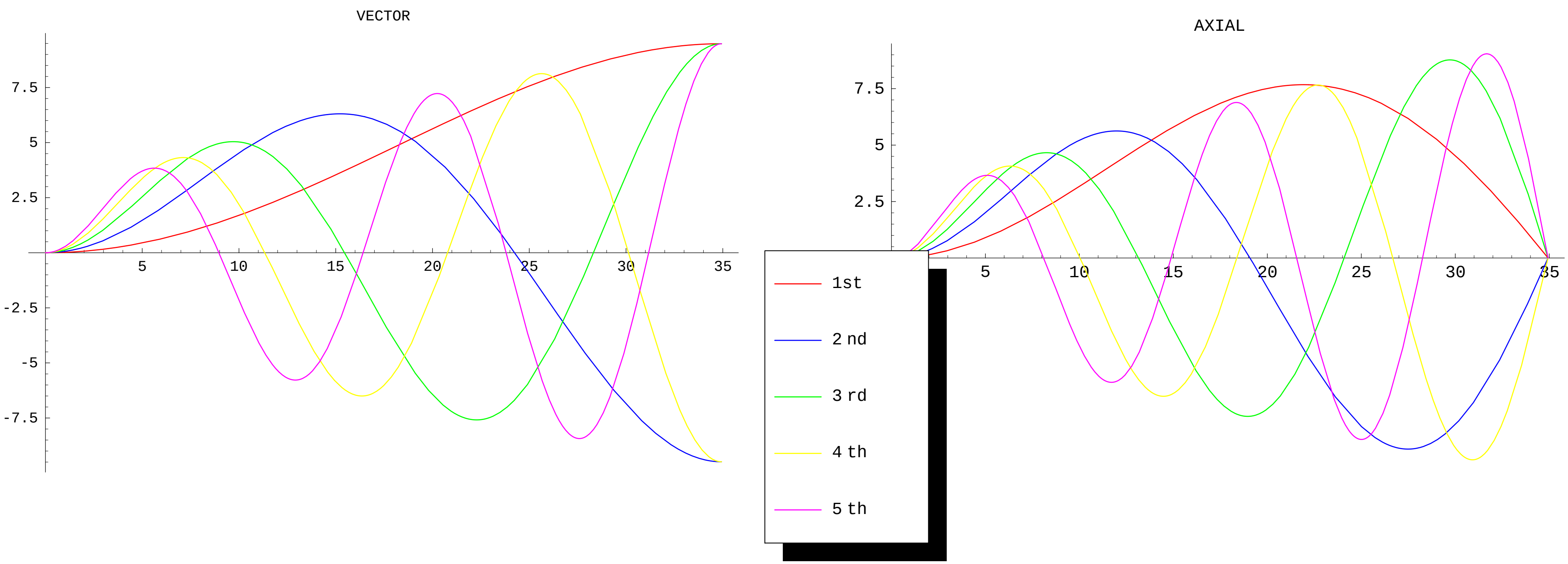
  - Other L_i's: relations

$$L_2 \;=\; 2\, L_1 \qquad\qquad L_3 \;=\; -6\, L_1 \qquad\qquad L_{10} \;=\; -L_9$$

Vector resonances: AdS$_5$ background

$w(z) = L_0/z$

- Resonances are the KKs of $V_\mu(z)$, $A_\mu(z)$
  - In Randall-Sundrum, their wave-functions are Bessel functions



- Masses are given by

$$\eta^{\mu\nu}\,\partial_5\left(\sqrt{g}\;g^{\rho\sigma}\,g^{55}\,\partial_5\right)\varphi_n(z) \;=\; \sqrt{g}\;g^{\mu\nu}\,g^{\rho\sigma}\,M_n^2\,\varphi_n(z)$$

- Approximately, zeroes of Bessel functions $\Longrightarrow$ For $M_n^{V,A} \ll 1/L_0$

$$M_n^{V,A} \;\simeq\; (n \mp 1/4)\,\frac{\pi}{L_1}$$

  - $F_{Vn}^2 \sim n \;\Longrightarrow\;$ should get $\Pi_V(Q^2) \underset{Q^2 \gg M_\rho^2}{\propto} \ln Q^2$

Numbers

Send UV regulator $1/L_0 \longrightarrow +\infty$: two parameters, as QCD! (but see below)

- Use low-energy quantities: $M_\rho = 776\,\mathrm{MeV}$ and $f_0 = 87\,\mathrm{MeV}$

$$\begin{aligned} L_1 &\simeq 2.4/M_\rho \\ \frac{L_0}{g_5^2} &\simeq L_1^2 f_0^2/4 \end{aligned}$$

- Plug back into the model (for more recent L_i fits, see Bijnens et al.)

	Model	Experiment
$10^3\,(2\,L_1 - L_2)(\mu = M_\rho?)$	0	-0.6 ± 0.6
$10^3\,L_2(\mu = M_\rho?)$	1.1	1.4 ± 0.3
$10^3\,L_3(\mu = M_\rho?)$	-3.1	-3.5 ± 1.1
$10^3\,L_9(\mu = M_\rho?)$	6.8	6.9 ± 0.7
$10^3\,L_{10}(\mu = M_\rho?)$	-6.8	-5.5 ± 0.7

	Model	Experiment
M_{a_1}(GeV)	1.2	1.230 ± 0.040
$M_{\rho'}$(GeV)	1.8	1.465 ± 0.025
$M_{\rho''}$(GeV)	2.8	1.720 ± 0.020

log's, or 'why Randall-Sundrum again ?'

Apart from the virtue of being analytically computable ?

- Compute using 5D propagator from UV brane and back
  - For $Q^2 \gg 1/L_1^2 \sim M_\rho^2$ but $Q^2 L_0^2 \ll 1$

$$\Pi_V(Q^2) \sim -\frac{L_0}{g_5^2} \ln\left(Q^2 L_0^2\right) + \mathrm{cst} + \mathcal{O}(\mathrm{e}^{-QL_1})$$

  - $\Longrightarrow$ No $1/Q^{2n}$ terms $\Longrightarrow$ no condensates in the OPE

- Presence of log depends only on metric near $z = L_0$

Son, Stephanov '03

  - Away from UV brane, need not be AdS_5. Then
    - Other terms in Π_V
    - Light resonances: $M_n^2 \not\propto n^2$

Locality:

- e^{-QL_1}: effect of the IR brane (suppressed at HE)
- Since BCs $\neq$ for A & for V
  - At HE $\Pi_{LR} = \Pi_V - \Pi_A \propto \mathrm{e}^{-QL_1} \Longrightarrow$ ∞ number of WSRs

Outlook: SχSB without local order parameters ?

(Non-local) order parameters $\Longrightarrow$ SχSB

$$\lim_{Q^2 \longrightarrow +\infty} Q^2 \, \Pi_{LR}(Q^2) \;=\; -\, f_0^2 \qquad\qquad \& \qquad\qquad L_i\text{'s}$$

Local order parameters:

- OPE for Π_{LR} Shifman, Vainshtein, Zakharov '79

$$\Pi_{LR}(Q^2) \underset{Q^2 \longrightarrow +\infty}{\sim} \sum_n c_n \frac{\langle \mathcal{O} \rangle^{(2n)}}{(Q^2)^n}$$

In our model, WSRs follow from locality in 5th D
Barbieri, Pomarol, Rattazzi '03; Hirn, Stern '04

- This is not QCD: $\langle \mathcal{O} \rangle^{(6)} \propto \langle \bar{q} q \rangle^2 \neq 0$ Knecht, de Rafael '98
  - Tell the model about quarks (condensing) $\Longrightarrow$ bulk spin-0 fields
    - $\Longrightarrow$ Introduces (pseudo-)scalar resonances

Differences with recent models

Erlich, Katz, Son, Stephanov '05; Da Rold, Pomarol '05

- Different BCs for gauge fields on the IR brane (where SχSB 'occurs')
- $\Longrightarrow$ Cannot describe SχSB without local order parameters
  - In QCD, we indeed have $\langle \bar{q} q \rangle \neq 0 \Longrightarrow$ Theoretical question

In our case:

- Pion as the A_5: essential for first WSR

$$\sum F_{Vn}^2 - \sum F_{An}^2 \;=\; f_0^2$$

  - π comes from the A_5 $\Longrightarrow$ related to resonances
- Chiral symmetry explicit. Recover $\mathcal{O}(p^4)$ from full tower of resonances

Note:

- Using our result for the $\ln Q^2$ in Π_V, we get, with QCD data

$$N_c \;\simeq\; 2.2$$

This shouldn't be taken too seriously, since

  - We still have to include scalar fields and destroy GWSRs
    - This will modify $\alpha(z)$ and hence the LECs: f_π & L_i's

Also, introduce condensates in Π_V, not only in $\Pi_V - \Pi_A$: different background ?