VNIVERSITAT DE VALÈNCIA

CSIC

Three-point Green Functions in the resonance region: LEC's

Jorge Portolés

Instituto de Física Corpuscular

CSIC-UVEG, Valencia (Spain)

Euridice Network

Summary

- LEC's in Chiral Perturbation Theory :

  how do we get them?
- The role of Resonance Chiral Theory
- Three-point Green Functions of QCD currents:
  - Resonance Chiral Theory approach
  - Meromorphic Function approach
- Conclusions

Task force : Vincenzo Cirigliano, Gerhard Ecker, Markus Eidemüller,
Roland Kaiser, Antonio Pich, J.P.

LEC's in Chiral Perturbation Theory

$$L_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \left\langle u_\mu u^\mu + \chi_+ \right\rangle$$

($N_F = 3$)

Even-intrinsic-parity sector only

$$\mathcal{L}^{(4)} = L_1 \left\langle u_\mu u^\mu \right\rangle^2 + \ldots = \sum_{i=1}^{10} L_i \; O_i^4$$

[Gasser & Leutwyler, 1985]

$$\mathcal{L}^{(6)} = C_1 \left\langle u_\alpha u^\alpha h_{\mu\nu} h^{\mu\nu} \right\rangle + \ldots = \sum_{i=1}^{90} C_i \; O_i^6$$

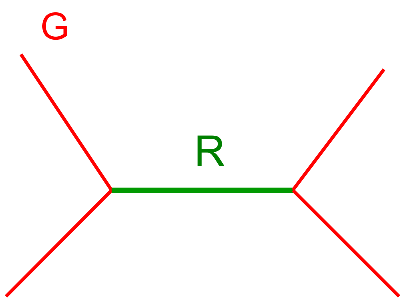
[Fearing & Scherer, 1996]

[Bijnens, Colangelo & Ecker, 1999,2000]

High-precision predictions within χPT : $F^\pi_{V,S}$, K_{l3}, … → L_i , C_i ?

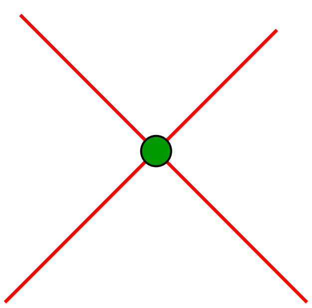
How do we get them?

[Gasser & Leutwyler, 1984] [Donoghue et al., 1989] [Ecker et al., 1989]



$$\int [dR]\ \exp\left[i \int L_{R\chi T}\left(R, U; \lambda_j\right)\right] = \exp\left[i \int L_{\text{No Local}}\left(U; \lambda_j\right)\right]$$

$q^2 \ll M_R^2$



$$L_{Local}\left(U; \lambda_j\right) = L_{Local}\left(U, L_i, C_i\right)$$

The role of Resonance Chiral Theory

$O(p^4)$ [Ecker et al, 1989]

$$L_{R\chi T} = \mathcal{L}^{(2)} + L_{\text{kin}}^{R} + L_{\text{int}}^{1}(s=0,1)$$

$$L_{\text{int}}^{1}(s=1) = \frac{F_V}{2\sqrt{2}}\left\langle V_{\mu\nu} f_{+}^{\mu\nu}\right\rangle + i\,\frac{G_V}{\sqrt{2}}\left\langle V_{\mu\nu} u^{\mu} u^{\nu}\right\rangle + \frac{F_A}{2\sqrt{2}}\left\langle A_{\mu\nu} f_{-}^{\mu\nu}\right\rangle$$

$$L_{\text{int}}^{1}(s=0) = c_d \left\langle S u_{\mu} u^{\mu}\right\rangle + c_m \left\langle S \chi_{+}\right\rangle + i d_m \left\langle P \chi_{-}\right\rangle$$

L_1	L_2	L_3	L_4	L_5	....
$\frac{G_V^2}{8M_V^2} - \frac{c_d^2}{6M_S^2}$	$\frac{G_V^2}{4M_V^2}$	$-\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2}$	$-\frac{c_d c_m}{3M_S^2}$	$\frac{c_d c_m}{M_S^2}$	

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$$L_{\text{int}} \sim \langle R\ \chi(p^2) \rangle$$

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$O(p^6)$ [Cirigliano et al, 2005]

$$L_{\text{int}} = L^1_{\text{int}} + L^2_{\text{int}}$$

R = V,A,S,P

$$L^2_{\text{int}} \begin{cases} \langle R\ \chi(p^4)\rangle \longrightarrow \boxed{\text{70 operators}} \longrightarrow \lambda_i^R \\ \langle R_1 R_2\ \chi(p^2)\rangle \longrightarrow \boxed{\text{38 operators}} \longrightarrow \lambda_i^{R_1 R_2} \\ \langle R_1 R_2 R_3\ \chi(p^0)\rangle \longrightarrow \boxed{\text{7 operators}} \longrightarrow \lambda_i^{R_1 R_2 R_3} \end{cases}$$

e.g.

$$C_{12}^{R} = -\frac{c_d c_m}{2M_S^4}$$

$$C_{34}^{R} = \frac{1}{2}\left[\frac{c_m(c_m - c_d)}{M_S^4} + \frac{d_m^2}{M_P^4} + 2\frac{d_m}{M_P^2}\lambda_6^{P} - 2\frac{c_m}{M_S^2}\lambda_8^{S} + 2\frac{c_d c_m}{M_S^2 M_P^2}\lambda_1^{SP}\right]$$

$$C_{88}^{R} = -\frac{F_V G_V}{4M_V^4} + C_{90}^{R}$$

$$C_{90}^{R} = -\frac{d_m}{M_V^2 M_P^2}\left[\frac{F_V}{\sqrt{2}}\lambda_1^{PV} + M_V^2\lambda_9^{P}\right]$$

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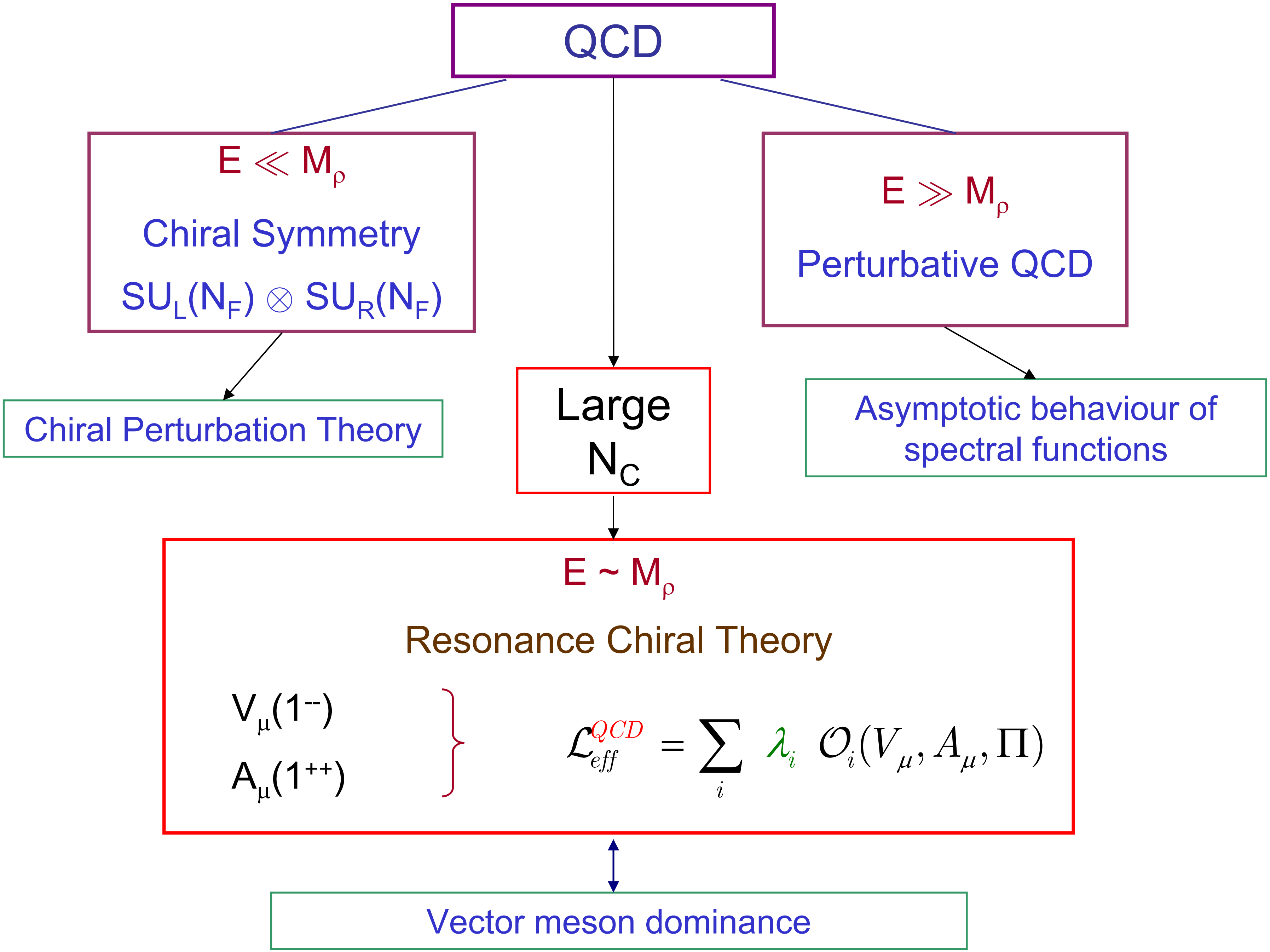
$L_{R\chi T}$ is NOT QCD for arbitrary values of the couplings


QCD
$E \ll M_\rho$
Chiral Symmetry
$SU_L(N_F) \otimes SU_R(N_F)$
Chiral Perturbation Theory
Large N_C
$E \gg M_\rho$
Perturbative QCD
Asymptotic behaviour of spectral functions
$E \sim M_\rho$
Resonance Chiral Theory
$V_\mu(1^{--})$
$A_\mu(1^{++})$
$\mathcal{L}_{eff}^{QCD} = \sum_i \lambda_i \, \mathcal{O}_i(V_\mu, A_\mu, \Pi)$
Vector meson dominance

What QCD tells us on Green Functions of QCD currents ?

1) Low energy expansion of Green Functions (Ward Id.)
2) Large N_C limit of QCD:
   - Infinite tower of non-decaying hadronic states
   - Description through a meromorphic function
3) Operator Product Expansion (OPE) at high energies
4) Brodsky-Lepage behaviour of form factors of hadron currents : $F_H \sim \frac{1}{Q^2} \quad , \; Q^2 \to \infty$

[Ecker, Gasser et al, 1989]

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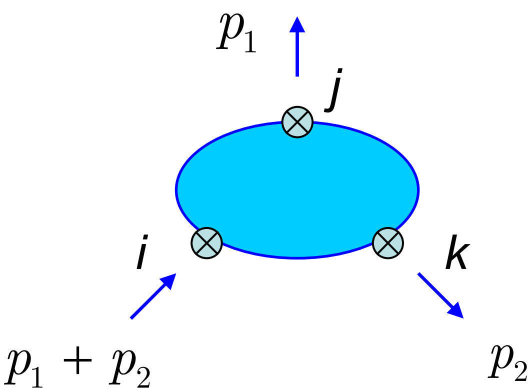
[Bijnens et al, 2003]

Three-point Green Functions of QCD currents

[Bijnens & Prades, 1994] [Moussallam, 1997] [Peris et al, 1998]

[Knecht & Nyffeler, 2001] [Ruiz-Femenía et al, 2003] [Bijnens et al, 2003]

$$\Pi_{123}^{ijk}(p_1,p_2) = i^2 \int d^4x d^4y \; e^{i(p_1\cdot x + p_2\cdot y)} \langle 0 | T\left\{ \left(\overline{\psi}\Gamma_1 \frac{\lambda^i}{2}\psi \right)(0) \left(\overline{\psi}\Gamma_2 \frac{\lambda^j}{2}\psi \right)(x) \left(\overline{\psi}\Gamma_3 \frac{\lambda^k}{2}\psi \right)(y) \right\} |0\rangle$$



$$\lim_{\lambda\to\infty} \Pi_{123}^{ijk}(\lambda p_1, \lambda p_2)$$

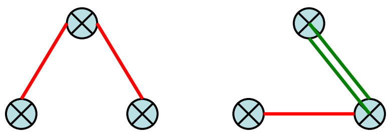
$$\lim_{\lambda\to\infty} \Pi_{123}^{ijk}(\lambda p_1, p_2)$$

$$\lim_{\lambda\to\infty} \Pi_{123}^{ijk}(p_1, \lambda p_2)$$

$$\lim_{\lambda\to\infty} \Pi_{123}^{ijk}(\lambda p_1, (1-\lambda)p_1 + p_2)$$

OPE + 1/λ expansion

1) Resonance Chiral Theory approach



$$\Pi_{123}^{ijk}\left(p_1, p_2\right)\Big|_{R\chi T} \xrightarrow{p_1, p_2 \gg M_R} \Pi_{123}^{ijk}\left(p_1, p_2\right)\Big|_{OPE}$$

…

Direct information on the couplings of $L_{R\chi T}$

- Hadronic decays of the tau lepton, decays of resonances,
- Hadronic cross-section, etc.

2) Meromorphic Function approach (<SPP>)

$$\Pi_{SPP}^{ijk} = d^{ijk}\mathcal{N}_{\chi}\frac{P_0 + P_1 + P_2 + P_3 + P_4}{[M_S^2 - s][-t][-u][M_P^2 - t][M_P^2 - u]} \xrightarrow{s,t,u \gg M_R^2} \Pi_{SPP}^{ijk}\Big|_{OPE}$$

Large N_C

$$P_n = \sum_{k=0}^{n}\sum_{l=0}^{k} c_{n-k,k-l,l}\; s^{n-k}t^{k-l}u^{l}$$

$$s = p_1^2,\;\; t = p_2^2,\;\; u = (p_1 + p_2)^2$$

On the constraints from Form Factors

Brodsky-Lepage behaviour of two-body Form Factors of Hadron Currents

$$F_H \sim \frac{1}{Q^2} \quad , \quad Q^2 \to \infty$$

3-point Green Functions + $\langle \pi, \gamma | \mathcal{J}_{QCD} | \pi \rangle$ ✓

3-point Green Functions + $\left\{ \begin{array}{l} \langle \pi | \mathcal{J}_{QCD} | R \rangle \\ \\ \langle R | \mathcal{J}_{QCD} | R \rangle \end{array} \right\}$ Not always !

↑

[Bijnens et al, 2003]

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Does phenomenology support this procedure?

E.g.

1) [Ruiz-Femenía et al, 2003] <VVP>

$$\Gamma(\omega\to\pi\gamma) = \frac{\alpha}{192} M_\omega \left(1-\frac{m_\pi^2}{M_\omega^2}\right)^3 \left[\frac{N_C}{4\pi^2}\frac{M_\omega^2}{F^2} - \frac{M_\omega^2}{M_V^2}\left(1+\frac{m_\pi^2}{M_\omega^2}\right)\right]^2$$

$$\Gamma(\omega\to\pi\gamma)\big|_{th} = (0.703 - 0.524)\ \mathrm{MeV} \quad [F = 87 - 92.4\ \mathrm{MeV}]$$

$$\Gamma(\omega\to\pi\gamma)\big|_{\exp} = (0.734 \pm 0.035)\ \mathrm{MeV}$$

2) [Cirigliano et al, 2004] <VAP>

$$\Gamma\left(a_1(1260) \rightarrow \pi\gamma\right) = \frac{\alpha}{24} M_A \left(\frac{M_A^2}{M_V^2} - 1\right)^3 \left(1 - \frac{m_\pi^2}{M_A^2}\right)^3$$

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[Moussallam, 1997]

	[Cirigliano et al, 2004]	[Moussallam, 1997]	Experiment
$\Gamma(a_1\rightarrow\pi\gamma)$ (MeV)	1.02	0.02	0.640 ± 0.246

$M_V = 0.77$ GeV , $M_A = 1.2$ GeV

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One multiplet of pseudoescalar resonances

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Results

<VAP> [Cirigliano et al, 2004]

$$C_{78}^R = \frac{F^2}{8M_V^4M_A^2}\left(3M_A^2+4M_V^2\right) - \frac{F^2}{16M_V^2M_P^2}$$

$$C_{82}^R = -\frac{F^2}{32M_V^4M_A^2}\left(4M_A^2+5M_V^2\right) - \frac{F^2}{32M_A^2M_P^2}$$

$\Big\}\ \pi \to \ell \nu_\ell \gamma$

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$$C_{90}^R = \frac{F^2}{8M_V^2M_P^2}$$

$\Big\}\ F_V^\pi(q^2)$, $K_{\ell 3}$

also C_{87}^R , C_{89}^R

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$F_V^\pi(q^2)$

$K_{\ell 3}$

ROBUST!

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$$C_{12}^{R} = -\frac{F^2}{8M_S^4}$$

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$$C_{38}^{R} = \frac{F^2}{16M_S^4} + \frac{F^2}{16}\left(\frac{1}{M_S^4} - \frac{1}{M_P^4}\right)$$

$K_{\ell 3}$

$f_+^{K^0\pi^-}(0)\Big|_{O(p^6)}$

All of these are

ROBUST !!

Conclusions

LEC's in χPT

Couplings λ_i in the Resonance Chiral Theory

- High-precision χPT predictions ($O(p^6)$)
- Applications of Resonance Chiral Theory
  - Hadronic cross-section
  - Resonance decays
  - Hadronic decays of the tau lepton
  - $(g\text{-}2)_\mu$......

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Procedure

Green Functions of QCD currents ⟷ Asymptotic constraints of QCD

<VAP>,<SPP>,….(still a lot to do!) and phenomenologically supported