RG evolution
of the MSSM with MFV

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Emanuel Nikolidakis

University of Bern

Aim

We are studying the influence of a supersymmetric extension of the SM on low-energy physics and are considering in this framework models with Minimal Flavour Violation (MFV) Ciuchini et al., Nucl. Phys. B534 (1998) 3

One part of this work is the analysis of the RG evolution of the parameters describing the model.

Work in progress with G. Colangelo, E. Lunghi and W. Porod.

Overview

- Supersymmetry and the Minimal Supersymmetric SM (MSSM)
  - Positive and Negative Aspects
- Minimal Flavour Violation (MFV)
  - Addressing the Flavour Problem in Low-Energy SUSY
  - Concept and Implementation
- RGE
  - Motivation and Use
  - Assumptions and Calculation
  - Results
- Summary
  - Summary and Outlook

We consider a minimal extension of the SM with softly broken Supersymmetry, the MSSM.

- Lagrangean: $\mathcal{L} = \mathcal{L}_{SUSY-gauge} + \mathcal{L}_{soft}$

$$
\begin{aligned}
\mathcal{L}_{soft} \quad = \quad & -(\text{gaugino masses}) \\
& -(\tilde{u}\mathbf{a_u^\dagger}\tilde{Q}H_u - \tilde{d}\mathbf{a_d^\dagger}\tilde{Q}H_d - \tilde{e}\mathbf{a_e^\dagger}\tilde{L}H_d) + c.c. \\
& -\tilde{Q}^\dagger\mathbf{m_Q^2}\tilde{Q} - \tilde{L}^\dagger\mathbf{m_L^2}\tilde{L} - \tilde{u}\mathbf{m_u^2}\tilde{u}^\dagger - \tilde{d}\mathbf{m_d^2}\tilde{d}^\dagger \\
& -\tilde{e}\mathbf{m_e^2}\tilde{e}^\dagger \\
& -(\text{Higgs masses})
\end{aligned}
$$

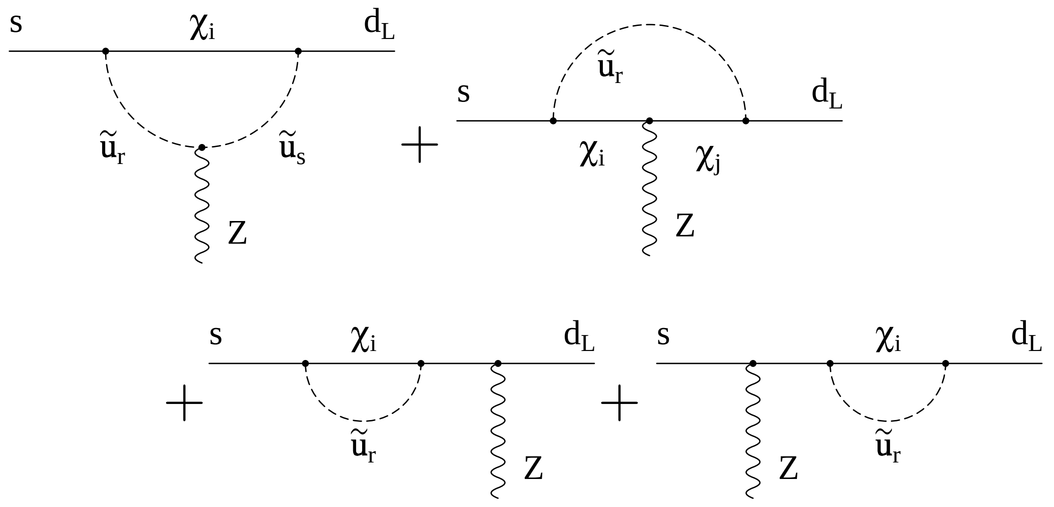
- Positive aspects:
  - Supersymmetry provides for a solution of the Hierarchy-Problem (if the SUSY-breaking scale is not much larger than a TeV).

Supersymmetry

- More positive aspects:
  - The gauge couplings unify at some high scale.
  - The LSP is a good Dark Matter candidate.
- Negative aspects:
  - A large number of new parameters due to the parametrization of the unknown mechanism of SUSY-breaking.
  - Flavour: Unconstrained SUSY breaking terms in the Lagrangean are potential sources of large Flavour Violation which is conflict with phenomenology.

An Example

Contributions to the effective Zsd-Vertex from Squark-Chargino-loops:



– Mass Eigenstates $\chi_{i,j}$ and $\tilde{u}_{r,s}$. The Vertices contain the matrices needed to diagonalize the mass matrices.

– Evaluation in the limit of zero external momenta.

– Divergences in the sum cancel.

Solutions to the Flavour Problem

The Parameterspace of the MSSM has to be constrained in order to agree with phenomenology. Assumptions about how this can be done:

- Universality: Mass matrices proportional to the unit matrix, trilinear couplings proportional to the Yukawas, no new phases. Minimal SUGRA.
- MFV: The new terms are chosen in such a way, that the CKM-matrix remains the only source of flavour violation. Ciuchini et al., Nucl. Phys. B534 (1998) 3

We will follow the second proposal.

Minimal Flavour Violation

We use an implementation of MFV introduced by D' Ambrosio, Giudice, Isidori and Strumia (Nucl. Phys. B645 (2002) 155). They identify MFV-allowed terms by means of a symmetry:

- The largest Group of unitary (MSSM-) field transformations that commutes with the gauge group is $U(3)^5$. Chivukula and Georgi, Phys. Lett. B 188 (1987) 99

  It can be decomposed as:

$$G_F \equiv U(3)^5 = SU(3)_q^3 \times SU(3)_l^2 \times U(1)^5$$

with

$$\begin{aligned} SU(3)_q^3 &= SU(3)_Q \times SU(3)_u \times SU(3)_d \\ SU(3)_l^2 &= SU(3)_L \times SU(3)_e. \end{aligned}$$

Minimal Flavour Violation

- Introduce auxiliary fields Y_u, Y_d and Y_e which transform under $SU(3)_q^3 \times SU(3)_l^2$ as

$$(3, \bar{3}, 1)_{SU(3)_q^3}, \quad (3, 1, \bar{3})_{SU(3)_q^3}, \quad (3, \bar{3})_{SU(3)_l^2},$$

respectively.

- Build new operators respecting G_F with the help of the Y.

- Identify the Y with the Yukawas.

- One can redefine the fields using $SU(3)_q^3 \times SU(3)_l^2$ such that

$$Y_d = \lambda_d, \quad Y_l = \lambda_l \text{ and } Y_u = V^\dagger \lambda_u,$$

with diagonal λ's and V being the CKM-matrix.

Minimal Flavour Violation

Following this approach, one writes the soft square masses and trilinear couplings as

D' Ambrosio et al., Nucl. Phys. B645 (2002) 155

$$\begin{aligned}
\mathbf{m_Q^2} &= m^2(a_1\mathbf{1} + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger \\
&\quad + b_3 Y_d Y_d^\dagger Y_u Y_u^\dagger + b_4 Y_u Y_u^\dagger Y_d Y_d^\dagger), \\
\mathbf{m_u^2} &= m^2(a_2\mathbf{1} + b_5 Y_u^\dagger Y_u), \\
\mathbf{m_d^2} &= m^2(a_3\mathbf{1} + b_6 Y_d^\dagger Y_d), \\
\mathbf{a_u} &= a(a_4\mathbf{1} + b_7 Y_d Y_d^\dagger) Y_u, \\
\mathbf{a_d} &= a(a_5\mathbf{1} + b_8 Y_u Y_u^\dagger) Y_d,
\end{aligned}$$

where m^2 and a define the scale and the a_i and b_i form a new set of parameters describing the soft breaking in the squark sector. Higher-order terms of the first two families can be neglected: The above equations are complete.

RGE: Motivation

- Relate the low-energy model to things that occur at the unification scale.
- Relate different energy scales below the SUSY-breaking scale.
- It is possible to rewrite the RG equations for the parameters a_i and b_i.$\longrightarrow$ The model respects MFV at every scale.
- W. Porod, SPheno (Comput. Phys.Commun. 153:275 (2003)): FV RGE's now included.

RGE: Derivation

The basis of the calculation is the article of Martin and Vaughn (Phys. Rev. D50 (1994) 2282) on the two-loop RGE's in the MSSM.

- We assume that the relevant flavour-changing structures can be written in terms of the following four matrices ($V = V_{CKM}$):

$$M_1 = V_{3i}^* V_{3j}, \qquad M_2 = V_{3i}^* \delta_{3j},$$
$$M_3 = \delta_{3i} V_{3j}, \qquad M_4 = \delta_{3i}\delta_{3j}.$$

- The product of two M_i's is again an M_i (times a factor ≈ 1).

- The Yukawas, as defined above, can then approximately be written as

$$Y_d \approx y_b M_4 \qquad Y_e \approx y_e M_4 \qquad Y_u \approx y_t M_2.$$

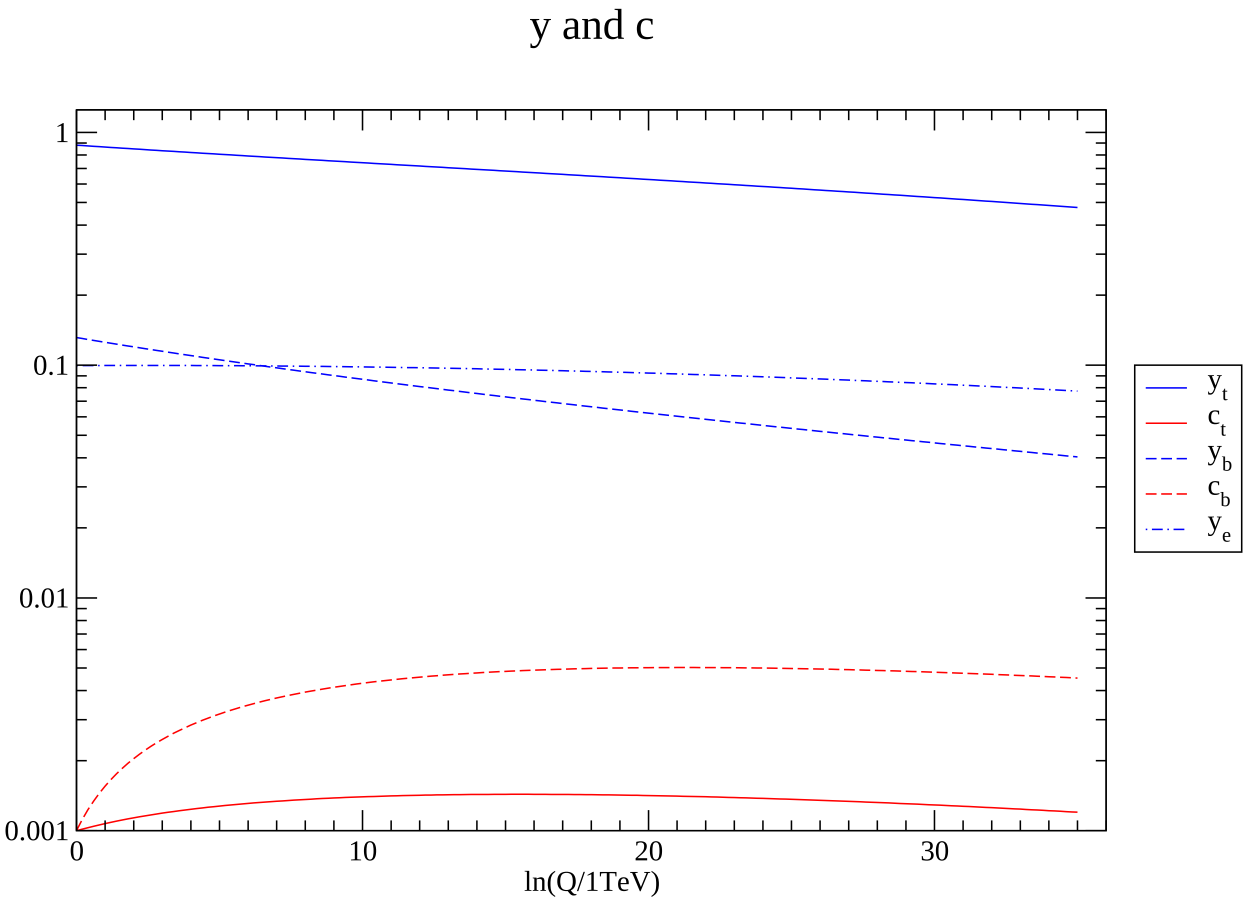
RGE: Derivation

- The set y_u, y_b and y_τ is not RGE-invariant. The β-function of Y_d contains also Y_u and vice versa.

  $\longrightarrow$ Add a correction:

$$Y_d \approx y_b M_4 + c_b M_2 \qquad Y_u \approx y_t M_2 + c_t M_4.$$

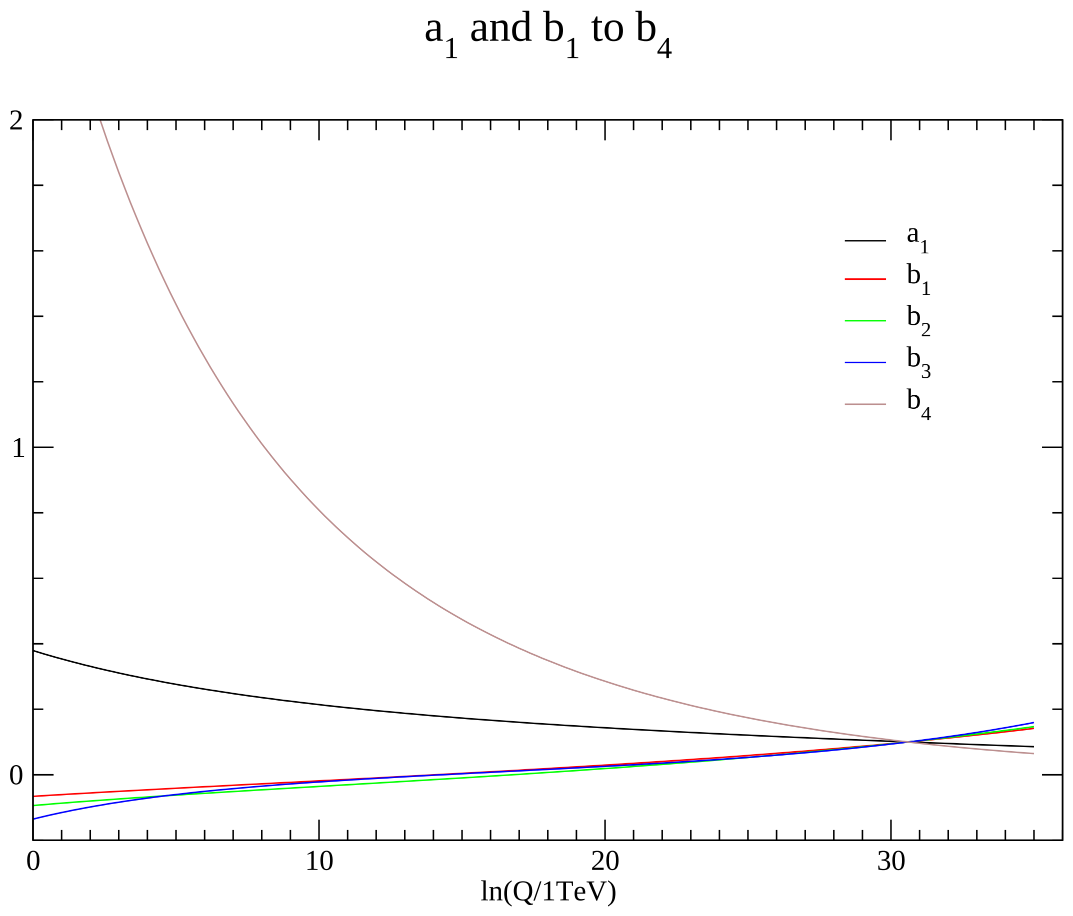
- The resulting set of the y_i and the c_i is RGE-invariant and evolves together with the gauge couplings independently from the other parameters.
- Substituting the Y in the β-functions and projecting out the coefficients of the respective M yields the β's for the y and c.
- Set the $c_i = 0$ at low energy.
- Small c_i remain small.


y and c
1
0.1
0.01
0.001
0
10
20
30
ln(Q/1TeV)
y_t
c_t
y_b
c_b
y_e

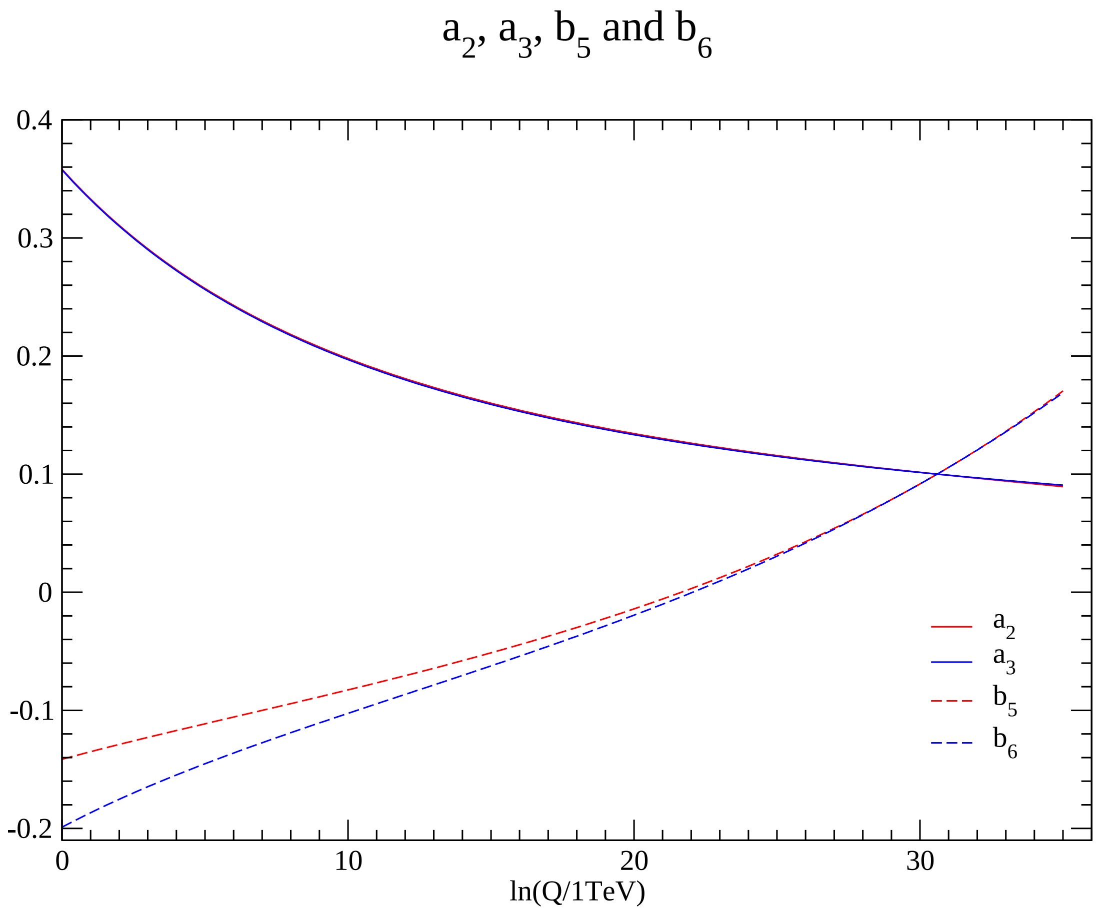
RGE: Derivation

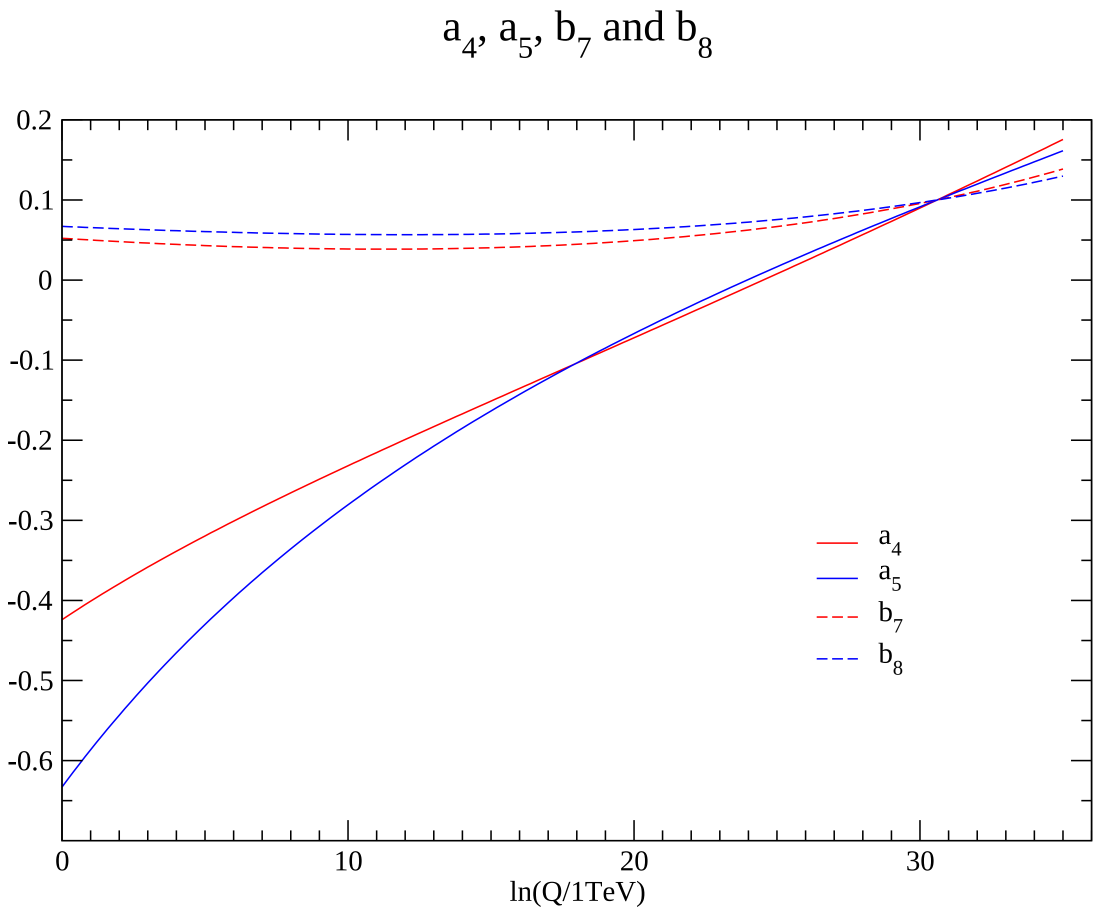
- The RGE's for the a_i and the b_i are obtained in a similar way:
  - Differentiate the equations on page 9 and replace the MSSM-parameters.
  - Project out the coefficient of the *combination* of M's which comes with the respective a' or b'.
- In the following plots the parameters are tuned to meet at the unification scale.
- Setting the b's to zero at some scale does not prevent them from being driven to nonzero values by the RGE's: Universality ($b_i = 0$) does not survive RG evolution.
- Example:

$$\begin{aligned} \beta^1_{b1} &= b_1(13/15 g_1^2 + 3 g_2^2 + 16/3 g_3^2) + \cdots \\ &\quad \cdots + a_1 + 2a_2 + 2a_4^2 + \cdots \end{aligned}$$


a1 and b1 to b4
2
1
0
0
10
20
30
ln(Q/1TeV)
a1
b1
b2
b3
b4

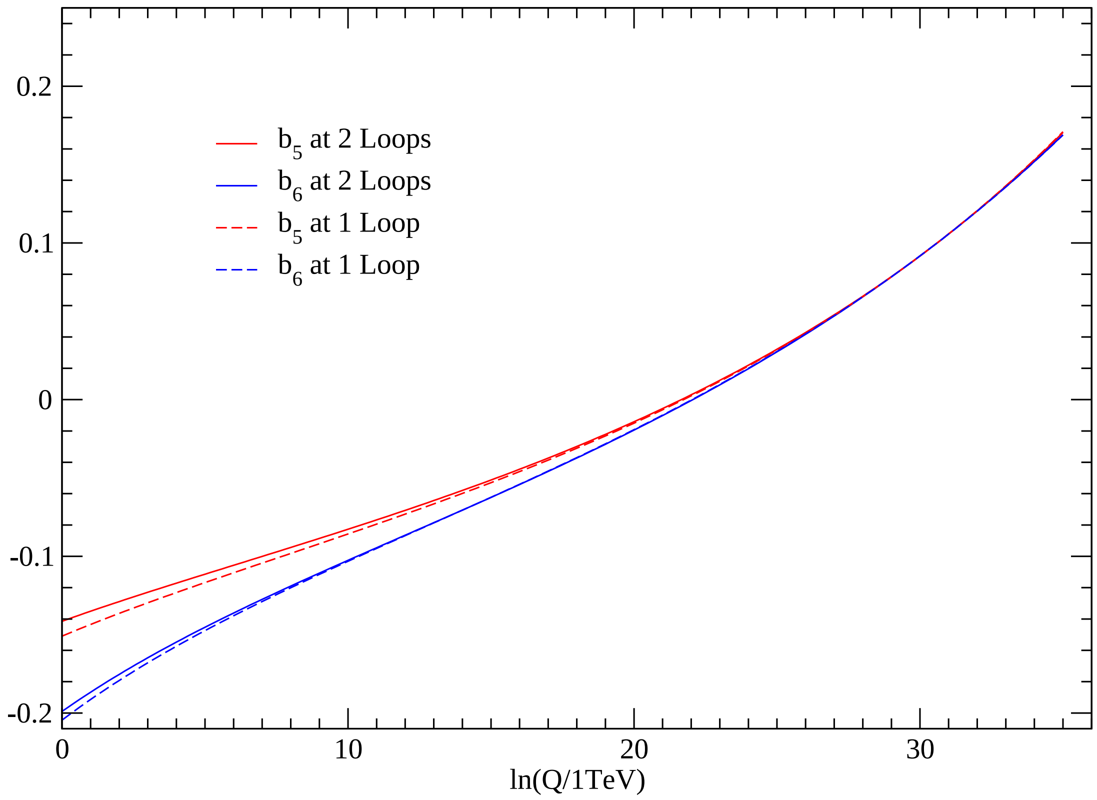
RGE




a4, a5, b7 and b8
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
10
20
30
ln(Q/1TeV)
a4
a5
b7
b8

RGE

1 Loop vs. 2 Loops



Summary

- I presented some relevant topics of the MSSM and MFV.
- I described the derivation of the RGE's for the parameters a and b which define FV.
- Outlook: Derivation of low-energy results and comparison with phenomenology in order to constrain the parameter space of the MSSM in the context of MFV.
- The Flavour problem can be evaded too in supersymmetric models with very heavy scalars, often called "Split" SUSY. In such a model for instance the FCNC in the example above is suppressed by the large mass of the particles in the loop.