Open problems in g-2 and related topics.

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Outline of Talk:

① **Hadronic Light-by-Light**

② **Isospin breaking: τ vs. e^+e^-**

③ **CMD-2 vs. KLOE**

④ **Final state radiation of hadrons**

⑤ **4-loop QED**

⑥ **Outlook**

1) The $(g-2)_\mu$ problem:

(Experiment (BNL 2004))

$$a_\mu^\pm = 11659208(6) \times 10^{-10}$$

(Theory)

$$a_\mu^\pm = 11659182.7(7.3) \times 10^{-10}$$

$$a_\mu^{\rm Exp} - a_\mu^{\rm The} = 25.3 \pm 9.4 \times 10^{-10} \quad \boxed{2.7\sigma}$$

Note: $\delta a_\mu^{\rm Exp} = 6.0 \times 10^{-10}$ $\quad \delta a_\mu^{\rm HVP} = 6.4 \times 10^{-10}$

2) The $\alpha(M_Z)$ problem: input for electroweak precision physics

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$

$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$

$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4} \quad \text{(present)}$$

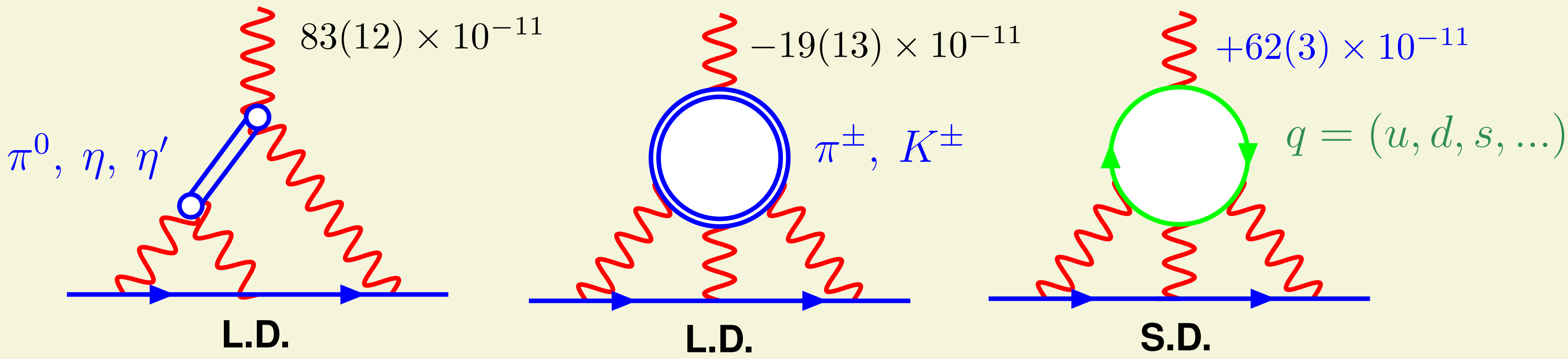
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad (\texttt{ILC requirement})$$

① Hadronic Light-by-Light scattering contribution

Melnikov-Vainshtein improvement of EJLN/HGS approach:

- **Hadronic light–by–light scattering** $a_\mu^{\rm lbl} = (\,80 \pm 40)\ \times 10^{-11}$ **(Knecht & Nyffeler 02)**
  $a_\mu^{\rm lbl} = (136 \pm 25)\ \times 10^{-11}$ **(Melnikov & Vainshtein 03)**

(Kinoshita et al., Bijnens et al.) **shift by** $+56 \times 10^{-11}$



Low energy effective theory: e.g. ENJL

MV and KN utilize the same model LMD+V form factor:

$$F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{4\pi^2F_\pi^2}{N_c}\,\frac{q_1^2q_2^2(q_1^2+q_2^2)-h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+(N_cM_1^4M_2^4/4\pi^2F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_1^2)(q_2^2+M_2^2)},$$

where $M_1 = 769\ \mathrm{MeV}$, $M_2 = 1465\ \mathrm{MeV}$, $h_5 = 6.93\ \mathrm{GeV}^4$.

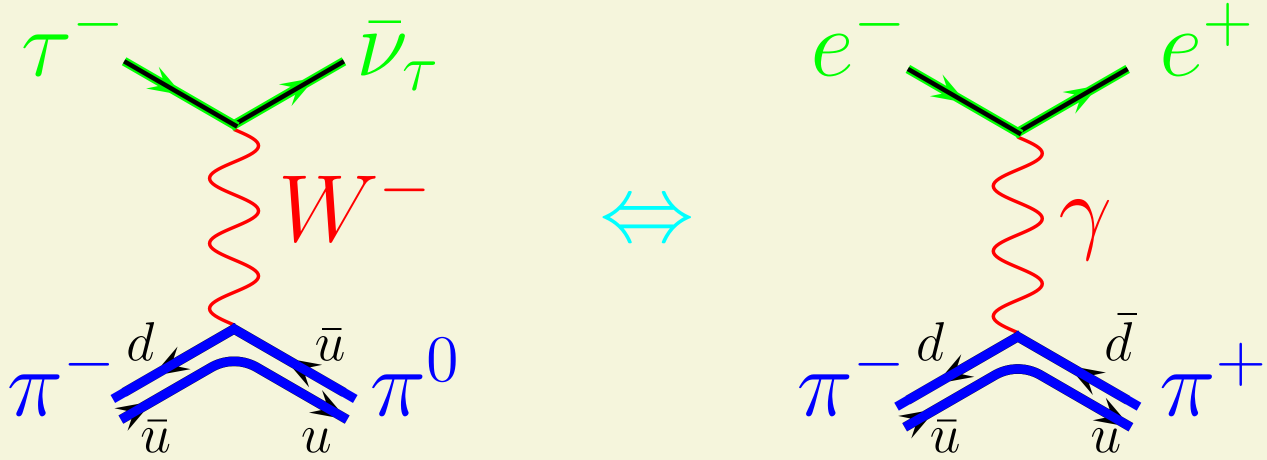
with two modifications:

- **form factor: undressed soft photon (non-renormalization of ABJ)**
- $h_2 = 0 \pm 20$ **GeV^2 (KN) vs.** $h_2 = -10$ **GeV^2 (MV) fixed by twist 4 in OPE** $(1/q^4)$

	π^0,η,η'[π^0]	a_1[f_1,f_1^*]	$\pi^\pm$	pQCD/QPM	tot
HK	83(06)	1.7 [a_1	-4.5(8.5)	10(11)	90(15)
BPP	85(13)	-4(3) [a_1+f_0]	-19(5)	21(3)	83(32)
KN	83(12)			80(40)	
MV	114.5[76.5]	22[7]	0	0	136(25)

② Isospin breaking: τ vs. e^+e^-

The iso-vector part of $\sigma(e^+e^- \to \mathrm{hadrons})$ may be calculated by a iso-spin rotation from τ–decay spectra (to the extend that CVC is valid)



$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$

X^- and X^0 are hadronic states related by iso-spin rotation.

The e^+e^- cross–section is then given by

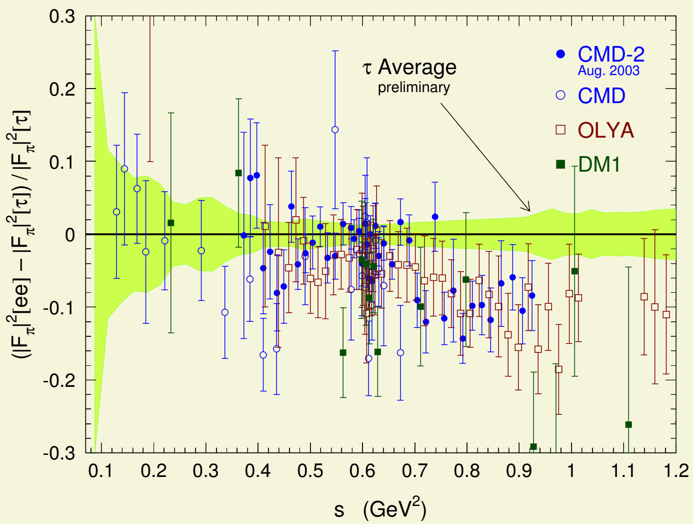
$$\sigma^{I=1}_{e^+e^-\to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

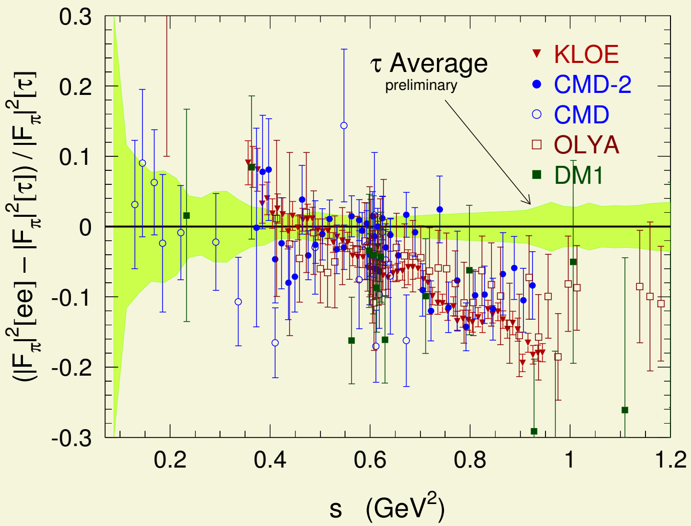
in terms of the τ spectral function v_1.

All kind of isospin breaking effects have to be taken into account !!!

(V. Cirigliano, G. Ecker and H. Neufeld)

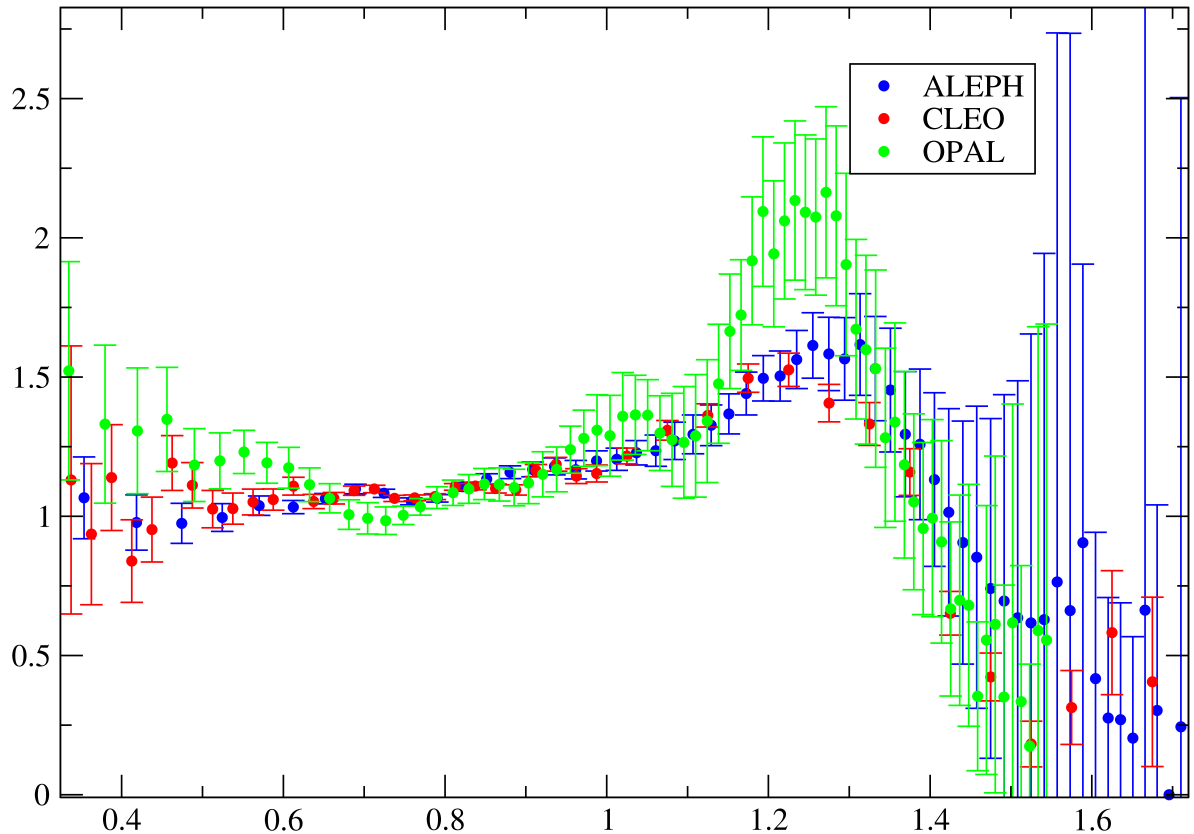
After known isospin corrections:





Experimental problems?

- **Comparison of τ–data:**



τ–data may be not so easy; DELPHI, L3 could not measure τ spectral–functions; ALEPH vs. OPAL no good agreement.

S. Ghozzi, F. J

Fit data by same Gounaris-Sakurai formula: Only parameters differ

in first place mass and width of ρ ! Two parameter fit (crude)

ALEP vs. CMD-2: $\Delta m_\rho = 2.7 \pm 0.8$ and $\Delta \Gamma_\rho = 1.3 \pm 1.0$ (S. Ghozzi, F. J.)

[$\Delta m_\rho = 3.1 \pm 0.9$ and $\Delta \Gamma_\rho = 2.3 \pm 1.6$] (M. Davier (Pisa))

Problem with theory: usual argument

$$\Delta m_\rho^2 = \Delta m_\pi^2$$

from a sum rule yields $m_{\rho^-} - m_{\rho^0} = \frac{1}{2}\frac{\Delta m_\pi^2}{m_{\rho^0}} \sim 0.82$MeV !. Too large by factor 2! width??

A. Höcker at ICHEP Bejing August 2004:

An empirical isospin-breaking correction of the ρ resonance lineshape (mass and width) improves but does not restore the agreement between the two data sets. It is a consequence of this confirmation that, until the CVC puzzle is solved, only e^+e^-data should be used for the evaluation of the dispersion integral. Doing so, and including the KLOE data, we find that the Standard Model prediction of a_μ differs from the experimental value by 2.7 standard deviations.

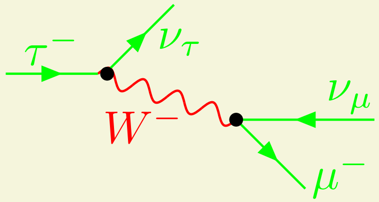
W. Morse, BNL:

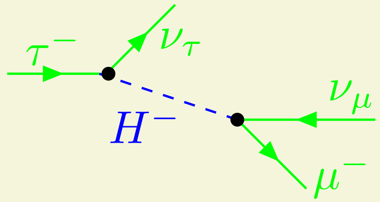
Is the source of isospin breaking a charged Higgs exchange?

Discusses how a charged Higgs propagator would modify the form factor in $\tau^- \to \pi^-\pi^0\nu_\tau$ decays.

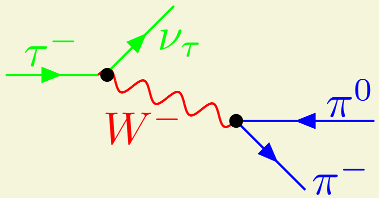
Isospin breaking in τ–decays via charged Higgs exchange

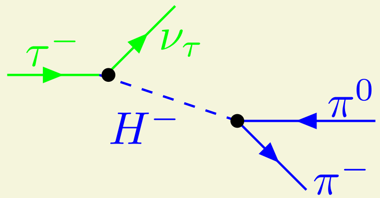
Typically in 2HDM: Large one-loop RC (M. Krawzcyk, D. Temes) in leptonic τ–decays see talk by M. Krawzcyk

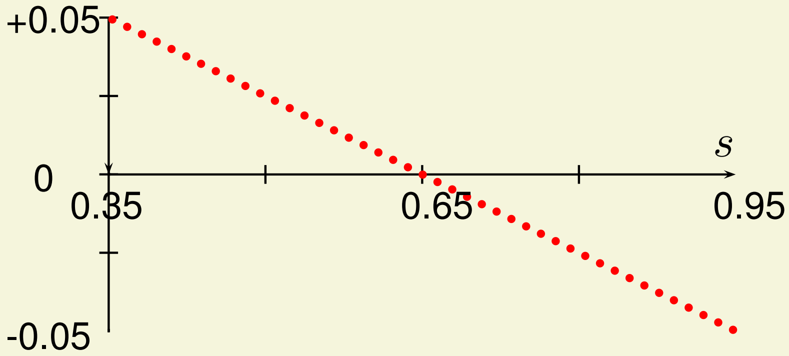




Large isopin violating H^- exchange in hadronic τ–decays (W. Morse)?



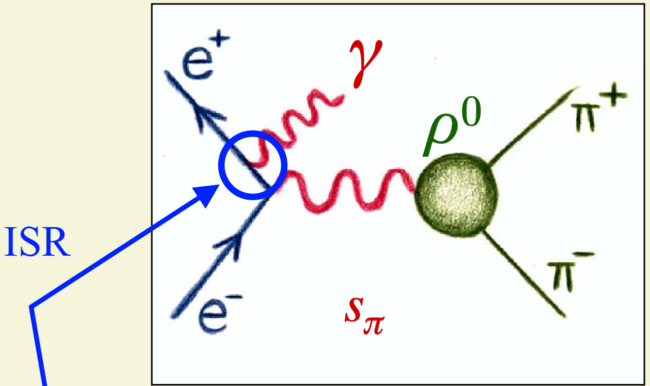




$$R = \frac{|\Psi_W + \Psi_H|^2 - |\Psi_W|^2}{|\Psi_W|^2}$$

Correctly normalized effect by far too small!

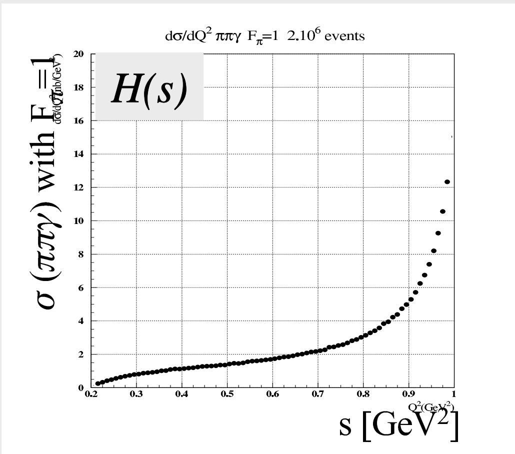
③ KLOE vs. CMD-2 Hadronic Cross Sections



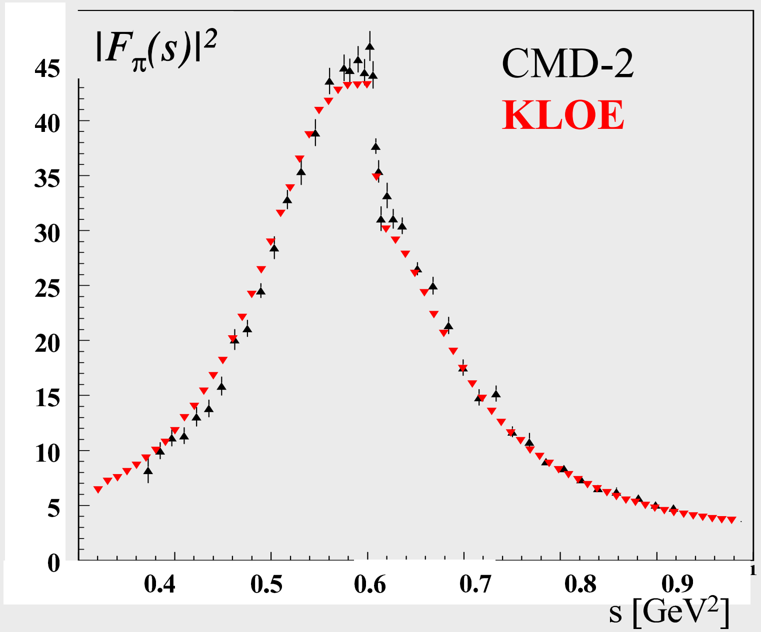
Radiative Return @ DAΦNE:
KLOE-Measurement of the hadronic cross section $\sigma(e^+e^- \to \pi^+\pi^-)$ below 1 GeV

Radiator-Function H(s) from Phokhara-Generator

$$M_{\pi\pi}^2 \frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \sigma_{\pi\pi}(s) \times H(s)$$



KLOE Measurement: Pion Formfactor



Note on new KLOE result: see talk by S. Müller

my old value:	694.75 (5.15) (6.83) [8.56]	
subtract cmd2:	389.36 (2.75) (2.59) [3.78]	extended to KLOE range
	305.39 (4.35) (6.32) [7.67]	
KLOE:	388.75 (0.52) (5.05) [5.08]	KLOE range: 591.6-969.5 MeV
add weighted	389.24 (1.14) (2.39) [2.65]	
my new value	694.63 (4.50) (6.76) [8.12]	

Theory of Pion FF: unitarity, analyticity and χPT (G. Colangelo (Pisa))

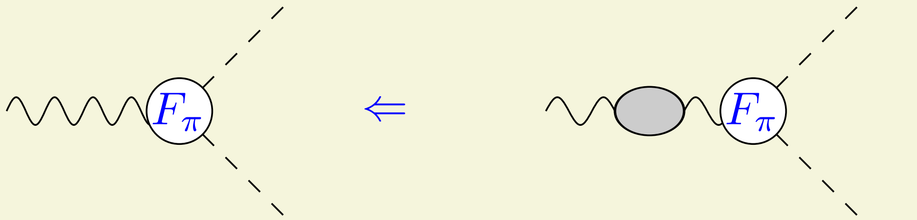
P	χ^2/d.o.f.	$\chi^2_{\mathrm{CMD2/NA7}}$	$10^{10} a_\rho$	$10^{10} a_{2M_K}$	$\langle r^2 \rangle$(fm^2)
0	84.9/83	43.6 / 43.7	420.1 ± 2.1	489.5 ± 2.2	0.4254 ± 0.0020
5	78.4/82	35.9 / 42.6	423.8 ± 2.6	494.1 ± 2.7	0.4300 ± 0.0024
6	78.1/81	36.0 / 42.2	424.4 ± 2.8	494.7 ± 2.9	0.4339 ± 0.0051
7	73.5/80	31.7 / 42.2	423.4 ± 2.9	493.2 ± 3.0	0.4350 ± 0.0051
8	73.5/79	31.6 / 42.2	423.5 ± 5.7	493.4 ± 7.4	0.4347 ± 0.0052

Numerical results for fits to CMD-2 and (spacelike) NA7 data. The errors given are purely statistical.

To be compared with: 429.02 ± 4.95 (stat) from trapezoidal rule. Gain factor of 2 in precision in stat error!

④ Final state radiation of hadrons

Remainder: Need 1pi "blob" in dispersion integrals: VP undressed cross-section



$$|F_\pi^{(0)}(s)|^2 = |F_\pi(s)|^2\ (\alpha/\alpha(s))^2$$

VP effects in physical quantities must include photonic corrections to the hadronic 1pi blob:

Add theoretical prediction for FS radiation (including full photon phase space):

$$|F_\pi^{(\gamma)}(s)|^2 = |F_\pi^{(0)}(s)|^2\ \left(1+\eta(s)\frac{\alpha}{\pi}\right)$$

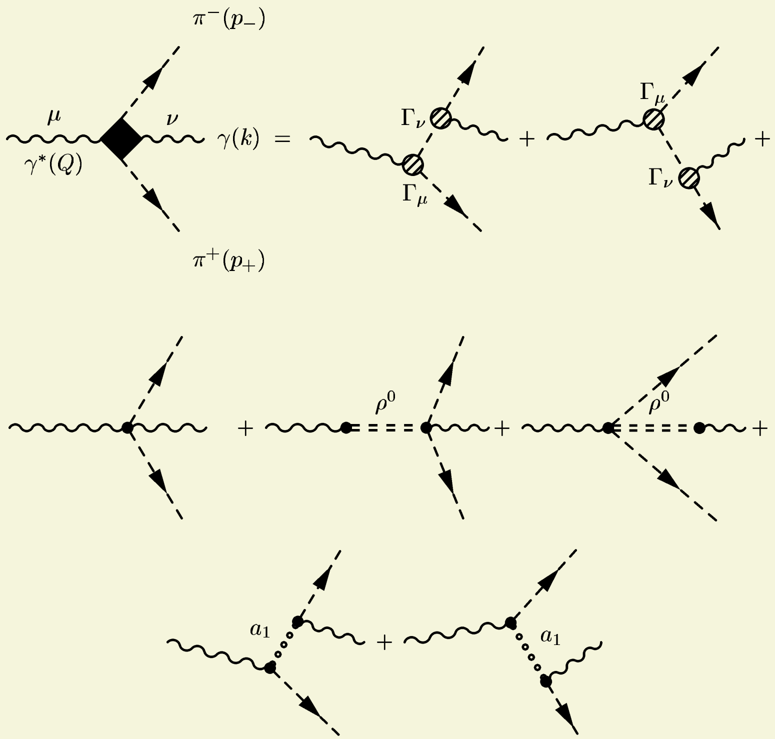
to order $O(\alpha)$, where $\eta(s)$ is a known correction factor (Schwinger 1989). The

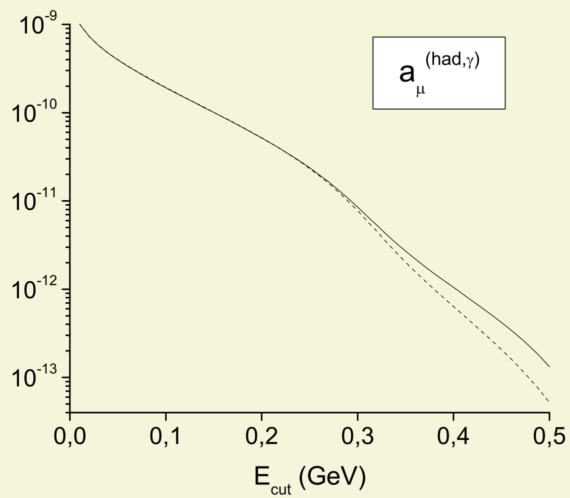
corresponding $O(\alpha)$ contribution to the anomalous magnetic moment of the muon is

$$\delta^\gamma a_\mu^{\rm had} = (38.6 \pm 1.0) \times 10^{-11}$$

Final–state radiation in electron–positron annihilation into a pion pair

(S. Dubinsky, A. Korchin, N. Merenkov, G. Pancheri, O. Shekhovtsova) $\Rightarrow$ talk by O. Shekhovtsova





⑤ 4-loop QED

Recent: Corrections due to internal e- and τ-loops updated

$$a_\mu = a_e^{\rm uni} + a_\mu(m_\mu/m_e) + a_\mu(m_\mu/m_\tau) + a_\mu(m_\mu/m_e, m_\mu/m_\tau)$$

$$\begin{aligned} a_\mu(m_\mu/m_e) &= 1.094\,258\,282\,8\,(98)\left(\frac{\alpha}{\pi}\right)^2 + 22.868\,379\,36\,(23)\left(\frac{\alpha}{\pi}\right)^3 \\ &+ \boxed{132.682\,3\,(72)}\left(\frac{\alpha}{\pi}\right)^4 \\ a_\mu(m_\mu/m_\tau) &= 7.8059\,(25)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^2 + 36.054\,(21)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^3 \\ &+ \boxed{127.50\,(41)}\left(\frac{\alpha}{\pi}\right)^4 \\ a_\mu(m_\mu/m_e, m_\mu/m_\tau) &= 52.763\,(17)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^3 \\ &+ \boxed{0.037\,594\,(83)}\left(\frac{\alpha}{\pi}\right)^4 \end{aligned}$$

with $\alpha^{-1}(\text{a.i.}) = 137.036\,000\,3\,(10)$ **[7.4 ppb]**

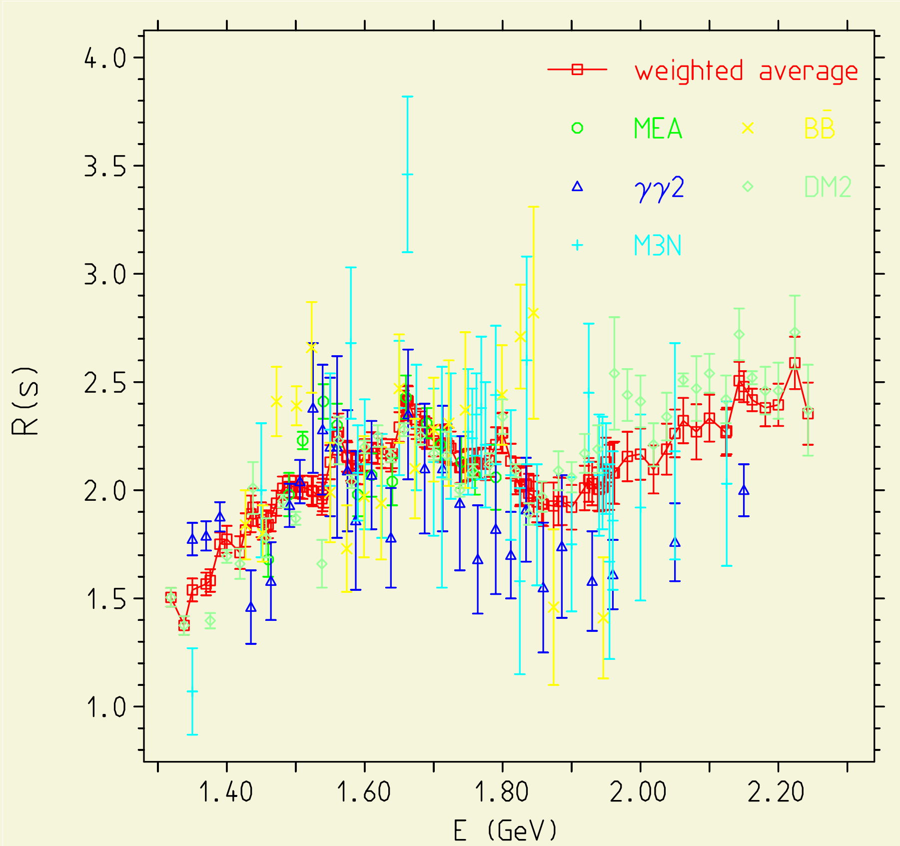
$$a_\mu^{\rm QED} = 116\,584\,719.35\,\underbrace{(0.03)}_{\alpha^4}\underbrace{(1.15)}_{\alpha^5}\underbrace{(0.85)}_{\alpha_{\rm inp}}\times 10^{-11}$$

shift by $+13.7\times 10^{-11}$

Kinoshita, Nio 04

⑥ Outlook

- **Most problematic region now 1.4-2.0 GeV**



Contributions: ρ 72% rel. error 2.1 % contribution to final error 1.3%

Region 1.4-2.0 GeV: 20% rel. error contribution to final error 1% !

In progress :

- R from ISR:
  - KLOE collect and analyze more data for $\pi\pi$
  - BABAR in progress
- R from scan:
  - R from CLEO will resolve Mark I vs. CB "discrepancy" check soon, run at 7.0, 7.4, 8.4, 9.4 10.0 and 10.3 GeV
- future plans:
  - BEPCII/BESIII: run at 2.2, 2.6, 3.0 GeV at precision 5.5, 3.4, 3.4 % ([7.6, 7.0, 5.6] % now)
  - VEPP-2000: 0.4-2.0 GeV [2005-2010] factor of 10 in Lumi, CMD-2,SND factor of 2 in precision ... 1.4 - 2 GeV in progress

For precision physics at an ILC, at least a τ–charm factory is required 1% hadronic cross section measurements up to 3 GeV mandatory!

Previously unaccounted contributions ?:

$e^+e^- \to \sigma\gamma,\ f_0\gamma$ **to** $a_\mu^{\rm had}$ **(data very poor)**

$\Rightarrow \delta a_\mu^S = 1.0(0.6)[13.0(11.0)]\ \times 10^{-10}$**, i.e.,** $\sim +0.1[1]\sigma$

(Narison 03, Dubnikova et al) TH based[PDG based] $\delta a_\mu(0.6-2.0$**GeV**$) < 0.7\ \times 10^{-10}$

form $\pi\pi\gamma, \pi\eta\gamma$ **which include decay products from** $\pi^0\gamma, \sigma\gamma, f\gamma, a_1\gamma$ **(Eidelman 03)**

(A. Dubniĉková et al.)

$$
\begin{aligned}
a_\mu(\pi^0\gamma) &= 17.2 \times 10^{-11} \\
a_\mu(\eta\gamma) &= 2.2 \times 10^{-11} \\
a_\mu(\eta'\gamma) &= 1.5 \times 10^{-11} \\
a_\mu(\sigma\gamma) &= 12.5 \times 10^{-11} \\
a_\mu(a^0\gamma) &= 0.9 \times 10^{-11}
\end{aligned}
$$

Controversial!

Theory:

- Radiative corrections for

  1) Radiative return calculations: continuing progress $\Rightarrow$ Talks by J. Kühn and H. Czyz

  2) Bhabha (small/wide) angle
     progress towards full two-loop: Dixon et al., Penin, Czakon et al.
     two-loop virtual + soft complete including collinear mass logs (Penin)
     two additional hard real photons exist (Jadach et al.)
     missing: one-loop + additional hard real photon I think in 1 year from now complete $O(\alpha^2)$ result available. However: at $O(\alpha)$ still 0.5% disagreement between existing calculations! Talk by F. Nguyen

  3) $\mu^+\mu^-$ for normalization/cross check/event separation

  4) Hadronic light-by-light ?, FSR in hadro production ? 4–loop QED ?

Check: in all calculations presently aimed precision requires e.g. check in your program proper treatment of vacuum polarization (VP) subtraction: now need time–like $\alpha(s)$ in s–channels: Bhabha: t–channel $\alpha(t)$, s–channel $\alpha(s)$

Challenge for the future:

- $(g-2)_\mu$ **new experiment under discussion, up to factor 10 improvement**
- $\alpha(M_Z)$ **for ILC factor 5 improvement required !**

The show will go on!