Radiative Upsilon decays

$$\Upsilon \to X\gamma$$

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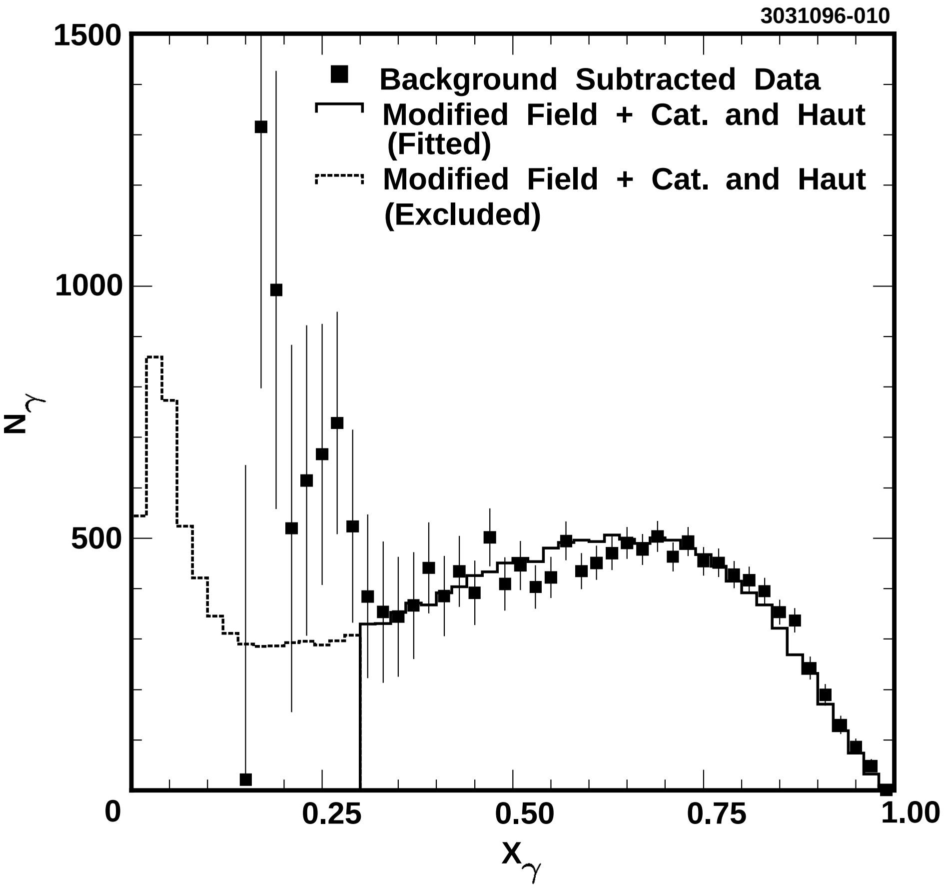
(work done with Joan Soto)

`Phys.Rev.D69, 114006 (2004) [hep-ph/0401233] + work in progress`

Introduction and motivation



From B. Nemati *et al.* [CLEO Collaboration], Phys. Rev. D **55** (1997) 5273 (hep-ex/9611020)

$$x_\gamma \equiv z = \frac{E_\gamma}{M/2}$$

Heavy Quarkonium decays

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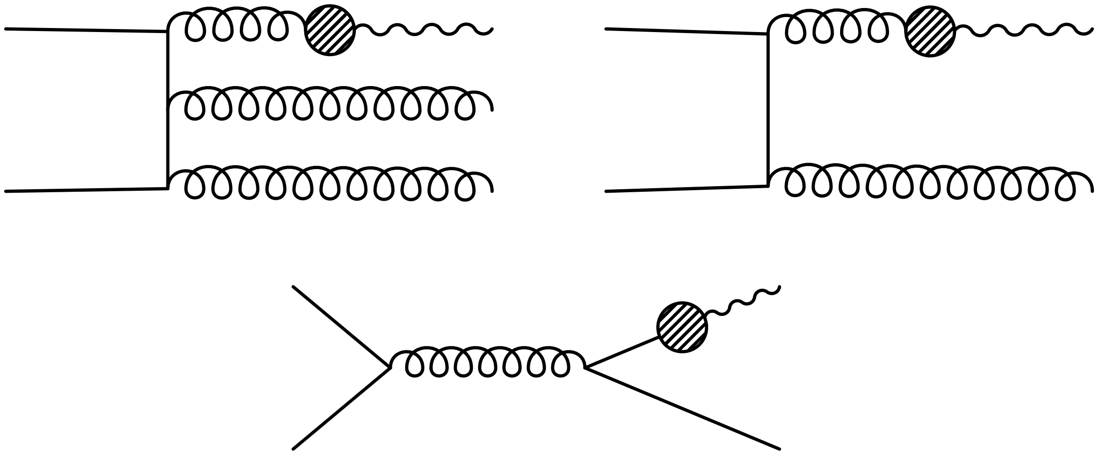
$$\frac{d\Gamma}{dz} = \frac{d\Gamma^{frag}}{dz} + \frac{d\Gamma^{dir}}{dz}$$

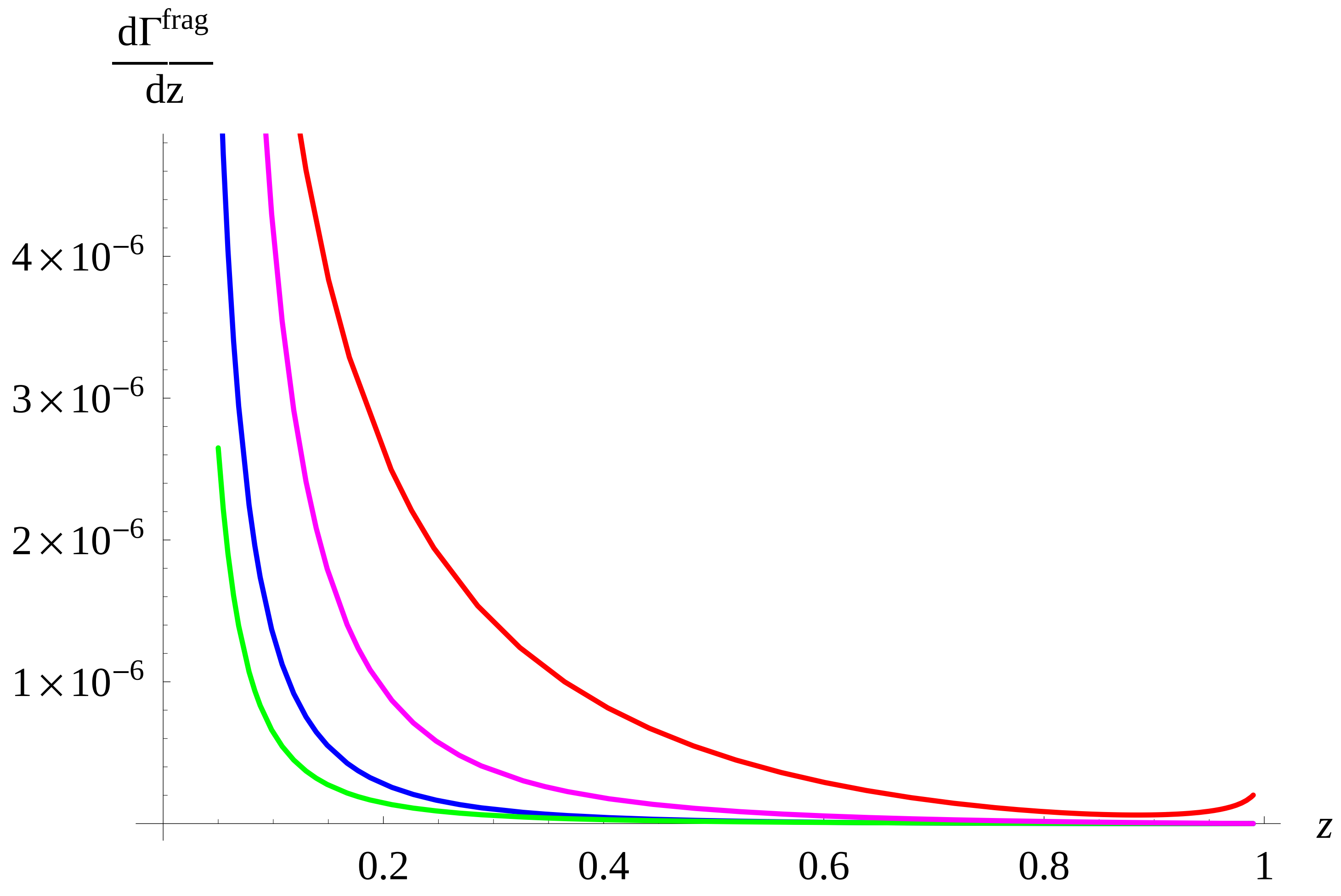
Fragmentation Contributions

Electromagnetic couplings to light quarks.

$$\frac{d\Gamma^{frag}}{dz} = \sum_a C_a \otimes D_{a\to\gamma}$$

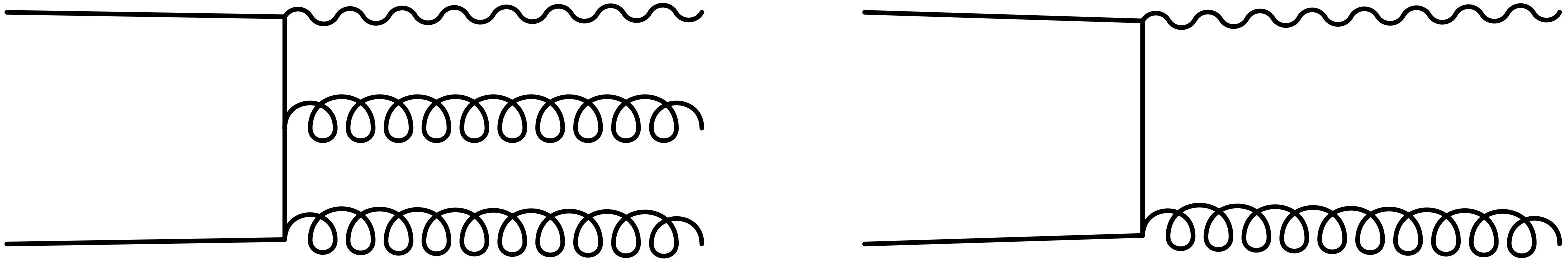


This type of contributions become important at low z



Direct contributions

Electromagnetic couplings to heavy quarks.



The NRQCD formalism organizes the decay as:

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- Leading Order (LO) operator (Color Singlet Model)

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- Next-to-Leading Order (NLO) (v^4 supressed)

$$\mathcal{O}_8\left({}^1S_0\right), \mathcal{O}_8\left({}^3P_J\right) \;\; \alpha \; \delta(1-z)$$

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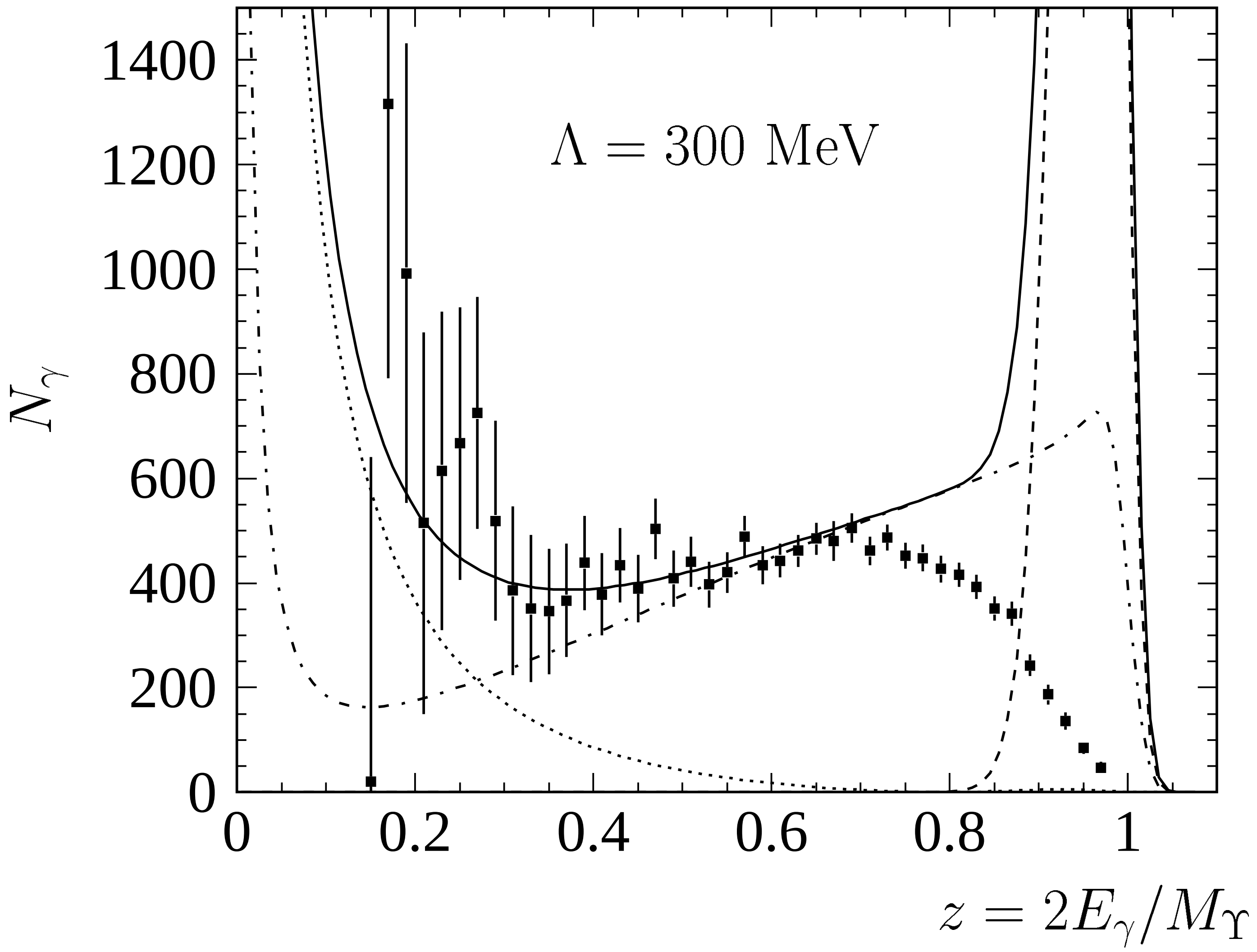
  - Higher order corrections to the coefficients have large logs $\log(1 - z)$ Krämer '99; Maltoni and Petrelli '98
  - NRQCD Operator Product Expansion breaks down
    - Shape functions resumming a certain class of operators must be introduced

      Rothstein and Wise '97



From S. Wolf, Phys. Rev. D **63** (2001) 074020 (arXiv:hep-ph/0010217)

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Soft-Collinear Effective Theory (SCET) is the theory that include these modes.

- SCET describe the interactions of collinear modes with (ultra)soft modes
- A combination of NRQCD and SCET is needed to study the end-point region

NRQCD+SCET approach

- The decay rate has been expressed in the factorized form:

$$\frac{d\Gamma}{dz} = \sum_{\omega} H(M, \omega, \mu) \int dk^+ S(k^+, \mu) \mathrm{Im} J_\omega(k^+ + M(1-z), \mu)$$

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- Sudakov logarithms have been resummed for both the color singlet and the color octet operators

Bauer et al. '01; Fleming, Leibovich '02

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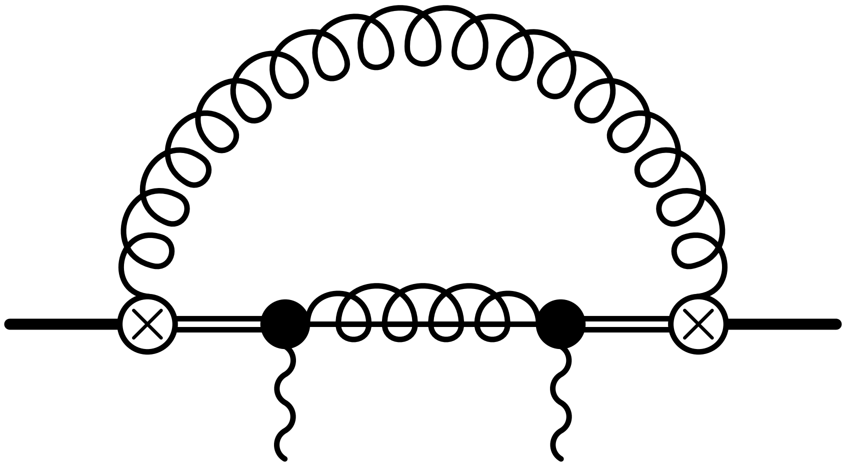
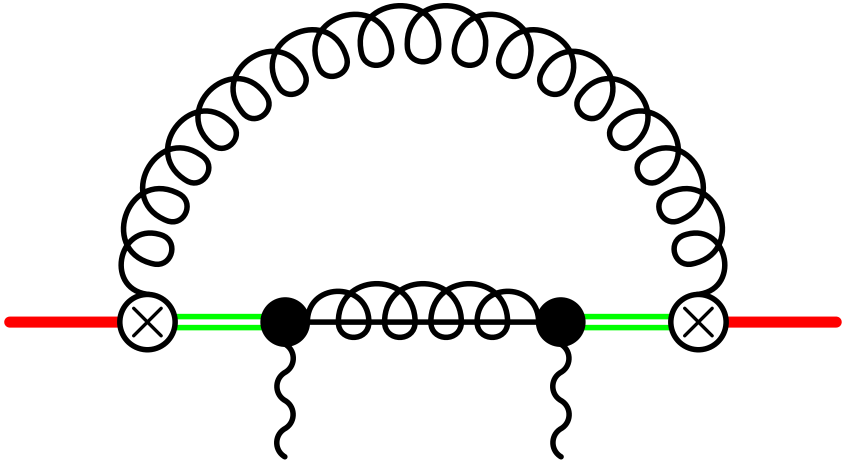
- Our calculation will be valid in the end-point region (i.e. as long as $M \gg M\sqrt{(1-z)}$)
  - At the very very end-point ($z > 0.92$) the calculation is no longer reliable

X.G.T. and Soto '04

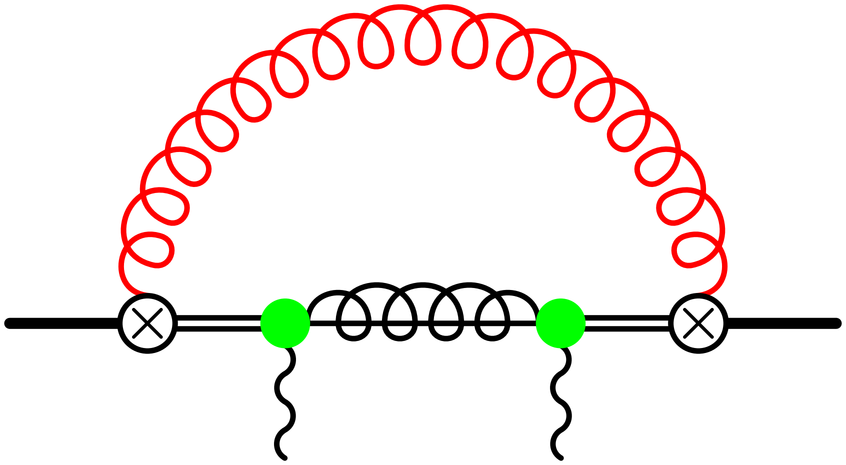
The calculation is a combination of potential NRQCD (pNRQCD) and SCET

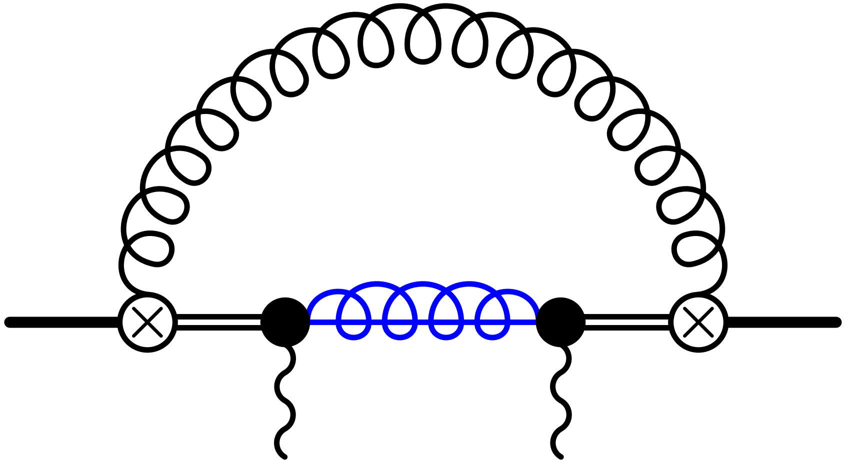


- Singlet field
- Octet field



- SCET octet operators
- ultrasoft gluon $p \sim (m\alpha_s^2, m\alpha_s^2, m\alpha_s^2)$





Collinear gluon $p_+, p_\perp \ll M\sqrt{1-z}$

$$\frac{1}{M^2(1-z) - Mk_+ - \mathbf{k}_\perp^2 + i\varepsilon}$$

$\mathbf{k}_\perp$ must be expanded to be consistent

$$T^{\alpha\alpha'}_{(8,{}^1S_0)}(z) = -i\eta_\perp^{\alpha\alpha'}(4\pi)\frac{32}{3}T_F^2\left(\frac{c_F}{2m}\right)^2\alpha_{\rm s}(\mu_u)C_f\times$$

$$\times\int d^3\mathbf{x}\int d^3\mathbf{x}'\psi^*_{n0}(\mathbf{x}')\psi_{n0}(\mathbf{x})\int\frac{d^4k}{(2\pi)^4}\frac{\mathbf{k}^2}{k^2+i\epsilon}\times$$

$$\times\left(\frac{1}{-k_0+E_n-h_o+i\epsilon}\right)_{\mathbf{x}',\mathbf{0}}\frac{1}{(M(1-z)-k_+)M-\mathbf{k}_\perp^2+i\epsilon}\left(\frac{1}{-k_0+E_n-h_o+i\epsilon}\right)_{\mathbf{0},\mathbf{x}}$$

$$\mathrm{Im}\left(T^{\alpha\alpha'}_{(8,{}^1S_0)}(z)\right) = -\eta_\perp^{\alpha\alpha'}\frac{16}{3}T_F^2\left(\frac{c_F}{2m}\right)^2\alpha_s(\mu_u)C_f\frac{1}{M}\int_0^\infty dk_+\delta(M(1-z)-k_+)\times$$

$$\times\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{k_+}{2}+x)-I_S^2(\frac{k_+}{2}+x)\right)$$

$$\mathrm{Im}\left(T^{\alpha\alpha'}_{(8,{}^1S_0)}(z)\right) = -\eta_\perp^{\alpha\alpha'}\int dl_+ S_S(l_+)\mathrm{Im}J_M(l_+-M(1-z))$$

$$\mathrm{Im}J_M(l_+-M(1-z)) = T_F^2\left(N_c^2-1\right)\frac{2\pi}{M}\delta(M(1-z)-l_+)$$

$$S_S(l_+) = \frac{4\alpha_s(\mu_u)}{3\pi N_c}\left(\frac{c_F}{2m}\right)^2\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{l_+}{2}+x)-I_S^2(\frac{l_+}{2}+x)\right)$$

We obtain the shape functions

$$S_S(l_+) = \frac{4\alpha_s(\mu_u)}{3\pi N_c}\left(\frac{c_F}{2m}\right)^2\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_S(\frac{l_+}{2}+x) - I_S^2(\frac{l_+}{2}+x)\right)$$

$$S_{P1}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c}\int_0^\infty dx\left(2\psi_{10}(\mathbf{0})I_P(\frac{l_+}{2}+x) - I_P^2(\frac{l_+}{2}+x)\right)$$

$$S_{P2}(l_+) := \frac{\alpha_s(\mu_u)}{6\pi N_c}\int_0^\infty dx\frac{8l_+x}{(l_++2x)^2}\left(\psi_{10}^2(\mathbf{0}) - 2\psi_{10}(\mathbf{0})I_P(\frac{l_+}{2}+x) + I_P^2(\frac{l_+}{2}+x)\right)$$

where

$$I_S(\frac{k_+}{2}+x) := m\sqrt{\frac{\gamma}{\pi}}\frac{\alpha_s N_c}{2}\frac{1}{1-z'}\left(1-\frac{2z'}{1+z'}\,{}_2F_1\left(-\frac{\lambda}{z'},1,1-\frac{\lambda}{z'},\frac{1-z'}{1+z'}\right)\right)$$

$$I_P(\frac{k_+}{2}+x) := \sqrt{\frac{\gamma^3}{\pi}\frac{8}{3}}(2-\lambda)\frac{1}{4(1+z')^3}\Big(2(1+z')(2+z') + (5+3z')(-1+\lambda) + 2(-1+\lambda)^2 +$$

$$+\frac{1}{(1-z')^2}\left(4z'(1+z')(z'^2-\lambda^2)\left(-1+\frac{\lambda(1-z')}{(1+z')(z'-\lambda)} + {}_2F_1\left(-\frac{\lambda}{z'},1,1-\frac{\lambda}{z'},\frac{1-z'}{1+z'}\right)\right)\right)\Big)$$

$$\gamma = \frac{mC_f\alpha_s}{2} \quad z' = \frac{\kappa}{\gamma} \quad -\frac{\kappa^2}{m} = E_1 - \frac{k_+}{2} - x \quad \lambda = -\frac{1}{2N_cC_f}$$

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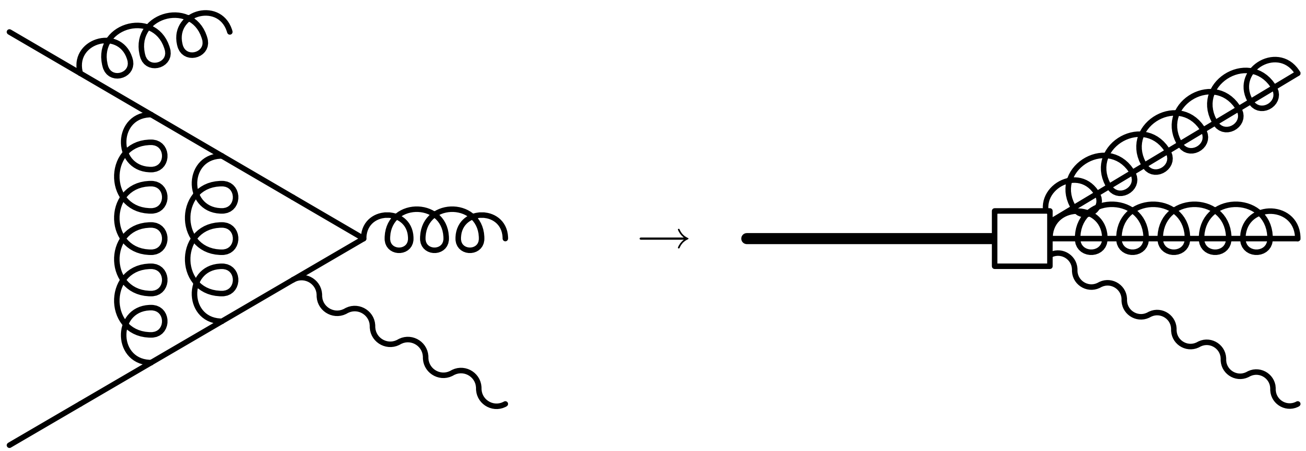
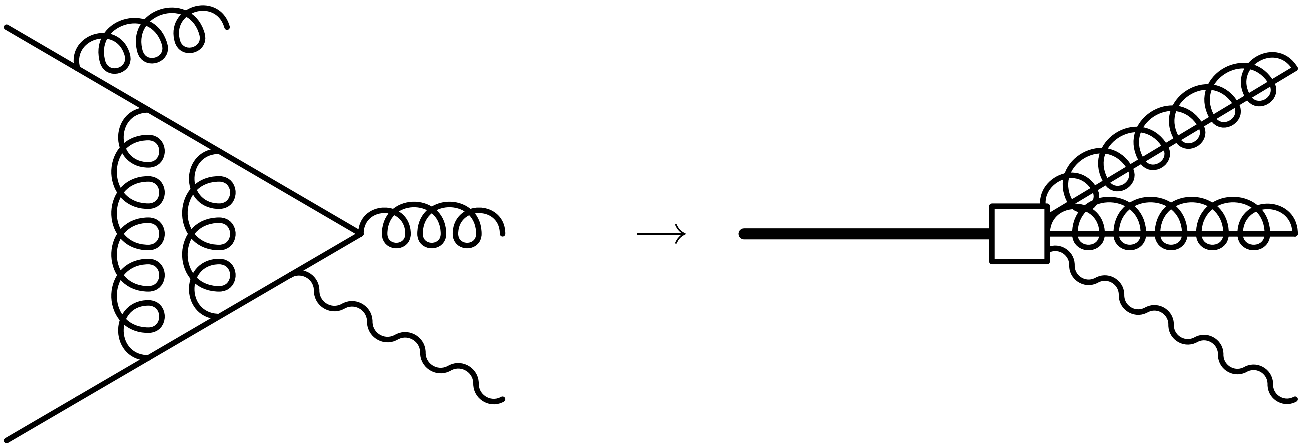
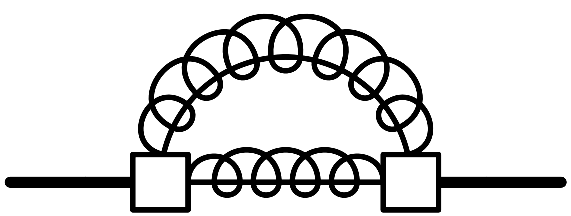
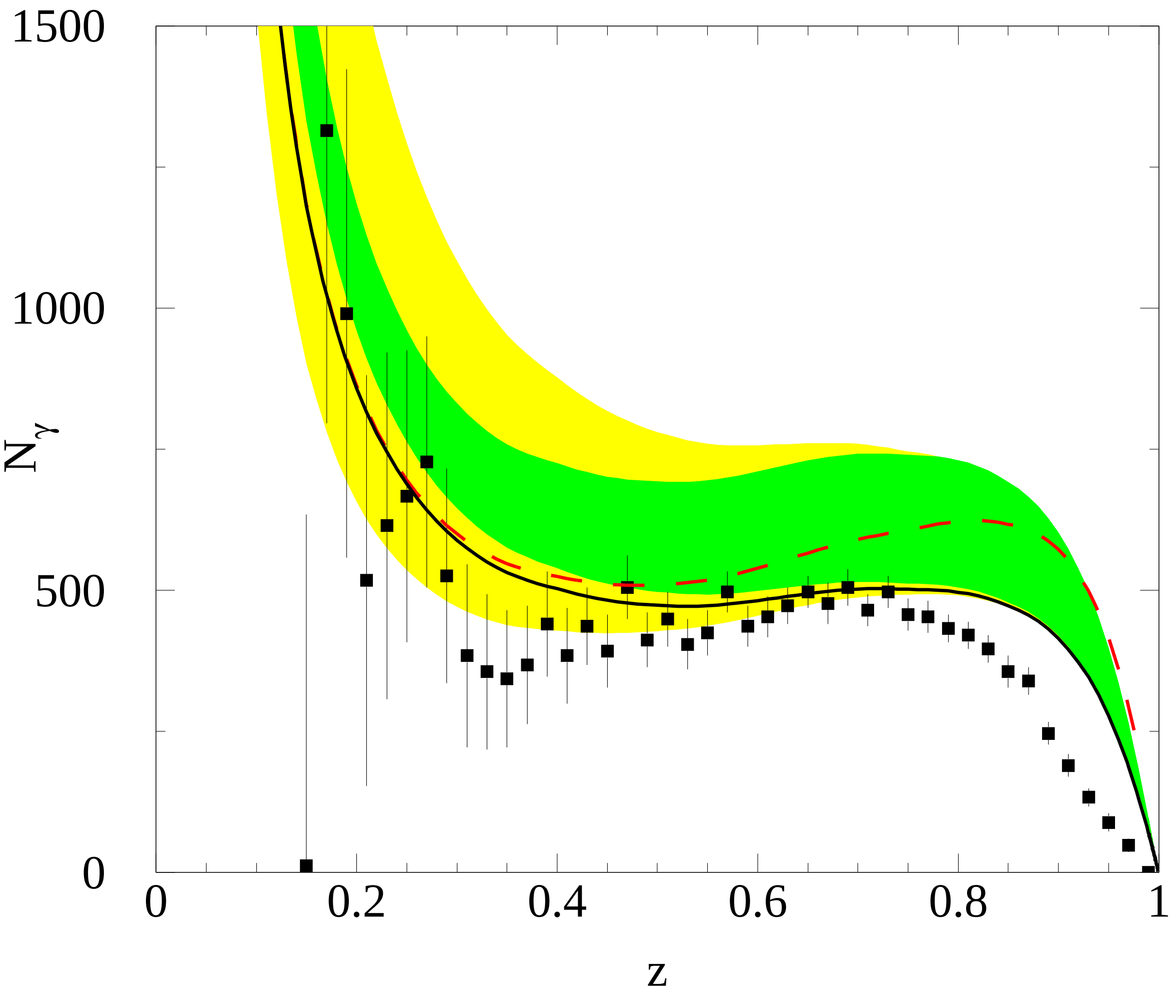
- Following the idea of the expansion by regions
  - compensated by IR divergence induced by keeping that term and expanding the ultrasoft scales accordingly







Results



From S. Fleming and A. K. Leibovich, Phys. Rev. D **67** (2003) 074035 (arXiv:hep-ph/0212094)

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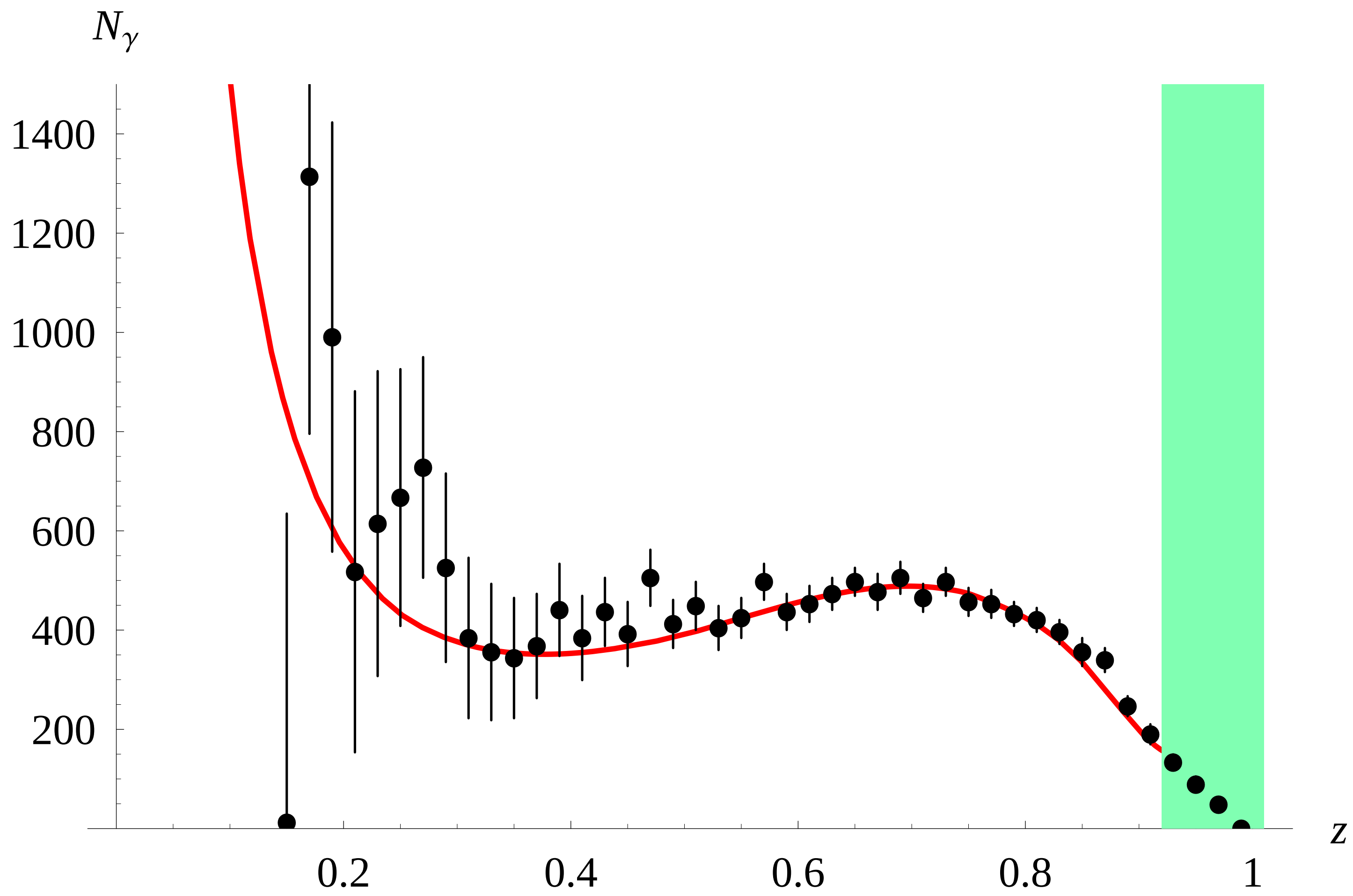

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- We've calculated the shape functions (in the perturbative regime)
  - Adding these contributions to the previous results makes the agreement with data almost perfect
- SCET opens the possibility for the study of exclusive heavy quarkonium decays from an effective field theory point of view