Study of the η-η' system in the two mixing angle scheme

Work in collaboration with J.-M. Frère (ULB), hep-ph/0501072

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Motivation:

- To perform an updated phenomenological analysis of various decay processes using the two mixing angle description of the η-η' system
- The analysis will serve us to check the validity of the two mixing angle scheme and its improvement over the standard one angle picture
- The analysis also tests the sensitivity to the mixing angle schemes: octet-singlet basis vs. quark-flavour basis

Notation

The decay constants of the η-η' system in the octet-singlet basis f_P^a $(a=8,0; P=\eta,\eta')$ are defined as

$$\langle 0|A_\mu^a|P(p)\rangle = i f_P^a p_\mu \ ,$$

where $A_\mu^{8,0}$ are the octet and singlet axial-vector currents with

$$\partial^\mu A_\mu^8 = \frac{2}{\sqrt{6}}(m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d - 2m_s\bar{s}i\gamma_5 s) \ ,$$

$$\partial^\mu A_\mu^0 = \frac{2}{\sqrt{3}}(m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d + m_s\bar{s}i\gamma_5 s) + \frac{1}{\sqrt{3}}\frac{3\alpha_s}{4\pi}G_{\mu\nu}^a\tilde{G}^{a,\mu\nu} \ .$$

The divergence of the matrix elements are then written as

$$\langle 0|\partial^\mu A_\mu^a|P\rangle = f_P^a m_P^2 \ .$$

The decay constants are parameterized in terms of f_8, f_0 and θ_8, θ_0 as

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 & -f_0\sin\theta_0 \\ f_8\sin\theta_8 & f_0\cos\theta_0 \end{pmatrix} \ .$$

Neglecting the contribution of the *up* and *down* quark masses, the matrix elements of the chiral anomaly between the vacuum and (η,η') states are

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle = \sqrt{\frac{3}{2}}m_\eta^2(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0) \ ,$$

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle = \sqrt{\frac{3}{2}}m_{\eta'}^2(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0) \ .$$

Notation

In the quark-flavour basis the decay constants are parameterized in terms of f_q, f_s and ϕ_q, ϕ_s as

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix} ,$$

and the non-strange and strange axial-vector currents are defined as

$$A_\mu^q = \tfrac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) = \tfrac{1}{\sqrt{3}}(A_\mu^8 + \sqrt{2}A_\mu^0) ,$$

$$A_\mu^s = \bar{s}\gamma_\mu\gamma_5 s = \tfrac{1}{\sqrt{3}}(A_\mu^0 - \sqrt{2}A_\mu^8) .$$

The divergences of these currents are

$$\partial^\mu A_\mu^q = \sqrt{2}(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d) + \tfrac{\sqrt{2}}{3}\tfrac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} ,$$

$$\partial^\mu A_\mu^s = 2m_s \bar{s} i\gamma_5 s + \tfrac{1}{3}\tfrac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} ,$$

and therefore the matrix elements of the chiral anomaly in this basis are

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle = \frac{3}{\sqrt{2}} m_\eta^2 f_q \cos\phi_q , \quad \langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle = \frac{3}{\sqrt{2}} m_{\eta'}^2 f_q \sin\phi_q .$$

Experimental values for $\theta_{8,0}$ and $\phi_{q,s}$

Experimental constrains:

$$\Gamma(\eta\to\gamma\gamma)=(0.510\pm 0.026)\ \text{keV}\ ,\qquad \Gamma(\eta'\to\gamma\gamma)=(4.29\pm 0.15)\ \text{keV}\ .$$

$$R_{J/\psi}\equiv\frac{\Gamma(J/\psi\to\eta'\gamma)}{\Gamma(J/\psi\to\eta\gamma)}=5.0\pm 0.6\ .$$

The interpolating fields η and η' are related with the axial-vector currents

$$\eta(x)=\frac{1}{m_\eta^2}\frac{f_{\eta'}^0\partial^\mu A_\mu^8(x)-f_{\eta'}^8\partial^\mu A_\mu^0(x)}{f_{\eta'}^0 f_\eta^8-f_{\eta'}^8 f_\eta^0}\ ,\qquad \eta'(x)=\frac{1}{m_{\eta'}^2}\frac{f_\eta^0\partial^\mu A_\mu^8(x)-f_\eta^8\partial^\mu A_\mu^0(x)}{f_\eta^0 f_{\eta'}^8-f_\eta^8 f_{\eta'}^0}\ .$$

This leads to

$$\Gamma(\eta\to\gamma\gamma)=\frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{f_{\eta'}^0-2\sqrt{2}f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8-f_{\eta'}^8 f_\eta^0}\right)^2=\frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{c\theta_0/f_8-2\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}\right)^2\ ,$$

$$\Gamma(\eta'\to\gamma\gamma)=\frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{f_\eta^0-2\sqrt{2}f_\eta^8}{f_\eta^0 f_{\eta'}^8-f_\eta^8 f_{\eta'}^0}\right)^2=\frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{s\theta_0/f_8+2\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}\right)^2\ .$$

Theoretical constrain: $f_8=1.28 f_\pi$ (χPT)

The radiative $J/\psi\to P\gamma$ decays are dominated by non-perturbative gluonic matrix elements:

$$R_{J/\psi}\propto\left|\frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle}\right|^2=\left|\frac{m_{\eta'}^2(f_{\eta'}^8+\sqrt{2}f_{\eta'}^0)}{m_\eta^2(f_\eta^8+\sqrt{2}f_\eta^0)}\right|^2=\left[\frac{m_{\eta'}^2(f_8\sin\theta_8+\sqrt{2}f_0\cos\theta_0)}{m_\eta^2(f_8\cos\theta_8-\sqrt{2}f_0\sin\theta_0)}\right]^2\ .$$

Experimental values for $\theta_{8,0}$ and $\phi_{q,s}$

octet-singlet basis

for $f_8 = 1.28 f_\pi$ (χPT):

$$\theta_8 = (-22.2 \pm 1.8)^\circ \ , \quad \theta_0 = (-8.7 \pm 2.1)^\circ \ , \quad f_0 = (1.18 \pm 0.04) f_\pi \ .$$

for $f_8 = 1.34 f_\pi$ (Large N_c χPT):

$$\theta_8 = (-22.9 \pm 1.8)^\circ \ , \quad \theta_0 = (-6.9 \pm 2.0)^\circ \ , \quad f_0 = (1.20 \pm 0.04) f_\pi \ .$$

quark-flavour basis for $f_q = f_\pi$: $\phi_q \simeq 39.8^\circ$ and $\phi_s \simeq 38.6^\circ$

$$f_q = (1.07 \pm 0.03) f_\pi \ , \quad f_s = (1.37 \pm 0.27) f_\pi \ , \quad \phi = (39.0 \pm 1.7)^\circ \ .$$

For comparison:

Large N_c χPT (H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223):

$$f_8 = 1.28 f_\pi \ , \quad \theta_8 = -20.5^\circ \ , \quad f_0 \simeq 1.25 \ , \quad \theta_0 \simeq -4^\circ$$

phenomenological analysis (T. Feldmann *et. al.*, Phys. Rev. D **58** (1998) 114006):

$$f_q = (1.07 \pm 0.02) f_\pi \ , \quad f_s = (1.34 \pm 0.06) f_\pi \ , \quad \phi = (39.3 \pm 1.0)^\circ$$

V-P electromagnetic form factors

We extend our analysis to the couplings of the radiative decays $V \to (\eta, \eta')\gamma$ and $\eta' \to V\gamma$ with $V = \rho, \omega, \phi$.

The form factors $F_{VP\gamma}(0,0)$ are fixed by the AVV triangle anomaly

$$F_{\rho\eta\gamma}(0,0) = \frac{\sqrt{3}}{4\pi^2}\frac{f_{\eta'}^0 - \sqrt{2} f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \ ,$$
$$F_{\rho\eta'\gamma}(0,0) = \frac{\sqrt{3}}{4\pi^2}\frac{f_\eta^0 - \sqrt{2} f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \ ,$$
$$F_{\omega\eta\gamma}(0,0) = \frac{1}{2\sqrt{2}\pi^2}\frac{(c\theta_V - s\theta_V/\sqrt{2}) f_{\eta'}^0 - s\theta_V f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \ ,$$
$$F_{\omega\eta'\gamma}(0,0) = \frac{1}{2\sqrt{2}\pi^2}\frac{(c\theta_V - s\theta_V/\sqrt{2}) f_\eta^0 - s\theta_V f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \ ,$$
$$F_{\phi\eta\gamma}(0,0) = -\frac{1}{2\sqrt{2}\pi^2}\frac{(s\theta_V + c\theta_V/\sqrt{2}) f_{\eta'}^0 + c\theta_V f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \ ,$$
$$F_{\phi\eta'\gamma}(0,0) = -\frac{1}{2\sqrt{2}\pi^2}\frac{(s\theta_V + c\theta_V/\sqrt{2}) f_\eta^0 + c\theta_V f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \ .$$

Using their analytic properties

$$F_{VP\gamma}(0,0) = \frac{f_V}{m_V} g_{VP\gamma} + \cdots \qquad \text{(Vector Meson Dominance)}$$

where the vertex couplings $g_{VP\gamma}$ are the on-shell V-P electromagnetic form factors

$$\langle P(p_P)|J_\mu^{\rm EM}|V(p_V,\lambda)\rangle|_{(p_V-p_P)^2=0} = -g_{VP\gamma}\epsilon_{\mu\nu\alpha\beta}p_P^\nu p_V^\alpha \varepsilon_V^\beta(\lambda) \ .$$

V-P electromagnetic form factors

V	P	$g_{VP\gamma}$ (th.)		$g_{VP\gamma}$ (exp.)
ρ	η	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{c\theta_0/f_8-\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.48\pm 0.08)$ GeV^{-1}	(1.59 ± 0.11) GeV^{-1}
ρ	η'	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{s\theta_0/f_8+\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.23\pm 0.08)$ GeV^{-1}	(1.35 ± 0.06) GeV^{-1}
ω	η	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})c\theta_0/f_8-s\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.57\pm 0.04)$ GeV^{-1}	(0.46 ± 0.02) GeV^{-1}
ω	η'	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})s\theta_0/f_8+s\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.56\pm 0.04)$ GeV^{-1}	(0.46 ± 0.03) GeV^{-1}
ϕ	η	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})c\theta_0/f_8+c\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(-0.76\pm 0.04)$ GeV^{-1}	(-0.690 ± 0.008) GeV^{-1}
ϕ	η'	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})s\theta_0/f_8-c\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.86\pm 0.05)$ GeV^{-1}	(0.71 ± 0.04) GeV^{-1}

Table 1: Theoretical and experimental values of the on-shell V-(η,η') electromagnetic vertex couplings in the octet-singlet η-η' mixing angle scheme. For $g_{VP\gamma}$ (th.) we give the experimental errors coming from the decay constants $f_{P,V}$ and the mixing angle values θ_8 and θ_0. We use $\theta_V=(38.7\pm 0.2)^\circ$ for the ϕ-ω mixing angle. Experimental values are taken from the Particle Data Group Coll., Phys. Lett. B **592** (2004) 1.

- quite remarkable agreement except for the $\omega\eta\gamma$ and $\omega\eta'\gamma$ couplings
  $\omega\eta\gamma$: (0.53 ± 0.05) GeV^{-1} (PDG'02) $\to$ (0.46 ± 0.02) GeV^{-1} (PDG'04) because of $e^+e^-\to\eta\gamma$ exclusion
  $\omega\eta'\gamma$: rather sensitive to θ_V, for instance $\theta_V=35.3^\circ \Longrightarrow$ 10% reduction
- for $f_8=1.28 f_\pi$: $g_{\phi\eta\gamma}=(-0.80\pm 0.04)$ GeV^{-1} and $g_{\phi\eta'\gamma}=(0.91\pm 0.06)$ GeV^{-1}
- fixing $\theta_8=\theta_0\equiv\theta$ implies an increase of $\chi^2/\mathrm{d.o.f.}$ by a factor of 3, and $g_{\phi\eta'\gamma}=(1.20\pm 0.06)$ GeV^{-1}, in clear contradiction with data.

Results for the mixing parameters

octet-singlet basis

Assumptions	Results	χ^2/d.o.f.	Assumptions	Results	χ^2/d.o.f.
$\theta_8 \neq \theta_0$ $f_8 = 1.28 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-22.5 \pm 1.3)^\circ$ $\theta_0 = (-8.0 \pm 1.4)^\circ$ $f_0 = (1.21 \pm 0.03) f_\pi$	42.3/6	$\theta_8 = \theta_0 \equiv \theta$ $f_8 = 1.28 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.9 \pm 1.2)^\circ$ $f_0 = (1.15 \pm 0.03) f_\pi$	81.4/7
$\theta_8 \neq \theta_0$ $f_8 = 1.34 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-22.9 \pm 1.3)^\circ$ $\theta_0 = (-6.6 \pm 1.4)^\circ$ $f_0 = (1.23 \pm 0.03) f_\pi$	31.2/6	$\theta_8 = \theta_0 \equiv \theta$ $f_8 = 1.34 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.7 \pm 1.1)^\circ$ $f_0 = (1.16 \pm 0.03) f_\pi$	77.8/7
$\theta_8 \neq \theta_0$ f_8 free $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-23.8 \pm 1.4)^\circ$ $\theta_0 = (-2.4 \pm 1.9)^\circ$ $f_8 = (1.51 \pm 0.05) f_\pi$ $f_0 = (1.29 \pm 0.04) f_\pi$	18.8/5	$\theta_8 = \theta_0 \equiv \theta$ f_8 free $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.5 \pm 1.2)^\circ$ $f_8 = (1.37 \pm 0.05) f_\pi$ $f_0 = (1.17 \pm 0.04) f_\pi$	77.4/6
$\theta_8 \neq \theta_0$ f_8 free θ_V free	$\theta_8 = (-24.0 \pm 1.6)^\circ$ $\theta_0 = (-2.5 \pm 1.9)^\circ$ $f_8 = (1.51 \pm 0.05) f_\pi$ $f_0 = (1.29 \pm 0.04) f_\pi$ $\theta_V = (39.4 \pm 2.2)^\circ$	18.6/4	$\theta_8 = \theta_0 \equiv \theta$ f_8 free θ_V free	$\theta = (-15.7 \pm 1.4)^\circ$ $f_8 = (1.37 \pm 0.05) f_\pi$ $f_0 = (1.17 \pm 0.03) f_\pi$ $\theta_V = (36.5 \pm 1.8)^\circ$	76.1/5

Table 2: Results for the η-η' mixing angles and decay constants in the octet-singlet basis of the two mixing angle scheme *(left)* and in the one mixing angle scheme *(right)*. For every fit, the theoretical assumptions taken, the set of numerical results, and the value of the χ^2/d.o.f. are shown in the first, second and third column respectively. The fitted experimental data includes the decay widths of $(\eta, \eta') \to \gamma\gamma$, $V \to P\gamma$, $P \to V\gamma$, and the ratio $R_{J/\psi}$.

Results for the mixing parameters

quark-flavour basis

Assumptions	Results	$\chi^2/\text{d.o.f.}$	Assumptions	Results	$\chi^2/\text{d.o.f.}$
$\phi_q \neq \phi_s$ $f_q = f_\pi$ $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi_q = (40.4 \pm 1.2)^\circ$ $\phi_s = (41.3 \pm 1.3)^\circ$ $f_s = (1.66 \pm 0.06) f_\pi$	34.6/6	$\phi_q = \phi_s \equiv \phi$ $f_q = f_\pi$ $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi = (40.8 \pm 0.9)^\circ$ $f_s = (1.66 \pm 0.06) f_\pi$	34.9/7
$\phi_q \neq \phi_s$ f_q free $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi_q = (39.9 \pm 1.3)^\circ$ $\phi_s = (41.4 \pm 1.4)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.06) f_\pi$	18.8/5	$\phi_q = \phi_s \equiv \phi$ f_q free $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi = (40.6 \pm 0.9)^\circ$ $f_q = (1.10 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.06) f_\pi$	19.4/6
$\phi_q \neq \phi_s$ f_q free ϕ_V free	$\phi_q = (39.8 \pm 1.3)^\circ$ $\phi_s = (41.2 \pm 1.5)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.67 \pm 0.07) f_\pi$ $\phi_V = (4.2 \pm 2.1)^\circ$	18.6/4	$\phi_q = \phi_s \equiv \phi$ f_q free ϕ_V free	$\phi = (40.4 \pm 1.0)^\circ$ $f_q = (1.10 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.07) f_\pi$ $\phi_V = (4.1 \pm 2.2)^\circ$	19.3/5

Table 3: Results for the η-η' mixing angles and decay constants in the quark-flavour basis of the two mixing angle scheme *(left)* and in the one mixing angle scheme *(right)*. The conventions are the same as in Table 2.

- in the octet-singlet basis a two mixing angle scheme is needed to describe experimental data in a better way;
- in the quark-flavour basis a one mixing angle description of data is enough at the current experimental accuracy.

Discussion about the mixing parameters

Our best results for the mixing parameters are

$$f_8 = (1.51 \pm 0.05) f_\pi \ , \qquad \theta_8 = (-23.8 \pm 1.4)^\circ \ ,$$
$$f_0 = (1.29 \pm 0.04) f_\pi \ , \qquad \theta_0 = (-2.4 \pm 1.9)^\circ \ ,$$

in the octet-singlet basis, and

$$f_q = (1.09 \pm 0.03) f_\pi \ , \qquad \phi_q = (39.9 \pm 1.3)^\circ \ ,$$
$$f_s = (1.66 \pm 0.06) f_\pi \ , \qquad \phi_s = (41.4 \pm 1.4)^\circ \ ,$$

in the quark-flavour basis.

At the present accuracy, our results satisfy the approximate relations

$$f_8 = \sqrt{1/3 f_q^2 + 2/3 f_s^2} \ , \qquad \theta_8 = \phi - \arctan(\sqrt{2} f_s / f_q) \ ,$$
$$f_0 = \sqrt{2/3 f_q^2 + 1/3 f_s^2} \ , \qquad \theta_0 = \phi - \arctan(\sqrt{2} f_q / f_s) \ .$$

For comparison:

Large N_c χPT (H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223):
$f_8 = 1.28 f_\pi$, $\theta_8 = -20.5^\circ$, $f_0 \simeq 1.25$, and $\theta_0 \simeq -4^\circ$
phenomenological analysis (T. Feldmann *et. al.*, Phys. Rev. D **58** (1998) 114006):
$f_q = (1.07 \pm 0.02) f_\pi$, $f_s = (1.34 \pm 0.06) f_\pi$ and $\phi = (39.3 \pm 1.0)^\circ$

Discussion about the mixing parameters

Large N_c χPT

H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223
R. Kaiser and H. Leutwyler, arXiv:hep-ph/9806336

octet-singlet basis

$$f_8^2 = \frac{4f_K^2 - f_\pi^2}{3} ,$$
$$f_0^2 = \frac{2f_K^2 + f_\pi^2}{3} + f_\pi^2 \Lambda_1 ,$$
$$f_8 f_0 \sin(\theta_8 - \theta_0) = -\frac{2\sqrt{2}}{3}(f_K^2 - f_\pi^2) .$$

quark-flavour basis

$$f_q^2 = f_\pi^2 + \frac{2}{3} f_\pi^2 \Lambda_1 ,$$
$$f_s^2 = 2f_K^2 - f_\pi^2 + \frac{1}{3} f_\pi^2 \Lambda_1 ,$$
$$f_q f_s \sin(\phi_q - \phi_s) = \frac{\sqrt{2}}{3} f_\pi^2 \Lambda_1 .$$

Our best results for the mixing parameters can be used to check the consistency of the former equations and therefore to test the reliability on the Large N_c χPT framework.

- our fitted values for f_8 and $f_0 \Longrightarrow \theta_8 - \theta_0 = (-13.7 \pm 0.6)^\circ$, to be compared with our prediction $\theta_8 - \theta_0 = (-21.4 \pm 2.4)^\circ$
- our fitted value for $f_0 \Longrightarrow \Lambda_1 = 0.34 \pm 0.10$ for the OZI-rule violating parameter Λ_1
- the fitted values for $f_{q,s}$ and $\phi_{q,s} \Longrightarrow \Lambda_1 = 0.32 \pm 0.10$, 2.34 ± 0.60 and -0.10 ± 0.13

The wide range of different values for Λ_1 as well as the discrepancies obtained for $\theta_8 - \theta_0$ and f_8 seem to indicate that the above equations are not experimentally consistent with each other.

Conclusions

- we have performed a phenomenological analysis on various decay processes using a two mixing angle scheme for the η-η' system
- the agreement between our theoretical predictions and the experimental values is very good and can be considered as a consistency check of the whole approach
- we have shown that a two mixing angle description in the octet-singlet basis is definitely required in order to achieve good agreement with experimental data
- on the contrary, in the quark-flavour basis and with the present experimental accuracy a one mixing angle description of the processes is still enough to reach agreement
- this behaviour gives experimental support to the fact that the difference of the two mixing angles in the octet-singlet basis is a $SU(3)$-breaking effect while in the quark-flavour basis is a OZI-rule violating effect which appears to be smaller
- finally, we have shown that our best fitted mixing parameters are in disaccord with the Large N_c χPT predictions thus indicating a possible discrepancy with this framework