$K_{l3\gamma}$ decays: present theoretical status and perspectives

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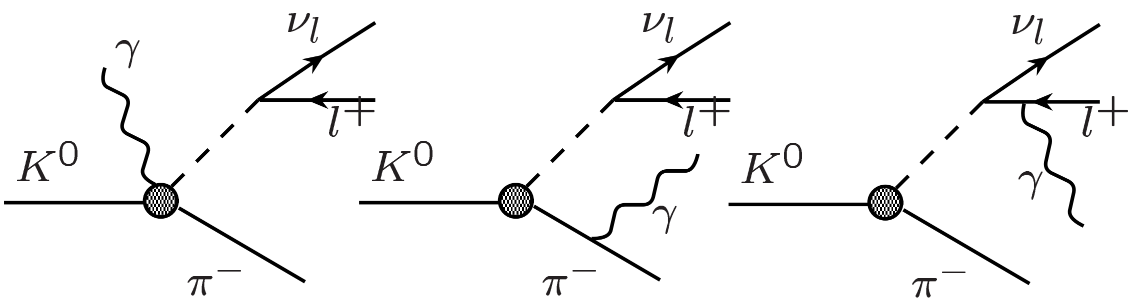
$\boxed{K_{l3\gamma} \text{ decays}}$

$$K^0(p) \to \pi^-(p')\, l^+(p_l)\, \nu_l(p_\nu)\, \gamma(q) \qquad [K^0_{l3\gamma}]$$



Matrix element

$$T = \frac{G_F}{\sqrt{2}}\, e\, V^*_{us}\, \epsilon^\mu(q)^* \Big\{ \overbrace{(\hat{V}^0_{\mu\nu} - A^0_{\mu\nu})\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, v(p_l)}^{\text{SD}}$$
$$\underbrace{+\, F^0_\nu\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5) \left[\frac{p_\mu}{pq} - \frac{(\not{p}_l + \not{q} - m_l)\,\gamma_\mu}{2 p_l q} \right] v(p_l)}_{\text{IB}} \Big\}$$

- SD amplitude:

$$V^0_{\mu\nu} = i \int \mathrm{d}^4x\, \langle \pi^-(p') \mid T\{V^{em}_\mu(x)\, V^{4-i5}_\nu(0)\} \mid K^0(p) \rangle$$
$$= \hat{V}^0_{\mu\nu} + \frac{p'_\mu}{p'q}\, F^0_\nu$$

$$\left. \begin{array}{l} \hat{V}^0_{\mu\nu} : V^0_1, \ldots, V^0_4 \\ A^0_{\mu\nu} : A^0_1, \ldots, A^0_4 \end{array} \right\} \text{infrared finite tensors}$$

- IB amplitude

$$F_\nu^0 = \langle \pi^-(p') \mid V_\nu^{4-i5}(0) \mid K^0(p) \rangle$$
$$F_\nu^0 : C_1^0,\ C_2^0$$

Infrared and collinear behaviour of the IB ampl.

- Infrared singularities: cut on photon energy

$$E_\gamma > 30 \text{ MeV} \equiv E_{cut}$$

- Collinear singularities: cut on emission angle
  $\theta_{l\gamma} > 20^\circ$

Consequences:

- most of the contribution to the rate comes from the region of small $E_\gamma,\ \theta_{l\gamma}$

- region of small $E_\gamma,\ \theta_{l\gamma}$ dominated by IB amplitude (especially if $l = e$) due to $1/E_\gamma$ behaviour.

Decay rates

$$\Gamma(K \longrightarrow \pi l \nu \gamma) = \frac{1}{2\, M_K\, (2\pi)^8} \int d_{LIPS}(p; p', p_l, p_\nu, q) \sum_{spins} \mid T \mid^2$$

$\sum_{spins} \mid T \mid^2$ is a bilinear form in the invariant amplitudes

- Fischbach and Smith (1969), Fearing et al. (1970): current algebra techniques.

- Holstein (1990): Chiral Perturbation Theory

- Bijnens, Ecker and Gasser (1993):
  Chiral Perturbation Theory at next-to-leading order.

$K^0_{l3\gamma}$ matrix element at $O(p^4)$ in CHPT

(Bijnens, Ecker, Gasser, 1993)

$O(p^2)$: tree-level diagrams from $\mathcal{L}_2$

$$
\begin{aligned}
V^0_{\mu\nu} - g_{\mu\nu} &+ \frac{p'_\mu\,(2p' + 2q + W)_\nu}{p'q} \\
A^0_{\mu\nu} &= 0 \\
F^0_\nu &= (p + p')_\nu
\end{aligned}
$$

$O(p^4)$

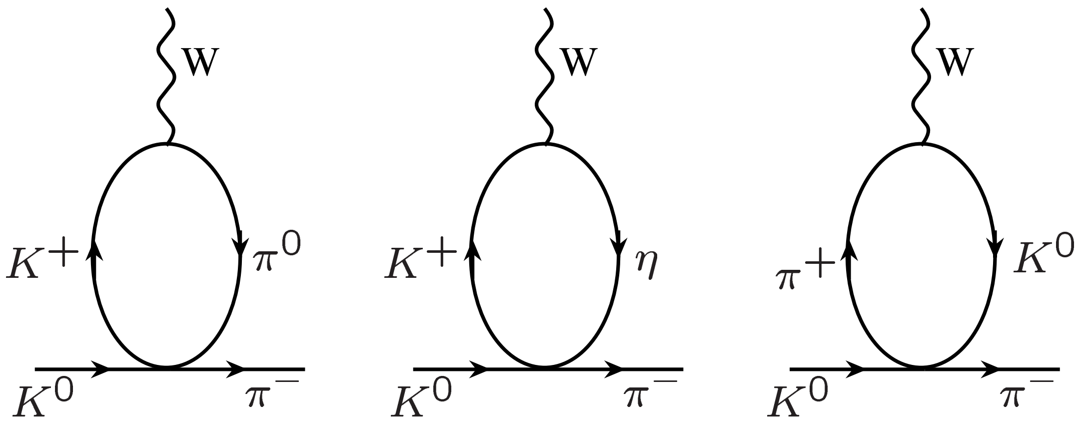
- Chiral anomaly yields axial amplitudes

$$A^0_{\mu\nu} = -\frac{i}{8\pi^2 F^2}\varepsilon_{\mu\nu\rho\sigma} q^\rho W^\sigma$$

- one-loop diagrams generated by $\mathcal{L}_2$

- tree-level diagrams generated by $\mathcal{L}_4$

One-loop diagrams



- Inner-Bremsstrahlung amplitude

$$F^0_\nu = -f^0_1(t)p'_\nu$$
$$+\left[-f^0_1(t)+\frac{1}{2}(M_K^2-M_\pi^2-t)f^0_2(t)+f^0_3(t)\right](p-p')_\nu$$

- Structure Dependent amplitude
  Invariant amplitudes V^0_i built of f^0_i and amplitudes I_1, I_2, I_3 coming from loop diagrams with photon appended on the charged lines inside the loops and on the vertices
  (non-Bremsstrahlung diagrams).

- Matrix element for $K_{l3\gamma}$ depend only on low-energy constants L_9^r, L_{10}^r (their value fixed by independent measurements).
  $\Rightarrow$ Possible to make precise predictions on $K_{l3\gamma}$ decays.
  Then careful measurement of radiative K_{l3} spectra could allow to test the standard model at the next-to-leading order in CHPT.

- KTeV result (2001):
  $$BR(K^0_{e3\gamma}) = (3.52 \pm 0.06) \times 10^{-3}$$
  "It is significantly lower than all published theoretical results" $\Rightarrow$ both theoretical and experimental checks are needed

- Measurements of $K_{l3\gamma}$ decays could be used to have another independent determination of the low-energy coupling constants L_9^r, L_{10}^r. $\Rightarrow$ it would be very useful to have a simplified version of the $O(p^4)$ contribution for better comparison with data (work in progress).

Simplification of matrix element

Approximation of loop functions

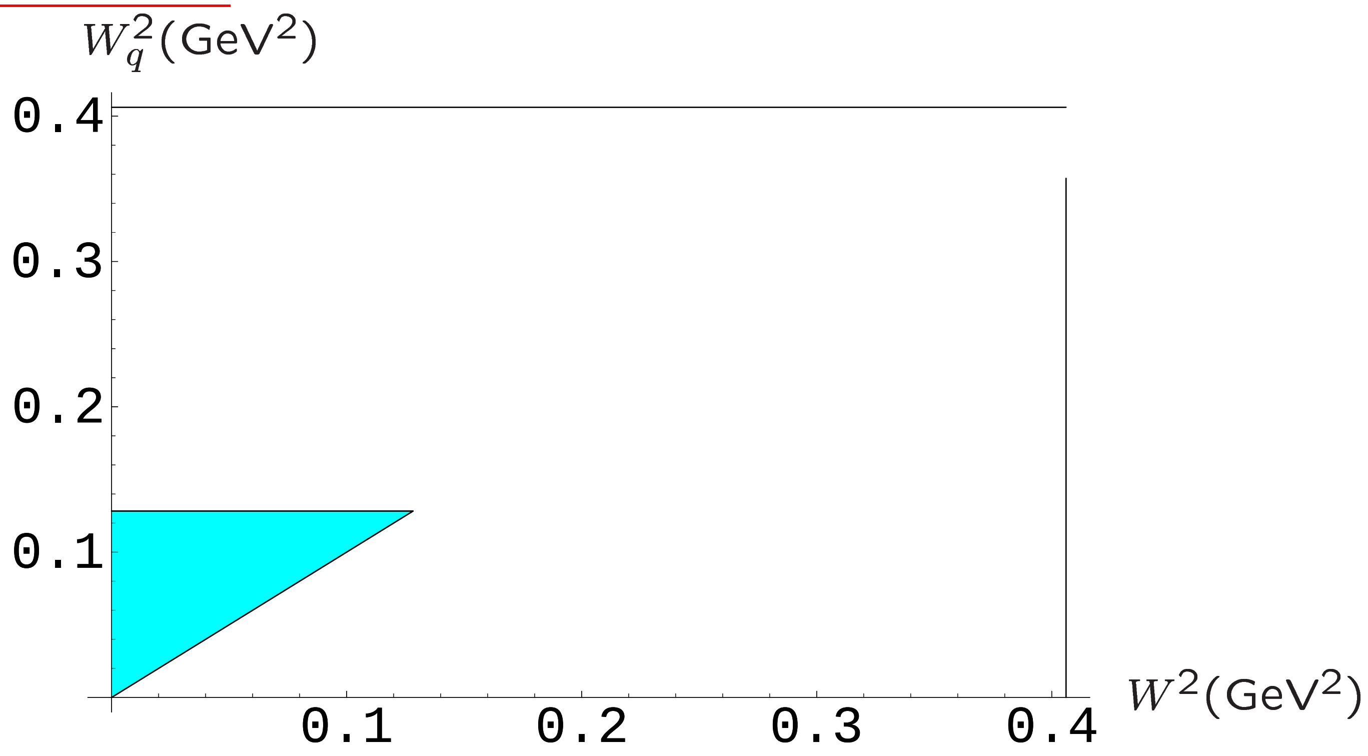
$$\bar{J}(W^2, M, m) = J(W^2, M, m) - J(0, M, m)$$
$$\bar{J}(W_q^2, M, m) = J(W_q^2, M, m) - J(0, M, m)$$

$$W^2 = (p_l + p_\nu)^2 \qquad W_q^2 = (p - p')^2$$

Branching points:

$$W_q^2,\ W^2 = (M+m)^2 \geq (M_K + M_\pi)^2$$

Dalitz Plot



Branching points far away from physical region of W^2, W_q^2

$\Rightarrow$ $\bar{J}(W^2)$, $\bar{J}(W_q^2)$ real in the physical region

$\Rightarrow$ Possible Taylor expansion around $W^2 = 0$, $W_q^2 = 0$

$$\bar{J}(t, M, m) = t\,\frac{1}{32\pi^2\,\Delta^3}\left[\Delta\,\Sigma + 2\,m^2\,M^2 \log\frac{m^2}{M^2}\right] + \ldots$$

$$t = W^2, W_q^2 \quad \Delta = M^2 - m^2 \quad \Sigma = M^2 + m^2$$

Numerical results show the validity of this approximation

A similar treatment is possible for the three-point function $C(W_q^2, W^2)$

Numerical results

FORTRAN code (Bijnens, Ecker, Gasser (1992)): provides matrix element squared SM

- Monte Carlo integration: branching ratios

$K^0_{e3\gamma}$	BR
full $O(p^4)$ amplitude	3.8095×10^{-3}
tree level	3.6039×10^{-3}

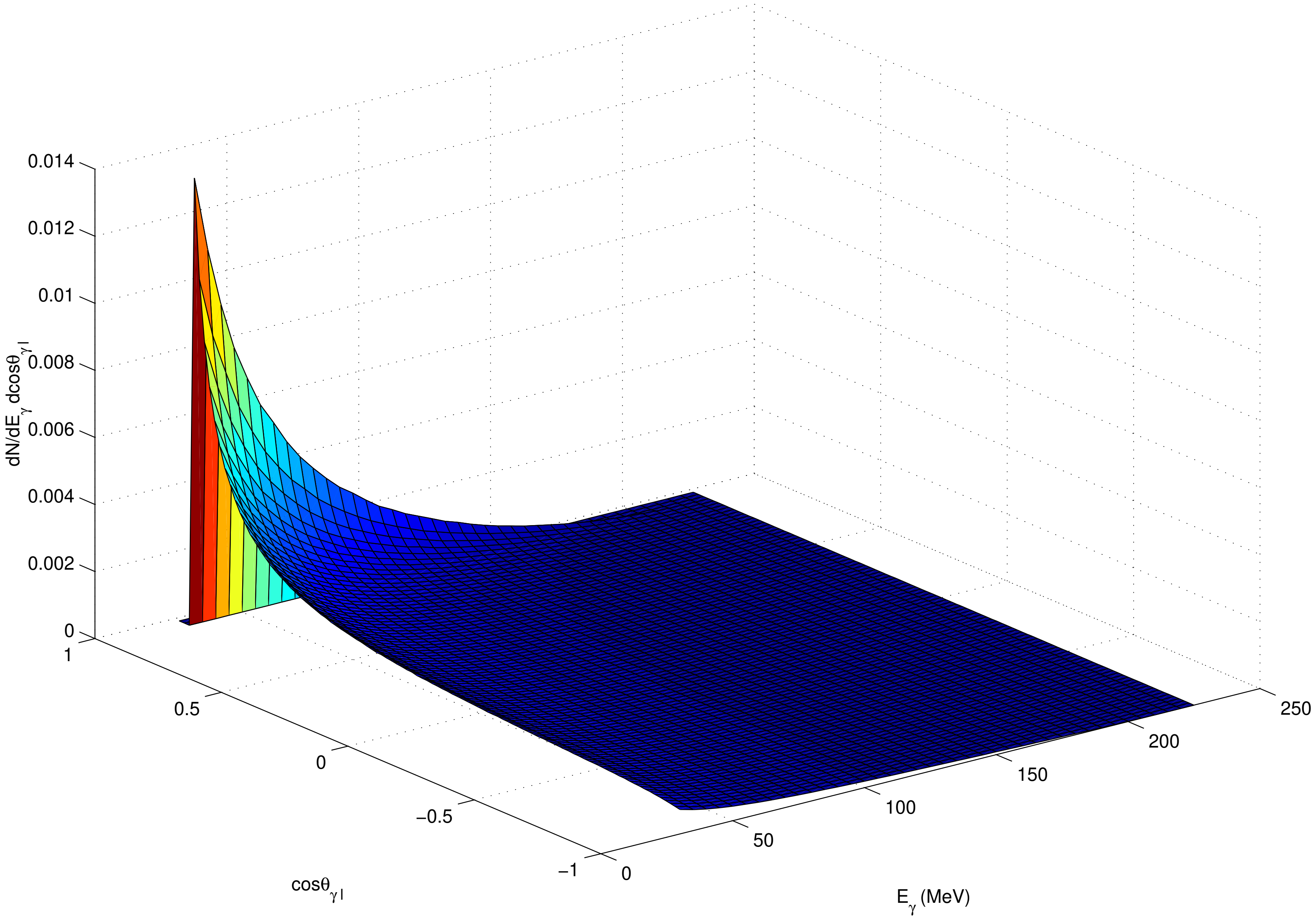
- Are $K_{e3\gamma}$ decays sensitive to low-energy coupling const. L_9^r and L_{10}^r?

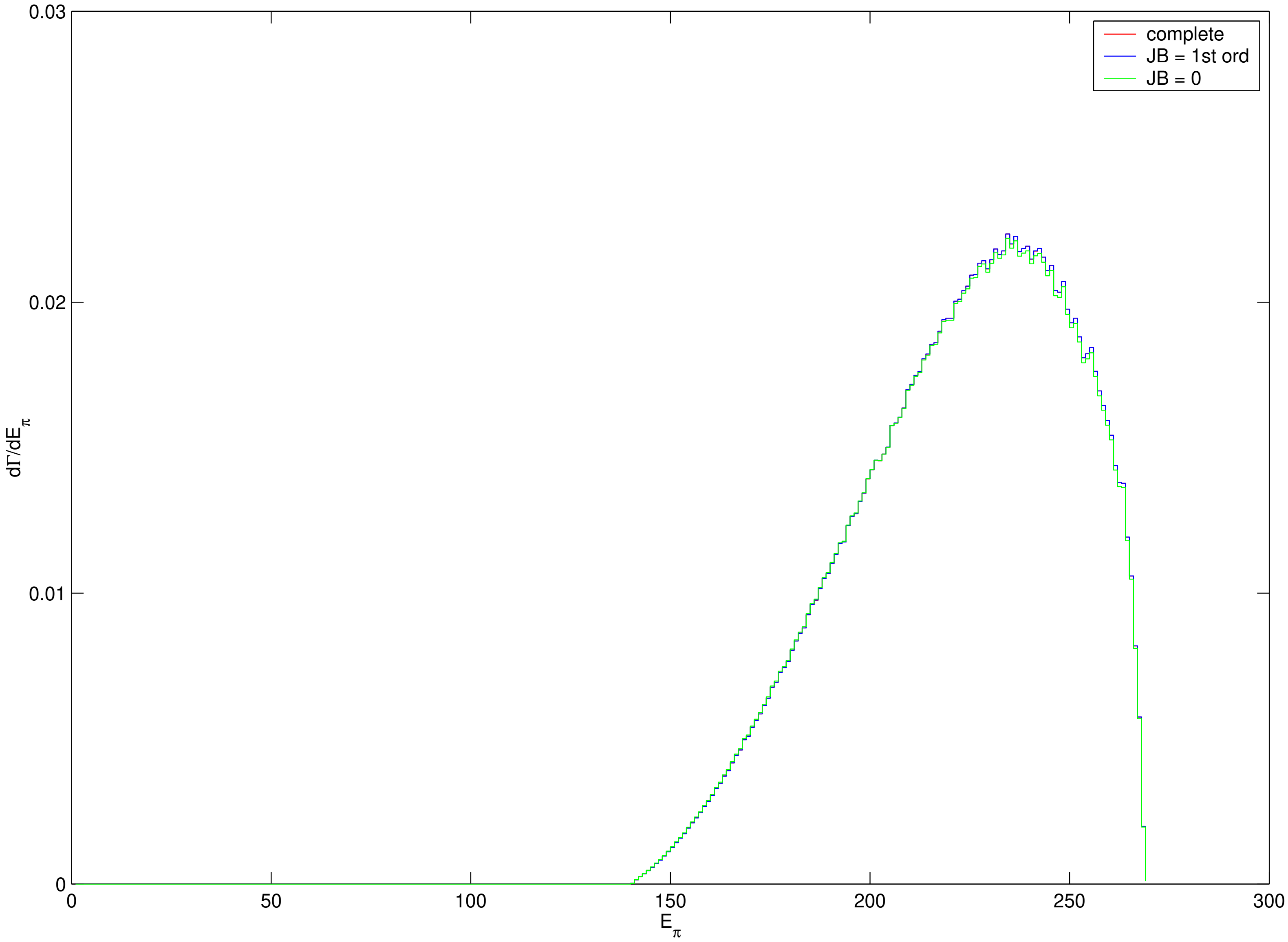
$$\Delta BR_9/BR \equiv (BR(\bar{L}_9 = 0) - BR)/BR \approx 14\% \text{ yes!}$$
$$\Delta BR_{10}/BR \equiv (BR(\bar{L}_{10} = 0) - BR)/BR \approx 0.1\% \text{ no?}$$

- Test of our approximation for the $K_{l3\gamma}$ matrix element

$K^0_{e3\gamma}$	BR
J_I at 1st order, $C = 0$	3.8096×10^{-3}
$J_I = 0,\ C = 0$	3.7931×10^{-3}

- Various single and double distributions
  - The biggest contribution comes from the region of small E_γ, $\cos\theta_{l\gamma}$.
  - The approximation of $\bar{J}$ and C is good.


0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
dN/dE$_\gamma$ dcos$\theta_{\gamma l}$
1
0.5
0
−0.5
−1
cos$\theta_{\gamma l}$
0
50
100
150
200
250
E$_\gamma$ (MeV)


complete
JB = 1st ord
JB = 0
dΓ/dE_π
E_π
0.03
0.02
0.01
0
0
50
100
150
200
250
300

Future developments

- Separation of Inner Bremsstrahlung and Structure Dependent contributions:
  - what is the their relative size?
    (We suspect that it is rather small...)
  - Is it possible to find an observable sensitive to the SD amplitude?

- Estimate of theoretical error

- Study of the cuts
  - A different choice can improve the relative size of SD/IB?

- Simplification of the $K_{l3\gamma}$ matrix element in order to facilitate the extraction of low-energy coupling constants.

- Is it possible to see the effect of axial anomaly?