AN OPEN QUANTUM SYSTEM APPROACH FOR THE B^0-$\overline{B}^0$ SYSTEM

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SUMMARY

- The B^0- $\overline{B}^0$ system
- Introduction to open quantum systems
- Time evolution of the B^0- $\overline{B}^0$ system taking into account dissipative effects
- Observables for a single meson
- Observables for two entangled mesons
- Conclusions

F. Benatti, R. Floreanini, R. R.: Nucl.Phys. B602 (2001) 541-571

THE B^0- $\overline{B}^0$ SYSTEM

It can be described in a 2 dimensional Hilbert-space: $\mid B^0\rangle$, $\mid \overline{B}^0\rangle$

Arbitrary state:

$\rho = \sum_{i=0}^{3} \rho_i \sigma_i$, where σ_i = Pauli matrices + identity

Effective hamiltonian (Weisskopf-Wigner):

$H = \begin{pmatrix} h_1 & h_3 \\ h_4 & h_2 \end{pmatrix}$ is written as:

$$H = M - \frac{i}{2}\Gamma;\ M = M^\dagger, \Gamma = \Gamma^\dagger$$

M is the mass term, Γ is the decay term

- eigenvectors: $\mid B_H\rangle$, $\mid B_L\rangle$
- eigenvalues: $\lambda_{H,L} = m_{H,L} - \frac{i}{2}\gamma_{H,L}$

$$H|B_H\rangle = \lambda_H|B_H\rangle,\ \ H|B_L\rangle = \lambda_L|B_L\rangle$$

where "H" and "L" mean *heavy* and *light*

Physical properties: B^0- $\overline{B}^0$ oscillations, CP violation

B^0- $\overline{B}^0$ OSCILLATIONS

$$|B_H\rangle \to e^{-i\lambda_H t}|B_H\rangle, \;\; |B_L\rangle \to e^{-i\lambda_L t}|B_L\rangle$$

$$|B^0\rangle \to c_H e^{-i\lambda_H t}|B_H\rangle + c_L e^{-i\lambda_L t}|B_L\rangle = c_1(t)|B^0\rangle + c_2(t)|\overline{B}^0\rangle$$

CP VIOLATION

$$CP\ |B^0\rangle = e^{i\phi}|\overline{B}^0\rangle, \;\; CP\ |\overline{B}^0\rangle = e^{-i\phi}|B^0\rangle$$

The B^0-$\overline{B}^0$ system behaves similarly to the K^0-$\overline{K}^0$ system, however:

- The CP violation is expected to be larger:
  for K mesons $\sim 0.1\%$; for B mesons $\sim 10\%$
- $\Delta\Gamma_B \ll \Delta\Gamma_K;\;\; \Delta m_B \gg \Delta m_K$ where
  $\Delta m = m_H - m_L, \;\; \Delta\Gamma = \gamma_L - \gamma_H$

Two kinds of experiments:

- B-factories (BaBar, Belle). Decay of single B-mesons
- colliders (CDF-II, HERA-B, LHC-b).
  Decay of entangled B-mesons, from: $e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0$

Probability of decay for $B^0 \to f$: $\mathcal{P}_f(B^0, t)$

T-invariance: $\mathcal{P}_{\overline{f}}(B^0, t) = \mathcal{P}_f(\overline{B}^0, t)$

CPT-invariance: $\mathcal{P}_f(B^0, t) = \mathcal{P}_{\overline{f}}(\overline{B}^0, t)$

Evaluation of asymmetries:

$$\mathcal{A}_{CPT}(t) \equiv \frac{\mathcal{P}_{\overline{f}}(\overline{B}^0,t)-\mathcal{P}_f(B^0,t)}{\mathcal{P}_{\overline{f}}(\overline{B}^0,t)+\mathcal{P}_f(B^0,t)},$$

$$\mathcal{A}_T(t) \equiv \frac{\mathcal{P}_{\overline{f}}(B^0,t)-\mathcal{P}_f(\overline{B}^0,t)}{\mathcal{P}_{\overline{f}}(B^0,t)+\mathcal{P}_f(\overline{B}^0,t)}$$

Relevant asymmetry for the study of dissipative effects:

$$\mathcal{A}_{\Delta m}(t) \equiv \frac{[\mathcal{P}_f(B^0,t)-\mathcal{P}_f(\overline{B}^0,t)]-[\mathcal{P}_{\overline{f}}(B^0,t)-\mathcal{P}_{\overline{f}}(\overline{B}^0,t)]}{\mathcal{P}_f(B^0,t)+\mathcal{P}_f(\overline{B}^0,t)+\mathcal{P}_{\overline{f}}(B^0,t)+\mathcal{P}_{\overline{f}}(\overline{B}^0,t)}$$

if f is a CP eigenstate ($\mathrm{CP}|f\rangle = \xi_f|f\rangle$, $\xi_f^2 = 1$):

$$\mathcal{A}_f(t) \equiv \frac{\mathcal{P}_f(B^0,t)-\mathcal{P}_f(\overline{B}^0,t)}{\mathcal{P}_f(B^0,t)+\mathcal{P}_f(\overline{B}^0,t)},$$

This asymmetry is relevant because it allows to determine the angle β appearing in the parametrization of the CKM matrix

In absence of direct CP violation $\mathcal{A}_f(t) = \sin(\Delta m\, t)\sin 2\beta$

Correlated mesons:

decay rate for the pair of B-mesons: $\mathcal{P}_{f_1,f_2}(t)$

where f_1 and f_2 are the products of the decay of $\mathrm{B}^0$and $\overline{\mathrm{B}}^0$respectively; t is the time difference between decays

Relevant property: $\mathcal{P}_{f,f}(0) = 0$

TIME EVOLUTION OF A Q. M. SYSTEM

described by a density matrix ρ

for a CLOSED SYSTEM

- stable: $H = H^\dagger$

$$\partial_t \rho(t) = -i\,[H,\, \rho(t)]$$

- unstable: $H \neq H^\dagger$

$$\partial_t \rho(t) = -i\, H\, \rho(t) \; + \; i\, \rho(t)\, H^\dagger$$

for an OPEN SYSTEM

Assumptions: weak interaction, absence of initial correlations

- stable

$$\partial_t \rho(t) = -i\,[H,\, \rho(t)] \; + \; L\Big[\rho(t)\Big]$$

- unstable

$$\partial_t \rho(t) = -i\, H\, \rho(t) \; + \; i\, \rho(t)\, H^\dagger \; + \; L\Big[\rho(t)\Big]$$

L is the well-known "Kossakowski-Lindblad form":

$$L\Big[\rho(t)\Big] = -\tfrac{1}{2}\{\textstyle\sum_i A_i^\dagger A_i, \rho(t)\} + \sum_i A_i \rho(t) A_i^\dagger$$

where A_i is a set of bounded operators characterizing the environment

$$\rho(0) \to \rho(t) = \Lambda_t\left[\rho(0)\right]$$

It generalizes the standard evolution: $\Lambda_t\Big[\rho(0)\Big] = e^{-iHt}\rho(0)\, e^{iH^\dagger t}$

Properties of the one-parameter linear maps Λ_t:

- semigroup composition law: $\Lambda_{t+s} = \Lambda_t \circ \Lambda_s$, with $t, s \geq 0$
- loss of coherence: pure states $\to$ mixed states
- completely positive evolution

NOTES:

- if the interaction is not weak, the evolution equation is not linear in time (memory effects)
- H is really redefined
- in an open system the entropy can increase or decrease

Request: increase of the von Neumann entropy

Change of notation:

$$\rho = \sum_{i=0}^{3} \rho_i \sigma_i \ \rightarrow \ |\rho\rangle \ = \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}$$

The time evolution becomes: $\partial_t |\rho(t)\rangle \ = \ (\mathcal{H} \ + \mathcal{L}) \, |\rho(t)\rangle$

with $\mathcal{H}$ and $\mathcal{L}$ 4×4 matrices:

$$\mathcal{H} = \begin{pmatrix} -\Im(h_1+h_2) & \Im(h_2-h_1) & \Re(h_3-h_4) & -\Im(h_3+h_4) \\ \Im(h_2-h_1) & -\Im(h_1+h_2) & \Re(h_3+h_4) & \Im(h_4-h_3) \\ \Re(h_3-h_4) & -\Re(h_3+h_4) & \Im(h_2-h_1) & \Re(h_1-h_2) \\ -\Im(h_3+h_4) & \Im(h_3-h_4) & \Re(h_2-h_1) & \Im(h_2-h_1) \end{pmatrix}$$

$$\mathcal{L} = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}$$

The Complete Positivity is expressed by the inequalities:

$$\begin{gathered} 2R \equiv \alpha + \gamma - a \ \geq \ 0 \ , \ \ RS \geq b^2 \\ 2S \equiv a + \gamma - \alpha \ \geq \ 0 \ , \ \ RT \geq c^2 \\ 2T \equiv a + \alpha - \gamma \ \geq \ 0 \ , \ \ ST \geq \beta^2 \\ RST \ \geq \ 2bc\beta + R\beta^2 + Sc^2 + Tb^2 \end{gathered}$$

$\mathcal{L}$ can be considered as a small perturbation of $\mathcal{H}$

$\Rightarrow \rho(t)$ can be obtained using perturbation theory:

$$|\rho(t)\rangle = e^{\mathcal{H}t}|\rho(0)\rangle + \int_0^t ds e^{\mathcal{H}(t-s)}\mathcal{L}\, e^{\mathcal{H}s}|\rho(0)\rangle + ...$$

NOTES:

- rough estimate of the dissipative terms: $\mathcal{L}_{ij} \sim \frac{E_s^2}{E_e}$, where E_s and E_e are the characteristic energies of the system and of the environment respectively
- only first order in perturbation theory
- the computation is simpler in the $|\mathrm{B_L}\rangle$ - $|\mathrm{B_H}\rangle$ base
- in this base the evolution is manifestly phase independent

DECAY RATE FOR B^0 AND $\overline{\mathrm{B}}^0$

In the density matrix formalism, the mean value of an observable $\mathcal{O}$ is obtained from:

$$\langle \mathcal{O}(t) \rangle = \mathrm{Tr}[\mathcal{O}\rho(t)]$$

The decay rates for B^0,$\overline{\mathrm{B}}^0 \rightarrow f$ are:

$$\mathcal{P}_f(\mathrm{B}^0, t) = \mathrm{Tr}(\mathcal{O}_f)\rho(t), \quad \text{with } \rho(0) = |\mathrm{B}^0\rangle\langle \mathrm{B}^0|$$

$$\mathcal{P}_f(\overline{\mathrm{B}}^0, t) = \mathrm{Tr}(\mathcal{O}_f)\rho(t), \quad \text{with } \rho(0) = |\overline{\mathrm{B}}^0\rangle\langle \overline{\mathrm{B}}^0|$$

where the operator $\mathcal{O}_f$ is defined by:

$$\mathcal{O}_f = \begin{pmatrix} o_1 & o_3 \\ o_4 & o_2 \end{pmatrix},$$

$$o_1 = |\mathcal{A}(\mathrm{B}^0 \rightarrow f)|^2, \; o_3 = [\mathcal{A}(\mathrm{B}^0 \rightarrow f)]^* \mathcal{A}(\overline{\mathrm{B}}^0 \rightarrow f),$$
$$o_2 = |\mathcal{A}(\overline{\mathrm{B}}^0 \rightarrow f)|^2, \; o_4 = [\mathcal{A}(\overline{\mathrm{B}}^0 \rightarrow f)]^* \mathcal{A}(\mathrm{B}^0 \rightarrow f)$$

$\mathcal{A}$ is the decay amplitude for the considered processes

The standard dynamics is characterized by:

$$\Delta m = m_H - m_L, \ \ \Delta\Gamma = \gamma_L - \gamma_H, \ \ \Gamma = \frac{\gamma_L+\gamma_H}{2}$$

$$\omega = \frac{\Delta m}{\Gamma}, \ \ \delta = \frac{\Delta\Gamma}{2\Gamma}, \ \ \tau = t\,\Gamma$$

$$\lambda_f = e^{i\psi}\,\frac{\mathcal{A}(\overline{B^0}\to f)}{\mathcal{A}(B^0\to f)}$$

The dissipative dynamics is characterized by:

$A, B, C, D =$ functions of $(a, b, c, \alpha, \beta, \gamma)$

Of particolar interest are the semileptonic decays and the decays in CP eigenstates

SEMILEPTONIC DECAYS: $\mathrm{B}^0(\overline{\mathrm{B}}^0)\to h^- l^+ \nu\,(h^+ l^- \overline{\nu})$

$$\mathcal{A}(\mathrm{B}^0\to h^- l^+ \nu) = \mathcal{M}_h(1-y_h),$$
$$\mathcal{A}(\overline{\mathrm{B}}^0\to h^+ l^- \overline{\nu}) = \mathcal{M}_h^*(1+y_h^*),$$
$$\mathcal{A}(\mathrm{B}^0\to h^+ l^- \overline{\nu}) = z_h\,\mathcal{A}(\overline{\mathrm{B}}^0\to h^+ l^- \overline{\nu}),$$
$$\mathcal{A}(\overline{\mathrm{B}}^0\to h^- l^+ \nu) = x_h\,\mathcal{A}(\mathrm{B}^0\to h^- l^+ \nu)$$

x_h and z_h measure the violations of the rule $\Delta B = \Delta Q$;

y_h measures the deviations from CPT invariance

Some results assuming:

- CPT invariance ($y_h = 0$)
- no violation of the $\Delta B = \Delta Q$ rule ($x_h = z_h = 0$)

$$
\begin{aligned}
\mathcal{P}_{h^+}(B^0, \tau) \quad = \quad & \frac{|\mathcal{M}_h|^2}{2} \, e^{-\tau} \Big[- \cos \omega \tau \; e^{-(A-D)\tau} + \\
& + \; \sin \omega \tau (\frac{4\,\delta}{\delta^2 + \omega^2} \, \Im\,(C) - \Re\,(B)) + \\
& + \cosh \delta \tau + \\
& + \sinh \delta \tau (\frac{D}{\delta} \; - \frac{4\,\omega}{\delta^2 + \omega^2} \, \Im\,(C)) \Big]
\end{aligned}
$$

$$
\mathcal{A}_{\Delta m}(\tau) \equiv \frac{[\mathcal{P}_{h^-}(B^0,\tau) - \mathcal{P}_{h^-}(\overline{B^0},\tau)] - [\mathcal{P}_{h^+}(B^0,\tau) - \mathcal{P}_{h^+}(\overline{B^0},\tau)]}{\mathcal{P}_{h^-}(B^0,\tau) + \mathcal{P}_{h^-}(\overline{B^0},\tau) + \mathcal{P}_{h^+}(B^0,\tau) + \mathcal{P}_{h^+}(\overline{B^0},\tau)},
$$

$$
\mathcal{A}_{\Delta m}(\tau) = e^{-A\,\tau} \cos \omega \tau + \Re\,(B) \; \sin \omega \tau
$$

Time integrated rates: $\mathcal{P}_f(B) = \frac{1}{\Gamma} \int_0^\infty d\tau \mathcal{P}_f(B,\tau)$

$$
\mathcal{A}'_{\Delta m} = \frac{1}{1+\omega^2} \left\{ 1 + \omega\, \Re\,(B) + \frac{1}{1+\omega^2} \left[(\omega^2 - 1)\, A - 2\omega^2 D \right] \right\}
$$

DECAYS IN CP EIGENSTATES: $B^0, \overline{B}^0 \to f$; $CP|f\rangle = \xi_f |f\rangle$

(for example: f = D^+D^-, $\pi^+\pi^-$, J/ψ K states)

$$\mathcal{A}_f(t) \equiv \frac{\mathcal{P}_f(B^0,t) - \mathcal{P}_f(\overline{B^0},t)}{\mathcal{P}_f(B^0,t) + \mathcal{P}_f(\overline{B^0},t)},$$

$$\mathcal{A}_f(t)_{CPconserved} = 0$$

$$\begin{aligned}\mathcal{A}_f(t) \quad = \quad & \frac{2\xi_f}{\omega}\Im(C) - \sin\omega\tau\left(\frac{2\Im(\lambda_f)}{1+|\lambda_f|^2} + \frac{2\xi_f}{\omega}\Re(C)\right) + \\ & + \cos\omega\tau\left(\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} - \frac{2\xi_f}{\omega}\Im(C)\right)\end{aligned}$$

NOTE:

In absence of direct CP violation ($|\lambda_f| = 1$):

$\mathcal{A}_f(t)_{standard} = \sin\omega\tau \sin 2\beta$, where $\sin 2\beta \equiv -\Im(\lambda_f)$

Then this parameter allows the determination of β, parameter of the CKM matrix

CORRELATED MESONS

Production: $\Upsilon(4S) \rightarrow B^0\overline{B}^0$

Initial state: $\rho(0) = |\psi_B\rangle\langle\psi_B|$

$$|\psi_B\rangle = \frac{1}{\sqrt{2}}\left(|\,B^0, -p\rangle \otimes |\overline{B}^0, p\rangle - |\overline{B}^0, -p\rangle \otimes |B^0, p\rangle\right)$$

Time evolution: $\rho(t_1, t_2) = (\Lambda_{t_1} \otimes \Lambda_{t_2})\rho(0)$

Decay rate for f_1, f_2 final states:

$$\mathcal{P}(f_1, t_1; f_2, t_2) = \mathrm{Tr}[(\mathcal{O}_{f_1} \otimes \mathcal{O}_{f_2})\, \rho(t_1, t_2)]$$

Single time distributions (with $t_1 = t' + t$, $t_2 = t'$):

$$\mathcal{P}_{f_1,f_2}(t) = \int_0^{+\infty} dt'\, \mathcal{P}(f_1, t' + t; f_2, t')$$

For negative t:

$$\mathcal{P}_{f_1,f_2}(-|t|) = \int_0^{+\infty} dt'\, \mathcal{P}(f_1, t' + t; f_2, t')\, \theta(t' - |t|)$$

DOUBLE SEMILEPTONIC DECAY:

Without dissipation: $\mathcal{P}_{h^+,h^+}(0) = \mathcal{P}_{h^-,h^-}(0) = 0$

$$\mathcal{R}(t) \equiv \frac{\mathcal{P}_{h^+,h^+}(t) + \mathcal{P}_{h^-,h^-}(t)}{\mathcal{P}_{h^+,h^-}(t) + \mathcal{P}_{h^-,h^+}(t)}$$

$$\mathcal{R}(0)_{standard} = 0$$

$$\mathcal{R}(0) = \frac{1}{2}\left[A + \frac{\omega}{1+\omega^2}\left(\omega\,\Im\left(B\right) - \Re\left(B\right)\right)\right]$$

$$\mathcal{A}_{\Delta m}(\tau) \equiv \frac{\left[\mathcal{P}_{h^+,h^-}(\tau) + \mathcal{P}_{h^-,h^+}(\tau)\right] - \left[\mathcal{P}_{h^+,h^+}(\tau) + \mathcal{P}_{h^-,h^-}(\tau)\right]}{\mathcal{P}_{h^+,h^+}(\tau) + \mathcal{P}_{h^-,h^-}(\tau) + \mathcal{P}_{h^+,h^-}(\tau) + \mathcal{P}_{h^-,h^+}(\tau)}$$

$$\mathcal{A}_{\Delta m}(\tau) = \tfrac{e^{-A\tau}}{1+A}\cos\omega\tau + \left(\omega\cos\omega\tau + \sin\omega\tau\right)\Re\left(\tfrac{B}{1-i\,\omega}\right)$$

NOTE:

$$\mathcal{A}_{\Delta m}(\tau)_{standard} = \cos\omega\tau$$

This parameter allows the determination of Δm

CONCLUSIONS

- We have treated the B^0- $\overline{B}^0$ system in the framework of the open quantum systems. Considering the nature of the interaction, we only assumed entropy increase during the time evolution.
- Considering the physical quantities describing this system, we have outlined the main differences between the open quantum systems approach and the standard one.
- Present and future planned experiments should give us some upper bounds to the introduced parameters. If a fit should show them non compatible with zero, the inequalities expressing the Complete Positivity could be tested.