Ground-state energy of pionic hydrogen to one loop

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Outline

- Introduction
- Experiment
- Current theoretical status
- What we are going to calculate?
- πN scattering amplitude at threshold
- Evaluation of S-matrix
- Ground state energy shift
- Comparision with model calculations
- Conclusion

Introduction

- Experiments:

  i) Muon anomalous magnetic moment (Brookhaven);

  ii) DIRAC experiment at CERN;

  iii) $\pi^- p$ atom ground state energy shift and width at PSI at the percent level.

- Theoretical side:

  i) Potential Model Approach;

  ii) Effective Theory of QED+QCD.

Pionic hydrogen

- The Pionic Hydrogen has some unique features:

  i) $\pi^- p$ atom is one of the simplest hadronic systems bound electromagnetically.

  ii) The electromagnetic interaction in exotic hydrogen atom is well understood. Any deviation caused by another interaction can therefore be studied with higher precision.

  iii) Scattering experiments are restricted to energies above $10 MeV$, whereas the exotic atom's method directly measures linear combinations of the isospin separated scattering lengths at almost zero energies and offers the highest intrinsic accuracy of all methods.

Recent theoretical investigations

- Potential Model: Gashi et. al.

$$\delta_\epsilon = (-2.1 \pm 0.5) \cdot 10^{-2}.$$

- Chiral perturbation theory(ChPT):
  1) A. Rusetsky & v. Lyubovitskij at order $O(p^2)$,
  Phys. Lett. **B 494**, 9 (2000)

$$\delta_\epsilon = (-4.8 \pm 2.0) \cdot 10^{-2},$$

2) N. Fettes & U.-G. Meißner HBChPT,
Nucl. Phys. **A 693** 693 (2001)

No explicit expression!

Energy shift

- The experiment has measured the transition energy E_{3p-1s}^{meas}

$$\epsilon_{1s} = E_{3p-1s}^{em} - E_{3p-1s}^{meas}$$

$$E_{3p-1s}^{em} = 2.8788 Kev$$

The aim of the new experiment at PSI is to measure ϵ_{1s} to a better than 1% precision.

- ϵ_{1s} is given by the following expression:

$$\epsilon_{1s} = -2\alpha^3\mu_c^2(a_{0+}^{+} + a_{0+}^{-})(1 + \underline{\delta_\epsilon})$$

- We introduce a common counting rule for the IS breaking effects $\delta \sim \alpha \sim (m_u - m_d)$

πN scattering amplitude at threshold

- Let us introduce the following splitting for threshold amplitude:

$$\mathcal{T}_{\pi N} = \mathcal{T}^0_{\pi N} + \delta\mathcal{T}$$

where $\mathcal{T}^0_{\pi N}$ — $M_{\pi^+} = M_{\pi^0}$, $m_p = m_n$, pure QCD!

$$\delta_\epsilon = \frac{\delta\mathcal{T}}{4\pi(1 + M_{\pi^+}/m_p)(a^+_{0+} + a^-_{0+})} + K + \delta^{vac}_\epsilon$$

$$K = -2\alpha(\ln\alpha - 1)\mu_c(a^+_{0+} + a^-_{0+})$$

- we need ϵ_{1s} at $O(\delta^4) \Rightarrow \delta\mathcal{T}$ must be known at $O(\delta)$. $\delta\mathcal{T}$ — up to $O(r)$ $\quad r = M_{\pi^+}/m_p$.

$$\delta\mathcal{T} = \delta\mathcal{T}_2 + \delta\mathcal{T}_3 + O(p^4)$$

One-loop and VP corrections

- We split $\delta\mathcal{T}$ into leading and one-loop parts

$$\delta\mathcal{T} = \delta\mathcal{T}_2 + \delta\mathcal{T}_3$$

- The leading term $\delta\mathcal{T}_2$ has been determined (Rusetsky&Lyubovitskij).
- Here we evaluate $\delta\mathcal{T}_3$, which amounts to a one-loop calculations of the amplitude.
- The VP correction is known (Eiras&Soto)

$$\delta_\epsilon^{vac} = 2\frac{\delta\Psi(0)}{\Psi(0)} = 0.48\%$$

$\Psi(0)$ — $\pi^- p$ atom WF at the origin, $\delta\Psi(0)$ — VP corrections to WF.

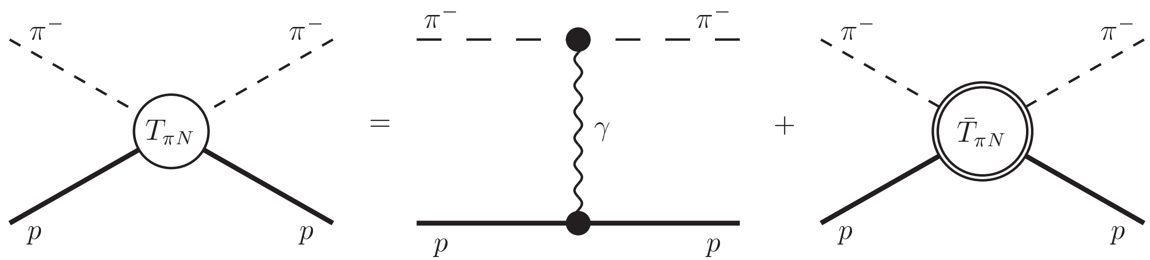
Pion proton scattering

- Let us consider the process

$$\pi^-(q) + p(p) \to \pi^-(q') + p(p')$$

- We are working at threshold

$$p^2 = p'^2 = m_p^2, \quad q^2 = q'^2 = M_{\pi^+}^2,$$
$$(s + t + u) = 2(m_p^2 + M_{\pi^+}^2).$$

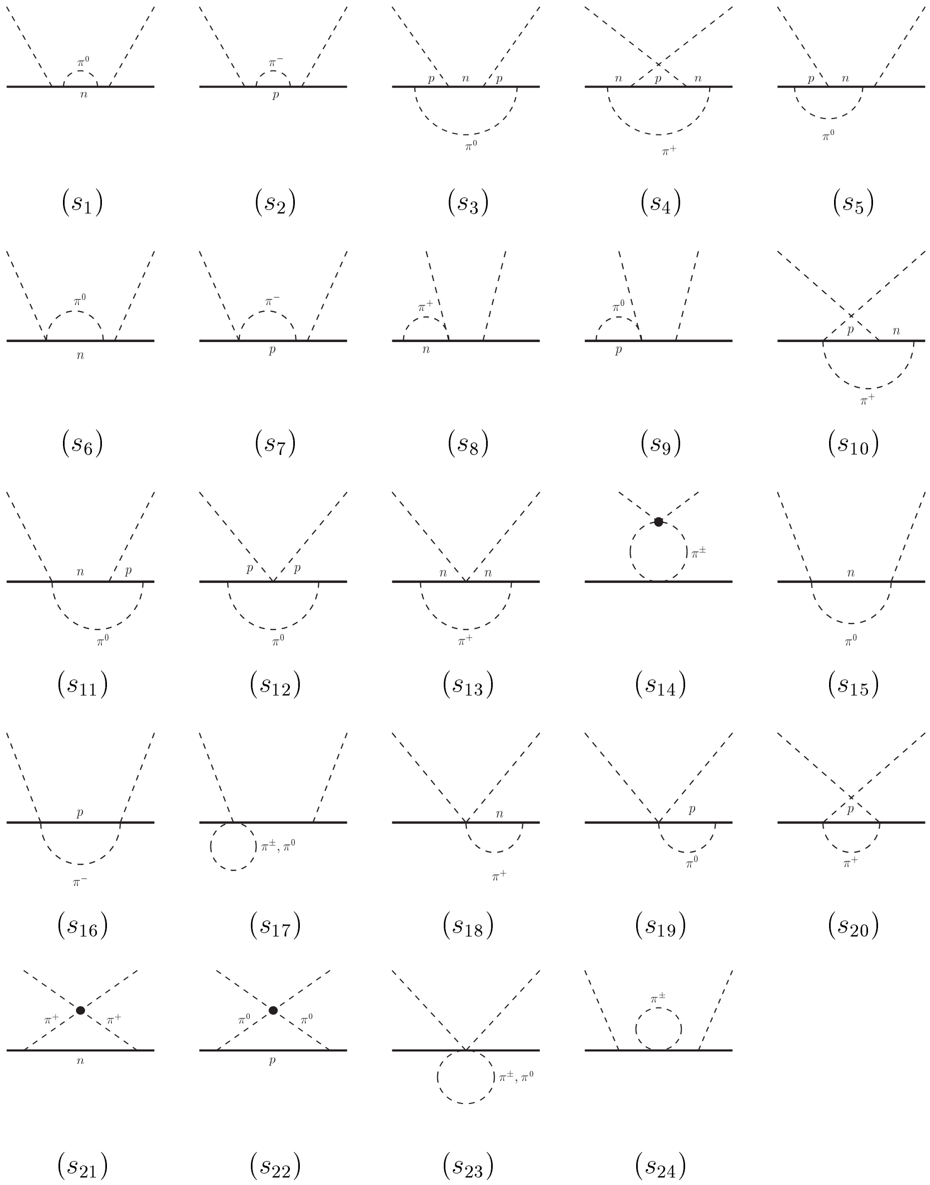


Lagrangians

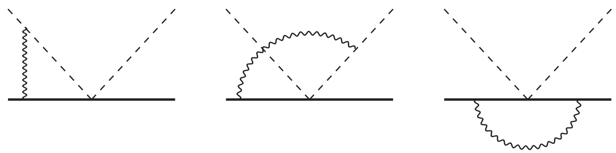
- The full Lagrangian consists of meson and nucleon parts, as well as the free photon Lagrangian together with gauge-fixing term:

$$
\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}_\pi + \mathcal{L}_N + \mathcal{L}_\gamma \,, \\
\mathcal{L}_\pi &= \mathcal{L}_\pi^{(p^2)} + \mathcal{L}_\pi^{(e^2)} + \mathcal{L}_\pi^{(p^4)} + \mathcal{L}_\pi^{(e^2p^2)} + \cdots \,, \\
\mathcal{L}_N &= \mathcal{L}_N^{(p)} + \mathcal{L}_N^{(p^2)} + \mathcal{L}_N^{(e^2)} + \mathcal{L}_N^{(p^3)} + \mathcal{L}_N^{(e^2p)} + \cdots \,.
\end{aligned}
$$

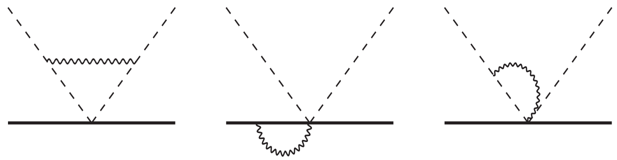
The strong one loop diagrams



The electromagnetic vector-type & axial-type diagrams

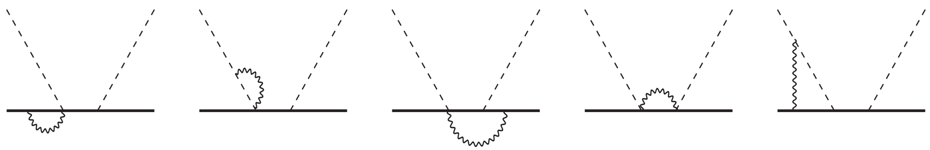


(v_1) (v_2) (v_3)

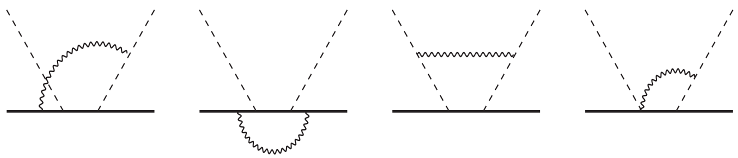


(v_4) (v_5) (v_6)

&



(a_1) (a_2) (a_3) (a_4) (a_5)



(a_6) (a_7) (a_8) (a_9)

Becher-Leutwyler method

- Infrared regularization

$$M^I = \int_0^\infty dx F = M^S - M^R,$$
$$M^S = \int_0^1 dx F, M^R = -\int_1^\infty dx F$$

- Heavy and Light lines in the diagrams are combined together separately by using Feynman parameterization.
- The pertinent denominators are then combined with a single Feynman parameter.
- All divergences in M^R at $d \to 4$ are ultraviolet.

Becher-Leutwyler method

- At the end, all infrared divergences cancel in $\mathcal{T}_{\pi\mathcal{N}}$.
- Ultraviolet divergences of the one-loop diagrams cancel against the divergent parts of the LECs, as a result of which the quantity$\mathcal{T}_{\pi n}$ IR and UV finite.
- Graphs with no virtual nucleons need no infrared regularization.
- Graphs with closed nucleon lines vanish in infrared regularization.

The threshold amplitude at order p^3

- The final expression for the threshold scattering amplitude is given by:

$$\mathcal{T}_{\pi N} = Z_\pi Z_N \mathcal{T}^B + \mathcal{T}^V + \mathcal{T}^A + \mathcal{T}^S + \mathcal{T}^{ct}$$

- Not all diagrams contribute to the threshold amplitude at $O(p^3)$.
- In final result UV divergences cancel with divergent parts of the LECs, whereas the IR divergences cancel among themselves.
- The final result is thus given in terms of renormalized LECs $l_i^r, k_i^r, d_i^r, g_i^r$.

The low energy constants

- The leading order contribution $\delta\mathcal{T}_2$ contains $f_1,\ f_2,\ c_1$

$$c_1 = -(0.93 \pm 0.07) Gev^{-1}$$

- f_2 we determine from the electromagnetic part of proton-neutron mass difference

$$-e^2 F^2 f_2 = (m_p - m_n) \qquad f_2 = -(0.97 \pm 0.38) Gev^{-1}$$

- f_1 we define from the combination $X = T^{\pi^+ p \to \pi^+ p} + T^{\pi^- p \to \pi^- p} - 2T^{\pi^0 p \to \pi^0 p}$.

$$F^2 e^2 |f_1| \simeq \frac{\alpha}{2\pi} m_p \qquad |f_1| \simeq 1.4 Gev^{-1}$$

The low energy constants

- The next-to-leading order contribution $\delta\mathcal{T}_3$ contains $d_5^r,\ k_1^r + k_2^r,\ g_6^r + g_8^r$
- For the combinations $k_1^r + k_2^r$ we use standard dimensional arguments for an estimate of its size, $k_i^r \simeq \frac{1}{16\pi^2}$, and apply the same rule to $g_6^r + g_8^r$

$$|k_1^r + k_2^r| \simeq \frac{2}{16\pi^2} \qquad |g_6^r + g_8^r| \simeq \frac{2}{16\pi^2}$$

- The value for d_5^r is given in Ref. T. Becher and H. Leutwyler, JHEP (2001)

$$16 M_\pi^2 d_5^r(\mu) = 0.04 \pm 0.06 - \frac{2M_\pi^2}{F^2} l_4^r(\mu), \quad \mu = 1Gev$$

- $l_4^r(1Gev) = 2.9 \cdot 10^{-3}$ Colangelo et al. $(2000)\&(2001)$.

Ground-state energy-level shift

- For the scattering length $a_{0+}^{\pm}$

$$a_{0+}^{+} + a_{0+}^{-} = 0.0883 M_{\pi}^{-1}$$

- Individual terms are scale dependent, whereas full result is not.
- The main part of strong contributions are due to the three graphs $(s_{19}), (s_{21}), (s_{22})$
- $\delta\mathcal{T}^{ct}$ — suppressed!
- the final numerical result is

$$\delta_{\epsilon} = (-7.2 \pm 2.9) \cdot 10^{-2}$$

- The large uncertainty is dominated by the coupling f_1.

Comparison with model calculations

- We compare our result with the calculations in the potential model and in the quark model.
- The Potential model predicts

$$\delta_\epsilon = (-2.1 \pm 0.5) \cdot 10^{-2}$$

- Estimation of low energy constants in quark model calculations leads finally to

$$\delta_\epsilon = -4.3 \cdot 10^{-2}$$

Summary and conclusions

- We have calculated the $\pi^{-}p$ elastic scattering amplitude at threshold at $O(p^3)$, in the presence of isospin breaking.
- We have generalized the procedure of IR regularization of relativistic ChPT in the presence of the virtual photons in the theory.
- The loop contributions, which contain mass splitting effects from diagrams with strong loops, turn out to be sizeable.
- A precise determination of scattering Lengths $a_{0+}^{+} + a_{0+}^{-}$ from precise measurement of ground-state energy-level shift of pionic hydrogen has to await a more precise determination of f_1.