Generating functional for $\mathrm{SU(2)_R \times SU(2)_L \times U(1)_V}$ with virtual photons

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Effective Lagrangian

One loop functional

$\pi - \pi$ scattering amplitude

Summary and Outlook

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Work done in collaboration with J. Gasser

Effective Lagrangian

- Integration variables: $U \in \mathrm{SU}(2)$, A_μ
- External fields: v_μ, a_μ, p, s
- Leading order

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\langle d^\mu U^\dagger d_\mu U + \chi^\dagger U + U^\dagger \chi\rangle$$
$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}(\partial^\mu A_\mu)^2 + C\langle QUQU^\dagger\rangle$$

Ecker, Gasser, Pich & de Rafael 89

$$\begin{aligned} d_\mu U &= \partial_\mu U - iA_\mu[Q,U] - iR_\mu U + iUL_\mu \\ \chi &= 2B\mathrm{diag}(m_u, m_d) + \ldots \\ Q &= \frac{e}{3}\mathrm{diag}(2,-1) \end{aligned}$$

- Power counting: $m_q = \mathcal{O}(p^2)$, $e = \mathcal{O}(p)$

Urech 95

- Pion masses at leading order

$$\begin{aligned} M_+^2 &= B(m_u + m_d) + \frac{2Ce^2}{F^2} \\ M_0^2 &= B(m_u + m_d) \end{aligned}$$

Equation of motion

- Field equations for pions $\bar{U} = \exp i/F\bar{\phi}^i\tau^i$ and photon field $\bar{A}_\mu$

$$
\begin{aligned}
(\Box + M_i^2)\bar{\phi}^i &= 2BFp^i + F\partial^\mu a^i_\mu + \ldots \\
\Box \bar{A}_\mu &= j_\mu
\end{aligned}
$$

- Counting scheme

$$
\begin{aligned}
a_\mu, p &\sim \mathcal{O}(\phi) \\
v_\mu, s &\sim \mathcal{O}(\phi^2)
\end{aligned}
$$

Gasser & Leutwyler 84

$\Rightarrow$ classical solutions $\bar{\phi} = \mathcal{O}(\phi)$, $\bar{A}_\mu = \mathcal{O}(\phi^2)$

One loop functional

- Expansion around classical solutions $\bar{U} = u^2$ and $\bar{A}_\mu$

$$U = u(1 + i\xi - \tfrac{1}{2}\xi^2 + \cdots)u, \qquad \xi = \frac{1}{F}\xi^i\tau^i$$
$$A_\mu = \bar{A}_\mu + \epsilon_\mu$$

- Collecting fluctuations ξ^i, ϵ_μ in real vector $\eta = (\xi^1, \ldots, \xi^3, \epsilon^1, \ldots, \epsilon^4)$
- Euclidean action to $\mathcal{O}(\xi^2, \epsilon^2, \epsilon\xi)$

$$\int dx_E \mathcal{L}_E^{(2)} = \int dx_E \bar{\mathcal{L}}_E^{(2)} + \tfrac{1}{2}\int dx\, \eta D_E \eta$$

Urech 95

- Generating functional $Z_E = Z_E(\bar{U}, \bar{A}_\mu, v_\mu, a_\mu, s, p)$

$$Z_{E\text{oneloop}} = \frac{1}{2}\ln \det D_E.$$

- Differential operator $D_E = D_0 + \delta$

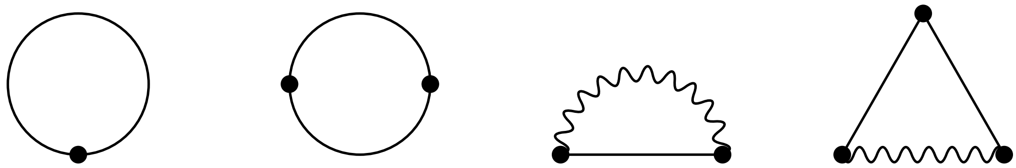
$$D_0 = \begin{pmatrix} (-\partial_\mu\partial_\mu + M_i^2)\delta^{ik} & \\ & -\partial_\mu\partial_\mu\delta^{\sigma\rho} \end{pmatrix}$$

$\delta = \delta(\bar{U}, \bar{A}_\mu, v_\mu, a_\mu, s, p) \to 0$ if external fields are switched off

- Expansion in powers of δ

$$
\begin{aligned}
Z_{E\text{oneloop}} &= \tfrac{1}{2}\text{lndet}D_0 + \tfrac{1}{2}\langle D_0^{-1}\delta\rangle \\
&\quad -\tfrac{1}{4}\langle D_0^{-1}\delta D_0^{-1}\delta\rangle \\
&\quad +\tfrac{1}{6}\langle D_0^{-1}\delta D_0^{-1}\delta D_0^{-1}\delta\rangle + \cdots
\end{aligned}
$$

- One loop diagrams ($\mathcal{O}(e^4)$ neglected)



The dots denote polynomials in $\bar{\phi}, \bar{A}_\mu$ and in the external fields

- Explicit generating functional to one loop

$$Z = Z_{\text{oneloop}} + \int dx(\bar{\mathcal{L}}^{(2)} + \bar{\mathcal{L}}^{(4)})$$

- Counterterms at $\mathcal{O}(p^4, e^2p^2)$

$$\mathcal{L}^{(4)} = \mathcal{L}_{p^4} + \mathcal{L}_{e^2p^2}$$

Gasser & Leutwyler 84

Meissner, Müller & Steininger 97

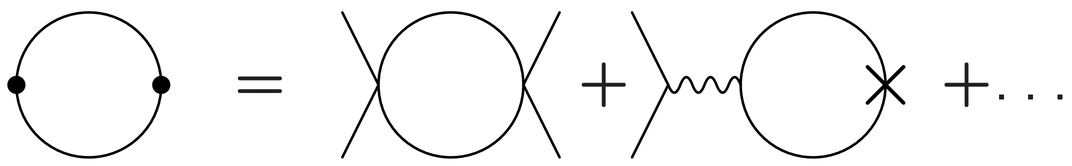
Knecht & Urech 98

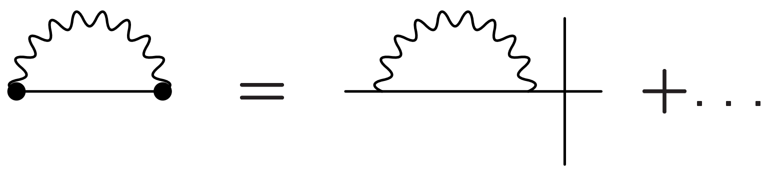
Kaiser 01

Illustration

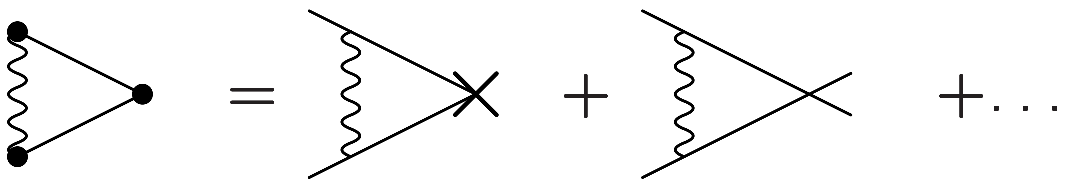
Explicit one loop functional Z_{oneloop} includes:

- One loop graphs with two vertices



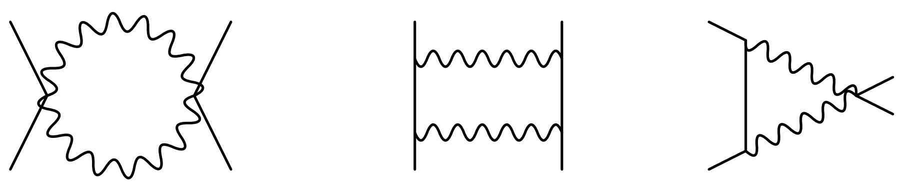


- Triangle vertex diagram



The cross stands for the vector current

- What is not included: Loops containing two photon propagators



$\pi - \pi$ scattering amplitude

- External fields $v_\mu = a_\mu = 0$, $s = \mathrm{diag}(m_u, m_d)$
- Pseudoscalar four-point function: coefficient of $p^{i_1}, \ldots, p^{i_4}$ in the expansion of $Z = Z(\bar{\phi}, \bar{A}_\mu, p)$ in powers of ϕ
- Scattering matrix element determined by on-shell residue of the fourfold pole
- Expansion in ϕ

$$Z = \text{const} + Z^{(2)} + Z^{(4)} + \ldots$$

- Rescaling $\bar{\phi} \to \phi_r$ such that

$$Z^{(2)} = \int dx \frac{1}{2} \left(\partial_\mu {\phi_r^i}^2 - M_{\pi_i}^2 {\phi_r^i}^2 \right) + G_{\pi_i} p^i \phi_r^i + \cdots$$

$\Rightarrow$ physical pion masses $M_{\pi_+}^2, M_{\pi_0}^2$

Knecht & Urech 98

$\Rightarrow$ matrix element $\langle 0 | \, i\bar{q}\gamma_5 \tau^i q \, | \pi^k \rangle = \delta^{ik} G_{\pi_i}$

- We may identify ϕ_r with the extremum of $Z^{(2)}$

$$(\Box + M_{\pi_i}^2)\phi_r^i = G_{\pi_i} p^i$$

- Scattering matrix element can be read off from terms of order ϕ_r^4

$\pi^+(p_1)\pi^-(p_2) \to \pi^0(p_3)\pi^0(p_4)$ scattering:

$$\Gamma_4(-p_1,-p_2,p_3,p_4) = G_{\pi_+}^2 G_{\pi_0}^2 \frac{\langle p_3,p_4 \text{out}|p_1,p_2 \text{in}\rangle_c}{(M_{\pi_+}^2-p_1^2)\dots(M_{\pi_0}^2-p_4^2)} + \dots$$

- G_{π_+} is infrared divergent

$$G_{\pi_+} = G_\pi + 2BF\Big[2e^2\Lambda_{\text{IR}}(M_{\pi_+}) + \frac{e^2 C}{16\pi^2 F^4}\bar{l}_3 + e^2\sum_i d_i\bar{k}_i + \mathcal{O}(e^2 m_q)\Big]$$

G_π pseudoscalar decay constant at $e=0$

$$G_\pi = 2BF\left[1 - \frac{M_{\pi_+}^2}{32\pi^2F^2}\left(\bar{l}_3 - 2\bar{l}_4\right) + \mathcal{O}(m_q^2)\right]$$

Gasser & Leutwyler 84

- The $\pi^+\pi^- \to \pi^0\pi^0$ scattering amplitude - extracted in this manner - agrees with the result of

Knecht & Urech 98

Summary and Outlook

- We constructed the explicit one loop functional for $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_V}$ with virtual photons

- This allows one to read off for instance the $\pi - \pi$ amplitude and the vector form factor

- As a check we extracted the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude

- Extension to $\mathrm{SU(3)_R} \times \mathrm{SU(3)_L}$ in progress

$\Rightarrow$ will allow a "simple" check of the $\pi - K$ scattering amplitude available in the literature

Kubis, Meissner 02

Nehme, Talavera 02