Next-to-next-to-leading corrections to $B \to X_s \ell^+ \ell^-$ in the standard model

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Outline

- Introduction
- Theoretical framework
- Corrections to the matrix elements
- Corrections to the branching ratio
- Corrections to the forward-backward asymmetry
- Summary

based on:

H. Asatryan, H. Asatrian, C. Greub, M. Walker, PLB 507 (2001) 162; PRD 65 (2002) 074004; PRD 66 (2002) 034004;

H. Asatrian, K. Bieri, C. Greub, A. Hovhannisyan hep-ph/0209006

A. Ghinculov, T. Hurth, G. Isidori, Y.-P. Yao hep-ph/0208088

Introduction

$B \to X_s \ell^+ \ell^-$; crucial variable: $s = (\ell^+ + \ell^-)^2$.

In this talk: We consider the region of $\sqrt{s}$ below the J/ψ resonance, more precisely we concentrate on the region

$$0.05 \leq \hat{s} = s/m_b^2 \leq 0.25 \,.$$

This window is interesting for two reasons:

1) Inv. mass distr. and the FB-asymmetries very sensitive to new physics in this window (Lunghi et al. ($B \to X_s \ell^+ \ell^-$); Ali et al. ($B \to K^* \mu^+ \mu^-$), etc.).

2) The process can be calculated in the framework of HQE in this window.

$$\Gamma(B \to X_s \ell^+ \ell^-) = \Gamma(b \to s \ell^+ \ell^- (g)) + \mathcal{O}(\Lambda_{QCD}^2/m_b^2)$$

- The leading term in the HQE consists of the quark decay $b \to s \ell^+ \ell^- (g)$ which can be calculated in renormalization group improved perturbation theory.
- $1/m_b^2$ corrections (Falk et al., Ali et al., Krüger, Sehgal,...)

- $1/m_c^2$ corrections (Voloshin, Ligeti et al, Grant et al., Buchalla et al.)

All studies indicate that the $1/m_b^2$ and $1/m_c^2$ corrections are well below 10%.

$\rightarrow$ For $\hat{s} \in [0.05, 0.25]$ the process is dominated by the free quark decay. $\rightarrow$ Try to improve on the perturbative part.

The situation before our work:

- NLL QCD corrections available (Misiak; Buras,Münz).
- NLL matching cond. have a large $\pm 16\%$ matching scale (μ_W) dep.
- Could be removed by Bobeth, Misiak, Urban (1999) , by NNLL matching (two-loop).
- But prediction for the BR in the window $s/m_b^2 \in [0.05, 0.25]$ has a $\pm 13\%$ (μ_b) renorm. scale dep. $\rightarrow$ two-loop matrix elements needed. $\rightarrow$ Topics of this talk!

Theoretical framework

$b \to s\ell^+\ell^-$ is known to be enhanced by large logs of the ratio m_b^2/M^2.

Exchange n gluons ($n = 0, 1, 2, 3, \ldots$):

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n+1} \frac{m_b^2}{M^2} \qquad \text{LL}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n} \frac{m_b^2}{M^2} \qquad \text{NLL}$$

LL and NLL terms must be resummed for all n.

Useful machinery to achieve resummation: construct effective Hamiltonian and resum logs using RGE techniques.

For $b \to s\ell^+\ell^-$ one gets the Hamiltonian $\mathcal{H}_{eff}$:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \quad .$$

$$\begin{aligned} O_1 &= (\bar{c}_L \gamma^\mu T^a b_L)(\bar{s}_L \gamma_\mu T^a c_L) \\ O_2 &= (\bar{c}_L \gamma^\mu b_L)(\bar{s}_L \gamma_\mu c_L) \end{aligned}$$

$$
\begin{aligned}
O_7 &= \frac{e}{g_s^2} m_b(\mu)\,(\bar{s}\sigma_{\mu\nu} R b)\,F^{\mu\nu} \quad \text{phot. dipole} \\
O_8 &= \frac{1}{g_s} m_b(\mu)\,(\bar{s}\sigma_{\mu\nu} T^a R b)\,G^{\mu\nu,a} \quad \text{gluonic dipole} \\
O_9 &= \frac{e^2}{g_s^2} (\bar{s}_L \gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell) \\
O_{10} &= \frac{e^2}{g_s^2} (\bar{s}_L \gamma^\mu b_L)(\bar{\ell}\gamma_\mu \gamma_5 \ell).
\end{aligned}
$$

$$\mathcal{H}_{eff} \sim \sum_i C_i(\mu) O_i(\mu)$$

$\mathcal{H}_{eff}$ independent of μ, while C_i and O_i depend on μ:

$\rightarrow$ RGE for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^T C_j(\mu)\,; \quad \gamma_{ij} : \text{anomalous dim. matrix}$$

Calculation of $BR(B \to X_s \ell^+ \ell^-)$ consists of three steps:

- matching at $\mu = \mu_W$: $\rightarrow C_i(\mu_W)$ $[O(\alpha_s)]$
- RGE: $\rightarrow$ $C_i(\mu_b)$ [with $\mu_b = O(m_b)$] $\rightarrow$ resums logs! $[O(\alpha_s^2)]$
- calc. of matrix element $\langle X_s \gamma | O_i(\mu_b) | B \rangle$ $[O(\alpha_s)]$

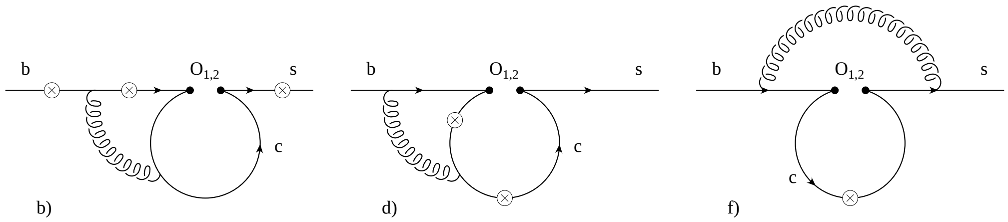
$\rightarrow$ Last step discussed in detail in the following.

Corrections to the matrix elements

Gauge invariance implies that the QCD-corrected $O(\alpha_s)$ matrix element can be written as

$$\langle s\ell^+\ell^-|O_i|b\rangle = \hat{F}_i^{(7)}\langle O_7\rangle_{\text{tree}} + \hat{F}_i^{(9)}\langle O_9\rangle_{\text{tree}}\,.$$

For O_1 and O_2 the $O(\alpha_s)$ diagrams are two-loop diagrams!



3 mass scales involved! ($s=(\ell^+ + \ell^-)^2$, m_b and m_c).

In our restricted s-window we have the hierarchies

$$\frac{s}{m_b^2} < 1\,; \quad \frac{s}{4m_c^2} < 1\,; \quad \frac{m_c^2}{m_b^2} < 1\,.$$

which allow to do expansions.

Depending on the diagram: heavy mass expansion; normal Taylor series expansion; Mellin-Barnes techniques.

The result is of the form

$$\hat{F}^{(7,9)} = \sum_{i,j,\ell,m} c_{ijlm}^{(7,9)} \, \hat{s}^{i} \, \ln^{j}(\hat{s}) \, \left(\hat{m}_c^2\right)^{\ell} \ln^{m}(\hat{m}_c) \, , \qquad (1)$$

where $\hat{s} = s/m_b^2$ and $\hat{m}_c = m_c/m_b$.

We kept all terms with i and ℓ up to 3.

Note: Work is in progress to get full range of $\hat{s}$ (A. Ghinculov, T. Hurth, G. Isidori, Y.-P. Yao).

The $O(\alpha_s)$ diagrams of O_7, O_8, O_9 and O_{10} are one-loop and therefore simpler.

Some of them suffer from infrared sing., which get cancelled by including gluon bremsstrahlung corrections.

Finally, there are various counterterm diagrams, some of them involving evanescent operators.

Corrections to the branching ratio

Consider the ratio

$$R_{\text{quark}}(\hat{s}) = \frac{d\Gamma(b \to X_s \ell^+ \ell^-)/d\hat{s}}{\Gamma(b \to X_c e \bar{\nu})} \tag{2}$$

where the factor $m_{b,pole}^5$ drops out.

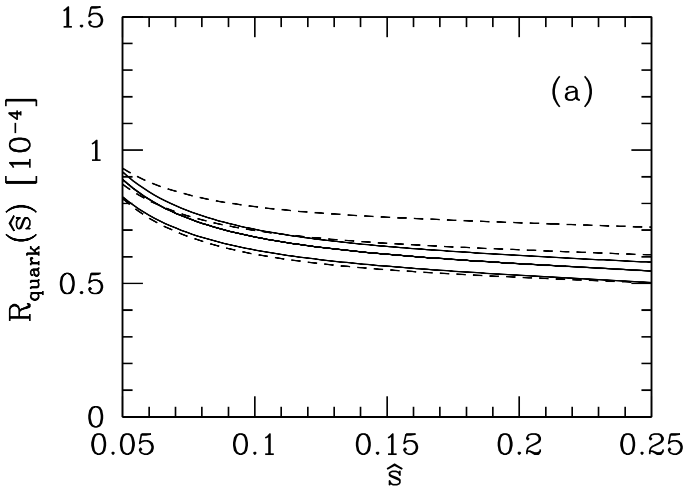


Figure 1: μ dependence: Solid lines include new corrections to the matrix elements; the dashed lines are without the new corrections. (2.5 GeV $\leq \mu \leq$ 10 GeV; $m_c/m_b = 0.29$.)

$\rightarrow$ For most values of $\hat{s}$ the μ-dependence gets reduced by more than a factor of 2!

For the integrated quantity we obtain

$$R_{\rm quark} = \int_{0.05}^{0.25} d\hat{s}\, R_{\rm quark}(\hat{s}) = (1.25 \pm 0.08) \times 10^{-5}\,, \quad (3)$$

when μ varying between 2.5 GeV and 10 GeV. Before our corrections, the result was $R_{\rm quark} = (1.36 \pm 0.18) \times 10^{-5}$.

$\rightarrow$ μ-dependence drops from $\sim \pm 13\%$ to $\sim \pm 6.5\%$!

Errors due to input parameters: The value of m_c/m_b dominates the parametric uncertainties. Questions concerning the definition of m_c less severe than in $b \to s\gamma$, because m_c has to be renorm. at 1-loop.

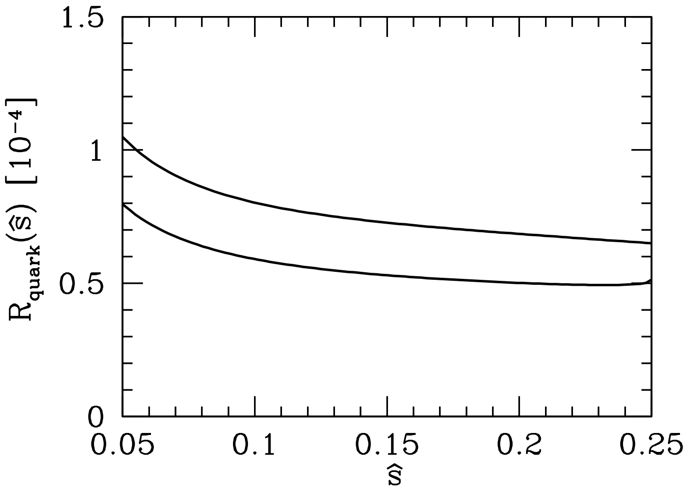


Figure 2: $\hat{m}_c = m_c/m_b$ dependence. $\hat{m}_c = 0.33$ (upper), $\hat{m}_c = 0.25$ (lower).

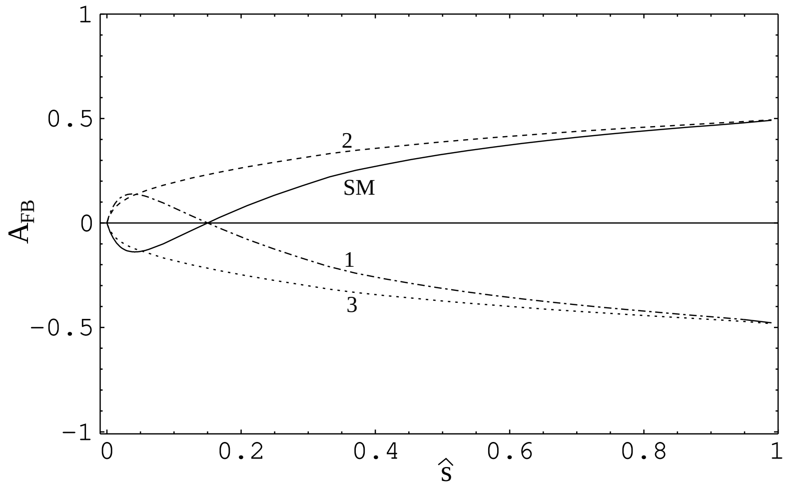
$\rightarrow$ Error due to m_c/m_b larger than left-over μ-dependence!

Corrections to the forward-backward asymmetry

$$\frac{d\bar{A}_{\rm FB}}{d\hat{s}} = \frac{\int_{-1}^{+1} d\cos\theta \, \frac{d^2\Gamma}{d\cos\theta d\hat{s}} \, \mathrm{sgn}(\cos\theta)}{\int_{-1}^{+1} d\cos\theta \, \frac{d^2\Gamma}{d\cos\theta d\hat{s}}}$$

θ: angle (ℓ^+, B) measured in rest frame of lepton-pair.

There are new physics scenarios, compatible with $B \to X_s\gamma$, limits on $\mathrm{BR(B \to X_s}\ell^+\ell^-)$, and measurements of the corr. exclusive decays, with have a totally different forward-backward asymmetry:



→ forward-backward asymmetry is a complementary observable in the search of new physics.

→ should be calc. as precisely as possible in SM.

Recently done to NNLL precision by A. Ghinculov, T. Hurth, G. Isidori, Y.-P. Yao hep-ph/0208088; H. Asatrian, K. Bieri, C. Greub, A. Hovhannisyan hep-ph/0209006.

For the calc. of $\mathrm{BR}(\mathrm{B} \to \mathrm{X_s} \ell^+ \ell^-)$ one could completely integrate lepton variables by tensor techniques. After this step: Angular correlations between leptons and hadrons lost.

Parts of the calc. had to be redone. γ_5 has to be treated carefully.

Results of the two groups were obtained by different methods (different treatment of IR sensitive part); Results agree!

Structure of the double diff. decay width:

$$\frac{d^2\Gamma(b \to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz} = \left(\frac{\alpha_{\rm em}}{4\,\pi}\right)^2 \frac{G_F^2\, m_{b,\rm pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\,\pi^3} (1-\hat{s})^2$$

$$\times \left\{ \frac{3}{4}[(1-z^2)+\hat{s}(1+z^2)] \left(\left|\widetilde{C}_9^{\rm eff}\right|^2 + \left|\widetilde{C}_{10}^{\rm eff}\right|^2 \right) \left(1 + \frac{2\alpha_s}{\pi}\, f_{99}(\hat{s},z)\right) \right.$$

$$+ \frac{3}{\hat{s}}[(1+z^2)+\hat{s}(1-z^2)] \left|\widetilde{C}_7^{\rm eff}\right|^2 \left(1 + \frac{2\alpha_s}{\pi}\, f_{77}(\hat{s},z)\right)$$

$$-3\,\hat{s}\,z\,\mathrm{Re}(\widetilde{C}_9^{\rm eff}\,\widetilde{C}_{10}^{\rm eff\,*}) \left(1 + \frac{2\alpha_s}{\pi}\, f_{910}(\hat{s})\right)$$

$$+6\,\mathrm{Re}(\widetilde{C}_7^{\rm eff}\,\widetilde{C}_9^{\rm eff\,*}) \left(1 + \frac{2\alpha_s}{\pi}\, f_{79}(\hat{s},z)\right)$$

$$\left. -6\,z\,\mathrm{Re}(\widetilde{C}_7^{\rm eff}\,\widetilde{C}_{10}^{\rm eff\,*}) \left(1 + \frac{2\alpha_s}{\pi}\, f_{710}(\hat{s})\right) \right\}.$$

In our work all function $f_{ij}(\hat{s})$ are given analytically.

From the double differential decay width it is straightforward to get the forward-backward asymmetry.

Results:

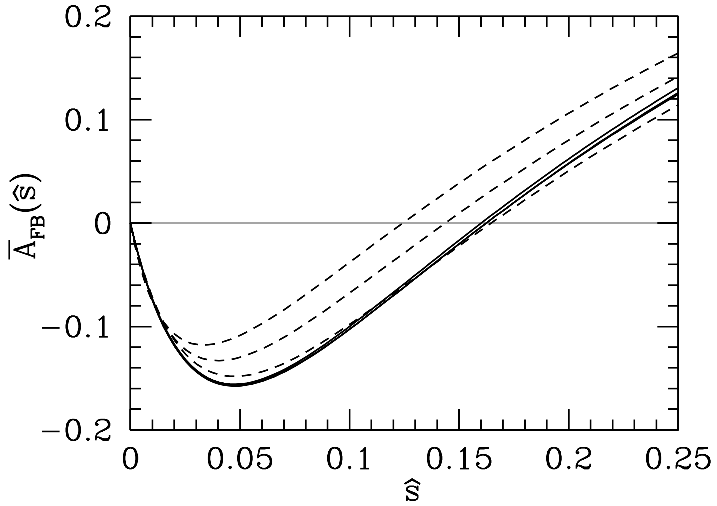


Figure 3: Solid lines: NNLL prediction for $\mu = 2.5, 5.0, 10.0$ GeV; Dashed lines: Corresponding NLL approximation. $m_c/m_b = 0.29$.

$\longrightarrow$ renormalization scale dependence drastically reduced.

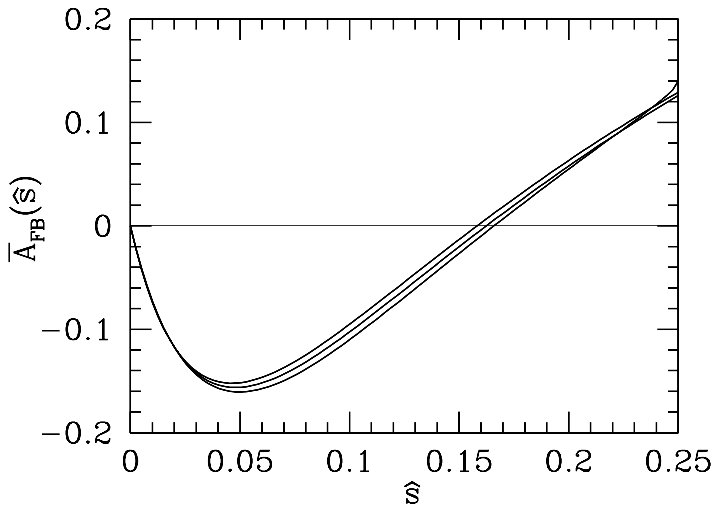


Figure 4: NNLL prediction for $m_c^{pole}/m_b^{pole} = 0.25,\ 0.29,\ 0.33.$ $\mu = 5$ GeV.

Crucial parameter is $\hat{s}_0$: Position, where the forward-backward asymm. vanishes. We get:

$$\hat{s}_0^{\rm NNLL} = 0.162 \pm 0.005 \quad ; \quad \hat{s}_0^{\rm NLL} = 0.144 \pm 0.020\,.$$

Summary

NNLL matching corrections to $B \to X_s \ell^+ \ell^-$ were calculated by Bobeth, Misiak and Urban. These corr. removed the $\pm 16\%$ matching scale dependence.

However, the result suffered from a $\pm 13\%$ renormalization scale dependence.

To remove this scale dependence, virtual- and bremsstrahlung corrections to the matrix elements of the relevant operators was calculated.

$\rightarrow$ The renormalization scale dependence of the (differential) branching ratio gets reduced by a factor of 2!

The forward-backward asymmetry is a crucial variable for new-physics studies.

Recently, the forward-backward asymmetry has been worked out at NNLL in the SM.

$\rightarrow$ Also here the renormalization scale dependence is drastically reduced.

In particular, the position $\hat{s}_0$, where the asymmetry vanishes, is precisely calculable.