LEADING CHIRAL LOGS TO THE HYPERFINE SPLITTING OF THE HYDROGEN AND MUONIC HYDROGEN

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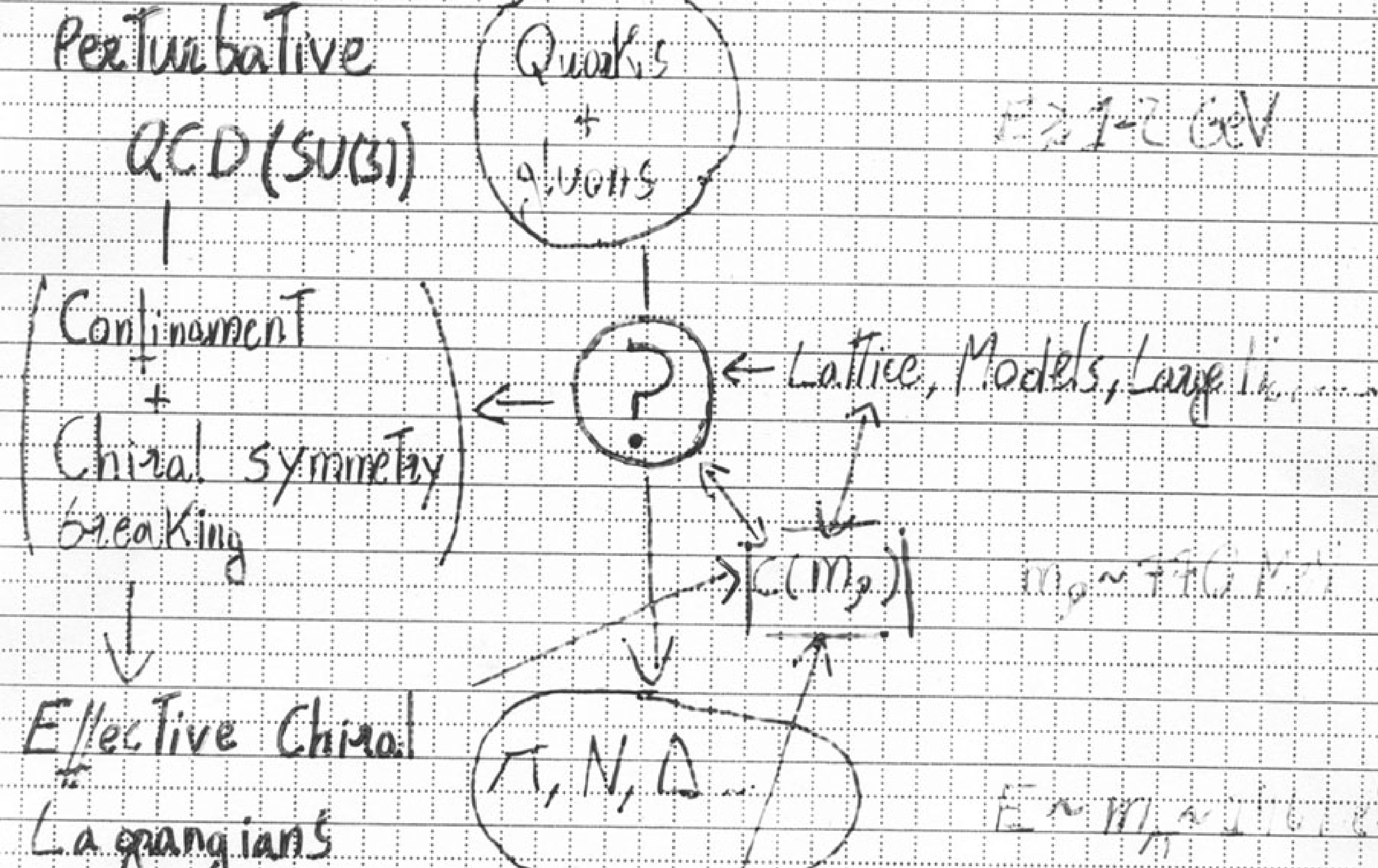
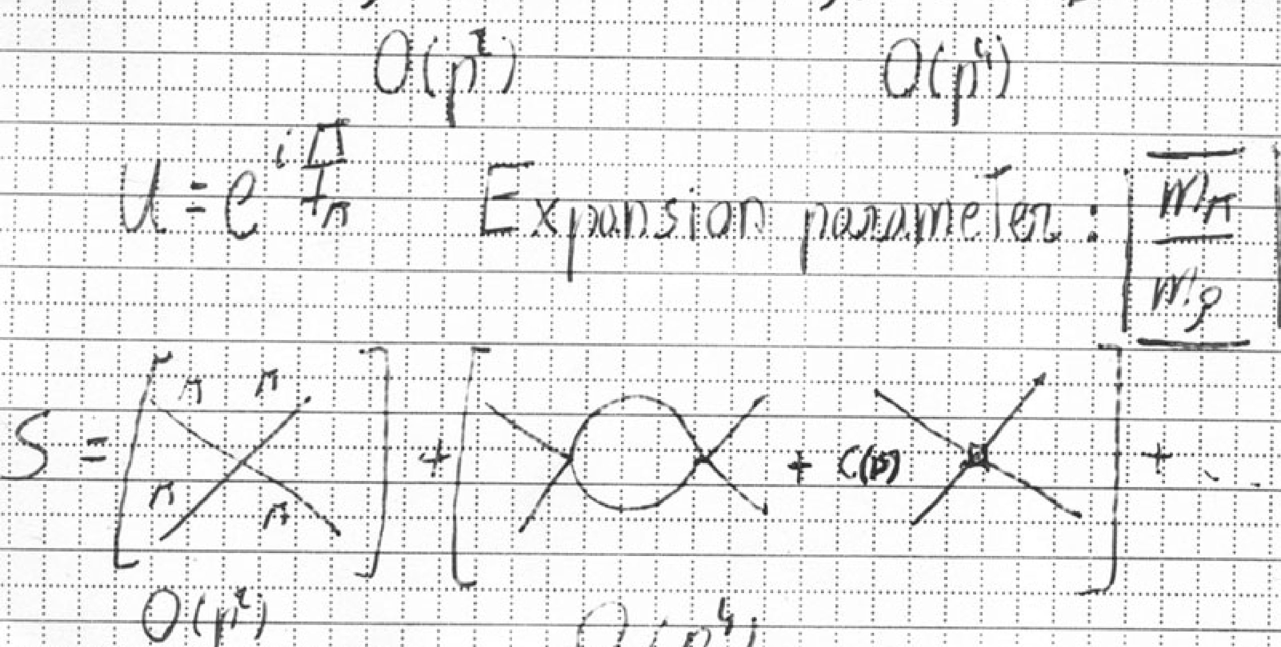
Particle Physics versus Nuclear Physics versus Atomic Physics



$$\underline{\pi}/\ \mathcal{L} = \underbrace{F_\pi^2\, \partial_\mu U \partial^\mu U}_{O(p^2)} + \underbrace{C(\nu)\, \partial_\mu U^\dagger \partial^\mu U\, \partial_\nu U^\dagger \partial^\nu U}_{O(p^4)} + \ldots$$

$$U = e^{i\frac{\pi}{F_\pi}} \qquad \text{Expansion parameter: } \boxed{\frac{m_\pi}{m_\rho}}$$

$S =$  $+ \ldots$

$\underline{1N}$ $M_N \gg m_\pi, M_\Delta - M_N$ $\quad$ $U = u^2 \quad u_\mu = i u^\dagger \nabla_\mu U$

$$\mathcal{L} = \bar{N}(i v\cdot D + g_A S\cdot u)N + \frac{1}{2M_N} N^\dagger \vec{D}^2 N + \ldots + (\Delta's)$$

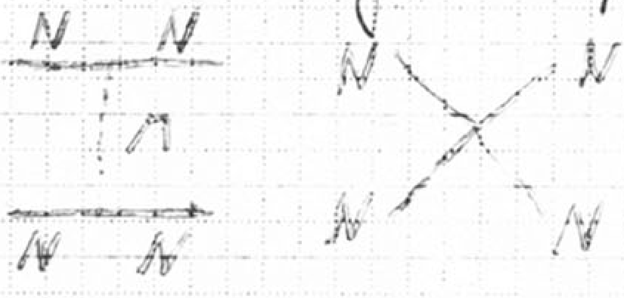
HBET $\quad O(p) \quad\quad O(p^2)$

$\left.\begin{array}{l} \text{Masses of the Baryons} \\ \pi N \text{ scattering} \\ \text{Electromagnetic Form Factors} \\ \text{(if the couplings with photons \& leptons} \\ \text{are introduced)} \end{array}\right\}$ Observables

Expansion parameter: $\boxed{\dfrac{m_\pi}{M_N}}$ $\quad$ $\underline{E \sim m_\pi}$

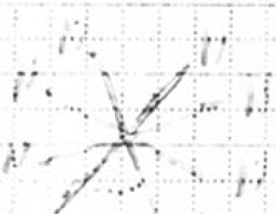
$\underline{2N}$ $\delta\mathcal{L} = \frac{C_S}{\Lambda_\chi^2} N^T N N^T N + \frac{C_V}{\Lambda_\chi^2} N^T \vec{\sigma} N N^T \vec{\sigma} N + \ldots$

NN scattering (with pions)



Deuteron

. . . .

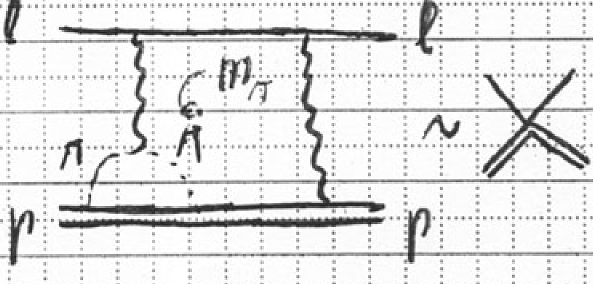
$\underline{nN}$ $n \to \infty$ $\quad$ $\boxed{\text{nuclear matter}}$ 

1N + leptons + photons

$\delta\mathcal{L} \sim \frac{C_R(\nu)}{M_N^2} N^\dagger \vec{\sigma} N \bar{\ell} \vec{\sigma} \ell$ "HBET" $C_R(m_p)$ (!) $\sim \alpha^2$

$E \sim m_\pi \sim 140$ MeV

Integrate out pions mass $\boxed{m_\pi}$



$$\delta\mathcal{L} \sim \frac{C_{NR}}{M_N^2} N^\dagger \vec{\sigma} N \bar{\ell} \vec{\sigma} \ell$$

$|m_e|$

$$C_{NR} \sim C_R(\nu) + \boxed{K} \alpha^2 \ln \frac{\nu^2}{m_\pi^2}$$

★ !!

"NRQED" (non-relativistic leptons) Caswell, Lepage

Integrating out $|m_e \alpha|$ scale

$$\delta V \sim C_{NR} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{\delta(\vec{r})}{M_N^2}$$

"Potential NRQED" $E \sim m\alpha^2$ Soto-Pineda

$(e^- p)$ Hydrogen atom

Hyperfine Splitting

$$\Delta E_{HF}(QED) - \Delta E_{HF}(exp) \sim -0.046 \text{ MHz}$$

(Muonic Hydrogen) $(\mu^- p)$

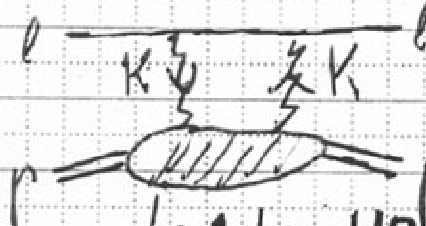
$$\Delta E^{(had)} \sim \left[m_e \alpha^4 \frac{m_e}{M_N}\right]\left[\frac{m_e}{M_N} \frac{C_{NR}^{(had)}}{\alpha}\right] \sim \left|-0.031 \text{ MHz}\right|$$

$$\Rightarrow \left|C_R(m_p) \sim -16\alpha^2\right| \qquad O\left(\alpha \frac{m_e}{M_N}\right)$$

Lamb shift $\longleftrightarrow$ $\delta\mathcal{L} \sim \frac{C}{M_N^2} N^\dagger N \ell^\dagger \ell$ $\longleftrightarrow$ proton radius (whatever it means)

2N, 3N ... nN + leptons $\rightarrow$ Atomic Physics $\Leftrightarrow$ $|C(m_p)|$

$$C_{4,NR} = -i\frac{g^4}{3}\int\frac{d^DK}{(2\pi)^D}\frac{1}{K^2}\frac{1}{K^4-4m_\ell^2K_0^2}\Big\{A_1(K_0,K^2)(K_0^2+2K^2)+$$
$$+3K^2\frac{K_0}{m_p}A_2(K_0,K^2)\Big\}$$



off-shell photons

$\rightarrow$ 1-loop HBET

$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|T\{J^\mu(x)\,J^\nu(0)\}|p,s\rangle$$

$$= -\frac{i}{m_p}\epsilon^{\mu\nu\rho\sigma}\Big(q_\rho s_\sigma A_1(\nu,q^2) + \frac{q_\rho}{m_p^2}\big((m_p\nu)s_\sigma-(q\cdot s)p_\sigma\big)A_2(\nu,q^2)\Big)$$

$$C_{4,NR} = C_{4,R} + \delta C_{4,\text{point-like}} + \delta C_{4,\text{Zemach}} + \delta C_{4,\text{pol}}$$

$$\delta C^{4,d}_{4,\text{Zemach}} = (4\pi\alpha)^2 m_p\frac{2}{3}\int\frac{d^{D-1}K_1}{(2\pi)^{D-1}}\frac{1}{K^4}G_E^{(0)}G_M^{(2)}$$

$$= \frac{m_p^2}{(4\pi F_\pi)^2}\alpha^2\frac{2}{3}\pi^2 g_A^2\ln\frac{m_p^2}{\nu^2}$$

$m_p\frac{\#}{(4\pi F_\pi)^2}|\vec{K}_1|$

e

$m_p \gg |\vec{K}_1| \gg m_\pi$

non-analytical behavior in $|\vec{K}_1|^2$

$\sqrt{\vec{K}_1^2}$

$A_i \rightarrow A_i^{pole} + \bar{A}_i$ (Ji-Osborne)

Zemach

polarizability

$$\delta C_{1,HR}(\nu) = \left(1 - \frac{m_p^2}{4}\right)\alpha^2 \ln \frac{m_\ell^2}{\nu^2}$$

oint-like

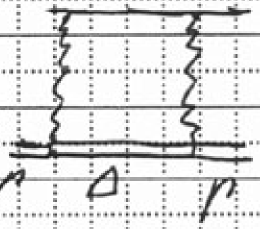
emach

$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{3}\pi^2 g_A^2 \ln \frac{m_\pi^2}{\nu^2}$$

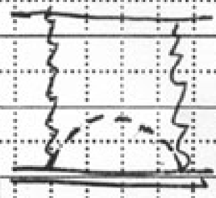
$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{8}{27}\pi^2 g_{\pi N\Delta}^2 \ln \frac{\Delta^2}{\nu^2}$$

larization

$$+ \frac{b_{1,F}^2}{18}\alpha^2 \ln \frac{\Delta^2}{\nu^2}$$



$$- \frac{m_p^2}{(4\pi F_0)^2} g_A^2 \frac{\alpha^2}{\pi} \frac{8}{3} C \ln \frac{m_\pi^2}{\nu^2}$$



$$+ \frac{m_p^2}{(4\pi F_0)^2} g_{\pi N\Delta}^2 \frac{\alpha^2}{\pi} \frac{64}{27} C \ln \frac{\Delta^2}{\nu^2}$$

SU(2)

emach

$$+ \frac{m_p^2}{(4\pi F_0)^2}\alpha^2 \frac{2}{9}\pi^2(-5D^2+6DF-9F^2) \ln \frac{m_K^2}{\nu^2}$$

SU(3)

$$+ (\text{pol. } SU(3) \text{ ?})$$

$$C = 2\int_0^1\int_0^1 dy\,dx\,\sqrt{1-y^2}\Bigg(-2x(2+y^2) + + \frac{1}{y}\left(2(1-x)x(2+y^2)\sqrt{\frac{1}{x-x^2+x^2y^2}} - 3(1-2x)y^2\sqrt{\frac{x}{1-x(1-y^2)}}\right)\sinh^{-1}\left[\sqrt{\frac{x}{1-x}}\,y\right]\Bigg)$$

$$= -0.165037$$

CONCLUSIONS

- $\boxed{\delta E_{HF} \sim \frac{m_l^3 \alpha^5}{m_p^2} \times (\ln m_q, \ln \Delta, \ln m_{l_i})}$

- $C_R \sim \alpha^2 \times (\ln m_q, \ln \Delta, \ln m_{l_i})$

- $C_R(m_p) \sim -16\,\alpha^2$

- $\delta E_{HF}(m_p) \simeq -0.031$ MHz
  $\left(\sim \frac{1}{3} \times \left[E_{HF}(QED) - E_{HF}(exp)\right]\right)$

- Methodology to connect Chiral Lagrangians with atomic physics: potential NRQED

$$\text{HBET} \rightarrow \text{QED} \rightarrow \text{NRQED} \rightarrow \text{pNRQED}$$

$$m_\pi \rightarrow m_e \rightarrow m_e\alpha \rightarrow m_e\alpha^2$$