Electroweak Penguin Contribution to ϵ'/ϵ

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Outline

- Introduction
- The "dispersive approach": summary
- Numerics: RWM & FESR
- Results and conclusions

V. C. , J. F. Donoghue, E. Golowich, K. Maltman:

Phys. Lett. **B 522** (2001), 245

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I EURIDICE Collaboration Meeting, Frascati, Oct 18-20 2002

The Electroweak Penguin Operators

$$\mathcal{Q}_7 = \frac{3}{2}\,\bar{s}(\Gamma_L)_\mu d \times \bar{q}\,Q\,(\Gamma_R)^\mu q$$
$$\mathcal{Q}_8 = \frac{3}{2}\,\bar{s}_\alpha(\Gamma_L)_\mu d_\beta \times \bar{q}_\beta\,Q\,(\Gamma_R)^\mu q_\alpha$$

$$\bar{q} = \left(\bar{u}, \bar{d}, \bar{s}\right) \qquad Q = \mathrm{Diag}(2/3, -1/3, -1/3) \qquad \Gamma_{L,R} = \frac{1 \mp \gamma_5}{2}$$

◇ Phenomenological relevance of $\langle(\pi\pi)_{I=2}|\mathcal{Q}_8|K^0\rangle$

Approximate Formula for ϵ'/ϵ [$\overline{\mathrm{MS}}$-NDR, $\mu = 2$ GeV]

$$\frac{\epsilon'}{\epsilon} = 20\times10^{-4}\left[-0.06 - 2.0\cdot\frac{\langle\mathcal{Q}_6\rangle_0}{\mathrm{GeV}^3}\,(1-\Omega_{\mathrm{IB}}) - 0.50\cdot\frac{\langle\mathcal{Q}_8\rangle_2}{\mathrm{GeV}^3}\right]$$

◇ Possibility of data-driven evaluation

$$\langle\mathcal{Q}_{7,8}\rangle_{p^0} \quad \Longleftrightarrow \quad \Pi^{33}_{V-A} \; ; \; \rho^{33}_{V-A}$$

$\updownarrow$ Leading term in chiral expansion

$\updownarrow$ Experimentally accessible

$\langle \pi\pi | \mathcal{Q}_{7,8} | K \rangle$ in χ-Limit

Chiral Symmetry

I) $\langle \pi\pi | \mathcal{Q}_{7,8} | K \rangle \longleftrightarrow \langle 0 | O_{1,8} | 0 \rangle$

up to $\mathcal{O}(M_K/\Lambda_\chi)^2$

$$O_{1,8} = V^{3,C} \cdot V^{3,C} - A^{3,C} \cdot A^{3,C}$$

II) $\langle O_{1,8} \rangle \longleftrightarrow \Pi^{ab}_{V-A} \ ; \ \rho^{ab}_{V-A}$

Central Tool: OPE ($Q^2 >> \Lambda^2_{QCD}$)

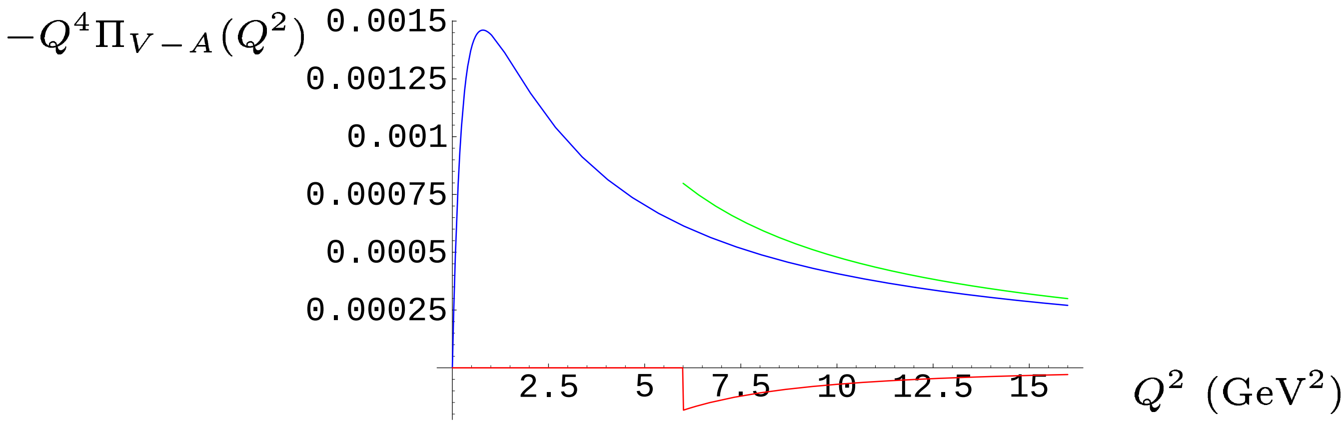
$$\Pi_{V-A}(Q^2) = \frac{a_6(\mu) + b_6 \log \frac{Q^2}{\mu^2}}{Q^6} + \underbrace{\sum_{d>6} \frac{a_d(\mu) + b_d \log \frac{Q^2}{\mu^2}}{Q^d}}_{\overline{\Pi}_{V-A}(Q^2)}$$

Rough answer:

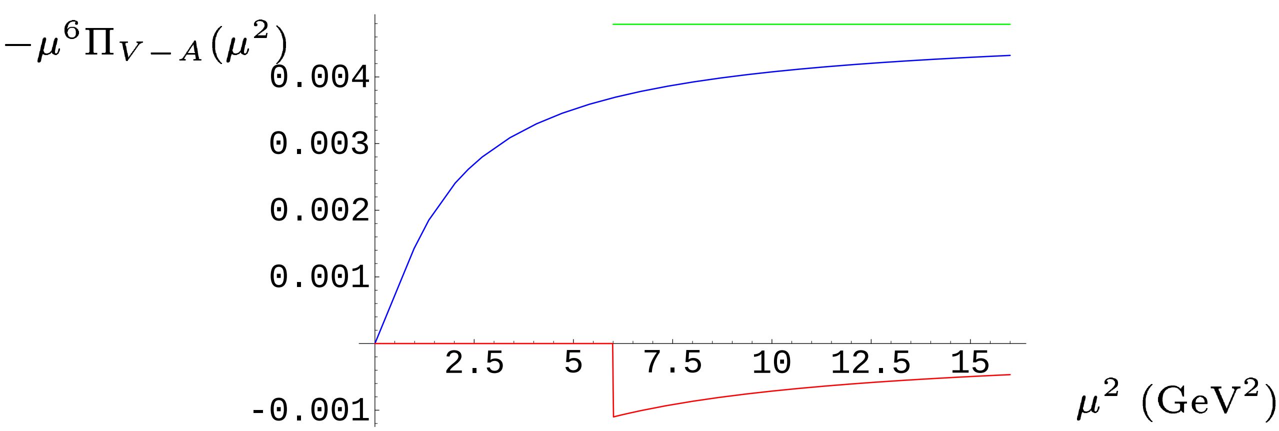
$\langle O_1 \rangle \sim \langle \mathcal{Q}_7 \rangle$: integral of $s^2 \Pi(s)$ over *all* spacelike momenta

$\langle O_8 \rangle \sim \langle \mathcal{Q}_8 \rangle$: leading short distance singularity of $\Pi(s)$ $[a_6]$

In Pictures:



$$I_1(\mu) = \int_0^{\mu^2} dQ^2\ Q^4 \Pi_{\text{V-A}}(Q^2) \qquad H_1(\mu) = \int_{\mu^2}^{\infty} dQ^2\ Q^4\ \overline{\Pi}_{\text{V-A}}(Q^2)$$



$$I_8(\mu) = \mu^6\ \Pi_{\text{V-A}}(\mu^2) \qquad H_8(\mu) = \mu^6\ \overline{\Pi}_{\text{V-A}}(\mu^2)$$

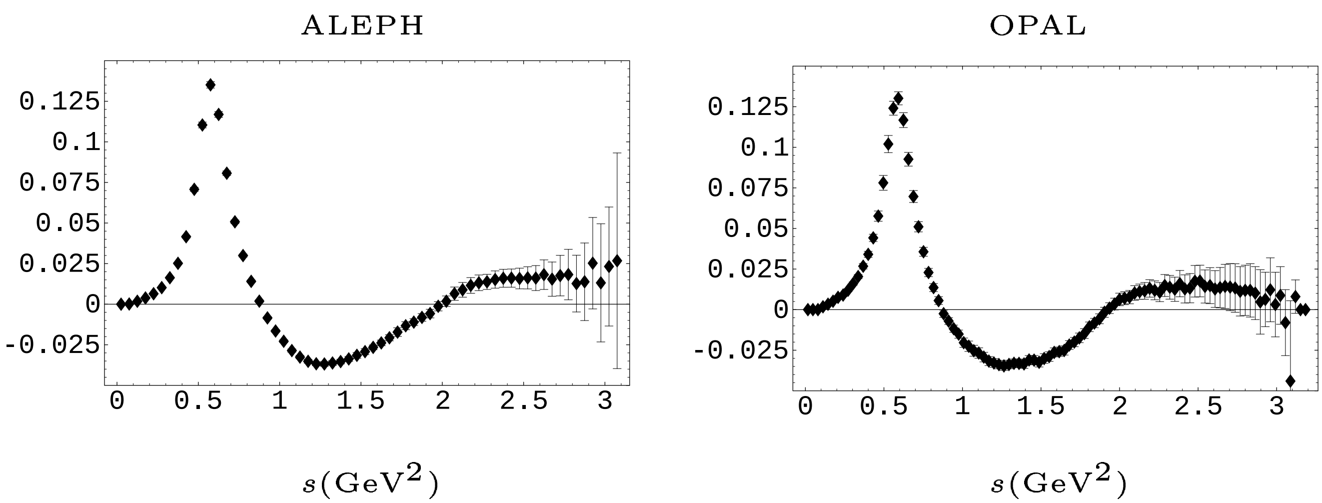
$$\begin{pmatrix} \langle O_1\rangle_\mu \\ \langle O_8\rangle_\mu \end{pmatrix} = \hat{M}(\alpha_s) \begin{pmatrix} I_1(\mu) + H_1(\mu) \\ I_8(\mu) - H_8(\mu) \end{pmatrix}$$

Relation to Data

◇ Dispersive Representation

$$\Pi_{V-A}(q^2) = \frac{F_\pi^2}{q^2} + \int_0^\infty ds \; \frac{\rho_{V-A}(s)}{s - q^2 - i\epsilon}$$

◇ Database: $\rho_{V-A}(s)$ from ALEPH, OPAL



◇ Use of data: two approaches

1. "Data + QCD constraints" (RWM) $\rightarrow I_{1,8}(\mu)$

2. "Data + QCD duality" (FESR) $\rightarrow$ $a_6 = I_8 - H_8 \quad \rightarrow \quad \langle \mathcal{Q}_8 \rangle$
   $a_d, \, d > 6 \quad \rightarrow \quad H_{1,8}$

Residual Weight Method

Enforce Chiral Constraints in calculation of:

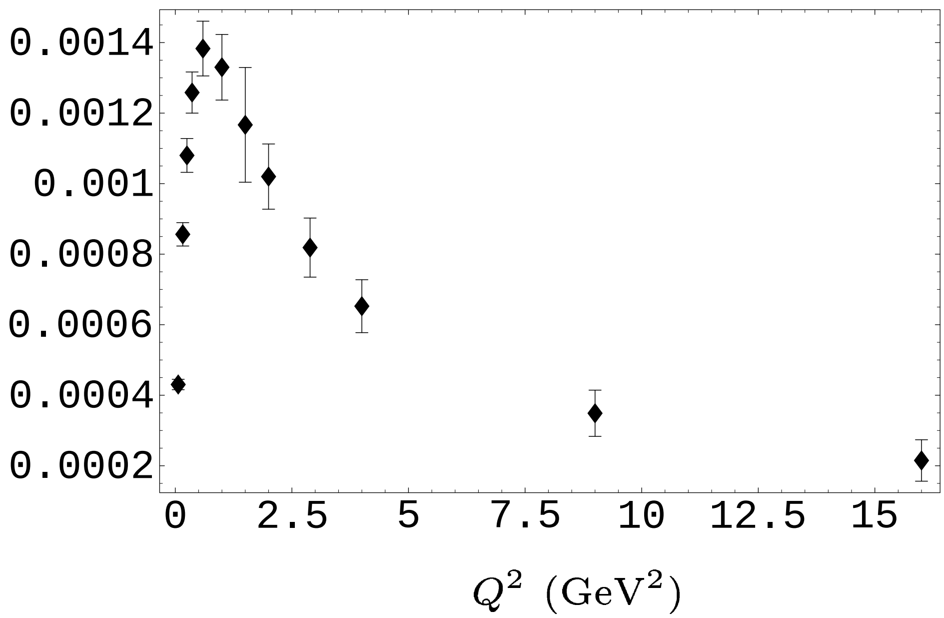
$$I(\mu) = \int_0^\infty ds \; K(s,\mu)\, \rho_{\mathrm{V-A}}(s)$$

- $K(s,\mu) = x\,1 \; + \; y\,s \; + \; z\,s\log s \; + \; r(s,\mu)$
- $I(\mu) = F_\pi^2 \left(x - z\frac{4\pi\delta M_\pi^2}{3\alpha} \right) + \int_0^\infty ds \; r(s,\mu)\, \rho_{\mathrm{V-A}}(s)$
- Choose (x, y, z) to minimize uncertainty due to
  - $F_\pi, \delta M_\pi$
  - Poor knowledge of $\rho_{\mathrm{V-A}}(s)$ for $s > 2.5$ GeV^2
- Realistic estimate of error bars

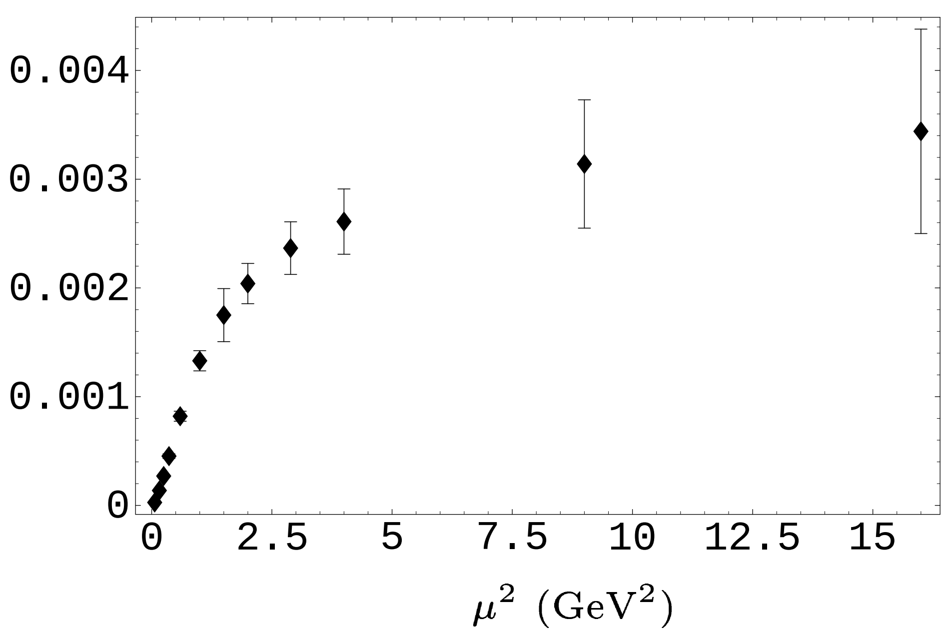
For detailed discussion and results see Phys. Lett. **B 522** (2001), 245

RWM: results in pictures

◇ Constraining $\langle O_1\rangle$: $-Q^4\Pi_{\text{V-A}}(Q^2)$



◇ Constraining $\langle O_8\rangle$: $-\mu^6\Pi_{\text{V-A}}(\mu^2)$

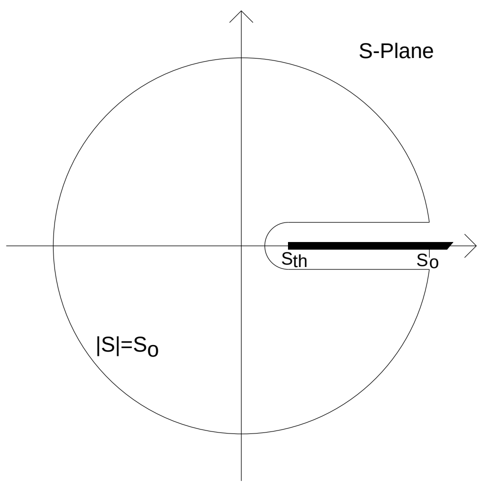


FESR Analysis: Generalities

Cauchy's theorem $\oplus$ $\Pi(s) \to \Pi_{\rm OPE}(s)$ $\oplus$ $R[w, s_0] \approx 0$

$$\underbrace{\int_{s_{th}}^{s_0} ds\ w(s)\rho(s)}_{J[w,s_0]} + R[w, s_0] = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\ w(s)\Pi_{\rm OPE}(s)$$

$$R[w, s_0] = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\ w(s)\left(\Pi_{\rm OPE}(s) - \Pi(s)\right)$$



Basic observation:

" $|s| \gg \Lambda^2_{QCD}$: $\Pi_{\rm OPE}(s) \sim \Pi(s)$ except near time-like axis "

$\Downarrow$

Pinched weights ($w(s_0) = 0$) produce

$R/J \ll 1$ at scales s_0 : $2 \to 3$ GeV^2

Choice of FESR weights

◇ Criteria

- TH: Use "pinched" weights ($w(s_0) = 0$)

- EXP: Maximize statistical signal in $J[w, s_0]$

  Avoid large V - A cancellations

- PRACTICAL: Maximize "constraining power" on a_d

  No more than two a_d on the OPE side of FESR

◇ Family of weights: $w(y) = (1-y)^2\, p(y) \qquad y = s/s_0$

$$
\begin{aligned}
w_1(y) &= (1-y)^2\,(1-3y) &\rightarrow\quad & a_6\,,\, a_8 \\
w_2(y) &= (1-y)^2\, y &\rightarrow\quad & a_6\,,\, a_8
\end{aligned}
$$

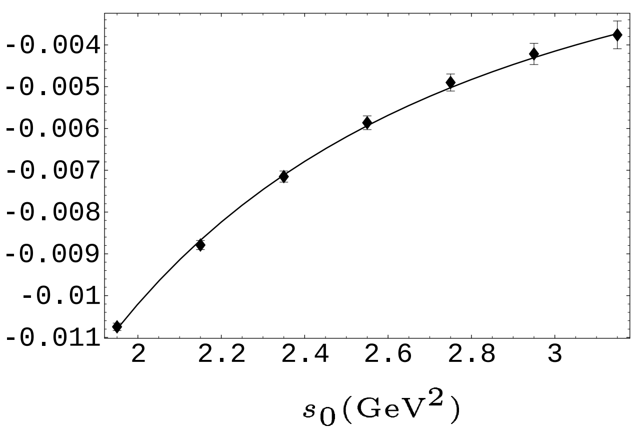
$$
\left.\begin{matrix} w_3\,, \ldots, w_6 \\ w_7\,, \ldots, w_{10} \end{matrix}\right\} \longrightarrow \begin{matrix} a_6\,,\, a_d \\ a_8\,,\, a_d \end{matrix} \qquad d = 10, 12, 14, 16
$$

FESR Analysis: Results

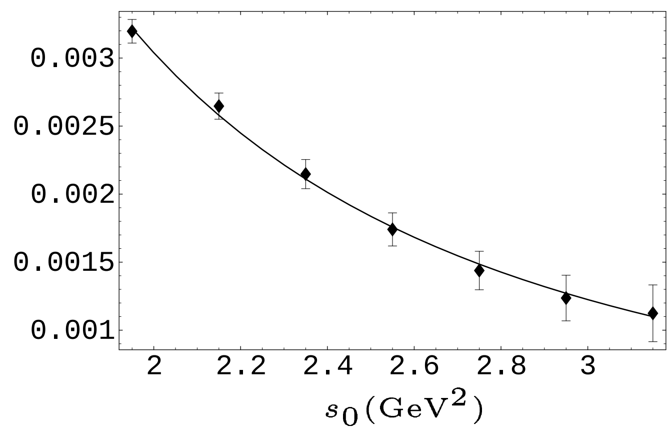
Fit to ALEPH data

$a_6 = -(2.25 \pm 0.35) \cdot 10^{-3}\,\mathrm{GeV}^6$ $\quad a_8 = -(3.0 \pm 1.7) \cdot 10^{-3}\,\mathrm{GeV}^8$

$J[w_1, s_0]$ vs $\mathrm{OPE}[a_6, a_8; s_0]$



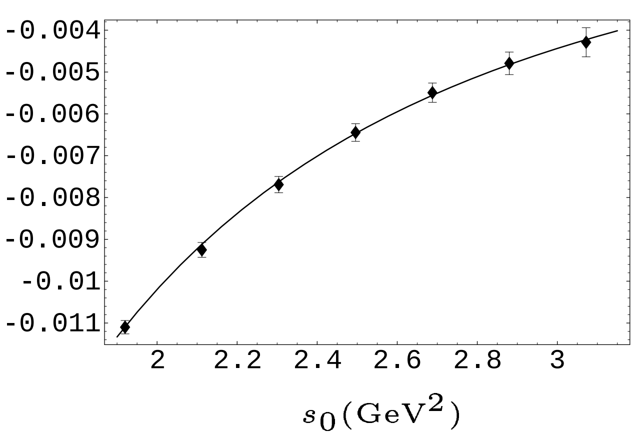
$J[w_2, s_0]$ vs $\mathrm{OPE}[a_6, a_8; s_0]$



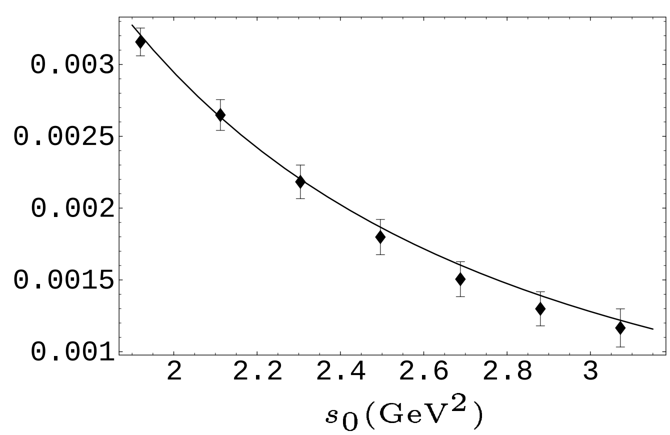
Fit to OPAL data

$a_6 = -(2.75 \pm 0.4) \cdot 10^{-3}\,\mathrm{GeV}^6$ $\quad a_8 = -(0.55 \pm 1.75) \cdot 10^{-3}\,\mathrm{GeV}^8$

$J[w_1, s_0]$ vs $\mathrm{OPE}[a_6, a_8; s_0]$

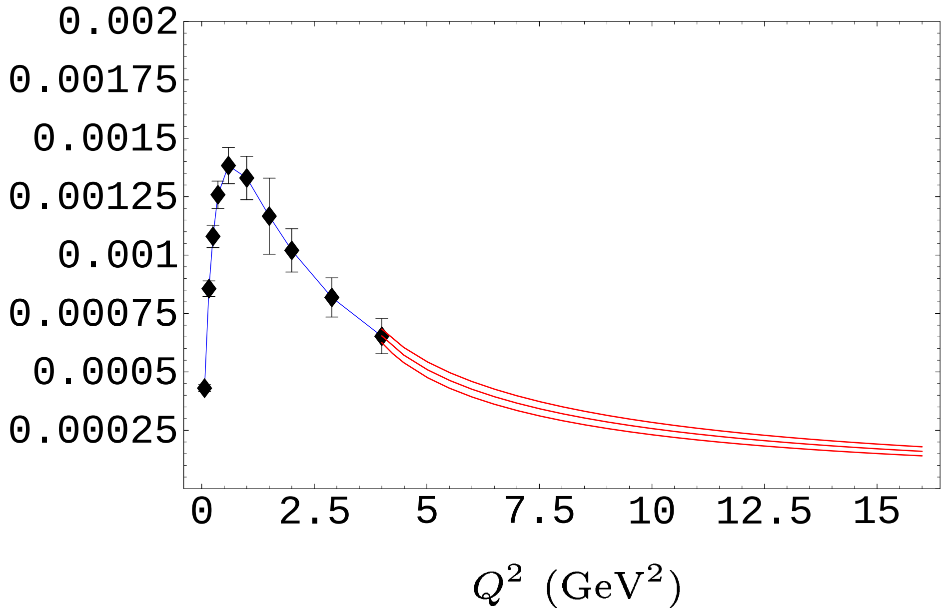


$J[w_2, s_0]$ vs $\mathrm{OPE}[a_6, a_8; s_0]$

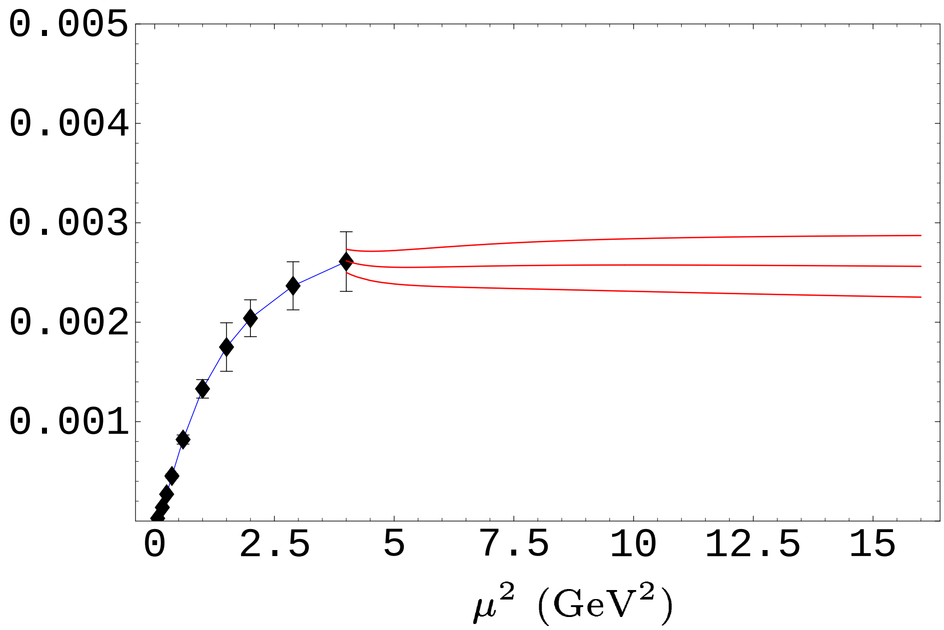


FESR: results in pictures

◇ Constraining $\langle O_1 \rangle$: $-Q^4 \Pi_{\text{V-A}}(Q^2)$



◇ Constraining $\langle O_8 \rangle$: $-\mu^6 \Pi_{\text{V-A}}(\mu^2)$



Back to $\langle \mathcal{Q}_{7,8} \rangle$

I. $\mu = 4$ GeV : $\begin{cases} \text{RWM}: & I_{1,8} \\ H_{1,8} = & 0 \end{cases}$ RG (μ : 4 $\to$ 2 GeV)

II. $\mu = 2$ GeV : $\begin{cases} \text{RWM}: & & I_1 \\ \text{FESR}: & a_6 = I_8 - H_8 & H_1 \end{cases}$

Results ($\overline{\text{MS}}$-NDR at $\mu = 2$ GeV)

	$\langle \mathcal{Q}_7 \rangle_{I=2}/\text{GeV}^3$	$\langle \mathcal{Q}_8 \rangle_{I=2}/\text{GeV}^3$
I	0.16 ± 0.10	2.2 ± 0.7
II	0.22 ± 0.05	1.5 ± 0.3
VSA*	0.32	0.94

* $(m_s + m_d)(2 \text{ GeV}) = 110$ MeV

- Procedures I and II lead to consistent results
- Procedure II has reduced uncertainty
- Appreciable violations to factorization

Comparisons ($\overline{\text{MS}}$-NDR at $\mu = 2$ GeV)

	$\langle \mathcal{Q}_7 \rangle_{I=2}/\text{GeV}^3$	$\langle \mathcal{Q}_8 \rangle_{I=2}/\text{GeV}^3$
This work	0.22 ± 0.05	1.5 ± 0.3
Bijnens et al.	0.24 ± 0.03	1.2 ± 0.7
Knecht et al.	0.11 ± 0.03	2.3 ± 0.7
Narison	0.21 ± 0.05	1.4 ± 0.4
RBC (DWF)	0.28 ± 0.04	1.1 ± 0.2
CP-PACS (DWF)	0.24 ± 0.03	1.0 ± 0.2
SPQ$_{\text{cd}}$R (Wilson)	0.24 ± 0.02	1.05 ± 0.1
VSA	0.32	0.94

⋆ Analytic results: directly in the chiral limit

⋆ Lattice results: extrapolation to the chiral limit:

- quenched approximation
- quoted error is statistical only