$\hat{B}_K$ in the $1/N_c$ expansion :

Impact of $d = 8$ operators in the ~~weak~~ OPE

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Frascati, 18 / 10 / 02

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Motivations

(de Rafael & Peris

$\hat{B}_K$ is one of the parameters which determine the $K^0-\bar{K}^0$ mixing. $K^0-\bar{K}^0$ mixing is normally studied with an EFT, valid at $E \lesssim m_c$.

The terms relevant for $K^0-\bar{K}^0$ mixing are

$$\mathcal{L}_{eff}^{\Delta S=2} = -\frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_c^2 F_1 + \lambda_t^2 F_2 + 2\lambda_c\lambda_t F_3\right]$$

$$G_{\Delta S=2}(\mu)\, Q_{\Delta S=2}(x)$$

$$\lambda_q = V_{qd}^* V_{qs} \quad , \quad q = u, c, t$$

$$F_{1,2,3} = f(m_t, m_{Z^0}, m_{W^\pm}, m_s, m_c)$$

$$Q_{\Delta S=2}(x) = \left[\bar{s}_L(x)\gamma^\mu d_L(x)\right]^2$$

It is conventional to define $\hat{B}_K$ through

$$\langle \bar{K}^0 | G_{\Delta S=2}(\mu)\, Q_{\Delta S=2}(0) | K^0 \rangle \equiv \frac{4}{3} f_K^2 M_K^2 \hat{B}_K$$

In the Large-N_c limit,

$$\bar{s}_L \gamma^\mu d_L \Leftrightarrow \frac{1}{\sqrt{2}} f_K \partial^\mu K^0$$

[bosonic realization in the chiral limit]

$$G_{\Delta S=2}(\mu)\Big|_{N_c\to\infty} = 1$$

$$\boxed{\hat{B}_K \Big|_{N_c\to\infty} = \frac{3}{4}}$$

B_K to NLO in the $1/N_c$ expansion

- We know the form of $\mathcal{L}_{eff}^{\Delta S=2}$ $(E \lesssim m_c)$, but we still have to connect it (match it) to the truly relevant EFT, valid at $E \sim M_K$.

Chiral symmetry constrains the EFT terms sharing the symmetries of $Q_{\Delta S=2}(x)$ to be of the form (to lowest order):

$$\mathcal{O}_\chi^{\Delta S=2} = -\frac{f_K^4}{4}\, g_{\Delta S=2}(\mu)\, tr\left[\lambda_{32}(D^\mu U^\dagger) U \lambda_{32}(D_\mu U^\dagger\right.$$

$$\lambda_{32} = \delta_{i3}\delta_{2j}$$

where $g_{\Delta S=2}(\mu)$ depends on the underlying dynamics of SχSB. Comparing $K^0 - \bar{K}^0$ matrix elements with $\mathcal{L}_{eff}^{\Delta S=2}$ and $\mathcal{O}_\chi^{\Delta S=2}$ one gets

$$\boxed{\hat{B}_K = \frac{3}{4} C_{\Delta S=2}(\mu)\, g_{\Delta S=2}(\mu)}$$

- What do we know about the Wilson coefficient $C_{\Delta S=2}(\mu)$?

From renormalization group-improved perturbation theory,

$$C_{\Delta S=2}(\mu) = \left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{1433}{1936} + \frac{1}{8}\kappa\right)\right]\left(\frac{1}{\alpha_s(\mu)}\right)$$

$\kappa = 0$ (naive dimensional regularization)

Notice that indeed $G_{\Delta S=2}(\mu)\Big|_{N_c\to\infty} = 1$.

- What do we know about the coupling $g_{\Delta S=2}(\mu)$?

It has to be determined by a matching condition.

$$\boxed{g_{\Delta S=2}(\mu,\varepsilon) = 1 - \frac{\mu_{had}^2}{32\pi^2 f_K^2}\left(\frac{4\pi\mu^2}{\mu_{had}^2}\right)^{\frac{\varepsilon}{2}}\frac{1}{\Gamma(2-\frac{\varepsilon}{2})}\int_0^\infty dz\, z^{-\frac{\varepsilon}{2}} W(z}$$

$$W(z) = z\frac{\mu_{had}^2}{f_K^2} W_{LRLR}^{(1)}(z\mu_{had}^2)$$

$$z \equiv \frac{Q^2}{\mu_{had}^2}$$

$$\int d\Omega_q\, g_{\mu\nu} W_{LRLR}^{\mu\alpha\nu\beta}(q,l) = \left(\frac{l^\alpha l^\beta}{l^2} - g^{\alpha\beta}\right) W_{LRLR}^{(1)}(Q^2$$

$$\boxed{\begin{aligned} W_{LRLR}^{\mu\alpha\nu\beta}(q,l) = i^3 \lim_{l\to 0}\int d^4x\, d^4y\, d^4z\, e^{i(q\cdot x + l\cdot(y-z))} \\ \langle 0|T\{L^\mu_{\bar s d}(x) R^\alpha_{\bar d s}(y) L^\nu_{\bar s d}(0) R^\beta_{\bar d s}(z)\}|0\rangle \end{aligned}}$$

$$L^\mu_{\bar s d}(x) = \bar s(x)\gamma^\mu \frac{1-\gamma_5}{2} d(x)$$

$$R^\mu_{\bar d s}(x) = \bar d(x)\gamma^\mu \frac{1+\gamma_5}{2} s(x)$$

- What do we know about $W(z)$?

In the Large-N_c limit it takes the form:

$$W(z) = 6 - \sum_{i=1} \frac{\alpha_i}{\rho_i} - \sum_{i=1} \frac{\beta_i}{\rho_i^2} + \sum_{i=1} \frac{\gamma_i}{\rho_i^3} +$$

$$+ \sum_{i=1}^{\infty} \left[\frac{\alpha_i}{z+\rho_i} + \frac{\beta_i}{(z+\rho_i)^2} + \frac{\gamma_i}{(z+\rho_i)^3} \right]$$

$$\rho_i \equiv \frac{M_i^2}{\mu_{had}^2}$$

where $\alpha_i, \beta_i, \gamma_i$ have to be determined by comparing with

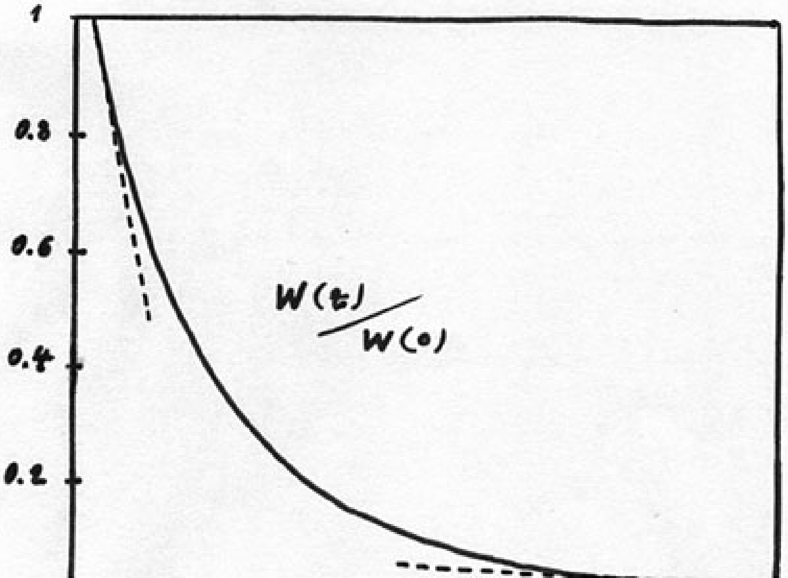
(a) χPT at low z

$$W(z) \approx 6 - \frac{24 \mu_{had}^2}{f_\pi^2} (2L_1 + 5L_2 + L_3 + L_9) z + \ldots$$

(b) the OPE at high z

$$W(z) \approx \frac{24 \pi \alpha_s f_\pi^2}{\mu_{had}^2} \left[1 + \frac{\varepsilon}{12} (5 + \kappa) + O(\alpha_s) \right] \frac{1}{z} + \ldots$$

The previous constraints allow to determine only one resonance (ρ), i.e., $\alpha_V, \beta_V, \gamma_V$. With this,



$f_\pi = 85.3$ MeV

$\mu_{had} = 1.4$ GeV

$M_V = 770$ MeV

$\hat{z} = 0.39$

Now we can plug in $W(z)$ and get $g_{\Delta S=2}(\mu)$ and finally $\hat{B}_K$.

$$\hat{B}_K = \left(\frac{1}{\alpha_s(\mu_{had})}\right)^{3/11} \frac{3}{4}\Big[1 - \frac{\alpha_s(\mu_{had})}{\pi}\left(\frac{1229}{1936} - \frac{3}{4}\log \hat{z}\right)$$

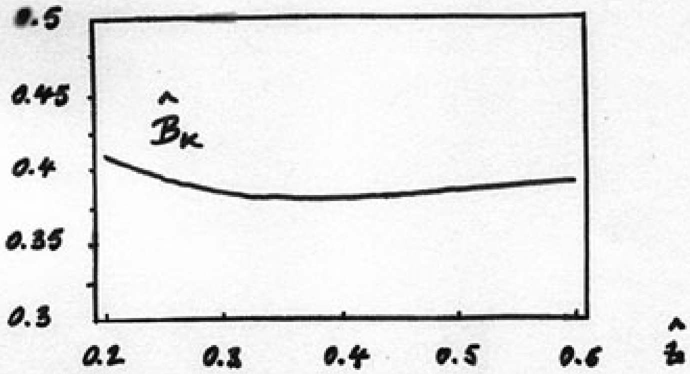
$$+ O\left(\frac{N_c\, \alpha_s^2(\mu_{had}^2)}{\pi^2}\right) - \frac{\mu_{had}^2}{32\pi^2 f_K^2}\Big(\alpha_V \log \frac{\hat{z}+\rho_V}{\rho_V} -$$

$$- \beta_V\left(\frac{1}{\hat{z}+\rho_V} - \frac{1}{\rho_V}\right) - \frac{\gamma_V}{2}\left(\frac{1}{(\hat{z}+\rho_V)^2} - \frac{1}{\rho_V^2}\right)\Big)\Big]$$

which results from integrating $W(z)$ in the following manner

$$\int_0^{\hat{z}} dz\, W(z) + \int_{\hat{z}}^{\Lambda^2/\mu_{had}^2} dz\, W_{OPE}(z)$$

Graphically,



$$\boxed{\hat{B}_K = 0.38 \pm 0.11}$$

Notice that the final expression for $\hat{B}_K$ is μ-independent and scheme-independent.

Impact of $d=8$ operators in the weak OPE

Recently, Cirigliano, Donoghue & Golowich suggested that $d=8$ operators may, in principles be of significance,

$$Q_1^{(8)} = \bar{s}\overleftarrow{D}_\mu \overleftarrow{D}^\mu T^a_\nu d\, \bar{s}\, T^\nu_a d + \ldots$$

$$Q_2^{(8)} = \bar{s}\, T^a_\nu \overrightarrow{D}_\mu d\, \bar{s}\, T^\nu_a \overleftarrow{D}^\mu d + \ldots$$

$$\vdots$$

$$Q_6^{(8)} = g\, \delta^{ab} \tilde{F}^{\mu\nu,b} \left(\bar{s}\, T^a_\mu d\, \bar{s}\, T_\nu d - \bar{s}\, T_\mu d\, \bar{s}\, T^a_\nu d \right)$$

with the corresponding Wilson coefficients, in agreement with C., D. & G.

Upon factorization, only $Q_1^{(8)}$ contributes. To evaluate such contribution, knowledge of

$$\langle 0 |\, \frac{g_s}{2}\, \bar{s}_L \gamma_\rho \tilde{G}^{\rho\nu} d_L \,| K \rangle$$

is needed. [Sum rules]. (Novikov et al.)

WORK IN PROGRESS...

Conclusions

. $\hat{B}_K^{\chi}$ has been determined, with a value $\hat{B}_K = 0.38 \pm 0.11$, and has been shown explicitly its $\underline{\mu}$ and <u>scheme</u>-independence. (de Rafael & Peris)

. Analysis of $d = 8$ operators is underway, but in that particular case ($\hat{B}_K$) we expect a small impact.