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Electromagnetic decays of vector mesons in lattice QCD

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Abstract

We present a lattice study of electromagnetic decays of vector mesons V into pseudoscalar mesons P, relevant to radiative decays $V \to P\gamma$ and conversion decays $V \to Pl^+l^-$. The form factor $f_q(q^2)$, associated with the electromagnetic current of the quark q, has been computed using Wilson fermions in the quenched approximation.

1 Introduction

The construction of the DAΦNE ϕ-factory [1] will enable, among others, the in-depth study of the electromagnetic decays of ϕ-mesons. The favoured electromagnetic decay modes of vector mesons are radiative decays $V \to P\gamma$ and conversion decays $V \to Pl^+l^-$, in which a pseudoscalar meson P is present in the final state. Actually, branching ratios for radiative decays have been measured with high accuracy, but we have scarce information about conversion decays [2]. An improvement of this situation is expected from measurements performed at DAΦNE: a large number of ϕ-meson conversion decays will be observed and precise measurements of branching ratios and q^2 distributions will be possible.

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From a theoretical point of view the electromagnetic decays of vector mesons have been studied so far using different phenomenological models [3] and recently the chiral perturbation theory [4]. The lattice QCD approach offers itself as a reliable quantitative tool to study these decays from first principles. In this contribution we present a preliminary lattice study of the electromagnetic decays of vector mesons [5]. Given the exploratory nature of our work, we have used the 15 gauge field configurations and the quark propagators of ref.[6], generated on a volume $20\times 10^2\times 40$, at $\beta = 6.0$, in the quenched approximation. Our results for radiative decays are in good agreement with the experimental data and provide strong support to the validity of the lattice calculation. However, a better determination of the q^2 dependence of the form factor, related to the matrix element of conversion decays, would be necessary for a more precise prediction of the relative branching ratios.

2 Form factors and decay rates

Consider the electromagnetic decays of a vector meson V in a pseudoscalar meson P. We may have radiative decays $V \to P\gamma$ when the emitted photon is real, and leptonic conversion decays $V \to Pl^+l^-$ when the photon is virtual and produces a lepton pair. These decays are both described by the hadronic matrix element of the electromagnetic current, $< P|J_\mu^{e.m.}|V >$. Using Lorentz and parity invariance, this matrix element can be parametrized in terms of a single form factor $f_{VP}(q^2)$ (with dimensions of an inverse energy) which is defined as follows:

$$< P|J_\mu^{e.m.}|V_r >= e f_{VP}(q^2)\epsilon_{\mu\alpha\beta\gamma}(p_V)^\alpha (p_P)^\beta \epsilon_r^\gamma \tag{1}$$

where ϵ_r^γ is the vector meson polarisation versor and q is the momentum transfer, $q = p_V - p_P$. $q^2 = 0$ for radiative decays and $4m_l^2 \leq q^2 \leq (m_V - m_P)^2$ for conversion decays. Using eq.(1), it is possible to determine the total decay rates. For radiative decays one finds:

$$\Gamma(V \to P\gamma) = \frac{1}{3}\alpha\left(\frac{m_V^2 - m_P^2}{2m_V}\right)^3 |f_{VP}(0)|^2 \tag{2}$$

For conversion decays we have:

$$\Gamma(V \to Pl^+l^-) = \frac{\alpha^2}{72\pi m_V^3}\int_{q^2_{Min}}^{q^2_{Max}} \frac{dq^2}{q^2}\lambda(q^2)^{3/2}\left(1 - \frac{4m_l^2}{q^2}\right)^{1/2}\left(1 + \frac{2m_l^2}{q^2}\right)|f_{VP}(q^2)|^2 \tag{3}$$

where $\lambda(q^2) = (m_V^2 + m_P^2 - q^2)^2 - 4m_V^2 m_P^2$.

We can express all the meson form factors f_{VP} ($f_{\rho\pi}$, $f_{\omega\pi}$, ... etc.) as linear combinations of quark current form factors $f_q(q^2)$. For the quark up we define $f_u(q^2)$ by the relation:

$$< \pi^+|\bar{u}\gamma_\mu u|\rho^+ >= f_u(q^2)\epsilon_{\mu\alpha\beta\gamma}(p_V)^\alpha (p_P)^\beta \epsilon_r^\gamma \tag{4}$$

Decay	$\lvert f_{VP}\rvert$	B. R.	$\lvert f_q(q^2=0)\rvert(\mathrm{Gev}^{-1})$
$\rho^{\pm}\to\pi^{\pm}\gamma$	$(1/3)f_u$	$(4.5\pm0.5)\ 10^{-4}$	$f_u=2.2\pm0.1$
$\rho^{0}\to\pi^{0}\gamma$	$(1/3)f_u$	$(7.9\pm2.0)\ 10^{-4}$	$f_u=2.9\pm0.4$
$\rho^{0}\to\eta\gamma$	$\sin\phi_P f_u$	$(3.8\pm0.7)\ 10^{-4}$	$f_u=2.3\pm0.2$
$\omega\to\pi^{0}\gamma$	f_u	$(8.5\pm0.5)\ 10^{-2}$	$f_u=2.32\pm0.08$
$\omega\to\eta\gamma$	$(1/3)\sin\phi_P f_u$	$(4.7^{+2.2}_{-1.8})\ 10^{-4}$	$f_u=1.7^{+0.4}_{-0.3}$
$\phi\to\eta\gamma$	$(2/3)\cos\phi_P f_s$	$(1.28\pm0.06)\ 10^{-2}$	$f_s=1.84\pm0.06$
$K^{*\pm}\to K^{\pm}\gamma$	$(2/3)f_u-(1/3)f_s$	$(1.01\pm0.09)\ 10^{-3}$	$\nearrow f_u=2.10\pm0.11$
$K^{*0}\to K^{0}\gamma$	$-(1/3)f_u-(1/3)f_s$	$(2.30\pm0.20)\ 10^{-3}$	$\searrow f_s=1.70\pm0.17$

Table 1: *Experimental branching ratios and form factors for radiative decays of mesons composed by two degenerate quarks. The angle ϕ_P is defined as $\phi_P=\theta_P-\theta_0$, where θ_P is the mixing angle between η and $\eta\prime$ and θ_0 is the ideal mixing angle.*

and $f_s(q^2)$ for the strange quark is defined by a similar equation. By assuming that the $V\to P$ decays proceed via two spectator processes in which the quark or the anti-quark of the $V-$meson emit the final photon, and assuming SU(2)-isospin symmetry, we simply obtain the relations between f_{VP} and f_u or f_s reported in table 1.

From an experimental point of view, by using eq.(2) the values of the form factors f_u and f_s at zero momentum transfer can be directly determined from the measurement of the branching ratio of radiative decays [2]. If we assume an ideal mixing angle for vector mesons ϕ and ω and the preferred experimental value $\theta_P=-20^{\circ}$ for the mixing between η and η', we obtain the results reported in table 1. The fact that the values of $f_u(0)$, obtained with different measurements, are compatible within the errors, indicates that its determination is reliable and justifies the implementation of the valence approximation in the analysis of these decays. The best experimental determination of $f_u(0)$, obtained from the $\omega\to\pi^0\gamma$ decay [2], is

$$f_u(0)=2.32\pm0.08\ \mathrm{GeV}^{-1} \tag{5}$$

For $f_s(0)$, from the $\phi\to\eta\gamma$ decay rate [2], we have:

$$f_s(0)=1.84\pm0.06\ \mathrm{GeV}^{-1} \tag{6}$$

In the following section, we will compare these results to our lattice predictions.

3 Lattice results

The results presented in this contribution have been obtained from the 15 gauge field configurations, generated in the quenched approximation at $\beta=6.0$, on a volume $20\times$

$f_u(q^2)$ (GeV^{-1})	$f_s(q^2)$ (GeV^{-1})	q^2 (GeV^2)
0.66 ± 0.07	0.58 ± 0.06	-1.76 ± 0.31
1.83 ± 0.27	1.49 ± 0.17	-0.46 ± 0.07

Table 2: *Values of the form factors $f_u(q^2)$ and $f_s(q^2)$.*

$10^2 \times 40$. All details of the lattice calculation are reported in ref.[5].

We have computed the quark form factors $f_u(q^2)$ and $f_s(q^2)$ for two different values of the momentum transfer. Because of the kinematic on our lattice, we obtain negative values of q^2, i.e. out of the accesible physical region. The results are reported in table 2.

The values of the form factors at zero momentum transfer and in the physical region can be deduced from the lattice results by making an assumption on the q^2 dependence. We have then considered both a linear dependence, which should hold at small q^2, and the vector meson dominance model. Fitting our data with the linear expression:

$$f_q(q^2) = f_q(0) + A_q \; q^2/m_V^2 \tag{7}$$

we obtain:

$$f_u(0) = (2.24 \pm 0.38) \text{ GeV}^{-1} \quad , \quad f_s(0) = (1.81 \pm 0.22) \text{ GeV}^{-1} \tag{8}$$

for the form factors at zero momentum transfer, and

$$A_u = (0.53 \pm 0.23) \text{ GeV}^{-1} \quad , \quad A_s = (0.73 \pm 0.24) \text{ GeV}^{-1} \tag{9}$$

for the slopes. The values in eq.(8) are in remarkable agreement with the experimental results reported in eqs.(5) and (6).

We have also considered the vector meson dominance model:

$$f_q(q^2) = \frac{f_q(0)}{1 - q^2/m_V^2} \tag{10}$$

Using this expression to fit our results, we find:

$$f_u(0) = (2.92 \pm 0.14) \text{ GeV}^{-1} \quad , \quad f_s(0) = (1.86 \pm 0.08) \text{ GeV}^{-1} \tag{11}$$

for the form factor at zero momentum transfer. The points are well fitted by the formula in eq.(10) but the resulting value of $f_u(0)$ in eq.(11) is larger then the experimental result, cfr.eq.(5). To obtain a more significative check of the validity of vector meson dominance, we have also considered the lattice values of $f_q(q^2)$ obtained at a given value of the hopping parameter, $K = 0.1515$, which are those affected by the smallest statistical errors in our calculation. We have fitted these points with eq. (10) using the value of the vector meson mass m_V computed on the lattice at the same value of the Wilson parameter. In fig.1 the results for $f_q(q^2)$ and the continuous curve representing the fit are shown. A χ^2 test gives $\chi^2 = 3.2$ per degree of freedom, which could be a signal of some deviation from the vector meson dominance prediction. We stress that some deviation is also indicated by the experimental results for the form factor $f_{\omega\pi}(q^2)$ [7].

From our calculation and using eq.(3) we can also estimate the widths of conversion decays. The theoretical predictions are strongly related to the dependence of the form factor on the momentum transfer q^2. In fig.2, we compare the behaviour of the form factor $f_u(q^2)$ in the physical region as indicated by the lattice calculation with the experimental data on $\omega \to \pi^0\mu^+\mu^-$ decay [7]. The curves in the figure are the lattice predictions obtained by assuming the linear dependence and the vector meson dominance model. The difference between the two curves gives an idea of the systematic uncertainties present in our determinations of the conversion decay widths.

The linear dependence, assumed in eq.(7), is reliable only in the small q^2 region. When an electron pair is produced in the final state, the lower limit for q^2 is $q^2 = 4m_e^2 \simeq 0$. The largest contribution to the width comes in this case precisely from the small q^2 region where eq.(7) is valid. For this reason, we have estimated the partial widths for the conversion decays $V \to Pe^+e^-$ by assuming both the linear dependence and the vector meson dominance model. We have used the values of the form factor at zero momentum transfer obtained on the lattice, eqs.(8) and (11). The resulting branching ratios are reported in table 3 together with the experimental values in the cases in which they are known. We observe that the predictions of the linear assumption are in good agreement with the experimental data. A large deviation from the experimental result is obtained from the vector meson dominance prediction for the $\omega \to \pi^0e^+e^-$ decay; this is essentially due to the large value of $f_u(0)$ in eq.(11).

A linear dependence on the momentum transfer of the form factors in the higher q^2 region cannot be assumed. For this reason, for the conversion decays in which a muon pair is produced in the final state, we have only considered the predictions of the vector meson dominance model. The corresponding branching ratios are also reported in table 3, together with the only avaliable experimental value.

In this contribution we have presented a preliminary lattice QCD study of electromagnetic $V \to P$ decays. The good agreement between our results and the experimental data for radiative decays provides strong support to the validity of the lattice calculation. However, a better determination of the q^2 dependence of the form factor would be very important for a more precise prediction of the conversion decay widths. This could be achieved via larger statistics and with a bigger lattice volume. An improvement of the lattice calculation would be also very interesting in view of the accurate experimental measurements of conversion decays that will be done at the DAΦNE ϕ-factory.

Decay	B.R. (lin.)	B.R. (vmd)	B.R. (exp.)
$\rho^0 \to \pi^0 e^+ e^-$	$(4.0 \pm 1.5)\ 10^{-6}$	$(7.2 \pm 0.7)\ 10^{-6}$	—
$\rho^0 \to \eta e^+ e^-$	$(2.6 \pm 0.9)\ 10^{-6}$	$(4.5 \pm 0.5)\ 10^{-6}$	—
$\omega \to \pi^0 e^+ e^-$	$(6.8 \pm 2.5)\ 10^{-4}$	$(12 \pm 1)\ 10^{-4}$	$(5.9 \pm 1.9)\ 10^{-4}$
$\omega \to \eta e^+ e^-$	$(6.1 \pm 2.2)\ 10^{-6}$	$(10 \pm 1)\ 10^{-6}$	—
$\phi \to \eta e^+ e^-$	$(1.0 \pm 0.3)\ 10^{-4}$	$(1.1 \pm 0.1)\ 10^{-4}$	$(1.3^{+0.8}_{-0.6})\ 10^{-4}$
$\phi \to \eta' e^+ e^-$	$(4.7 \pm 1.1)\ 10^{-7}$	$(6.3 \pm 0.6)\ 10^{-7}$	—
$\rho^0 \to \pi^0 \mu^+ \mu^-$	—	$(6.9 \pm 0.7)\ 10^{-7}$	—
$\rho^0 \to \eta \mu^+ \mu^-$	—	$(6.5 \pm 0.7)\ 10^{-11}$	—
$\omega \to \pi^0 \mu^+ \mu^-$	—	$(12 \pm 1)\ 10^{-5}$	$(9.6 \pm 2.3)\ 10^{-5}$
$\omega \to \eta \mu^+ \mu^-$	—	$(2.4 \pm 0.2)\ 10^{-9}$	—
$\phi \to \eta \mu^+ \mu^-$	—	$(5.2 \pm 0.5)\ 10^{-6}$	—

Table 3: *Theoretical and experimental values for the branching ratios of the conversion decays $V \to P l^+ l^-$.*

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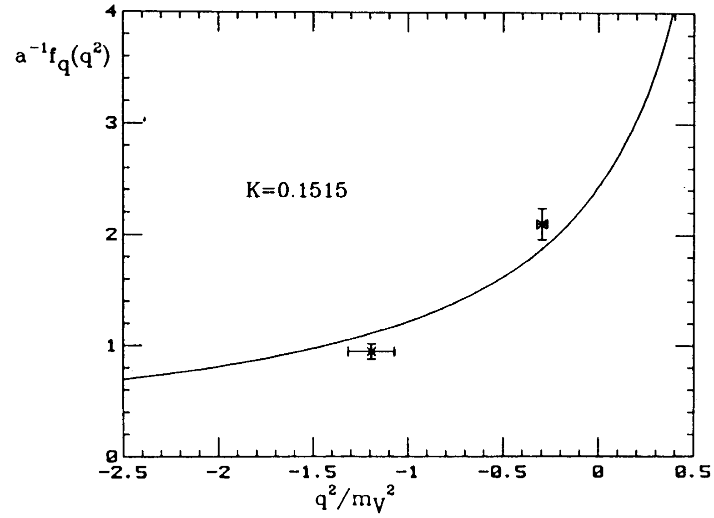

Figure 1: *Lattice results for the form factor $f_q(q^2)$ as a function of the dimensionless quantity q^2/m_V^2 for K=0.1515. The curve represents the fit with the vector meson dominance model.*

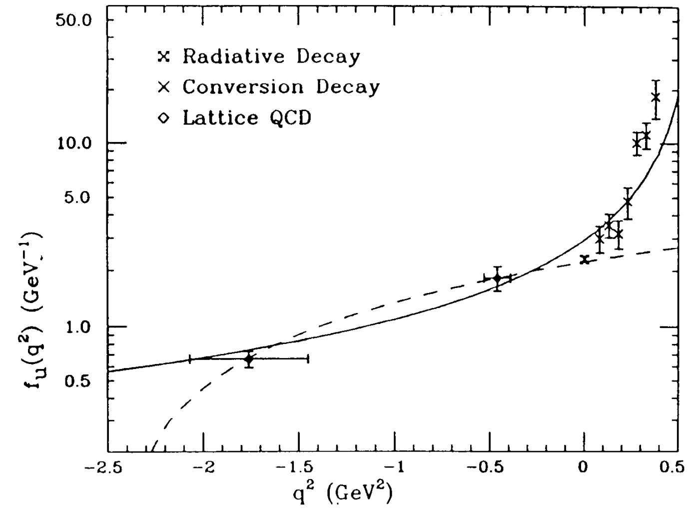

Figure 2: *Lattice values for the form factor $f_u(q^2)$ together with the experimental results from radiative decay $\omega \to \pi^0\gamma$ and conversion decay $\omega \to \pi^0\mu^+\mu^-$. The continuous and dashed curves are the lattice predictions obtained by assuming the vector meson dominance and the linear dependence respectively.*