INFN - Laboratori Nazionali di Frascati, May 1995
From: **The Second DaΦne Physics Handbook**[1]
Chapter 9: One Photon Initiated Processes
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The Muon Gyromagnetic Ratio and R_H at DAΦNE

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Abstract

The muon anomalous magnetic moment is in principle sensitive to electroweak effects and could be a probe for new phenomena at very large energy scales. Computations of the hadronic contributions to the muon anomaly are beyond present theoretical understanding. This can however be circumvented by good measurements of the e^+e^- annihilation cross section into hadrons. The purpose of this note is to discuss the feasibility of such measurements.

1. The Muon Gyromagnetic Ratio. The $g-2$ value of the muon is about 10^4 times more sensitive than the electron $g-2$ value to large mass states[1]. We can

[1]Supported by the INFN, by the EC under the HCM contract number CHRX-CT920026 and by the authors home institutions

therefore hope to experimentally verify the renormalizability of the electroweak interaction. Defining the anomaly as $a = (g-2)/2$, the weak contribution to the muon anomaly is a_μ=(19.5±0.1)×10^{-10}[2]. The pure QED anomaly[3], to $\mathcal{O}(\alpha^4)$, is $a_{QED} = (11\,658\,480 \pm 3)\times 10^{-10}$. Hadronic contributions to the anomaly are not computed from prime principles. The hadronic contributions to the photon propagator are obtained from e^+e^- annihilations into hadrons. Including next to leading QED corrections and light by light scattering, the authors of reference 3 estimate $a_{\mu,\ \mathrm{hadrons}} = (703 \pm 19) \times 10^{-10}$ introducing an error in the theory larger than the weak contributions.[2]

The complete value for the anomaly is given below compared to the experimental result, an average of the μ^+ and μ^- data:

$$\begin{aligned} a_\mu(\mathrm{Theory}) &= 11\,659\,203(20) \times 10^{-10} \\ a_\mu(\mathrm{Experiment}) &= 11\,659\,240(81) \times 10^{-10} \end{aligned}$$

An experiment whose goal is to measure the muon anomaly approximately 20 times better than previous efforts, is under construction at Brookhaven[1, 4]. Improved estimates of the hadronic contributions to a_μ are necessary to test the validity of the weak interaction correction as well as probe new phenomena at very high energy scales.

2. a_μ AND $e^+e^- \to$ HADRONS. The lowest order contribution to the muon anomaly $a_\mu = (g-2)/2$ due to hadronic corrections to the photon propagator is given by[3]

$$\delta a_\mu(hadr,\ lo) = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \sigma_{e^+e^-\to hadrons}(s) K(s) \mathrm{d}s \tag{1}$$

where

$$\begin{aligned} K(s) &= x^2\Big(1 - \frac{x^2}{2}\Big) \\ &+ (1+x)^2(1+x^{-2})\Big(\log(1+x) - x + \frac{x^2}{2}\Big) \\ &+ \frac{1+x}{1-x}x^3\log x \end{aligned}$$

with

$$x = \frac{1-\sqrt{1-4m_\mu^2/s}}{1+\sqrt{1-4m_\mu^2/s}}$$

where $W = s^{1/2}$ is the c.m. energy.

The authors of reference 3 have used all the available experimental data to obtain the value $\delta a_\mu(hadr,\ lo) = 706(6)(17) \times 10^{-10}$, where 6×10^{-10} and 17×10^{-10} are respectively statistical and systematic errors.

[2]There is actually an inconsistency in ref. 3. From the values given for the various contribution, eqs. 1.4, 1.6, 1.7b, one obtains a_μ=666±19 rather than 703. This is however of no consequence to the following discussion.

Several years ago it was suggested[5] that better measurements of $\sigma(e^+e^-\to\pi^+\pi^-)$ should be possible at e^+e^-colliders. Unfortunately such colliders did not appear to be available. With the advent of DAΦNE it will become very easy to perform such measurements.

3. THE REACTION $e^+e^-\to\pi^+\pi^-$. The cross section for $e^+e^-\to$hadrons is dominated by ρ production below 1.08 GeV. For simplicity, we consider in the following only the contribution to a_μ from $e^+e^-\to\rho\to\pi^+\pi^-$, which represents ~70% of the lowest order estimate of a_μ. For the ρ production cross section we use the expression[6]:

$$\sigma_{\pi\pi}(s) = \frac{877 \text{ nb}}{s}\beta^3\frac{(0.153/2)^2}{(\sqrt{s}-0.77)^2+(0.153/2)^2}$$

which peaks at about 765 MeV with a value of 1205 nb. Performing the integral up to W=1.08 GeV gives $\delta a_\mu(hadr,\ lo)$=0.019 GeV$^2\times$nb=488$\times10^{-10}$ to be compared with $\delta a_\mu(hadr,\ lo)$=506(2)(15)$\times10^{-10}$ given in reference 3 for $e^+e^-\to\rho,\ \omega\to\pi^+\pi^-$.

The ρ contribution should be measured to an accuracy of at least one half of the uncertainty in the pure QED contribution to a_μ, *i.e.* 1.5$\times10^{-10}$. This correspond to a measurement of the integral above to an accuracy of ~0.3%. Measuring $\sigma(e^+e^-\to\pi^+\pi^-)$ at 100 equally spaced values of W, for $2m_\pi < W <$1.08 GeV, each resulting in an accuracy of 1.5$\times10^{-10}\sqrt{100}$=1.5$\times10^{-11}$ in a_μ, gives the integral of eq. 1 to an accuracy of 1.5$\times10^{-10}$. This in turn requires measuring the $\pi^+\pi^-$ cross section with a fractional accuracy of ~0.9% per point around the ρ peak and considerably lower at other energies. An integrated luminosity of ~10 nb^{-1} per point is required for measurements around W = 765 MeV. The total integrated luminosity needed for the complete measurement is ~380 nb^{-1}.

4. THE MACHINE ENERGY SCALE. The absolute energy of the colliding electrons, for a contribution to the final error of the magnitude discussed above, must be known to a fractional accuracy of $\mathcal{O}(10^{-4})$. This can be easily achieved by the $g-2$ depolarizing resonance method[7]. Hopefully measurements at a few energies, combined with accurate magnetic measurements will be enough.

5. IDENTIFYING THE $\pi^+\pi^-$ FINAL STATE. The $\pi^+\pi^-$ channel must be distinguished from the competing two-body processes $e^+e^-\to\mu^+\mu^-$ and Bhabha scattering. The latter can easily be recognized by calorimetry, the former by momentum measurements, time of flight and calorimetry in KLOE. The helium drift chamber in KLOE gives excellent momentum resolution. Additional μ pair rejection is provided by the angular distribution of pions, $\propto \sin^2\theta$ and muons, $\propto 1+\cos^2\theta$. The KLOE shower detector is very effective in positively identifying Bhabha scattering (and converted $\gamma\gamma$ events) as necessary for measuring correctly $\sigma(e^+e^-\to\pi^+\pi^-)$ and to obtain good luminosity measurements as discussed later. The shower detector will provide detection of final states with π^0's.

6. A QUESTION OF NORMALIZATION. *Absolute cross sections* measurements are necessary and precise measurements of the luminosity are therefore required. For the error in luminosity to be negligible the luminosity should be measured to at least twice the accuracy of the pion yield measurements. Thus at the ρ peak the luminosity must be measured to an accuracy of at least 0.4%.

Cross section for $e^+e^-\to\mu^+\mu^-$, and $e^+e^-\to\rho\to\pi^+\pi^-$.

W GeV	s GeV^2	$\sigma_{\mu\mu}$ nb	$\sigma_{\pi\pi}$ nb
0.370	0.136	608.3	64.1
0.470	0.220	388.0	126.5
0.570	0.324	266.1	228.8
0.670	0.448	193.2	543.0
0.770	0.591	146.5	1200.0
0.870	0.755	114.8	364.2
0.970	0.939	92.4	104.8
1.070	1.143	75.9	42.1

The differential cross section for $e^+e^-\to\mu^+\mu^-$, to lowest order in QED, is given by[8]:

$$\frac{\mathrm{d}\sigma_{\mu\mu}}{\mathrm{d}\Omega}=\frac{\alpha^2}{4s}\beta(2-\beta^2+\beta^2\cos^2\theta)=\frac{5.18\ \mathrm{nb}}{s\ (\mathrm{GeV}^2)}\beta(2-\beta^2+\beta^2\cos^2\theta)$$

and the total cross section by

$$\sigma_{\mu\mu}=\frac{43.4\ \mathrm{nb}}{s\ (\mathrm{GeV}^2)}\beta(3-\beta^2)$$

As shown in the table, the dimuon yield cannot be used to measure the luminosity with the required accuracy in most of the energy range of interest, without requiring considerably more running time. At the ρ peak, for instance, the dipion cross section is about eight times the dimuon cross section. Together with the need of observing four times as many $\mu\mu$ events, the required luminosity is increased by a factor of 32 and severe systematic errors might be incurred.

7. Bhabha scattering as a measure of the luminosity. Bhabha scattering, even at medium large angles, say >20°, is much larger than muon pair production and can be used to measure the luminosity. The Bhabha scattering cross section, integrated over azimuth is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}=\frac{65.1\ \mathrm{nb}}{s\ (\mathrm{GeV}^2)}\Big(\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)}-\frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)}+\frac{1+\cos^2\theta}{2}\Big).$$

Thus, for instance, in the center of mass angular range $20°<\theta<160°$, which is well covered by the KLOE detector, the Bhabha cross section is given by $\sigma_{\mathrm{Bhabha}}\sim(10^4\ \mathrm{nb})/(s\ \mathrm{GeV}^2)$, more than one hundred times the muon pair production cross section. Radiative correction are ignored here, they become important in order to reach accuracies of $\mathcal{O}(10^{-3})$ and they are a job for theorist.

8. Luminosity requirements. While there are no statistical limitations in performing excellent luminosity measurements, the very strong angular dependence of the

Bhabha scattering and radiative Bhabha events, $e^+e^-\to e^+e^-\gamma$, requires great care in obtaining correct values. In practice this demands very good knowledge of beam and tracking system positions, of efficiency vs angle, of all materials responsible for multiple scattering and so on. Folding in all of these effects, and the effect of radiative corrections as well, requires extensive Monte Carlo calculations, quite beyond the scope of this note. 500 nb^{-1} corresponds to 14 hours of collisions at a luminosity $\mathcal{L}=10^{30}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$, well below the expected maximum DAΦNE luminosity of 10^{33}. In practice it will take considerably longer time, in order to perform repeated measurements, and the always necessary auxiliary measurements, in particular the determination of the absolute machine energy scale.

DAΦNE appears to be an ideal machine to achieve the goal of a definitive measurements of the contributions of hadronics states to the spectral function[9] of the photon propagator, $\Pi(\mu^2)$.[10] We have not discussed measurements of $\pi^+\pi^-\pi^0$. This channel contributes about 50×10^{-10} to the muon anomaly. Any luminosity adequate for the two pions case, will amply do for the three pions one. The next largest contribution is due to the ϕ meson. The two pions contribution becomes negligible beyond w=1.08 GeV.

9. Conclusion. We have shown that a very modest luminosity is necessary to perform measurements of $e^+e^-\to$ hadrons, to the accuracy required to evaluate the muon anomaly to the accuracy necessary to measure electroweak effects and/or signals from new physics. It appears quite feasible to identify the reactions $e^+e^-\to\pi^+\pi^-$and $e^+e^-\to 3$ or 4 pions in a large background of dimuon and Bhabha events by well established techniques. Measurements of the luminosity will require most care and possibly some help from expert theorists, but there is ample yield of Bhabha scattering.

Acknowledgements I wish to acknowledge discussions with T. Kinoshita, M. Greco, J. Lee- Franzini, P. J. Franzini and M. Baillargeon.

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[4] Brookhaven Experiment 821, proposed by a Boston, BNL, CCNY, Columbia, Cornell, Heidelberg, LANL, Michigan, Mississippi, Sheffield, Tokyo, KEK, Riken, Yale collaboration (V. Hughes et al.) is under construction and should be running soon.

[5] P. Franzini, Brookhaven Workshop on Muon $g-2$, June 11–16, 1984, unpublished.

[6] See reference [3] for an extensive list of references of measurements.

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