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Hadronic Contributions to the Muon g-2

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The obvious target in the domain of the precision measurements of the muon g-2 is the detection of the electroweak corrections due to vector and Higgs boson exchange. The present experimental value of the muon anomaly is [1]

$$\begin{aligned} a_\mu = \frac{\alpha}{2\pi} + ... = & \\ & = 116593700 \pm 1200 \times 10^{-11}(\mu^-) \\ & = 116591000 \pm 1100 \times 10^{-11}(\mu^+) \end{aligned} \tag{1}$$

with the electroweak contribution predicted to be:

$$a_\mu(EW) = (195 \pm 1) \times 10^{-11} \tag{2}$$

To identify the effect (2), we need to reduce the error in the theoretical prediction of the other contributions to a_μ, besides reducing the experimental error in (1) by more than one order of magnitude.

The main source of these errors are the hadronic vacuum polarization corrections, Fig. 1(a), given in terms of an appropriate integral over the hadronic (one-photon) e^+e^- cross section. With present data, one has [2, 3]

$$a_\mu(had, vac-pol) = (7030 \pm 59 \pm 164) \times 10^{-11} \tag{3}$$

(the first is the statistical, the second the systematic error).

The error in (3) is mainly due to the imperfect knowledge of $\sigma(e^+e^- \to hadrons)$ at low energy, as shown in Tab. 1 [2].

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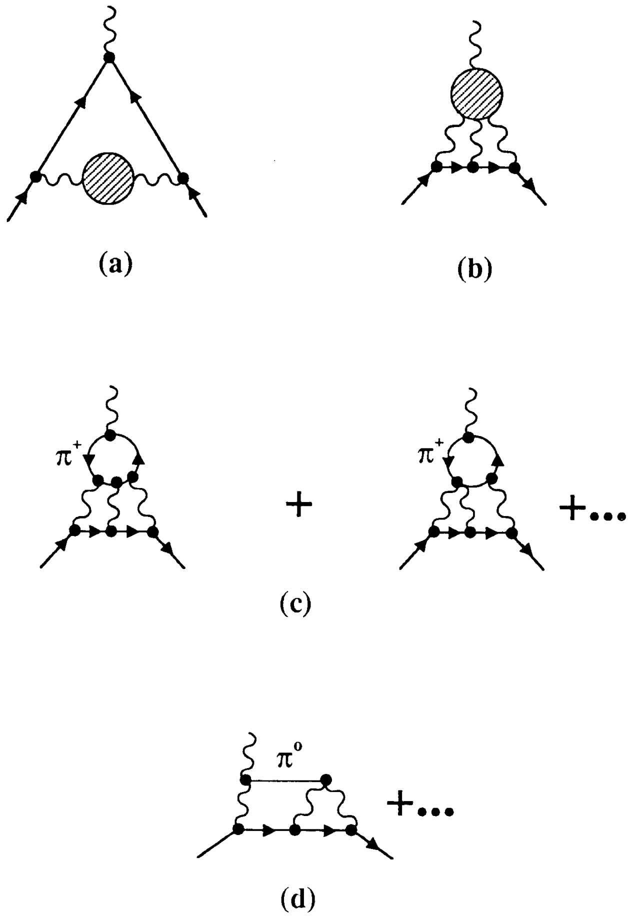

Figure 1: Feynman diagrams representing hadronic corrections to the muon anomaly . (a) lowest order contribution due to vacuum polarization; (b) next order contribution from light-by-light scattering; (c) charged and (d) neutral pion contributions to the corrections (b).

energy region	ρ	ω	ϕ	high energy	total
stat. error	21	48	18	21	59
sys. error	150	15	13	63	164
total				67	174

Table 1: Analysis [2] of the present statistical and systematic error of the hadronic contribution to the muon anomaly, in units of 10^{-11}. Totals are obtained in quadrature.

quark loop	charged pion loop (point like)	charged pion loop (ρ form-factor)	neutral pion (ρ form-factor)
60	-48(-521+473)	-16(-339+323)	+66

Table 2: Contributions to the muon anomaly in units of 10^{-11}, from different estimates of light-by-light scattering [2], see fig. 1. Numbers in parenthesis are the contributions from two different, gauge invariant set of diagrams, see ref. [2]. In the quark loop, ($m_u = m_d$=0.3, m_s=0.5, m_c=1.5 GeV)

DAΦNE can improve considerably on the statistical and systematic errors in the ρ, ω and ϕ regions, reducing the overall error to something like 70×10^{-11}, sufficient to identify the bulk of the electroweak correction.

This is not the full story, however.

Hadronic corrections, to the next order, enter via the light-by-light scattering diagram, Fig 1(b), and we have to make sure that the error on this contribution is also sufficiently small. This question was considered in ref.[2] , where various estimates are given, see Tab. 2, replacing the hadron bubble in Fig. 1(b) with a pointlike quark loop and adding the contribution arising from the $\pi^0 \to \gamma\ \gamma$ coupling, see Fig. 1(d).

For the charged pion loop, the pointlike case has been considered, as well as the case where ρ-dominated form factors are associated with the $\gamma - \pi\pi$ vertices (form factors softening is always necessary for the neutral pion contribution, which would be UV divergent otherwise).

The quark loop result, with the masses indicated in the Table, agrees remarkably well with the sum of the contributions of the neutral and charged pions (with form factors) leading the authors to quote, as best value:

$$a_\mu(had, light - light) = 49 \pm 5 \times 10^{-11} \tag{4}$$

The error is their estimate of the model dependence of the result.

There are reasons to think, however, that the error has been quite underestimated. For one, the pion result arises from the cancellation of different (gauge invariant) contributions, each of which is larger than the electroweak correction itself, see Tab. 2 and ref [2], so that the uncertainty of the result is unlikely to be as small as quoted in (4). Moreover, the quark model result is very sensitive to the values of the assumed light quark masses (it goes approximately like m^{-2}) and the agreement of the quark and meson results looks suspiciously close to being accidental. A quark loop estimate of $a_\mu(had, vac - pol)$, with the quark masses given in Tab. 2 yields

$$a_\mu(had, vac - pol)_{quark\ loop} = 3000 \times 10^{-11} \tag{5}$$

One could think to fix the light quark mass by requiring the quark loop to reproduce the value (3). This requires a light quark mass of about 0. 16 GeV (a not unreasonable value) and would give $a_\mu(had, light - light)_{quark\ loop} = 150 \times 10^{-11}$, a much less favorable result.

The problem appears still open. More thinking is needed at least to estimate the size of the contribution from $\gamma - \gamma$ scattering. In the meanwhile, a good measurement of the hadron cross-section at low energy, with DAΦNE, cannot but be welcome.

After the first writing of this brief report [4], some new contributions on the issue under discussion have appeared in the literature. They are:

- i) A careful reconsideration of the hadronic vacuum polarisation corrections [5];
- ii) A discussion on the uncertainties to be attributed to the light-light hadronic contribution [6, 7].

Their content can be summarised as follows.

From Ref. [5], the hadronic vacuum polarisation corrections are given by

$$a_\mu(had, vac - pol) = (7023.5 \pm 58.5 \pm 140.9)10^{-11}$$

to be compared with equation (3).

As to the light-light hadronic contribution , both Refs. [6, 7] agree on the statement that more work needs to be done in order to reduce the error below the expected sensitivity of the foreseen Brookhaven experiment [8] on the muon anomaly. This might be possible by resorting to theoretical models that are thought to represent a fairly good approximation of low energy QCD.[2]

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[2]Note from the Editors : After receiving this contribution, we became aware of a recent paper [9], where this problem is reexamined.